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Comparative study of one-bid versus two-bid auctions

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Abstract

The paper focuses on a comparative investigation of the standard one-bid first-price auction and a corresponding two-bid auction where each buyer may place two bids: a high-bid and a low one and the winner pays his low-bid if this was higher than all other bids. We characterize the equilibria of the two mechanisms and prove some results on the ranking of revenues for the symmetric case. We show that subjects in an experiment prefer the two-bid over the one-bid auction when given the possibility of choosing between the two and compare the observed behavior to the theoretical predictions.

JEL classification: D44; C91

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1. Introduction

The dramatic increase in auction-trade volume over the last few years has been followed by an impressive increase in the variety of auction mechanisms used by the sellers. Leading listing auction-sites (Lucking-Reiley, 1999) on the Web such as for example Ebay, Yahoo, and City Auction offer the chance to sell/buy products through (different variations on) standard English and Dutch auctions. Other auction-sites employ a large variety of different auction mechanisms such as sealed-bid auctions, auctions where players pay fixed participation fees and more. Indeed, it has been suggested (Monderer and Tennenholtz, 1998) that

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online auctioneers compete for the pool of potential buyers by choosing the auction type that would maximize their expected payoffs, while taking into account that buyers would choose the auction-site that maximizes their expected utilities.

This paper deals with a new type of auction that has recently appeared on the Web, a first-price auction where buyers may submit several bids for the same object. One of the largest auctions in Israel, "The State Auction", for example, allows each bidder to submit up to three different price proposals in each auction.[2] The rules state that "if more than one of your offers has won, then the highest winning offers would be canceled and you would only pay your lower winning offer". The same is true for "The Double Auction", another leading auction in Israel.[3] Here, participants may submit up to five offers for the same product but pay a per-offer participation fee so that as the number of offers players submit increases, they pay more.

With these examples as a motivation, this paper compares the standard (iid values; one indivisible object) first-price auction where each player may submit only one bid (henceforth, the one-bid auction) to a corresponding two-bid first-price auction (henceforth, the two-bid auction) where each player may submit two bids with the winner paying his low-bid if this is higher than the highest bid submitted by his opponents. We characterize the (constant relative risk aversion) equilibria of the two mechanisms and prove some results on the ranking of revenues and expected utilities across the two mechanisms for the symmetric case. Furthermore, we conduct an experiment where subjects play repeatedly one-bid auctions and two-bid auctions with randomly chosen partners and compare the experimental results with the equilibrium predictions.

One of the interesting features of our experiment is that, in the last phase of the experiment, subjects are repeatedly asked to choose between the two auction mechanisms. In particular, in the second phase of the experiment (after playing each auction type for 24 rounds in the first phase) subjects could choose (for 16 rounds) their favorite auction mechanism before observing their realized value. This enabled us to examine directly subjects' preferences across the two mechanisms and check the consistency of subjects' behavior with respect to the equilibrium benchmark.

Another non-standard feature of the experiment is that we allow subjects to submit bids that are lower than the minimum possible private value. Indeed, we find that many subjects bid less than the minimum value when given the opportunity.

The experimental literature on auction mechanisms in general and first-price private-value auctions in particular is too large to survey in this short introduction.[4] Therefore, we will mention just some of the contributions. Cox et al. (1982, 1985, 1988, 1992) ran a comprehensive set of experiments trying to explain and characterize subjects' behavior in different auctions. Some of the factors investigated are the impact of the chosen price rule (see also Coppinger et al., 1980; Güth et al., 2002), the number of bidders (see also Kagel and Levin, 1993), subjects' motivation (see also Kagel and Levin, 1985; Kagel and Roth, 1992), and the selected incentive scheme (see also Harrison, 1989). In an ef-

[2] "The State Auction" is both Web based at http://www.e-hamichraz.co.il/ and published in a booklet that is distributed with leading newspapers. Bidders may thus place bids through the Web or by calling a phone number.

[3] Again the auction is Web based at http://www.2bid.co.il, but the catalogue also appears as a printed booklet.

[4] For more details and references see the survey by Kagel (1995).

fort to explain significant heterogeneity in individual bidding patterns, Cox et al. develop formal models where subjects have the constant relative coefficient of risk aversion (CRRA) type of utility functions (the model is henceforth referred to as CRRAM). Chen and Plott (1998) investigate bidding behavior in auctions where the private valuations are drawn from non-uniform distributions. Pezanis-Christou (2002) and Güth et al. (2001) study different types of auctions with asymmetric bidders. However, all these studies focus on the traditional one-bid type of auctions. This paper is (to the best of our knowledge) the first study of multi-bid auctions where bidders may submit several bids for a given object.

Our main results can be summarized as follows:

(1) Subjects reveal a robust inclination to prefer the two-bid auction over the one-bid auction in the second phase of the experiment, although their average revenues were not significantly different across the two mechanisms in the first phase.
(2) In both auction formats, subjects behave as if they are risk-averse in the sense of bidding more than the risk-neutral equilibrium strategies in approximately 60–70 percent of the cases. Still, we find a robust inclination to "bargain-bid" (i.e., to bid prices that are below the lowest possible private value).

The remainder of the paper is organized as follows: the theoretical results are presented in Section 2. The experimental procedure is described in Section 3 and the experimental results in Section 4. Section 5 concludes.

2. A model

Consider an independent private-value two-bidder auction where $V_i \sim U[0, 1]$ for each bidder $i = 1, 2$. Let $v_i \in [0, 1]$ denote the realized value of bidder i. We compare the case of a one-bid auction where each bidder i may only submit one bid $b_i(v_i)$ to the case of a two-bid auction where each bidder i may submit two bids, a high-bid $h_i(v_i)$ and a low-bid $l_i(v_i)$ where $l_i(v_i) \leq h_i(v_i)$ for every $v_i \in [0, 1]$.

The rules of the one-bid auction are the standard rules for first-price sealed-bid auctions, the highest bidder wins the auction and pays his bid. The rules of the two-bid auction are as follows.

The bidder who has submitted the highest bid wins the auction. If both bidders have submitted the same highest offer, the winner is randomly selected (with probability 0.5 for each bidder). If the winner's low-bid was strictly higher than the high-bid of the other bidder, then the winner pays his low-bid. Otherwise, the winner pays his high-bid.

As a benchmark for analyzing the experimental results that follow, we would like to characterize and compare the equilibria of the one-bid auction and the two-bid auction. First, we restrict the analysis to the case of bidders with symmetric preferences and assume that the utility function of the representative bidder takes the constant relative coefficient of risk aversion form $U(x) = x^{\alpha}$ for some $\alpha > 0$ (see, however, Proposition 4 for the asymmetric case). Note that $\alpha < 1$ describes a case where the agents are risk-averse, $\alpha = 1$ is the case for risk-neutral agents, and $\alpha > 1$ is the case for risk-seeking agents. Proposition 1 gives the symmetric equilibrium bidding strategies of the two games.

