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INFLATION, THE LEVEL OF INVESTMENT, AND INTEREST RATES*

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Using a microeconomic model, the paper examines the impact of inflation on the level of investment. In the framework of the loanable-funds market, the behavior of a typical risk averse borrower and a risk averse lender is investigated. It is shown how inflation depresses the level of activity in the loanable-funds market and under what conditions the Fisherian rule for the relation between inflation and the interest rate holds.

1. Introduction

The long bout which the industrialized world had with inflation during the seventies was accompanied by much lower rates of investment growth than theretofore.[1] It stands to reason that there is a causal relationship between the two phenomena. But while it is generally accepted that the heightened degree of uncertainty that characterizes inflation is somehow to blame, we are not aware of many particular analyses that consider the assumed effects in detail.

It is perhaps not surprising that not much attention has been paid to the possibility of inflation having a retarding effect on investment.[2] For, not so long ago, when inflation used to be a typical business cycle phenomenon, it was generally understood to result from the investment boom that occurred near the peak of the cycle. That boom was accompanied by a lot of credit generation, i.e., creation of inside money. This also explains the preoccu-

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[1]Thus, for instance, the average real rate of growth of gross capital formation in France was 7.8% during 1956–1960, 9.6% in 1961–1965, 5.6% in 1966–1970, 3.9% in 1971–1975 and only 1.4% during 1976–1980. In 1975 and 1977 there were absolute declines, for the first time since 1958.

[2]One exception that we are aware of is Fama (1982), but his is mostly an empirical study using a macroeconomic framework.

pation of economists with the relationship between inflation and the rate of interest, which in turn dictated the use of macroeconomic models.[3]

The present inquiry constitutes a methodological departure in two respects. First, we are primarily interested in the impact of inflation on the level of investment. Results concerning the relation between inflation and the rate of interest are incidental and, regarding equilibrium rates, sometimes inconclusive. Secondly, the investigation uses as its vehicle a strictly microeconomic framework – the loanable-funds market. We consider the typical agent in each side of that market and center the analysis on its characterization.

The most important stipulation concerns what is not being assumed: in particular, we assume neither 'perfection' nor 'completeness'. To stress what is not assumed is a very unusual approach but also an unavoidable one in this context, for anyone used to portfolio analysis. The portfolio approach, in its general equilibrium setting,[4] created a situation where the two sides of the loanable funds market are undistinguishable, unless specific restrictions are imposed.

In the present case, we want to make sure that the borrower is an entrepreneur who contemplates investing in his plant. The opportunities he faces are 'incomplete' in two ways. First, we bar him from purchasing securities or real estate. For if he does so instead of investing in productive capacity, he becomes irrelevant to the problem at hand. Moreover, if he was driven by inflation into purchasing securities instead of investing in plant and equipment, the problem would be assumed away. Second, our entrepreneur is not allowed to issue indexed bonds. This is not an elegant restriction, but certainly conforms to casual observation.[5]

On the other side of the market, the lender is assumed to either face an imperfect market or to lack the qualities needed to be an entrepreneur. Consequently, his access to the capital market is restricted to purchasing securities and real estate. In particular, we concentrate on the debt component of his portfolio in a two-asset world.

The focus of our inquiry is the effect of an increase in inflation on the activity level of the loanable funds market. In our model, increased inflation is expressed as a decline in the purchasing power of money, rather than directly. This captures, as Eden (1976) has argued, what ultimately concerns

[3]See, e.g., Mundell (1963), Feldstein (1976), Makin (1978), Levi and Makin (1979) and Benninga and Protopapadakis (1983). The last is outstanding in that it uses a microeconomic general equilibrium framework.

[4]See, e.g., Grauer and Litzenberger (1979).

[5]For a discussion of why firms usually refrain from issuing indexed debt, see Liviatan and Levhari (1977). Chiefly, a firm would usually seek to index its debt to the price of its output, whereas lenders would be interested in the general purchasing power. Professor Levhari pointed out to us two exceptions where corporations did issue debt *indexed to the price of their own product*: Michelin, France's tyre producer, and Pemex, Mexico's national oil company.

the agents in the money market: purchasing power. But it also has an analytical advantage since we do not need to assume, as, e.g., Levi and Makin (1979) did, that increasing inflation is accompanied by an increase in price variability. As we will see, the latter is a consequence of the former.

Our agents are assumed to have complete knowledge of the pertinent probability distributions. Since lenders are conceived as individuals, they are assumed to be risk averse. Borrowers, on the other hand, can be firms, and may be regarded as either risk neutral or risk averse. With these underlying specifications, we are able to conclude that (i) inflation has a depressing effect on the loanable funds market: it retards the volume of transactions, (ii) if lenders alone are presumed to be risk averse, the Fisherian rule[6] concerning the relation between the inflation and the interest rate definitely does not hold: the real rate of interest *rises* in the face of inflation,[7] (iii) if borrowers, too, are risk averse, the Fisherian rule may hold, but no definite conclusion can be stated, since inflation affects both the supply and demand side of the loanable funds market.

