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SHLOMO YITZHAKI AND HAIM SHALIT

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# EFFICIENT PORTFOLIO SELECTION: APPLICATION TO THE TEL AVIV STOCK EXCHANGE

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## INTRODUCTION AND MAIN FINDINGS[1]

One of the conventional methods of analyzing investors' behavior under uncertainty is the mean-variance criterion, which has been used in several studies of the Israeli economy.[2] Under this method the distribution of investment prospects is summarized by two parameters: the expected rate of return, which represents the profitability of the investment, and the variance, which represents its risk. It is known that under certain conditions the variance does not correctly reflect the risk, and so mean-variance analysis may lead to unwarranted conclusions.

In this paper we present the mean-Gini (MG) approach as an alternative to the mean-variance (MV) method. In MG, the Gini index (a conventional measure of income inequality) replaces the variance as a measure of dispersion. This index provides a more reliable measure of risk, thus meeting some of the criticisms leveled against mean-variance analysis.

The mean-Gini approach is used here to analyze the performance of the Tel Aviv Stock Exchange during the period 1976-80. The results are compared with those obtained from a mean-variance analysis. Following are our main conclusions:

(a) Efficient portfolios selected by the mean-Gini method were very similar but not identical to those selected according to the mean-variance criterion.

(b) Since the increase in the consumer price index had a similar effect on most groups of securities traded on the exchange, portfolios constructed using nominal rates of return resembled those using real rates of return.

(c) Indexed bonds did not constitute an alternative to commercial bank stocks. These shares were riskier than bonds, but they also gave a higher re-

* Lecturers in the Agricultural Economics and Management Department, Hebrew University of Jerusalem. This study was funded by a grant from the Israeli government's National Council for Research and Development. We are grateful to the Council for its financial support, and to Shlomo Nimrodi and Amos Golan for assistance in processing the data.

[1] This study was completed in September 1982. Since then the Israeli stock market has undergone many changes, including the collapse of commercial bank stocks in October 1983. But as our main objective is methodological, we did not revise the study.

[2] See, for example, Levy and Sarnat (1975) on the Tel Aviv Stock Exchange, Ben-Bassat (1980) on the optimal composition of Israel's foreign exchange reserves, and Inbar and Peleg (1978) on agricultural investment.

turn; hence they made up, in varying proportions, the bulk of the efficient portfolios.

The validity of these conclusions depends on the degree to which the ex post data used here serve as a proxy for ex ante expectations. Moreover, if new information becomes available, or if some of the information does not find expression in the yield data, the conclusions may be misleading. This deficiency applies to every attempt to predict the future from past performance.

Section 1 presents the problem facing the investor in the stock market and describes the mean-variance method. The next section presents the mean-Gini approach and discusses its properties. Section 3 describes the data, and the final section gives the results of the analysis.

## 1. CONVENTIONAL METHODS OF DETERMINING EFFICIENT PORTFOLIOS

The problem confronting the investor in the stock market is how to allocate his wealth among the various types of securities so as to maximize his expected utility. Our basic assumptions are as follows: the investor has a given initial capital, $K_0$, a single-period investment horizon, and is faced with $J$ securities in which he can place his money. For simplicity we assume that he cannot borrow. Let the investor's utility function, $U$, which is defined on his terminal capital, have a positive and decreasing marginal utility. The investor's basic problem can then be written:

$$\max_{\alpha_1,\ldots,\alpha_J} E\{U(y)\} \tag{1}$$

subject to the constraints[3]

$$y = K_0 \sum_{j=1}^{J} \alpha_j x_j$$

$$\sum_{j=1}^{J} \alpha_j = 1$$

where $y$ = capital at end of period

$\alpha_j$ = share of capital invested in security $j$

$x_j$ = rate of return during the period (1 plus the rate of change in the value of the security)

We assume that $x_j$ is a random variable, whose probability distribution is invariant over the period investigated and known, and with mean $\mu_j$ and variance $\sigma_j^2$.