Proposition 1. *The bidding strategy*

$$b^*(v) = \frac{1}{1+\alpha} \cdot v \tag{1}$$

is a symmetric equilibrium of the one-bid auction.[5]

The bidding strategies

$$l^*(v) = L(\alpha) \cdot v \quad \textit{and} \quad h^*(v) = H(\alpha) \cdot v \tag{2}$$

with

$$L(\alpha) = \frac{1 - [\alpha/(1+\alpha)]^\alpha}{1 + \alpha - [\alpha/(1+\alpha)]^\alpha} \quad \textit{and} \quad H(\alpha) = \frac{1 + \alpha \cdot L(\alpha)}{1+\alpha}, \tag{3}$$

constitute a symmetric equilibrium of the two-bid auction. In equilibrium:

$$b^*(v) = \tfrac{1}{2}v, \qquad l^*(v) = \tfrac{1}{3}v \quad \textit{and} \quad h^*(v) = \tfrac{2}{3}v, \tag{4}$$

when the bidders are risk-neutral;

$$b^*(v) > \tfrac{1}{2}v, \qquad l^*(v) > \tfrac{1}{3}v \quad \textit{and} \quad h^*(v) > \tfrac{2}{3}v, \tag{5}$$

when the bidders are risk-averse;

$$b^*(v) < \tfrac{1}{2}v, \qquad l^*(v) < \tfrac{1}{3}v \quad \textit{and} \quad h^*(v) < \tfrac{2}{3}v, \tag{6}$$

when the bidders are risk-seeking.

The derivation of the equilibrium strategies can be found in Appendix A.[6]

Note that the risk-neutral equilibrium strategies for the two-bid auction when there are $n+1$ participating bidders are given by[7]

$$h^* \cdot (v) = \left(1 + \frac{1}{n}\left(1 - \left(\frac{n}{n+1}\right)^n\right)\right)^{-1} v \quad \text{and} \quad l^*(v) = \frac{n}{n+1} h^*(v). \tag{7}$$

However, we could not derive an explicit solution for the case of general risk-preferences (any $\alpha > 0$) with more than two bidders. See also Cox et al. (1988) for the generalized equilibrium strategies for the one-bid auctions (as a function of n and α). The results stated in the following Propositions 2–4 are thus restricted to the special case of two bidders.

From Proposition 1 it immediately follows that the expected revenue for the seller is equal across the two auction types when $\alpha = 1$.[8] Proposition 2 compares the expected revenues for the seller from both auction types for other risk-preferences.

Proposition 2. *In equilibrium, the expected revenue for the seller from the one-bid auction is higher than the expected revenue from the two-bid auction when the agents are risk-averse*;

5 For notational convenience let v rather than v_i denote the realized value of bidder i.

6 The appendix is available at the Elsevier website.

7 For the derivation of the first-order conditions see Appendix A.

8 By standard arguments, this also follows from the Revenue-Equivalence theorem.

the expected revenue for the seller from the two-bid auction is higher than the expected revenue from the one-bid auction when the bidders are risk-seeking.

The next proposition compares the expected utility of the bidder across the two mechanisms for different α types.

Proposition 3. *In equilibrium, for every* $v \in [0, 1]$, *the expected utility of the bidder with valuation* v *from the two-bid auction is higher than his expected utility from the one-bid auction when the bidder is risk-averse; the expected utility from the one-bid auction is higher than the expected utility from the two-bid auction when the bidder is risk-seeking; the expected utilities are equal across the two mechanisms when the bidder is risk-neutral.*

Our final proposition generalizes Proposition 1 by claiming that the corresponding results apply to the asymmetric CRRAM (see Cox et al., 1982) where the utility function of agent i takes the form $U_i(x_i) = x_i^{\alpha_i}$ and $\alpha_1 \neq \alpha_2$.[9]

Proposition 4. *Assume (w.l.g.) that* $\alpha_1 \geq \alpha_2$. *The bidding strategy*

$$b_i^*(v_i) = \frac{1}{1+\alpha_i} \cdot v_i \tag{8}$$

for $v_i \leq (1+\alpha_i)/(1+\alpha_1)$, *characterizes an equilibrium of the one-bid auction.*

The bidding strategies

$$l_i^*(v_i) = L(\alpha_i) \cdot v_i \quad \textit{and} \quad h_i^*(v_i) = H(\alpha_i) \cdot v_i \tag{9}$$

with

$$L(\alpha_i) = \frac{1 - [\alpha_i/(1+\alpha_i)]^{\alpha_i}}{1 + \alpha_i - [\alpha_i/(1+\alpha_i)]^{\alpha_i}} \quad \textit{and} \quad H(\alpha_i) = \frac{1 + \alpha_i \cdot L(\alpha_i)}{1+\alpha_i} \tag{10}$$

for $v_i \leq H(\alpha_1)/H(\alpha_i)$, *characterize an equilibrium of the two-bid auction.*

In equilibrium, for v_i *satisfying the conditions above*:

$$b_i^*(v_i) = \tfrac{1}{2} v_i, \qquad l^*(v_i) = \tfrac{1}{3} v_i \quad \textit{and} \quad h^*(v_i) = \tfrac{2}{3} v_i, \tag{11}$$

when bidder i is risk-neutral;

$$b_i^*(v_i) > \tfrac{1}{2} v_i, \qquad l^*(v_i) > \tfrac{1}{3} v_i \quad \textit{and} \quad h^*(v_i) > \tfrac{2}{3} v_i, \tag{12}$$

when bidder i is risk-averse;

$$b_i^*(v_i) < \tfrac{1}{2} v_i, \qquad l^*(v_i) < \tfrac{1}{3} v_i \quad \textit{and} \quad h^*(v_i) < \tfrac{2}{3} v_i, \tag{13}$$

when bidder i is risk-seeking.

[9] Note, however, that Propositions 2 and 3 cannot be directly generalized to the case of asymmetric bidders. In particular, note that in the asymmetric case, one bidder might be risk-averse while the other is risk-seeking so that the revenue for the seller and the expected utility of the buyer depend more complicatedly on the risk-preferences of both participants.

The proof (with all other proofs) can be found in Appendix A.

Finally note that by standard arguments the corresponding equilibrium strategies for the case where subjects' valuations are drawn from the interval $[a, b]$ are

$$b^*(v) = a + \frac{1}{1+\alpha} \cdot (v - a), \tag{14}$$

$$l^*(v) = a + L(\alpha) \cdot (v - a) \quad \text{and} \quad h^*(v) = a + H(\alpha) \cdot (v - a) \tag{15}$$

with $L(\alpha)$ and $H(\alpha)$ as defined in Eqs. (3).

Observe also that since

$$L(\alpha) \leq \frac{1}{1+\alpha} \leq H(\alpha) \tag{16}$$

independent of α,

$$l(v) \leq b(v) \leq h(v) \tag{17}$$

for every v independent of the risk-preferences of the agents.

In the proceeding analysis we use these equilibria and the propositions above as a benchmarks for analyzing the experimental results.[10]

3. Experimental design

The experimental sessions were subdivided into two distinct phases. In the first phase the subjects played repeatedly the two different auction types (one-bid and two-bid auctions) for 48 consecutive rounds. First, they played six rounds of the one-bid auction, then six rounds of the two-bid auction. These twelve games formed the first block of the experiment. It was followed by three other similar blocks where each auction type was played for six consecutive rounds in the same order as in block 1. The number of participants in each session was 8. In each round, the eight subjects were randomly divided into four pairs.