Although it may be self evident, it will nevertheless serve clarity to state what is exactly meant by the Fisherian rule in a world of uncertainty. The definition adopted is this: Consider two loan contracts. Define the purchasing power of money, on the date on which the contracts are concluded, to be one. Next, assume that the purchasing power of money is expected to remain unchanged over the life of the first of these contracts. Then if the (nominal) interest rate quoted on this contract is r, then it is also the expected real rate. In contrast, the second contract is concluded under inflationary expectations. On the date on which the debt is to be discharged, the *expected* purchasing power of money is $\mu<1$, so the expected change is $1-\mu$. If i, the nominal rate of interest quoted on this second contract, is chosen so that $1+i=(1+r)/\mu$, Fisher's rule is said to hold.

The discussion is organized as follows: first, the borrower's side of the market is considered. He is viewed as though he is risk averse, with risk neutrality as a special case. This is followed by a consideration of the lender's side of the market. We end by considering the market as a whole. In all cases, the two-period analysis method is used.

[6]See Fisher (1930).

[7]As a purely historical matter, Fisher himself does not seem to have regarded the rule bearing his name as applying except under very limited and unrealistic conditions. In his words: 'If the money rate of interest were perfectly adjusted to changes in the purchasing power of money – which means, in effect, if those changes were perfectly and universally foreseen – the relation of the rate of interest to those changes would have no practical importance' (1930, pp. 43–44). It is also clear that Fisher did have a clear view of what held in reality: 'When the cost of living is not stable, the rate of interest takes the appreciation and depreciation into account to some extent, but only slightly and, in general, indirectly. That is, when prices are rising, the rate of interest tends to be high but not so high as it should be to compensate for the rise; and when prices are falling, the rate of interest tends to be low, but not so low as it should be to compensate for the fall' (1930, p. 43).

2. The borrower

Assume that the borrower contemplates an investment of size K, expected to bear fruit in the real amount $f(K)$, where

$$f'>0, \quad f''<0. \tag{1}$$

To do so, he must borrow the sum K in the first period and, upon realizing $f(K)$ one period hence, he must pay back the loan plus interest charges in the nominal amount $K(1+i)$.

As is evident, no uncertainty is involved in the return on the investment. The reason is that, on the one hand, such uncertainty exists whether or not inflation is present; on the other hand, we do not wish to introduce a priori uncertainty due to inflation. Hence, $f(K)$ is presumed certain and the resulting loss of generality has nothing to do with the object of the present inquiry.

At the same time the loan is made, the real value of money is defined to be 1, while by the time the debt is discharged, the real value of money is $\pi=1/(1+g)$, with $g>-1$ denoting the rate of inflation. It is assumed that g, and so π, are random with π, the purchasing power of money, having the known probability distribution $H(\pi)$, and $\mathrm{E}\pi=\mu$. Let w_b denote the borrower's terminal wealth in real terms and $u(w_b)$ denote his utility function. From the mathematical standpoint, risk neutrality is a special case of risk aversion. Therefore, we shall develop the argument under the assumption

$$u'>0, \quad u''<0. \tag{2}$$

However, when the borrower is a corporation, the assumption of risk aversion constitutes a complication which, from the economic standpoint, may not be a generalization of risk neutrality.[8] Therefore, we shall also examine the case where

$$u''=0. \tag{2.1}$$

Next turn to the real rate of interest, given by the random expression $(1+i)\pi-1$. Henceforth, whenever reference is made to the real rate of interest, r, we shall mean the expected value thereof, i.e.,

$$r\equiv(1+i)\mu-1. \tag{3}$$

[8]This has to do with the fact that risk neutrality can be rationalized on the basis of its compatibility with shareholders' preferences. And, the latter can compensate in their personal portfolios, for any excess or deficiency of risk which they perceive.

With these, the borrower's problem is to select K so as to maximize

$$\mathrm{E}u[f(K)-K(1+i)\pi]. \tag{4}$$

Before proceeding, note that as a result of choosing π, rather than g, as the key stochastic variable, the expected purchasing power of money is $\mathrm{E}(\pi)$, not $1/[1+\mathrm{E}(g)]$. The use of the second expression would have yielded a certainty equivalent solution.