Our assumptions about the utility function, $U$, are not sufficient for solving the investor's problem. It is possible to specify two functions both of which

[3] As a rule there is an additional constraint: portfolios cannot be sold short, i.e. $\alpha_j \geq 0$. However, we ignore this here, as it does not alter the principles, but only the method of calculation. All the propositions presented here can be fitted to this constraint.

have a positive, decreasing marginal utility but give a different ranking: according to one, portfolio A is superior to portfolio B, and according to the second, the opposite is true. Since we are unable to rank all the possible portfolios, we cannot solve problem (1).[4] We therefore have to reduce the number of portfolios, discarding those that are dominated by others. This gives the efficient set (efficient frontier) — i.e. the group of portfolios that are in the solution set but cannot be reduced to a single optimal choice without additional assumptions.

The stochastic dominance (SSD) approach is the direct and correct way of determining the efficient set. However, it has several disadvantages:[5] its computation is complicated, it does not reduce the efficient set sufficiently, and it can solve only relatively simple problems. The SSD method is therefore not appropriate, as it stands, for determining the composition of efficient portfolios. Analysts thus have no choice but to use other methods, which yield outcomes that are valid only under specific conditions. The most widely used method is mean-variance,[6] according to which an efficient investment portfolio is one that maximizes the expected rate of return for a given return variance. This approach rests on the following proposition:

Proposition 1:[7] If the securities are distributed normally and/or the utility function is quadratic, then a necessary and sufficient condition for portfolio $l$ to be a possible solution to problem (1) is the absence of another portfolio, $k$, with a higher expected rate of return and lower return variance than the return and variance of portfolio $l$: $\mu_k \geq \mu_l$ and $\sigma_k^2 \leq \sigma_l^2$ (with at least one strict inequality).

According to this proposition, if the securities are normally distributed and/or the utility function is quadratic, the mean-variance method enables one to derive the set of efficient portfolios that would be obtained by the SSD criterion. To this end a certain expected return level, $\mu_0$, is given, and the problem is solved as follows:

$$(2) \qquad \min_{\alpha_1,\ldots,\alpha_J} \sigma^2(\alpha_1,\ldots,\alpha_J)$$

subject to the constraints

$$\mu_0 = \sum_{j=1}^{J} \alpha_j \mu_j$$

$$\sum_{j=1}^{J} \alpha_j = 1$$

where $\sigma^2(\alpha_1,\ldots,\alpha_J)$ is the variance of the portfolio's rate of return.

[4] For a fuller discussion of this problem see Atkinson (1970).

[5] For a review of the literature see Kroll and Levy (1980).

[6] For a discussion of this method and its use for analyzing securities traded on the Tel Aviv Stock Exchange, see Levy and Sarnat (1975).

[7] For a proof of this proposition see Rothschild and Stiglitz (1970).

By varying the values of the parameter $\mu_0$ and re-solving the problem, we obtain the entire efficient set. Since the variance is a quadratic function, we solve problem (2) with the help of quadratic programming. This is a simple method of computation, and it is a convenient alternative to the SSD approach. If the restrictive conditions are not satisfied, we can only hope that their violation does not significantly affect the optimal solution.

Violation of Proposition 1 conditions may lead to wrong choices, as can be seen from the following extreme example. Assume that an investor is confronted with two portfolios, one yielding a rate of return of between 1 and 2 percent, the other between 10 and 20 percent. For mean-variance analysis both portfolios will be efficient. However, despite its smaller return, the first portfolio may be preferred by the investigator, because the second has a higher return variance (even though the variance does not measure the risk). The second portfolio is, of course, preferable for all investors, as in every situation its return will be higher.

This example, as stated, is an extreme and probably unrealistic one. Nevertheless, investigators who use a computer and do not know the distribution of the rates of return are liable to make a similar erroneous choice. It is therefore desirable to use a method that is formally similar to mean-variance but is free of its deficiencies. We have chosen the mean-Gini approach, in which the Gini replaces the variance as the measure of risk.

## 2. MEAN-GINI ANALYSIS

The Gini index is a measure of dispersion which in its various formulations is applied to such diverse subjects as income distribution, geographic dispersion of the population, etc.[8] In investment analysis the Gini index is defined as the expected absolute difference between two random realizations of the rate of return of an investment.