In the second phase of the experiment, the participants played repeatedly for 16 rounds an extended auction-selection game where bidders may choose their favorite auction type before observing their realized private value. To guarantee that an even number of subjects chooses each mechanism, we have only let seven out of the eight participants choose their favorite auction type (one-bid or two-bid auction) in each round. The 8th participant was automatically assigned to one of the auction types accordingly. The identity of the "8th" player was changed in each round, so that each subject played the balancing role twice among the 16 rounds. After the auction-selection stage each auction was played just as in the first phase of the experiment.

The private values of the bidders in each round were randomly drawn from the set $V = \{50, 51, 52, \ldots, 148, 149, 150\}$ with all values $v_i \in V$ being equally likely. Subjects could choose integer bids between 0 and 200.[11] Thus bidders were allowed to under-bid the lowest

[10] Note, however, that (because of the obvious technical constraints) in the experiment v_i was drawn from the finite set $V = \{50, 51, 52, \ldots, 148, 149, 150\}$.

[11] When comparing the experimental results to the equilibrium benchmarks, we sometimes normalize the realized private values and the bids submitted in the experiment by subtracting 50 and dividing the difference by 100.

possible private value $v_i = 50$ as well as to over-bid the highest possible private value $v_i = 150$. All values were denoted in a fictitious currency termed ECU for Experimental Currency Unit. Actual payments were determined according to the rules of the corresponding auction. At the end of each round, the bidders observed a feedback-window specifying whether they had won the current auction, the final buying price, the bids of both participants, their own profit in the current round, their total profits up to the current round, and their average profit in each auction type.

All experimental sessions were computerized.[12] Most participants were students of economics or business administration at Humboldt University, Berlin. In total, an experimental session lasted about 2 h. The conversion rate of the ECU earned by each subject into cash was 1 ECU = 0.03 DM. In addition, subjects were paid a fixed participation fee of 10 DM. Subjects' total earnings ranged between 18.00 and 44.41 DM with a mean of 32.44 DM. Altogether, we ran six sessions resulting in 1152 (576 one-bid, 576 two-bid) auctions in the first phase and 384 (101 one-bid, 283 two-bid) auctions in the second phase.

4. Results

4.1. Bidding behavior

4.1.1. Over-bidding realized values

Bidding above one's private valuation always defines a weakly dominated strategy. In our sample, only 10 of the 1354 bids submitted in the one-bid auction were above value. Moreover, 9 of these 10 bids were submitted in the first block and are presumably due to initial problems in understanding the game rules.

Over-bidding was much more frequent in the two-bid auctions. The over-bidding rate for these auctions was 3.84 percent (66 bids out of the 1718 high-bids submitted in these auctions).[13] A possible explanation for the high over-bidding ratios observed in the two-bid auctions is a bidder's illusion that by submitting a large high-bid he increases his chances of winning the auction (i.e., submitting the largest high-bid) without affecting the actual buying price that will be equal to his low-bid.[14] The data indeed shows that in 40 percent of all over-bidding cases in the first phase of the experiment, the over-bidders ended up with positive payoffs.

4.1.2. Bargain-bidding

Recall that the minimal possible valuation for the buyer in our auctions was 50. It immediately follows that in equilibrium bidders should never bid less than 50.[15] The experimental

[12] The software for the computerized experiment was developed with the help of z-Tree (Fischbacher, 1998).

[13] The over-bidding rate was 4.8 percent for the first phase of the experiment and 1.9 percent for the second phase of the experiment. The differences in over-bidding ratios across the four blocks in the first phase were not statistically significant. In only three cases did bidders submit a low-bid higher than their valuation; all these cases occurred in the first block of the experiment.

[14] Kagel et al. (1987) similarly conjecture that over-bidding in second-price sealed-bid auctions follows from an illusion that bidding above values increases the chances of winning without affecting the buying price that will stay equal to the second-highest bid.

[15] Bids equal to 50 might be submitted in equilibrium when the realized valuation of the buyer is 50.

Table 1
Proportion of bargain-bidding, average bargain-bid, corresponding average value

	One-bid		Two-bid_high		Two-bid_low	
	Phase 1	Phase 2	Phase 1	Phase 2	Phase 1	Phase 2
Proportion (percent)						
Bargain-bid	7.7	7.4	6.2	4.9	16.9	16.8
Mean						
Bargain-bid	33	38	30	35	31	35
Value	63	58	60	58	66	66

data set, however, shows that 7.7 percent of the bids submitted in one-bid auctions, 16.9 percent of the low-bids submitted in two-bid auctions, and 5.8 percent of the high-bids submitted in two-bid auctions were lower than 50. We henceforth refer to these cases as "bargain-bidding". Table 1 gives the proportion of bargain-bidding, the average bid submitted by bargain-bidders and the corresponding average values of bargain-bidders for each type of bid.

The relatively high rates of extremely low bidding seem like an interesting feature of our experimental data set. A possible explanation could be the bidders' attempts to win the auction at special bargain-prices. Indeed the specific auctions mentioned in the introduction (and many other auction-sites on the Web) frequently close with prices that are significantly lower than the perceived fair-market price for similar items. The possibility of buying the merchandise in a special bargain-price seems to be one of the factors that drive potential buyers to these auctions.[16]

Note that under-bidding may be rationalized by claiming that bargain-bidders expect other bidders to go for such bargain-prices as well. Our data, however, shows that only 0.7 percent of the first-phase one-bid auctions (and 1.4 percent of the first-phase two-bid auctions) closed at prices lower than 50. Still, 7.4 percent of the one-bids, 16.8 percent of the low-bids and 4.9 percent of the high-bids submitted in the second phase of the experiment were lower than 50 (see Table 1). The proportions of lower-than-50 bids in the second phase of the experiment were not significantly different[17] than the corresponding rates for the first phase. The inclination to bid lower than 50 thus seems robust with respect to the unsuccessful experience it produced at the first phase of the experiment.

4.1.3. *Bidding relatively to RNE benchmark*

The formal results in Section 2 imply the following equilibrium benchmarks for risk-neutral agents (henceforth referred to as the RNE strategies):

$$b^*(v) = 50 + \tfrac{1}{2}(v - 50), \tag{18}$$

[16] An alternative explanation is that bidders submit bids lower than 50 to signal willingness to collude. This in particular may explain the observed zero-bids submitted by some subjects along the experiment. The proportion of zero-bids from all bids lower than 50, however, was only 9 percent for the one-bids, 3.1 percent for the low-bids and 8.5 percent for the high-bids. Zero-bidding (i.e., collusive attempts) thus accounts for only a small share of the observed bargain-bids in the sample.

[17] For each session.

$$l^*(v) = 50 + \tfrac{1}{3}(v - 50) \quad \text{and} \quad h^*(v) = 50 + \tfrac{2}{3}(v - 50). \tag{19}$$

We employed a Wilcoxon test to check the null hypothesis that the observed bids are not significantly different from the equilibrium benchmarks for risk-neutral agents (i.e., $b(v) - b^*(v) = 0$, $h(v) - h^*(v) = 0$ and $l(v) - l^*(v) = 0$, respectively). We ran the test for each subject and each type of bid separately. Following the benchmarks of Proposition 1, we say that the subject bids as if (s)he is risk-averse when the Wilcoxon statistic is positive. We similarly say that the subject bids as if (s)he is risk-seeking when the statistic is negative.