To proceed, differentiate eq. (4) with respect to K, obtaining the first-order condition

$$\mathrm{E}u'(w_b^*)[f'(K^*)-(1+i)\pi]=0, \tag{5}$$

where K^* is the solution and w_b^* is w_b evaluated at $K=K^*$. It follows that

$$f'(K^*)=(1+i)\mu\left[1+\frac{\mathrm{cov}\,(u'^*,\pi)}{\mu \mathrm{E}u'^*}\right], \tag{6}$$

where 'cov' denotes covariance and u'^* is the derivative of $u(\ )$ evaluated at $K=K^*$. Recall now that if $\phi(x)$ is a continuous, strictly increasing (decreasing) function of the random variable x, then the covariance between x and $\phi(x)$ is positive (negative). Since $u'(\ )$ as a function of π is indeed continuous and strictly increasing, $\mathrm{cov}\,(u'^*,\pi)>0$ and thus[9]

$$f'(K^*)>(1+i)\mu=1+r. \tag{7}$$

Turning to the second-order condition, one has by (1) and (2)

$$D\equiv \mathrm{E}u''^*[f'(K^*)-(1+i)\pi]^2+\mathrm{E}u'^*f''(K^*)<0. \tag{8}$$

Since eq. (8) holds for any K, not just K^*, the latter is a global maximizer.

As stated, the main objective is to evaluate the impact of increased inflation on the behavior of the agents in the market. The precise representation of increased inflation is a spread-preserving shift of the probability distribution of the purchasing power of money.[10] The first comparative-statics exercise is designed to evaluate the impact of increased inflation on investment, in the case where the nominal interest rate is set in such a way, that the real interest rate remains unchanged at r. That is, if the probability

[9]Note, that $\mu=1$, which in terms of the purchasing power of money represents the situation where no inflation is expected, constitutes no special case.

[10]Probabilistically speaking, we cannot use the rate of inflation in place of the purchasing power of money, since π is a strictly convex function of g and so, by Jensen's inequality $\mu>1/(1+\mathrm{E}g)$.

distribution $H(\pi)$ is shifted by α, we insist that

$$(1+i)(\mu+\alpha)=1+r. \tag{9}$$

In order to accommodate α, condition (5) is rewritten as

$$\mathrm{E}u'\left[f(K^*)-K^*(1+r)\frac{\pi+\alpha}{\mu+\alpha}\right]\left[f'(K^*)-(1+r)\frac{\pi+\alpha}{\mu+\alpha}\right]=0. \tag{5.1}$$

Differentiating the last expression with respect to K and α at $K=K^*$ and $\alpha=0$, one obtains

$$\frac{\partial K^*}{\partial\alpha}=\frac{1+r}{D}\mathrm{E}\left\{u''^*K^*\frac{\mu-\pi}{\mu^2}\left[f'(K^*)-(1+r)\frac{\pi}{\mu}\right]+u'^*\frac{\mu-\pi}{\mu^2}\right\}. \tag{10}$$

Consider first the case of perfect foresight in the deterministic sense, represented by $\pi=\mu$. Then eq. (10) vanishes and the amount of investment desired by the borrower remains unaffected. The same is true in the case of risk neutrality, represented by eq. (2.1). This leads us to state

Proposition 1. The amount a risk-neutral borrower wishes to borrow is unaffected by a spread-preserving shift of $H(\pi)$, as long as the nominal rate of interest is set so as to maintain the existing real rate.

We turn next to the more 'general' case. Taking up the last term of eq. (10) first one has, after multiplying by μ^2,

$$\mathrm{E}u'^*(\mu-\pi)=\mu\mathrm{E}u'^*-\mathrm{E}u'^*\pi=-\mathrm{cov}\,(u'^*,\pi)<0.$$

Next, assuming non-increasing absolute risk aversion and utilizing a proof due to Sandmo (1971)[11] (see appendix), it is possible to establish that the first term in eq. (10) is non-positive. Since $D<0$, it follows that

$$\frac{\partial K^*}{\partial\alpha}>0. \tag{11}$$

It is thus possible to state

Proposition 2. Even if the nominal rate of interest is chosen so that the expected real rate is unaffected by a fall in the expected purchasing power of money (which leaves the other moments of the distribution $H(\pi)$ unchanged), the

[11]See also Ishii (1977).

demand of a risk-averse borrower for funds declines, if he displays non-increasing absolute risk aversion.

This proposition would be intuitively obvious to most readers, had it been consequent on an increase in uncertainty (represented, say, by the variance of π). This not being the case, the result merits an explanation. The latter is found with the fact that although the spread of $H(\pi)$ has not changed, this is not true as regards the probability distribution of wealth, and hence utility. In fact, the shift of $H(\pi)$ causes an increase in the variance of wealth. The shift of $H(\pi)$ by α is reflected in real wealth by

$$w_b^* = f(K^*) - K^*(1+r)\frac{\pi+\alpha}{\mu+\alpha}.$$

Now, mean wealth is unaffected; it is still given by $f(K^*) - K^*(1+r)$. As for its variance, we have

$$\sigma^2 = \operatorname{var}(w_b^*) = \frac{[K^*(1+r)]^2}{(\mu+\alpha)^2}\operatorname{var}(\pi).$$