The Gini index is very similar to the variance, since it measures the dispersion of the random variable. Moreover, assuming a normal distribution, the ranking of prospects by this index will be identical to the ranking according to the variance, since in this case the Gini index is $\Gamma = \sigma\sqrt{\pi}$. The mean-Gini approach is in principle similar to the mean-variance method. For a given mean return, prospects with a smaller Gini index are preferred, whereas for a given Gini index, prospects with a larger mean return are preferred. Furthermore, the approach is justified by the following proposition:

Proposition 2:[9] A necessary condition for portfolio $l$ to be included in the

[8] In the case of population dispersion, this index indicates the expected distance between two randomly selected observations.

[9] The proof of Proposition 2 is based on Proposition 3. For a proof of the latter see Yitzhaki (1982).

solution set of problem (1) is the absence of another portfolio, $k$, where $\mu_k \geq \mu_l$ holds, and $\Gamma_k \leq \Gamma_l$ (with at least one strict inequality).

The similarity between Propositions 1 and 2 can be seen by restricting them to normal distributions. In this case Proposition 2 is a sufficient and necessary condition. The advantage of the Gini over the variance is, however, based on Proposition 3.

Proposition 3: A necessary condition for portfolio $l$ to be a possible solution of the investor's problem is the absence of another portfolio, $k$, such that $\mu_k \geq \mu_l$ and $\mu_k - \Gamma_k \geq \mu_l - \Gamma_l$.

Since Proposition 3 holds for all distributions, we do not have to restrict ourselves to normal distributions. This, as stated, is one of the advantages of the mean-Gini over the mean-variance. According to Proposition 3, a portfolio cannot be a possible solution to problem (1) even if there is another portfolio with a higher rate of return and higher dispersion, provided that the difference in their expected rates of return is larger than the difference in their Gini index. The investigator using the Gini index can therefore be sure that he will not be guilty of the error shown in the example in the previous section.

Since Proposition 3 presents a necessary condition for possible solutions to problem (1), the group of efficient portfolios derived from it is a subset of the efficient set constructed according to the SSD rule. But it is not very easy to construct efficient portfolios by this method (in the attempts made with single prospects the efficient set was found to be very large); the mean-Gini approach thus has a clear advantage also over SSD.

As for the actual solution of the investor's problem, in the case of MV we solve the following problem:

$$\min_{\alpha_1,\ldots,\alpha_J} \Gamma(\alpha_1,\ldots,\alpha_J) \tag{3}$$

subject to the constraints

$$\sum_{j=1}^{J} \alpha_j \mu_j = \mu_0$$

$$\sum_{j=1}^{J} \alpha_j = 1$$

By varying the values of $\mu_0$ we obtain the efficient set according to Proposition 2, and by applying Proposition 3 we obtain the efficient set that is a subset of all the possible solutions of problem (1). The objective function in problem (3) is a piecewise linear function, and so the algorithm for solving the problem is an adjusted formulation of the simplex method, which is used in linear programming.[10]

[10] The Gini index is not the only one to which Propositions 2 and 3 apply; there are many such indexes, all of which belong to the Gini family (see Yitzhaki, 1983). The differences between them

## 3. THE DATA

In this study we use 12 categories of securities for which Central Bureau of Statistics data were published during the entire period from December 1976 to December 1980. The data are monthly overall rate of return indexes for stocks and bonds listed on the Tel Aviv Stock Exchange. These rates reflect the total net return to the investor; i.e. the data have been adjusted for dividends, interest, rights, and an average marginal tax of 35 percent on dividends and 25 percent on bond interest.[11] We calculate the rate of return indexes for groups of securities instead of using individual securities, in order to reduce the number of observations and to avoid the exaggerated effect of extreme observations, for the variance is sensitive to such observations (this is one of the drawbacks of the mean-variance approach). In addition, the indexes are calculated in both nominal and real terms (the rates of return adjusted for monthly inflation).[12]

## 4. ANALYSIS OF EFFICIENT INVESTMENTS

The analysis first concentrates on the choice of a single group of securities,[13] and then examines the problem of investing in a diversified portfolio.

### Properties of a Single-Security Investment

Assuming that the investor can be satisfied with holding a single security group, what choice should he make? The analysis of a single-security investment disregards the correlations between the various investments, since its entire purpose is to present in a simple manner the components of the portfolios for convenience sake.