We find that 38 (10) subjects act as if risk-averse (risk-seeking) in the one-bid auctions; 36 (12) subjects act as if risk-averse (risk-seeking) when placing the low-bid in the two-bid auction. The same division (36 subjects acting as if risk-averse and 12 acting as if risk-seeking) is observed for the high-bids. Overall, the proportion of risk-averse behavior is not lower than 75 percent while the proportion of risk-seeking behavior is close to 25 percent. These proportions seem reminiscent of those observed in Cox et al. (1988) and other experimental studies of first-price auctions and support the general belief that subjects act as if risk-averse rather than risk-seeking in first-price auctions (see Kagel, 1995) independently of the applied auction format (one-bid or two-bid).[18]

4.1.4. Bidding ratios

Define the (revealed) *bidding ratio* of an agent with a realized valuation v in a one-bid auction as follows:[19]

$$r_b(v) = \frac{b(v) - 50}{v - 50}. \tag{20}$$

Similarly, define the (revealed) bidding ratios of the agent in a two-bid auction as

$$r_l(v) = \frac{l(v) - 50}{v - 50} \quad \text{and} \quad r_h(v) = \frac{h(v) - 50}{v - 50}. \tag{21}$$

Recall (see the last paragraph of Section 2) that in equilibrium $r_l(v) < r_b(v) < r_h(v)$, for each value v, independent of the risk-preferences of the subjects. In particular, for risk-neutral agents, $r_l(v) = 1/3 < r_b(v) = 1/2 < r_h(v) = 2/3$. Indeed, when we average the three bidding ratios for each subject we find that 79.3 percent of the subjects conform with the prediction $r_l(v) < r_b(v)$, while 83.7 percent conform with the conjectured $r_b(v) < r_h(v)$.[20]

[18] These proportions are also close to those observed in many "other" past experiments on choice and decision. Tversky and Kahneman (1986), to cite a classic reference, report that 72 percent of 126 subjects preferred a certain payoff of 100 over a 50 percent chance of obtaining a payoff of 200. In a recent investigation, Sonsino et al. (2002) find that 69 percent of 120 subjects prefer a certain payoff of 107 on a lottery that pays 150 with probability 30 percent, 80 with probability 40 percent and 100 with probability 30 percent.

[19] For simplification we suppress the agent-index i from the notation for r.

[20] The conformity rates for the first phase of the experiment were 72.9 and 83.3 percent, respectively; the corresponding rates for the second phase were 86.4 and 84.1 percent, respectively. A Wilcoxon test, $N = 6$, two-tailed, suggests that the differences in proportions across the two phases are statistically insignificant ($P > 0.1$).

Table 2
Median bidding ratios

Session	$r_b(v)$		$r_h(v)$		$r_l(v)$	
	Phase 1	Phase 2	Phase 1	Phase 2	Phase 1	Phase 2
1	0.56	0.62	0.74	0.71	0.39	0.19
2	0.60	0.66	0.77	0.78	0.41	0.47
3	0.59	0.47	0.77	0.85	0.43	0.46
4	0.70	0.81	0.86	0.88	0.44	0.46
5	0.61	0.68	0.79	0.83	0.45	0.50
6	0.58	0.55	0.77	0.70	0.46	0.41
All	0.61	0.63	0.79	0.80	0.44	0.44

Table 2 gives the median bidding ratios for each session and each auction type. The median ratios are higher than the benchmark RNE ratios, conforming with the equilibrium predictions for risk-averse agents.[21]

4.1.5. *Correlation between bids and values*

The coefficients of correlation between the three individual bidding ratios (r_b, r_l, r_h) and the corresponding realized values were found to be positive and statistically significant at 0.01 for both phases of the experiment. Moreover, the coefficients for the second phase are significantly higher than those for the first phase.[22] In particular, the Spearman's correlation coefficients for the bidding ratio in the one-bid auctions (r_b) were 0.174 (for the first phase of the experiment) and 0.299 (for the second phase). The corresponding coefficients for the high-bidding ratio (r_h) were 0.143 and 0.217 and for the low bidding ratio (r_l), 0.365 and 0.410, respectively. Note that in our benchmark equilibrium model the bidding ratios are constant; this implies zero correlation between bidding ratios and values, a prediction that is violated by the data. The observed positive correlations suggest that subjects tend to bid more aggressively as their realized value increases. Moreover, this inclination increases in the second phase of the experiment.

The correlation between bids and values becomes even more significant when we examine the three Relative Bidding Deviations (see Kagel and Roth, 1992):

$$\mathrm{RBD}_b = \frac{b(v) - b^*(v)}{v}, \tag{22}$$

$$\mathrm{RBD}_h = \frac{h(v) - h^*(v)}{v} \quad \text{and} \quad \mathrm{RBD}_l = \frac{l(v) - l^*(v)}{v}. \tag{23}$$

The coefficients described in Table 3 are positive and statistically significant. Recall that in Kagel and Roth, the coefficients were negative and used to support Harrison's "flat maximum

[21] The average bidding ratios are much lower than the median ratios because of the under-bidding phenomenon discussed above.

[22] Wilcoxon one-tailed test, $P < 0.05$ for r_b and r_h, $N = 6$. For r_l, the correlation coefficient in phase 2 is higher than in phase 1 in four (out of six) sessions.

Table 3
Spearman's correlation coefficients: RBDs and values (number of RBDs in parentheses)

Session	One-bid		Two-bid_high		Two-bid_low	
	Phase 1	Phase 2	Phase 1	Phase 2	Phase 1	Phase 2
1	0.319** (192)	0.618** (46)	0.402** (192)	0.470** (82)	0.661** (192)	0.661** (82)
2	0.275** (192)	0.522** (28)	0.328** (192)	0.466** (100)	0.544** (192)	0.536** (100)
3	0.296** (192)	0.395** (44)	0.187** (192)	0.321** (84)	0.335** (192)	0.239* (84)
4	0.456** (192)	0.851** (42)	0.301** (192)	0.423** (86)	0.376** (192)	0.406** (86)
5	0.172* (192)	0.194 (32)	0.281** (192)	0.601** (96)	0.313** (192)	0.644** (96)
6	0.428** (192)	0.830** (10)	0.370** (192)	0.394** (118)	0.539** (192)	0.641** (118)
All	0.312** (1152)	0.477** (202)	0.302** (1152)	0.427** (566)	0.462** (1152)	0.521** (566)

* Significant at 0.05 level (two-tailed).
** Significant at 0.01 level (two-tailed).

critique"[23] concerning the experimental evidence with first-price auctions.[24] The positive coefficients found in the current study may thus be used as counter-evidence to the flat maximum critique and (in this sense) seem to strengthen the weight of our evidence for risk-averse behavior in first-price auctions.

4.1.6. Dynamics

Fig. 1 describes the changes in median bidding ratios in the 48 rounds of the first phase of the experiment. The rather flat lines suggest that subjects' behavior does not change in any specific direction across these rounds. Furthermore, the results of a Spearman's correlation analysis between the relative bidding deviations (RBDs) at each round t and the corresponding round index t disclose no clear-cut trends; the correlation coefficients are both, positive and negative, and in almost all cases insignificant. Overall it seems that our subjects did not significantly modify their behavior during the experiment.

4.2. Efficiency, prices and payoffs

We say that an auction allocation is efficient if the bidder with the highest value wins the auction. Overall, the proportions of inefficient one-bid auctions (15.3 percent for phase 1 and 16.8 percent for phase 2) are considerably higher than the corresponding proportions (10.2 and 12.0 percent, respectively) for the two-bid auctions.