It follows that if the borrower does not adjust his investment in response to the α-shift of $H(\pi)$, i.e., leaves K at K^*, then

$$\left.\frac{\partial\sigma^2}{\partial\alpha}\right|_{\alpha=0} = -2\frac{[K^*(1+r)]^2}{\mu^3}\operatorname{var}(\pi) < 0.$$

Thus, a fall in the purchasing power of money leads to an increase in the variance of wealth, and so an increase in risk. Another way of explaining Proposition 2 is to note that inflation enters wealth in a multiplicative way. Hence, a rise in inflation 'stretches' the probability distribution of wealth. Another way of getting an intuitive feel for the result is to note that there exists a y such that

$$\pi + \alpha = \frac{1}{1+g+y}. \qquad \text{Hence,}$$

$$y = \frac{1}{\pi+\alpha} - (1+g), \qquad \text{and}$$

$$\left.\frac{\partial y}{\partial\pi}\right|_{\alpha=0} = -\frac{1}{\pi^2} < 0.$$

This means, that the smaller π, i.e. the larger g, the larger must be y. But this

implies that the distribution of $(g+y)$ is more stretched than that of g, indicating an increased variance of inflation which is what Levi and Makin (1979) assumed.

The validity of the Fisherian rule in regard to the borrower can be examined by asking what change in the rate of interest is necessary to induce the borrower to stand by his original plans – to invest K^*. For an answer, write in place of eq. (5.1)

$$\mathrm{E}u'[f(K^*)-K^*(1+i)(\pi+\alpha)][f'(K^*)-(1+i)(\pi+\alpha)]=0, \tag{5.2}$$

and differentiate with respect to i and α. Evaluating the result at $\alpha=0$, one gets

$$\frac{\partial i}{\partial \alpha}=-(1+i)\frac{\mathrm{E}z}{\mathrm{E}z\pi}, \tag{12}$$

where $z\equiv u''^*K^*[f'^*-(1+i)\pi]+u'^*$.
If perfect foresight is contemplated, then $\pi=\mu$; if risk neutrality is considered, then z is a constant. In either case eq. (12) implies

$$\frac{\partial i}{\partial \alpha}=-\frac{1+i}{\mu}. \tag{13}$$

Applying eq. (13) to eq. (9), we obtain

$$\left.\frac{\partial r}{\partial \alpha}\right|_{\alpha=0}=0. \tag{14}$$

Proposition 3. When the borrower is risk neutral, his behavior is consistent with the Fisherian rule: borrowing plans do not change if the real rate of interest is preserved.

As for risk aversion, it is shown in the appendix that $\mathrm{E}z\pi$, $\mathrm{E}z$ and $\mathrm{cov}(z,\pi)$ are all positive. Hence, if eq. (12) is rewritten as

$$\frac{\partial i}{\partial \alpha}=-\frac{1+i}{\mu}\left[1-\frac{\mathrm{cov}(z,\pi)}{\mathrm{E}z\pi}\right]\equiv-\frac{1+i}{\mu}(1-\beta), \tag{15}$$

we know that $0<\beta<1$. Differentiating eq. (9) once again with respect to α at $\alpha=0$, we get by eq. (15)

$$\frac{\partial r}{\partial \alpha}=\beta(1+i)>0.$$

We thus have

Proposition 4. If the borrower is risk averse, with non-increasing absolute risk aversion, his behavior is inconsistent with the Fisherian rule: a (spread preserving) fall in the expected purchasing power of money, will require a fall in the expected real rate of interest, if borrowing plans are to remain unaffected.

3. The lender

Unlike the borrower, the lender faces a portfolio problem, if one is to avoid the assumption that the only alternative to lending is consumption. The original source for loans consists of individual savers and banks. It will be assumed, as explained in the introduction, that both types of agents are not likely to assume the role of the investor when they refrain from lending. For, if they did, the distinction between lender and borrower, saver and investor would be blurred theoretically and unhelpful practically. The assumption which will be adopted here is that the alternative to lending is the purchase of a 'safe' asset, in the sense that it provides a perfect haven from inflation. In view of the introduction, we are tempted to think of real estate. This involves, however, all sorts of complications better avoided. The safe asset will thus be assumed to be an indexed (government) bond bearing a certain real rate of interest. Safe assets are common in portfolio literature and in this respect we adhere to tradition.

Let the lender possess a sum of M dollars, to be allocated between the purchase of one-period, risky corporate bonds, in the amount L, and one-period indexed bonds, in the amount $x=M-L$. Accordingly, the face value of each bond is one dollar. While the corporate bond pays a nominal rate of interest i, the indexed bond pays a real rate j. It is naturally assumed that

$$1+j \leqq 1+r=(1+i)\mu. \tag{16}$$

That is, the safe asset is not dominant.