Table 1 presents the options facing the individual during the period studied, i.e. the nominal and real monthly rates of return obtained on each of the investments. The higher the mean rate of return, the more sharply the security is expected to fluctuate, and hence the greater will be the risk. The group of land, construction, and development shares yielded the highest return, but its rate of dispersion — measured by both the variance and Gini indexes — was

are expressed in a single parameter, which reflects the investor's risk aversion. For a theoretical discussion of the properties of this family and its use in stock market analysis, see Shalit and Yitzhaki (1984).

[11] For explanations and definitions see the supplement to the Central Bureau of Statistics' *Monthly Bulletin of Statistics*, No. 2, 1977.

[12] For the first part of the period investigated the monthly overall rates of return measure the returns earned between the 23rd of a given month and the 23rd of the following month. Since the consumer price index is an average index for the month, its midpoint is the 15th of the month. We did not adjust the rates of return for the difference in midpoints.

[13] Henceforth an investment in a single group of securities will be referred to as an "investment in a single security" or a "single-security investment".

**Table 1**

**MONTHLY RATES OF RETURN[a] ON AN INVESTMENT IN A SINGLE-SECURITY GROUP, DECEMBER 1976 TO DECEMBER 1980**

(Percentages)

| | Nominal | | | Real | | |
|---|---|---|---|---|---|---|
| | Mean rate of return | Standard deviation | Gini index | Mean rate of return | Standard deviation | Gini index |
| **Stocks** | | | | | | |
| (1) Commercial banks | 7.2 | 9.1 | 4.6 | 1.88 | 7.9 | 4.13 |
| (2) Mortgage banks | 6.3 | 15.3 | 8.5 | 1.06 | 14.5 | 8.06 |
| (3) Specialized financial institutions | 6.1 | 15.7 | 8.2 | 0.90 | 15.0 | 7.87 |
| (4) Investment companies | 7.4 | 14.6 | 7.97 | 2.08 | 13.7 | 7.56 |
| (5) Industry | 6.3 | 14.1 | 8.01 | 0.99 | 13.0 | 7.38 |
| (6) Commerce and services | 6.4 | 16.8 | 9.4 | 1.17 | 15.7 | 8.87 |
| (7) Land, construction, and development | 7.7 | 16.8 | 9.2 | 2.37 | 16.0 | 8.89 |
| **Bonds** | | | | | | |
| (8) Linked to consumer price index | 5.3 | 4.5 | 2.3 | 0.06 | 3.2 | 1.80 |
| (9) Traded in foreign currency | 4.4 | 5.3 | 2.7 | -0.74 | 4.0 | 2.13 |
| (10) Linked to foreign currency | 4.5 | 7.1 | 3.4 | -0.70 | 5.7 | 2.90 |
| (11) Convertible bonds | 7.6 | 16.9 | 9.2 | 2.26 | 15.5 | 8.43 |
| (12) Capital notes | 6.5 | 11.8 | 6.0 | 1.25 | 10.5 | 5.33 |

[a] All rates of return in the tables are overall rates of return.

also the highest. Indexed bonds were the safest investment, since their dispersion index was the smallest, but they barely compensated the investor for inflation. As expected, the real rates of return fluctuated less sharply than the nominal rates, but the difference was surprisingly small. It follows that inflation does not greatly increase the riskiness of an investment.

Since we measure risk by both the variance and Gini indexes, which vary in a different range, there is no point in making a cardinal comparison of the results. It would be more appropriate to compare the rankings of the different groups.

The first three columns of Table 2 rank the alternative investments by their mean rate of return, variance (standard deviation), and Gini index. This table clearly shows that investments with a high rate of return have a higher risk. The rankings by these two indexes are similar but not identical: in five out of 12 groups the ranking is identical, and in only one group is there is a difference of more than one rank.

The last two columns of Table 2 show the construction of a subgroup of investments that are not dominated by other investments (according to the SSD method).[14] According to the MG criterion, security $i$ is not dominated by another security if there is no other security, $j$, such that $\mu_j \geq \mu_i$ and

[14] It should be reiterated that here we are discussing an investment in a single security; in the next section we deal with multisecurity portfolios.