To analyze further the efficiency of each auction type we define the efficiency ratio (ER):

$$\mathrm{ER} = \frac{v_{\mathrm{buyer}}}{\max\{v_1, v_2\}}, \tag{24}$$

where v_{buyer} denotes the valuation of the winner.

[23] Harrison's critique suggests that subjects who have low realized values and thus have low odds of winning the auction have little incentives to bid optimally. See chapter G in Kagel's survey on auctions in the Handbook of Experimental Economics (1995) for more details on this controversy.
[24] Negative correlations between subjects' valuations and the relative bidding ratios were also found by Pezanis-Christou in his study of asymmetric auctions.

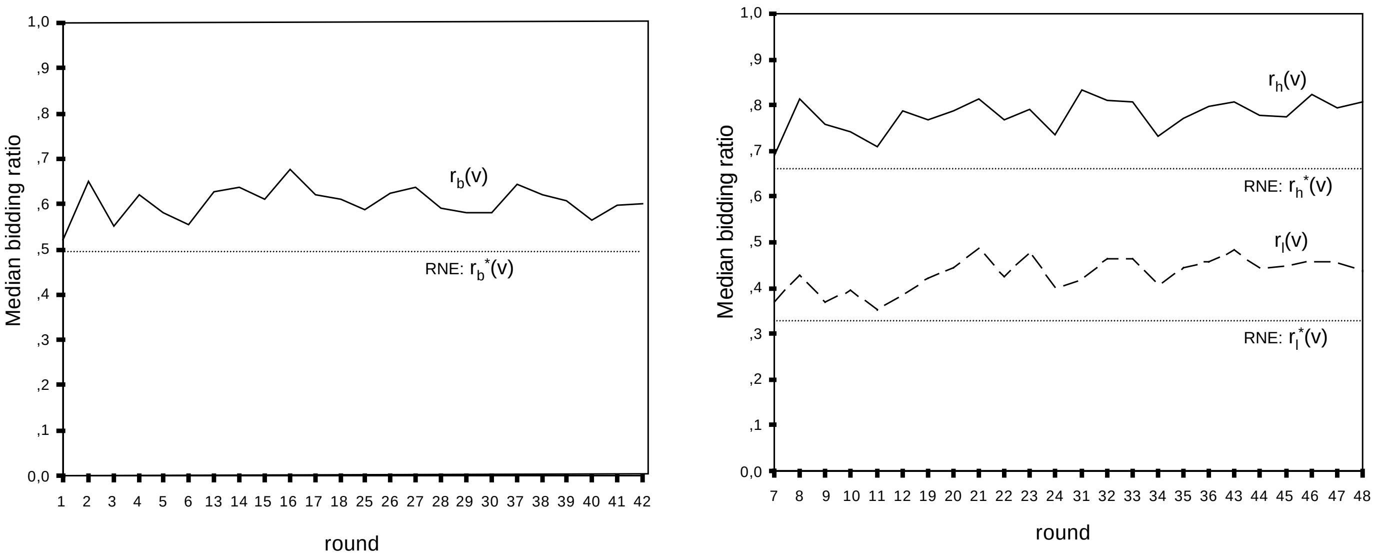

Fig. 1. Time paths of median bidding ratios (phase 1).

Table 4
Mean efficiency ratios ER (in percent)

Session	Phase 1		Phase 2	
	One-bid	Two-bid	One-bid	Two-bid
1	97.5	98.1	98.6	98.3
2	97.8	98.7	99.2	98.9
3	97.7	98.6	96.7	99.0
4	98.8	98.6	99.4	99.5
5	97.4	98.8	96.9	99.4
6	98.2	98.8	99.5	98.1
All	97.9	98.6	98.2	98.8

Table 5
Mean prices (S.D.)

Session	Phase 1		Phase 2	
	One-bid	Two-bid	One-bid	Two-bid
1	91.22 (18.54)	86.70 (21.79)	91.09 (21.52)	81.32 (23.15)
2	89.83 (17.26)	89.03 (22.61)	101.71 (17.59)	89.20 (25.51)
3	90.47 (21.40)	94.29 (20.47)	89.36 (15.91)	91.48 (17.16)
4	96.50 (19.05)	94.01 (22.58)	104.14 (24.58)	94.05 (18.85)
5	92.43 (18.34)	91.14 (18.40)	95.50 (14.05)	88.04 (18.46)
6	96.43 (19.80)	87.57 (18.66)	86.60 (18.00)	90.07 (19.96)
All	92.81 (19.26)	90.46 (20.99)	95.38 (20.13)	89.12 (21.00)

Note that the efficiency ratio equals 1 when the auction is won by the bidder with the highest valuation; however, the ratio is lower than 1 when the auction is inefficient, and it is monotonically decreasing with the gap between the highest valuation and the valuation of the buyer. Table 4 gives the average efficiency ratios for each auction. Again, the observed efficiency is slightly higher in the two-bid auction compared to the one-bid auction in both phases of the experiment. A Wilcoxon test suggests that the difference is statistically significant in the first phase ($P = 0.031$, $N = 6$, one-tailed). Efficiency increases slightly for both auction types in the second phase. The difference in favor of the two-bid auction, however, is no longer significant.

The average prices collected by the seller in each auction are presented in Table 5. The average prices obtained in the first phase of the experiment are quite similar across the two mechanisms. In the second phase, the one-bid auction seems to produce slightly higher prices but the differences are statistically insignificant (Wilcoxon test, $P = 0.219$ (0.156), for the first (second) phase, $N = 6$, two-tailed). Additional tests confirm that there are no significant differences between the cumulative distributions of prices and the average prices obtained in each block of the experiment, across the two auction types.[25]

The average payoffs of the winning bidder in each auction type in the first phase of the experiment are displayed in Table 6. Again, the results reveal no significant differences

[25] We have confirmed that the distribution of actually selected private values in both auction formats does not differ significantly.

Table 6
Average payoffs in phase 1 (S.D.)

	Session						
	1	2	3	4	5	6	All
One-bid	12.37	12.33	11.43	9.30	10.57	11.52	11.25
	(16.39)	(15.86)	(16.86)	(12.59)	(15.73)	(16.60)	(15.74)
Two-bid	13.31	11.83	12.70	9.22	13.63	13.82	12.42
	(17.83)	(18.12)	(18.74)	(17.22)	(19.34)	(18.16)	(18.27)

Table 7
Auction-selection proportions (in percent) (phase 2)

	Session						
	1	2	3	4	5	6	All
One-bid	33	21	31	28	21	6	23
Two-bid	67	79	69	72	79	94	77

across the two mechanisms (Wilcoxon test, $P = 0.156$, $N = 6$, two-tailed). Note, however, that the standard deviation is significantly higher in the two-bid auction (Binomial test, $P = 0.031$, $N = 6$, two-tailed).

Overall the results suggest that (on average) the experience gained with the two mechanisms during the first phase of the experiment did not provide a strong reason to prefer one of the two auction types in the second phase of the experiment. The similarity in average payoffs is maintained in the second phase of the experiment.[26]

4.3. *Choice of mechanism*

Recall that in the second phase of the experiment subjects could choose their favorite auction type before observing their realized value. Table 7 gives the distribution of choices for each session. Altogether, subjects preferred the two-bid auction in 516 out of 672 cases (77 percent). Moreover, subjects' strong preference for the two-bid auction holds in all six sessions.