Let $v(.)$ represent the lender's utility. We assume non-satiation and risk aversion, i.e.,

$$v'>0, \quad v''<0. \tag{17}$$

The lender's problem consists of choosing L so as to maximize

$$\mathrm{E}v[L(1+i)\pi+(1+j)(M-L)].$$

For L^* to be optimal, it is necessary that

$$\mathrm{E}v'^*[(1+i)\pi-(1+j)]=0. \tag{18}$$

Analogously to (6), one obtains from (17)

$$1+j=(1+i)\mu\left[1+\frac{\operatorname{cov}(v'^*,\pi)}{\mu \mathrm{E}v'^*}\right]. \tag{19}$$

Since $\partial v'/\partial\pi<0$ due to risk aversion, $\operatorname{cov}(v'^*,\pi)<0$. Hence,

$$1+j<(1+i)\mu=1+r,$$

which was assumed in eq. (16), in order to insure an interior solution. That eq. (16) constitutes an assumption and not a result, is manifested by the fact that all elements involved in eq. (16) are parameters; no decision variables are involved. As with the borrower, the difference between j and r constitutes a risk premium. One might add that in a general equilibrium context, the fact that the expected value r of the risky rate must exceed the certain rate j, would emerge as a result.

As regards the second-order condition, if the lender is risk averse, then

$$C\equiv \mathrm{E}v''^*[(1+i)\pi-(1+j)]^2<0. \tag{20}$$

As with the borrower, we first examine the impact of a spread-preserving change in the expected purchasing power of money on the supply of credit. The change is effected in such a way that eq. (9) holds, i.e., the real rate of interest remains unchanged. Reintroducing the shift parameter α, eq. (18) is rewritten, in analogy to eq. (5.1), as

$$\mathrm{E}\left\{v'\left[L^*(1+r)\frac{\pi+\alpha}{\mu+\alpha}+(1+j)(M-L)\right]\left[(1+r)\frac{\pi+\alpha}{\mu+\alpha}-(1+j)\right]\right\}=0. \tag{21}$$

Differentiating with respect to L and α, one gets

$$\frac{\partial L^*}{\partial\alpha}=-\frac{1+r}{C}\mathrm{E}\left\{v''^*L^*\frac{\mu-\pi}{\mu^2}\left[(1+r)\frac{\pi}{\mu}-(1+j)\right]+v'^*\frac{\mu-\pi}{\mu^2}\right\}. \tag{22}$$

In the unlikely case of perfect foresight in the deterministic sense, so that $\pi=\mu$, eq. (22) vanishes and the composition of the portfolio remains optimal. As regards the general case, notice first that

$$\frac{1}{\mu^2}\mathrm{E}v'^*(\mu-\pi)=-\frac{1}{\mu^2}\operatorname{cov}(v'^*,\pi)>0.$$

The positivity of the first term in the bracketed expression of eq. (22) can

be established in the same way it was for the corresponding term in eq. (10), provided that non-increasing absolute risk aversion is postulated. Because of eq. (21), it follows that

$$\frac{\partial L^*}{\partial \alpha} > 0. \tag{23}$$

One therefore concludes

Proposition 5. A fall in the expected purchasing power of money, which leaves the spread of $H(\pi)$ unaffected, will cause a decline in the supply of funds even if the nominal rate of interest is set so as to maintain the real rate.

As with the borrower, the explanation is to be found with the change in the probability distribution of wealth. Although $H(\pi)$ is uniformly shifted, the spread of the probability distribution of wealth increases and so, therefore, does the amount of risk.

Concerning the Fisherian rule, the question is what adjustment in the rate of interest is required, in the face of increased inflation, for lending to be kept up at its old pace, L^*. For an answer, replace eq. (18) with

$$\mathrm{E}\{v'[L^*(1+i)(\pi+\alpha)+(1+j)(M-L)][(1+i)(\pi+\alpha)-(1+j)]\}=0, \tag{18.1}$$

and differentiate with respect to i and α. This yields

$$\frac{\partial i}{\partial \alpha} = -(1+i)\frac{\mathrm{E}\eta}{\mathrm{E}\eta\pi}, \tag{24}$$

where $\eta \equiv v''^* L^*[(1+i)\pi-(1+j)]+v'^*$. (25)

In the case of perfect foresight which Fisher considered unlikely $\pi=\mu$ and so eq. (24) implies eqs. (13) and (14). The Fisherian rule then holds.