**Table 2**

**RANKING OF SINGLE-SECURITY INVESTMENTS, DECEMBER 1976 TO DECEMBER 1980**

(Percentages)

| | By nominal rate of return | | | By mean rate of return less Gini | |
|---|---|---|---|---|---|
| | Mean rate of return | Standard deviation | Gini index | Nominal | Real |
| **Stocks** | | | | | |
| (1) Commercial banks | 4 | 9 | 9 | 2.6 | -2.3 |
| (2) Mortgage banks | 8 | 5 | 4 | -2.2 | -7.0 |
| (3) Specialized financial institutions | 9 | 4 | 5 | -2.1 | -7.0 |
| (4) Investment companies | 3 | 6 | 7 | -0.5 | -5.5 |
| (5) Industry | 7 | 7 | 6 | -1.7 | -6.4 |
| (6) Commerce and services | 6 | 3 | 1 | -3.0 | -7.7 |
| (7) Land, construction, development | 1 | 2 | 2 | -1.3 | -6.5 |
| **Bonds** | | | | | |
| (8) Linked to consumer price index | 10 | 12 | 12 | 3.0 | -1.7 |
| (9) Traded in foreign currency | 12 | 11 | 11 | 1.7 | -2.9 |
| (10) Linked to foreign currency | 11 | 10 | 10 | 1.1 | -3.6 |
| (11) Convertible bonds | 2 | 1 | 3 | -1.6 | -6.2 |
| (12) Capital notes | 5 | 8 | 8 | 0.5 | -4.1 |

$\mu_j - \Gamma_j \geq \mu_i - \Gamma_i$, where $\mu$ is the mean rate of return and $\Gamma$ is the Gini index.

Column 4 in Table 2 presents the $\mu - \Gamma$ for the nominal rates of return. These data, together with the data on the mean rates of return in Table 1, show that during the period investigated commercial bank stocks (1) dominated investments (2), (3), (5), (6), (9), (10), and (12), while the land, construction, and development group (7) dominated (11). The efficient set thus consists of commercial bank (1), investment company (4), and land, construction, and development (7) stocks and index-linked bonds (8).

When real rates of return are used for building the efficient set, group (7) does not dominate group (11). Convertible bonds therefore join the efficient set, and this is the only difference.

If the investor is restricted to choosing a single security group, the investments that are worthwhile from the risk-return aspect are thus limited to four or five groups.

**Properties of an Efficient Portfolio**

If the investor is not constrained to a single security but can select a diversified portfolio, the question that arises is how to allocate his money among the different alternatives. The advantage of a mixed portfolio over a single security lies in the fact that the correlation between the various categories of securities is less than unity. The investor can therefore reduce the risk of his investment by spreading it over a number of securities. To show the magnitude of the difference between a single-security and a multisecurity portfolio, we present in Table 3 the coefficients of correlation between the various invest-

Table 3

CORRELATION COEFFICIENTS BETWEEN MONTHLY REAL RATES OF RETURN OF SECURITY GROUPS, DECEMBER 1976 TO DECEMBER 1980

(Percentages)

| | Stocks | | | | | | | Bonds | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| **Stocks** | | | | | | | | | | | |
| (1) Commercial banks | 100.0 | | | | | | | | | | |
| (2) Mortgage banks | 69.6 | 100.0 | | | | | | | | | |
| (3) Specialized financial institutions | 63.3 | 80.0 | 100.0 | | | | | | | | |
| (4) Investment companies | 63.5 | 80.1 | 78.1 | 100.0 | | | | | | | |
| (5) Industry | 54.0 | 77.9 | 71.4 | 79.6 | 100.0 | | | | | | |
| (6) Commerce and services | 48.8 | 73.9 | 72.0 | 79.7 | 87.0 | 100.0 | | | | | |
| (7) Land, construction, and development | 62.9 | 68.1 | 63.2 | 76.2 | 77.9 | 65.8 | 100.0 | | | | |
| **Bonds** | | | | | | | | | | | |
| (8) Linked to consumer price index | 4.5 | -13.5 | -22.5 | -16.6 | 8.3 | -0.1 | -0.5 | 100.0 | | | |
| (9) Traded in foreign currency | 35.6 | 14.7 | 7.9 | 18.8 | 22.4 | 20.4 | 10.6 | 30.9 | 100.0 | | |
| (10) Linked to foreign currency | 37.2 | 18.0 | -2.9 | 15.3 | 26.3 | 20.3 | 13.7 | 42.3 | 70.3 | 100.0 | |
| (11) Convertible bonds | 65.1 | 84.2 | 80.4 | 89.3 | 86.3 | 84.2 | 72.9 | -8.3 | 15.6 | 12.5 | 100.0 |
| (12) Capital notes | 89.5 | 68.9 | 73.6 | 70.7 | 64.9 | 56.2 | 62.8 | -9.1 | 27.8 | 25.6 | 72.4 |