This strong result is confirmed by the data at the individual level. Recall that each subject was allowed to choose his favorite auction type in 14 rounds (and acted as the balancing player at the other two rounds). Fig. 2 demonstrates that an overwhelming majority (77 percent) of the participants chose the two-bid auction at least nine times. The average number of times the two-bid auction was chosen was 10.75 with a median of 13 while the numbers were 3.25 and 1 for the one-bid auction.

Furthermore, when we count the number of cases (in the second phase of the experiment) where: (i) a subject that lost a two-bid auction chose the same auction type in the following round or (ii) a subject that won a one-bid auction chose the two-bid auction in the following round, we find that in 197 out of 246 cases (80 percent) subjects who had plausible reasons to select the one-bid auction voted for the two-bid auction.

[26] Wilcoxon test, $P = 0.219$, $N = 6$, two-tailed.

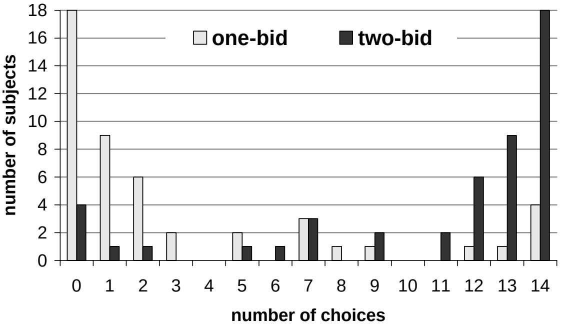

Fig. 2. Distribution of subjects' choices.

One possible explanation to the observed strong preference for two-bid auctions is risk-aversion of the participating subjects. Our benchmark equilibrium model indeed suggests that when subjects act risk-aversively their expected utility from the two-bid auction would be higher than the corresponding utility from the one-bid auction for each value v. However, risk aversion is expected to affect subjects' choices via revenues and prices, which as already shown do not significantly differ as predicted. So, risk aversion cannot be the only reason for the observed behavior.

Alternatively, one might invoke a context-dependent preferences type of argument to explain the results. In particular, it may be the case that the two-bid mechanism is perceived as more exciting than the standard one-bid auction. A variation on Conlisk's (1993) idea, for instance, would suggest that subjects who exhibit an "extra tiny utility" from the possibility to place two different bids might prefer the two-bid mechanism over the standard one-bid auction, as demonstrated in our data set.

Finally one might suggest that the two-bid auction is perceived as easier to solve than the one-bid auction. In particular, note that in the standard one-bid case, a submitted bid not only determines the probability of winning but also represents the price that the bidder is willing to pay for the object. In the two-bid case the bidders have two separate decision parameters. The probability of winning is determined by the high-bid alone. The actual price paid by the bidder upon winning, however, might also depend on the low-bid. This suggests that bidders might find the two-bid auction a cognitively easier problem to solve and thus reveal strong preferences for this mechanism.[27]

5. Concluding discussion

The constant increase in economic activity on the Web provides a serious challenge to basic economic research. Economic transactions on the Web might be quite different in many respects from the corresponding transactions in traditional environments. The costs

[27] In the post-experimental questionnaire subjects gave the following main reasons for preferring the two-bid mechanism: higher (perceived) probability of winning; higher payoffs; the possibility to "play" with more than one bid; easier decision-problem.

of conducting an auction on the Web, for instance, seem marginal compared to the costs of running the auction in a real auction house. Indeed, some of the largest auction-sites on the Web (e.g., *Ebay, Yahoo, City auction*) act as listing-sites (Lucking-Reiley) through which potential sellers may auction their merchandise with relatively low cost and effort. Note also that it is quite inexpensive to modify a given auction mechanism when the auction is mounted on the Web. A shift from using a one-bid auction to implementing a two-bid auction, for instance, seems like a technically trivial problem that should not impose any significant costs on the auctioneer. These conveniences of running auctions on the Web, together with the "global market" effect and the intensified competition on the electronic medium, create the possibility that sellers might compete at the level of the auction mechanism by trying to find the mechanism that attracts the most bidders and generates the highest revenue for the seller. Indeed, Monderer and Tennenholtz study an auction-selection game where multiple sellers of a homogeneous good compete for a given pool of buyers. Sellers may choose different auction mechanisms (first-price, second-price and in general kth price auction); buyers choose their favorite auction-sites accordingly. Monderer and Tennenholtz investigate the equilibria of the underlying games and show that the optimal auction mechanism depends on the risk-preferences of the potential bidders.

With this as a general motivation for comparative studies of different auction mechanisms on the Web, this paper focuses on a comparative experimental investigation of the one-bid and two-bid first-price auctions. We show that in both auction types subjects typically act as though they are risk-averse in the sense of bidding more than the Nash equilibria for risk-neutral agents and in the sense of preferring the two-bid auction over the one-bid auction when given the possibility to choose between the two. However, the average profits to the buyer and the average prices collected by the seller were not significantly different across the two mechanisms. These latter findings contradict the benchmark model predictions for risk-averse agents.

Thus our results suggest that subjects have clear-cut preferences among the two auction mechanisms that are not only driven by expected profitability or observed-monetary-gain type of considerations. This raises further questions, such as what drives subjects' behavior in such decision problems and how stable is the observed inclination to prefer the two-bid auction. The broader issue of characterizing and understanding decision-makers' preferences between different auction mechanisms seems like an interesting domain for further research.

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Appendix A. Proof of propositions

First we derive the equilibrium strategies presented in Proposition 1.

We start with the two-bid model.

Assume a symmetric equilibrium $h(\cdot)$, $l(\cdot)$ where both bidding functions are strictly increasing and continuous. Let $h^{-1}(\cdot)$ and $l^{-1}(\cdot)$ denote the corresponding inverse functions.

Let v denote the realized value of bidder i.

Let $y \equiv l(v)$ and $z \equiv h(v)$ denote the corresponding equilibrium bids.

Note that the probability that i's lowest bid would win is equal to

$$\text{Prob}[h(v_j) < y] = \text{Prob}[V_j < h^{-1}(y)] = h^{-1}(y).$$

Similarly, the probability that i's high-bid would be highest is equal to

$$\text{Prob}[y \le h_j(v_j) \le z] = \text{Prob}[h^{-1}(y) \le V_j \le h^{-1}(z)] = h^{-1}(z) - h^{-1}(y).$$

Assuming the utility function $u(x) = x^\alpha$, the expected utility of player i equals

$$h^{-1}(y) \cdot [v - y]^\alpha + [h^{-1}(z) - h^{-1}(y)] \cdot [v - z]^\alpha. \tag{A.1}$$

Differentiating (A.1) with respect to y gives the first-order condition:

$$-\alpha[v - y]^{\alpha-1} \cdot h^{-1}(y) + [v - y]^\alpha \cdot \frac{\mathrm{d}}{\mathrm{d}(y)} h^{-1}(y) - [v - z]^\alpha \cdot \frac{\mathrm{d}}{\mathrm{d}(y)} h^{-1}(y) = 0. \tag{A.2}$$

Differentiating (A.1) with respect to z gives the first-order condition:

$$-\alpha[v - z]^{\alpha-1} \cdot [h^{-1}(z) - h^{-1}(y)] + [v - z]^\alpha \cdot \frac{\mathrm{d}}{\mathrm{d}(z)} h^{-1}(z) = 0.$$