When uncertainty reigns, the case of the lender is a bit more complicated than that of the borrower. The complication comes in the form of conflicting income and substitution effects, owing to the limit on the size of the portfolio – the 'budget constraint'. When the nominal rate of interest on corporate bonds rises, there is an incentive to increase the amount of corporate bonds and reduce the amount of indexed bonds in the portfolio. At the same time, an increase in i causes an increase in terminal wealth even if no adjustment is undertaken. Hence, if one differentiates eq. (18) with respect to L^* and i to obtain

$$\frac{\partial L^*}{\partial i} = \frac{1}{C}\mathrm{E}\eta\pi,$$

it is impossible to tell whether the result is positive or negative. It will therefore be assumed that the supply of credit to corporations is an upwards-sloping function, i.e., it is increasing in i, so that

$$\mathrm{E}(\eta\pi) > 0.^{12} \tag{26}$$

Returning to the investigation of eq. (24), rewrite first eq. (22), with the aid of eqs. (3), (23) and (25), as

$$\frac{\partial L^*}{\partial \alpha} = -\frac{1+r}{\mu C}\mathrm{E}\eta + \frac{1+r}{\mu^2 C}\mathrm{E}\eta\pi > 0.$$

This implies, by eq. (20),

$$\mathrm{cov}(\eta, \pi) < 0. \tag{27}$$

Since (24) can be written as

$$\frac{\partial i}{\partial \alpha} = -\frac{1+i}{\mu}\left[1 - \frac{\mathrm{cov}(\eta, \pi)}{\mathrm{E}\eta\pi}\right] \equiv -\frac{1+i}{\mu}(1+\delta), \tag{28}$$

we conclude that $\delta > 0$. Upon differentiation of eq. (9) with respect to α at $\alpha = 0$, one gets with the aid of eq. (26),

$$\frac{\partial r}{\partial \alpha} = -\delta(1+i) < 0. \tag{29}$$

Proposition 6. In the face of a spread-preserving decline in the purchasing power of money, a lender with non-increasing absolute risk aversion will keep up his lending plans only if the real rate of interest is increased; this behavior is inconsistent with Fisher's Rule.

It seems instructive to point out that, in contrast to Propositions 2 and 4, Propositions 5 and 6 are not always dual results. While 2 implies 4 and vice-versa, Propositions 5 and 6 are subject to a similar kind of equivalence only under the assumption embodied in eq. (26). If the latter did not hold, then the amount of credit supplied to corporations would be a declining function in the nominal rate of interest. In that event, Proposition 6 would state that the expected real rate of interest must decrease, rather than increase.

[12]The set of admissible utility functions satisfying the assumption is not empty. One that does is the logarithmic function.

4. The market

Aggregation in the loanable-funds market is not as straightforward as in some other markets. This, because of the fact that participants, or transactors in this market consist partly of individuals and partly of institutions (borrowing corporations and lending banks). If one is willing to ignore such complications, then some conclusions concerning the market as a whole can be drawn. For the borrowers' side of the market differentiate eq. (5.2) with respect to K, i and α; for the lenders', differentiate eq. (18.1) with respect to L, i and α. This yields, respectively,

$$\frac{\mathrm{d}K^*}{\mathrm{d}\alpha}=\frac{1}{D}\left[\mathrm{E}z\pi\,\frac{\partial i}{\partial\alpha}+(1+i)\mathrm{E}z\right], \tag{30}$$

$$\frac{\mathrm{d}L^*}{\mathrm{d}\alpha}=-\frac{1}{C}\left[\mathrm{E}\eta\pi\frac{\partial i}{\partial\alpha}+(1+i)\mathrm{E}\eta\right]. \tag{31}$$

At equilibrium, eqs. (28) and (29) must be equal to each other. Exploiting this equality, we can solve for $\partial i/\partial\alpha$ to obtain

$$\frac{\partial i}{\partial\alpha}=-(1+i)\frac{\frac{1}{D}\mathrm{E}z+\frac{1}{C}\mathrm{E}\eta}{\frac{1}{D}\mathrm{E}z\pi+\frac{1}{C}\mathrm{E}\eta\pi}. \tag{32}$$

From part (b) of the appendix we know that $\mathrm{E}z$, $\mathrm{E}z\pi\geqq 0$; by eqs. (26) and (27), $\mathrm{E}\eta$, $\mathrm{E}\eta\pi>0$. Hence, eq. (32) is negative. Applying this conclusion to eqs. (30) and (31), it is shown in the appendix that they are both positive. Hence

Proposition 7. Let both lenders and borrowers be risk averse, with non-increasing risk aversion. Then if both classes of agents perceive a (spread-preserving) decline in the expected purchasing power of money, the volumes of transactions in the loanable-funds market will decrease irrespective of what happens to the real rate of interest.

The reference to the real rate of interest in Proposition 7 merits clarification. The reason for it is that in the present context, the rate of interest is an outcome of the interaction between lenders and borrowers. But regardless of what happens to the rate of interest, Proposition 7 asserts that the volume of transactions will definitely decline.