ments.[15] The following conclusions can be drawn from the table: (a) The correlations between all the stocks are positive and high, i.e. all the stocks tended to move up and down in step, and consequently selecting combinations of different groups of stocks does not greatly reduce the risk. (b) The correlation between indexed bonds and the various stocks is low and/or negative; hence efficient portfolios could be expected to consist of stocks and indexed bonds. (c) The correlation between indexed bonds and bonds traded in foreign currency is predictably high, while the correlation between bonds and stocks is lower than those between the various stocks. One could therefore expect to find bonds traded in foreign currency included in the efficient portfolios. (d) Finally, since most of the correlations are high, a diversified portfolio is not always far superior to an investment in a single security.

In constructing efficient portfolios by the mean-variance and mean-Gini methods we used two sets of data, the nominal rates of return and the real rates of return. We sought the allocations that minimized the index of dispersion for a given mean rate of return. By varying the mean rates of return we derived the efficiency curve, i.e. all the minimum dispersion combinations for the given rates of return. The efficiency curve can be analyzed in the same way as a single-security investment. For comparative purposes we required that the given rates of return be identical in both methods of measurement, and that the rates of return of the diversified portfolios be equal to those of the single-security portfolios. In addition, we ruled out short sales.

The upper panel of Table 4 sets out the efficient portfolios for the various nominal rates of return on individual securities according to the mean-Gini approach. Investors wanting the highest return, an average of 7.65 percent a month, had to invest all their money in the group with the highest rate of return — i.e. land, construction, and development stocks (7); those preferring a smaller return but with a lower risk could choose a portfolio composed mainly of commercial bank stocks, land, construction, and development stocks, and convertible bonds (11). Investors who were more risk-averse could choose portfolios giving a lower return, such as combinations of commercial bank stocks and bonds; and those with the greatest aversion to risk concentrated mainly on indexed bonds. The risk associated with a diversified portfolio was much lower than that of an individual security giving the same return. For example, portfolio (4) attained a 7.4 percent monthly return — which was identical to the return earned on investment company stocks — but the mean-Gini index for this portfolio was 6.016, compared with 7.97 for a single-security investment. This investigation thus confirms the adage that one should not put all his eggs in one basket.

Table 4 shows that commercial bank stocks also lie on the efficiency curve. This means that an efficient portfolio could be constructed solely from such securities (see portfolio 1, 97 percent of which consists of bank stocks). Presum-

[15] The correlations are between the real rates of return. The same picture emerges from the nominal rates of return.

**Table 4**

**EFFICIENT PORTFOLIOS,[a] NOMINAL VALUES, DECEMBER 1976 TO DECEMBER 1980[b]**

(Percentages)