That is

$$-\alpha \cdot [h^{-1}(z) - h^{-1}(y)] + [v - z] \cdot \frac{\mathrm{d}}{\mathrm{d}(z)} h^{-1}(z) = 0. \tag{A.3}$$

Note, however, that

$$\frac{\mathrm{d}}{\mathrm{d}(y)} h^{-1}(y) = \frac{1}{h'(y)}, \tag{A.4}$$

$$\frac{\mathrm{d}}{\mathrm{d}(z)} h^{-1}(z) = \frac{1}{h'(z)}. \tag{A.5}$$

Substituting (A.4) and (A.5) into (A.2) and (A.3) yields

$$-\alpha[v - y]^{\alpha-1} \cdot h^{-1}(y) + [v - y]^\alpha \cdot \frac{1}{h'(y)} - [v - z]^\alpha \cdot \frac{1}{h'(y)} = 0, \tag{A.6}$$

$$-\alpha \cdot [h^{-1}(z) - h^{-1}(y)] + [v - z] \cdot \frac{1}{h'(z)} = 0. \tag{A.7}$$

Trying

$$y = L(\alpha) \cdot v \quad \text{and} \quad z = H(\alpha) \cdot v \tag{A.8}$$

in (A.6) and (A.7) gives

$$L(\alpha) = \frac{1-[\alpha/(1+\alpha)]^{\alpha}}{1+\alpha-[\alpha/(1+\alpha)]^{\alpha}} \quad \text{and} \quad H(\alpha) = \frac{1+\alpha L}{1+\alpha}.$$

Direct calculations show that the Hessian matrix of the objective (A.1) is negative semi definite so that

$$l(v) = L(\alpha)\cdot v \quad \text{and} \quad h(v) = H(\alpha)\cdot v \tag{A.9a}$$

is a symmetric equilibrium of the two-bid auction.

Assuming (more generally) that there are $n+1$ participating bidders gives the objective

$$[h^{-1}(y)]^{n}\cdot(v-y)^{\alpha} + ([h^{-1}(z)]^{n} - [h^{-1}(y)]^{n})\cdot(v-z)^{\alpha}. \tag{A.9b}$$

Differentiation with respect to y and z and some algebra gives the general first-order conditions for H and L:[28]

$$n\left[\left(\frac{1-H}{1-L}\right)^{\alpha} - 1\right] = \frac{-\alpha L}{1-L} \quad \text{and} \quad \alpha\left(1-\left[\frac{L}{H}\right]^{n}\right) = (1-H)\frac{n}{H}. \tag{A.9c}$$

Consider next the one-bid first-price auction where the bidders' utility function is given by $u(x) = x^{\alpha}$. We derive the symmetric equilibrium of the model using $b(\cdot)$ to denote the equilibrium bidding strategy.[29]

Note that the expected utility of a bidder with realized value v from bidding $s \equiv b(v)$ is

$$b^{-1}(s)\cdot[v-s]^{\alpha}. \tag{A.10}$$

Differentiating with respect to s gives

$$\frac{\mathrm{d}}{\mathrm{d}(s)}b^{-1}(s)\cdot[v-s]^{\alpha} - \alpha[v-s]^{\alpha-1}\cdot b^{-1}(s) = 0.$$

This implies that

$$\frac{\mathrm{d}}{\mathrm{d}(s)}b^{-1}(s)\cdot[v-s] = \alpha\cdot b^{-1}(s).$$

Trying $s = B(\alpha)\cdot v$ gives $B(\alpha) = 1/(1+\alpha)$.

Checking the second-order conditions confirms that

$$b(v) = \frac{1}{1+\alpha}\cdot v \tag{A.11}$$

is an equilibrium of the one-bid auction.

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[28] For the case of risk neutrality ($\alpha = 1$) the equilibrium strategies for the two-bid auction are $h^{*}(v) = (1 + (1/n)(1-(n/(n+1))^{n}))^{-1}v$ and $l^{*}(v) = (n/(n+1))h^{*}(v)$.

[29] Vickrey (1961) solves the first-price auction model for the case where the bidders are risk-neutral; Holt (1980), Riley and Samuelson (1981) assume identically risk-averse bidders. Cox et al. (1982) characterize the equilibria of the asymmetric model when the agents may have different α's (CRRAM). For the completeness of the exposition, however, we outline the derivation of the symmetric equilibrium.

To prove Proposition 2 note that, by Bernoulli's inequality:

$$\left[\frac{\alpha}{1+\alpha}\right]^{\alpha} = \left[1-\frac{1}{1+\alpha}\right]^{\alpha} \le 1-\frac{\alpha}{1+\alpha} = \frac{1}{1+\alpha} \le \frac{2-\alpha}{2}$$

when $\alpha \le 1$ and the inequalities are reversed when $\alpha \ge 1$.

From the solution for $l(v)$ above, it follows that $l(v) \ge v/3$ iff $[\alpha/(1+\alpha)]^{\alpha} \le [(2-\alpha)/2]$ iff $\alpha \le 1$ and the inequalities are reversed when $\alpha \ge 1$.

It immediately follows that $l(v) = v/3$ [and $h(v) = 2v/3$] when the bidder is risk-neutral (i.e., when $\alpha = 1$); $l(v) > v/3$ [and $h(v) > 2v/3$] when the bidder is risk-averse (i.e., when $\alpha < 1$) and $l(v) < v/3$ [and $h(v) < 2v/3$] when the bidder is risk-seeking. This completes the proof of Proposition 1.

From Myerson's (1981) revenue equivalence principle, it should immediately be clear that the two-bid auction is revenue-equivalent to the standard one-bid first-price auction when the bidders are risk-neutral. To prove Proposition 2, we have to compare the revenues for the cases where $\alpha \ne 1$.

The expected revenue for the seller in the one-bid auction when the buyer with the highest valuation has a realized value of v is

$$\frac{1}{1+\alpha}\cdot v. \tag{A.12}$$

From Eq. (A.9a) it follows that the expected revenue for the seller in the two-bid auction when the buyer with the highest valuation has a realized value of v is

$$\frac{L(\alpha)}{H(\alpha)}\cdot L(\alpha)\cdot v + \frac{H(\alpha)-L(\alpha)}{H(\alpha)}\cdot H(\alpha)\cdot v. \tag{A.13}$$

We now show that the seller prefers the two-bid mechanism over the one-bid mechanism when $\alpha > 1$ and the inequality is reversed when $\alpha < 1$, proving Proposition 2.

From (A.12) and (A.13) above it is clear that it is enough to show that

$$\frac{L(\alpha)}{H(\alpha)}\cdot L(\alpha)\cdot v + \frac{H(\alpha)-L(\alpha)}{H(\alpha)}\cdot H(\alpha)\cdot v > \frac{1}{1+\alpha}\cdot v \tag{A.14}$$

for every $v \in [0, 1]$ when $\alpha > 1$ and the inequality is reversed when $\alpha < 1$. From the definitions of $H(\alpha)$ and $L(\alpha)$, however, we know that

$$\frac{L}{H} = \frac{(1+\alpha)L}{1+\alpha L} \tag{A.15}$$

and

$$1-\frac{L}{H} = \frac{1-L}{1+\alpha L}. \tag{A.16}$$

Substituting (A.15) and (A.16) into (A.14) and rearranging we get the inequality:

$$L > \frac{1}{1+\alpha+\alpha^2}.$$

Substituting the definition of L we get the inequality:

$$\left(\frac{\alpha}{1+\alpha}\right)^{\alpha} < \frac{\alpha}{1+\alpha}$$

which clearly holds when $\alpha > 1$ and is reversed when $\alpha < 1$. This completes the proof of Proposition 2.