This brings us to the question of what happens to the market real rate of

interest. A little algebra transforms eq. (32) into

$$\frac{\partial i}{\partial \alpha} = -\frac{1+i}{\mu}\left[1 - \frac{C\,\mathrm{cov}(z,\pi) + D\,\mathrm{cov}(\eta,\pi)}{C\mathrm{E}z\pi + D\mathrm{E}\eta\pi}\right] \equiv -\frac{1+i}{\mu}(1-\gamma). \tag{33}$$

For a risk-neutral borrower, $\mathrm{cov}(z,\pi)=0$, $\mathrm{E}z\pi=\mu b$, where b is a constant, and $D=bf''(K^*)$. Hence, γ reduces to

$$\gamma = \frac{f''\,\mathrm{cov}(\eta,\pi)}{\mu C + f''\mathrm{E}\eta\pi} < 0. \tag{34}$$

Utilizing eqs. (9) and (33), we obtain

$$\frac{\partial r}{\partial \alpha} = 1 + i + \mu\frac{\partial i}{\partial \alpha} = \gamma(1+i) \leqq 0$$

by eq. (34). This implies

Proposition 8. When borrowers are risk neutral, lenders risk averse and both groups expect a uniform decline in the purchasing power of money, then the resulting equilibrium real rate of interest will be higher than the old one. This implies that Fisher's rule for the market does not hold.

The situation is depicted in fig. 1, by the movement from the equilibrium (K^*, r) to the equilibrium (K', r'). In fig. 1, $L(\mu)$ and $K(\mu)$ represent supply of and demand for loans, respectively, before any changes in the purchasing power of money are perceived. Supply of loans after the change is represented by $L(\mu+\alpha)$.[13]

When the borrower is risk averse, it is evident, upon inspecting (33), that Fisher's rule for the market holds only if

$$C\,\mathrm{cov}(z,\pi) + D\,\mathrm{cov}(\eta,\pi) = 0. \tag{35}$$

This is a possibility, since $\mathrm{cov}(z,\pi)>0$, while $\mathrm{cov}(\eta,\pi)<0$. But there is no reason why eq. (35) should hold; it would be a mere coincidence if it did. Only one thing can be asserted on the basis of eq. (32), namely, $\partial i/\partial\alpha<0$. Thus, the market nominal rate of interest increases with a decline in the expected purchasing power of money. All this is summarized in fig. 1, where $K_1(\mu+\alpha)$ and $K_2(\mu+\alpha)$ are two possible demand curves for loans, after a change in the purchasing power of money is introduced. In the case of $K_1(\mu+\alpha)$, the equilibrium rate of interest is $r_1>r$; in the case of $K_2(\mu+\alpha)$, $r_2<r$

[13]Recall that α is negative expressing a decline in the purchasing power of money.

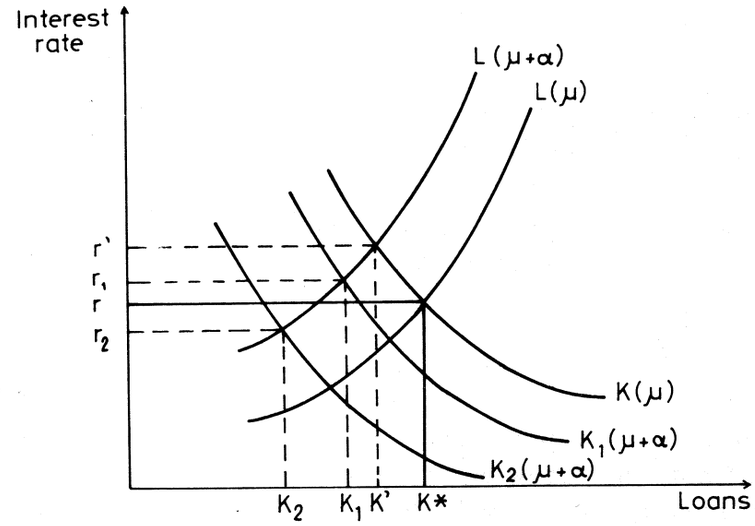

Fig. 1

is the equilibrium rate. One notes that although $r_2 < r_1$, $K_2 < K_1$: the lower rate of interest is associated with a lower level of investment activity.

5. Conclusion

Since the above analysis is cast entirely within a microeconomic framework, it is in all probability incomplete. For example, inflation may have a negative impact on output which is disconnected from its retarding effect on the money market. In that case investment will be also depressed through a reverse accelerator mechanism. Moreover, depending on the institutional arrangements in the economy, inflation may affect the level of savings, which will have a secondary effect in the loanable funds market.

In any case, reduced investment is enough to cause increased unemployment with the attendant macroeconomic effects. These, in turn, will have further effects on investment. Clearly, a lot remains to be done in this area and ours is, perhaps, only a modest beginning.