Next, we examine the expected utility of a buyer with valuation v in the two mechanisms.

Plugging (A.11) into the objective (A.10) gives the expected utility for a bidder with valuation v in the one-bid auction:

$$\left[\frac{\alpha}{1+\alpha}\right]^{\alpha} \cdot v^{1+\alpha}. \tag{A.17}$$

The corresponding equation for the two-bid auction mechanism is

$$\frac{L(\alpha)}{H(\alpha)} \cdot v \cdot [v - l(v)]^{\alpha} + \frac{H(\alpha) - L(\alpha)}{H(\alpha)} \cdot v \cdot [v - h(v)]^{\alpha}. \tag{A.18}$$

Formally, we will now show that

$$\frac{L(\alpha)}{H(\alpha)} \cdot v \cdot [v - l(v)]^{\alpha} + \frac{H(\alpha) - L(\alpha)}{H(\alpha)} \cdot v \cdot [v - h(v)]^{\alpha} > \left[\frac{\alpha}{1+\alpha}\right]^{\alpha} \cdot v^{1+\alpha}, \tag{A.19}$$

when $\alpha < 1$, and the reverse inequality holds when $\alpha > 1$, as proposed in Proposition 3. (Note immediately that the two sides are indeed equal when $\alpha = 1$.)

From the definitions of $H(\alpha)$ and $L(\alpha)$ we have

$$1 - H = \frac{\alpha}{1+\alpha} \cdot [1 - L], \tag{A.20}$$

$$\left[\frac{\alpha}{1+\alpha}\right]^{\alpha} = \frac{1 - (1+\alpha)L}{1 - L}. \tag{A.21}$$

Substituting (A.15) and (A.16), and (A.20) and (A.21) into (A.19) and rearranging, we get the inequality:

$$(1 - L)^{\alpha+1} > 1 - L - \alpha \cdot (1+\alpha) \cdot L^2. \tag{A.22}$$

Plugging in the definition of L and rearranging we get

$$(1+\alpha)^{\alpha} > \left[1 + 2\alpha - \left[\frac{\alpha}{1+\alpha}\right]^{\alpha} \cdot (1+\alpha)\right] \cdot \left[1 + \alpha - \left[\frac{\alpha}{1+\alpha}\right]^{\alpha}\right]^{\alpha-1}. \tag{A.23}$$

This implies that

$$\left[\frac{1 + \alpha - [\alpha/(1+\alpha)]^{\alpha}}{1+\alpha}\right]^{\alpha} < \frac{1 + \alpha - [\alpha/(1+\alpha)]^{\alpha}}{1 + 2\alpha - (1+\alpha)[\alpha/(1+\alpha)]^{\alpha}}. \tag{A.24}$$

Since however

$$\frac{1 + \alpha - [\alpha/(1+\alpha)]^{\alpha}}{1+\alpha} = 1 - \frac{1}{1+\alpha}\left[\frac{\alpha}{1+\alpha}\right]^{\alpha},$$

and for $\alpha < 1$:

$$1 - \frac{1}{1+\alpha}\left[\frac{\alpha}{1+\alpha}\right]^{\alpha} < 1 - \left[\frac{\alpha}{1+\alpha}\right]^{\alpha+1},$$

it immediately follows that

$$\left[\frac{1+\alpha-[\alpha/(1+\alpha)]^{\alpha}}{1+\alpha}\right]^{\alpha} < 1-\left[\frac{\alpha}{1+\alpha}\right]^{1+\alpha}. \tag{A.25}$$

To prove (A.24) it is thus enough to show that for $\alpha < 1$:

$$1-\left[\frac{\alpha}{1+\alpha}\right]^{1+\alpha} < \frac{1+\alpha-[\alpha/(1+\alpha)]^{\alpha}}{1+2\alpha-(1+\alpha)[\alpha/(1+\alpha)]^{\alpha}}. \tag{A.26}$$

Direct algebra shows that this is equivalent to

$$\frac{\alpha(1-[\alpha/(1+\alpha)]^{\alpha})}{1+2\alpha-(1+\alpha)[\alpha/(1+\alpha)]^{\alpha}} < \left[\frac{\alpha}{1+\alpha}\right]^{1+\alpha} \tag{A.27}$$

which under some additional manipulations becomes

$$\left[1-\left[\frac{\alpha}{1+\alpha}\right]^{\alpha}\right]^{2} < \left[\frac{\alpha}{1+\alpha}\right]^{1+\alpha}. \tag{A.28}$$

More algebra gives

$$(1+\alpha)^{\alpha}-\alpha^{\alpha} < \alpha^{(\alpha+1)/2}(1+\alpha)^{(\alpha-1)/2} \tag{A.29}$$

which in turn is equivalent to

$$\left[\frac{1+\alpha}{\alpha}\right]^{(\alpha-1)/2} > \left[\frac{1+\alpha}{\alpha}\right]^{\alpha}-1. \tag{A.30}$$

To prove (A.19) it is thus enough to show that inequality (A.30) holds when $\alpha < 1$. This, however, follows directly from the fact that the LHS and the RHS of the inequality increase in α, are equal when $\alpha = 1$, and the derivative of the LHS (with respect to α) is lower than the derivative of the RHS (with respect to α), for every α.

A symmetric argument shows that inequality (A.19) is reversed when $\alpha > 1$, which completes the proof of Proposition 3.

Finally, we need to prove Proposition 5.

This, however, follows directly from slight modifications on the arguments used in the proof of Proposition 1.

Using subscript i to denote the bidding strategy of agent i we get the following first-order conditions for the asymmetric case (see Eqs. (A.6) and (A.7)).

$$\begin{aligned}&-\alpha_i[v_i-l_i(v_i)]^{\alpha_i-1}\cdot h_j^{-1}(l_i(v_i))+[v_i-l_i(v_i)]^{\alpha_i}\cdot\frac{1}{h'_j(l_i(v_i))}\\&\quad-[v_i-h_i(v_i)]^{\alpha_i}\cdot\frac{1}{h'_j(l_i(v_i))}=0,\end{aligned} \tag{A.31}$$

$$-\alpha_i \cdot [h_j^{-1}(h_i(v_i)) - h_j^{-1}(l_i(v_i))] + [v_i - h_i(v_i)] \cdot \frac{1}{h'_j(h_i(v_i))} = 0. \tag{A.32}$$

Trying

$$l_k(v_k) = L_k(\alpha_k) \cdot v_k \quad \text{and} \quad h_k(v_k) = H_k(\alpha_k) \cdot v_k \tag{A.33}$$

for $k = i, j$ in (A.31) and (A.32) and multiplying both equations by the constant H_j gives the same first-order conditions as obtained for the symmetric model. This (together with the argument used to prove Proposition 1 for the symmetric model) completes the proof of Proposition 4.[30]

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[30] The bounds on v_i stated in the proposition is imposed for the "usual" reasons; see, for instance, Cox et al. (1982).

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