Appendix

(*a*) *Proof of eq. (11)*

The required relation is

$$(1+r)\mathrm{E}u''^{*}\frac{\mu-\pi}{\mu}\left[f'^{*}-(1+r)\frac{\pi}{\mu}\right]\leqq 0. \tag{A.1}$$

We have

$$(1+r)\mathrm{E}u''^*\frac{\mu-\pi}{\mu}\left[f'^*-(1+r)\frac{\pi}{\mu}\right]$$

$$=\mathrm{E}u''^*\left[1+r-f'^*+f'^*-(1+r)\frac{\pi}{\mu}\right]\left[f'^*-(1+r)\frac{\pi}{\mu}\right]$$

$$=\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]^2+(1+r-f'^*)\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right].$$

The first of the two expressions is clearly negative. As for the second expression, denote the Arrow–Pratt absolute risk-aversion coefficient, $R(w_b)$, evaluated at $\pi/\mu=f'^*/(1+r)$, by R_0. Since $R'\leqq 0$, it is true that

$$(1+r)\frac{\pi}{\mu}\geqq(\leqq)f'^*, \qquad R\geqq(\leqq)R_0.$$

This is to say,

$$-\frac{u''^*}{u'^*}\geqq(\leqq)R_0. \tag{A.2}$$

At the same time,

$$(1+r)\frac{\pi}{\mu}\geqq(\leqq)f'^*, \; -u'^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]\geqq(\leqq)0. \tag{A.3}$$

It follows from (A.2) and (A.3) that

$$u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]\geqq -R_0u'^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]. \tag{A.4}$$

Applying the expectation operator to both sides of eq. (A.4) and employing eq. (5), we find

$$\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]\geqq 0. \tag{A.5}$$

But by eq. (7),

$$(1+r-f'^*)\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]\leqq 0.$$

and so eq. (A.1) is established.

(*b*) *Proof that* $\mathrm{E}z\pi>0$, $\mathrm{cov}(z,\pi)>0$, $\mathrm{E}z>0$

By definition,

$$\mathrm{E}z\pi=\mathrm{E}\{[u''^*K^*(f'^*-(1+i)\pi)+u'^*]\pi\}.$$

First, since $\mathrm{cov}(u',\pi)\geqq 0$ and μ, $u'>0$, it follows that $\mathrm{E}u'^*\pi>0$. Next, using eq. (3),

$$\frac{1+r}{1+r}K^*\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]\pi$$

$$=\frac{K^*}{1+r}\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]\left[\mu(1+r)\frac{\pi}{\mu}-\mu f'^*+\mu f'^*\right]$$

$$=-\mu\frac{K^*}{1+r}\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]^2+\mu K^*\frac{f'^*}{1+r}\mathrm{E}u''^*\left[f'^*-(1+r)\frac{\pi}{\mu}\right]>0. \tag{A.6}$$

This follows from eqs. (2) and (A.5). Hence, $\mathrm{E}z\pi>0$.

Regarding the second part of the proof, since $\mathrm{cov}(u',\pi)>0$,

$$\mathrm{E}u'\pi>\mu\mathrm{E}u'. \tag{A.7}$$

Furthermore, from eqs. (A.1) and (7) it follows that

$$\mathrm{E}u''^*K\left[f'^*-(1+r)\frac{\pi}{\mu}\right]\pi>\mu\mathrm{E}u''^*K\left[f'^*-(1+r)\frac{\pi}{\mu}\right]. \tag{A.8}$$

Combining eqs. (A.7) and (A.8), we obtain

$$\mathrm{E}z\pi>\mu\mathrm{E}z.$$

With $\mathrm{E}z\pi>0$, this completes the proof that $\mathrm{cov}(z,\pi)>0$.

Furthermore, $\mathrm{E}z>0$ follows at once from eq. (A.5).

(c) *Proof of* $\mathrm{d}K^*/\mathrm{d}\alpha$, $\mathrm{d}L^*/\mathrm{d}\alpha > 0$

Substitute eq. (32) for $\partial i/\partial\alpha$ in eq. (30). This yields, after some simple arithmetics

$$\frac{\mathrm{d}K^*}{\mathrm{d}\alpha} = \frac{1+i}{CD}\left[\frac{(\mathrm{E}z/\mathrm{E}z\pi)\mathrm{E}\eta\pi - \mathrm{E}\eta}{\dfrac{1}{D} + \mathrm{E}\eta\pi/C\mathrm{E}z\pi}\right]. \tag{A.9}$$

Since $\mathrm{cov}(z, \pi) > 0$,

$$\frac{\mathrm{E}z}{\mathrm{E}z\pi} < \frac{1}{\mu}. \tag{A.10}$$

Since $\mathrm{cov}(\eta, \pi) < 0$,

$$\frac{1}{\mu}\mathrm{E}\eta\pi - \mathrm{E}\eta < 0. \tag{A.11}$$

Combining eqs. (A.10) and (A.11), it follows that

$$\frac{\mathrm{E}z}{\mathrm{E}z\pi}\mathrm{E}\eta\pi - \mathrm{E}\eta < 0.$$

Since the denominator of the bracketed term in eq. (A.9) is negative, the whole term is positive; so is also $(1+i)CD$, which implies $\mathrm{d}K^*/\mathrm{d}\alpha > 0$. As $\mathrm{d}L^*/\mathrm{d}\alpha = \mathrm{d}K^*/\mathrm{d}\alpha$, the proof is completed.

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