| Monthly rate of return | 7.654 (7) | 7.621 (11) | 7.394 (4) | 7.188 (1) | 6.546 (12) | 6.435 (5) | 6.310 (6) | 6.252 (2) | 6.119 (3) | 5.252 (8) | 6.04 Market portfolio[c] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Mean-Gini** | | | | | | | | | | | |
| Stocks | | | | | | | | | | | |
| (1) Commercial banks | | 4.44 | 53.75 | 96.63 | 63.65 | 57.61 | 51.06 | 47.99 | 41.02 | 0.11 | 30.6 |
| (3) Specialized financial institutions | | | | | | | | | | 5.01 | 4.9 |
| (4) Investment companies | | | | 2.94 | 2.89 | 3.19 | 3.26 | 3.32 | 3.60 | 3.44 | 5.9 |
| (7) Land, construction, development | 100.00 | 57.74 | 34.16 | | | | | | | | 1.6 |
| Bonds | | | | | | | | | | | |
| (8) Linked to consumer price index | | | | 0.43 | 33.46 | 39.20 | 45.68 | 48.69 | 55.38 | 76.80 | 47.1 |
| (9) Traded in foreign currency | | | | | | | | | | 14.64 | 3.0 |
| (11) Convertible bonds | | 37.81 | 12.09 | | | | | | | | 0.3 |
| Total | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** |
| Gini index | 9.230 | 8.530 | 6.021 | 4.564 | 3.375 | 3.192 | 3.000 | 2.915 | 2.739 | 2.116 | 2.987 |
| Gini index—optimal MV portfolio | 9.230 | 8.532 | 6.028 | 4.574 | 3.385 | 3.203 | 3.013 | 2.928 | 2.745 | 2.128 | |
| **Mean-variance** | | | | | | | | | | | |
| Stocks | | | | | | | | | | | |
| (1) Commercial banks | | 4.30 | 54.73 | 94.33 | 59.83 | 53.90 | 47.18 | 44.04 | 36.87 | | 30.6 |
| (3) Specialized financial institutions | | | | | | | | | | 7.60 | 4.9 |
| (4) Investment companies | | | | 1.53 | 3.58 | 3.93 | 4.33 | 4.52 | 4.94 | | 5.9 |
| (7) Land, construction, development | 100.00 | 55.89 | 30.30 | 3.21 | 2.46 | 2.33 | 2.19 | 2.12 | 1.97 | 1.36 | 1.6 |
| Bonds | | | | | | | | | | | |
| (8) Linked to consumer price index | | | | 0.94 | 34.13 | 39.84 | 46.31 | 49.32 | 56.22 | 78.92 | 47.1 |
| (9) Traded in foreign currency | | | | | | | | | | 12.11 | 3.0 |
| (11) Convertible bonds | | 39.82 | 14.97 | | | | | | | | 0.3 |
| Total | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | |
| Standard deviation | 16.831 | 15.119 | 11.111 | 9.056 | 6.592 | 6.221 | 5.800 | 5.617 | 5.227 | 3.980 | 5.630 |
| Standard deviation—optimal MG portfolio | 16.831 | 15.321 | 11.205 | 9.056 | 6.605 | 6.222 | 5.811 | 5.630 | 5.251 | 3.997 | |

[a] Securities that do not appear in any efficient portfolio (five securities, amounting to 7 percent of the value of the market portfolio) are excluded from the table.

[b] The numbers in parentheses designate the securities that are included in the efficient portfolios.

[c] Calculated at the middle of the period.

ably this is explained by the practice of most of the country's leading banking groups of supporting their equities during the period investigated. As opposed to the efficient performance of commercial bank stocks, the inefficiency of index-linked bonds is surprising: the investment that could earn the same rate of return (portfolio 8) consisted of only 77 percent of such bonds. This indicates that the Bank of Israel intervened in the market (to prevent sharp fluctuations in bond prices) to a lesser degree than the banks that supported their stocks.

The stocks of the mortgage bank, industry and commerce, and service groups, as well as capital notes do not appear in a single efficient portfolio. They accounted for only 7 percent of the total market value of all listed securities in the middle of the period investigated. Some of these results can probably be attributed to the shortness of the period investigated (only four years), and so no firm conclusions should be drawn regarding the inefficiency of these securities. (For example, if 1982 had been included in the data, we would probably get different results for industrial stocks.)

Bonds linked to or traded in foreign currency also do not appear in the efficient portfolios. There are combinations that gave a higher rate of return at a lower risk. The last line in the upper panel of Table 4 shows the Gini indexes for portfolios constructed by the mean-variance method. A comparison of these indexes with the Gini indexes for portfolios constructed according to the MG criterion gives some notion of the difference between the two methods. In the range of returns close to that of commercial bank stocks (1), the difference between the two Gini indexes is negligible (4.574 vs. 4.564, i.e. only 0.2 percent), and it increases the further we move away from this range. For example, in the case of a portfolio yielding the same rate of return as indexed bonds (8), the difference is 0.5 percent. On the whole, the differences are not great, but it should be borne in mind that we are dealing with indexes of groups of securities. Since the variance is sensitive to extreme observations, it can be assumed that if we had used data on individual securities we would have found larger differences between the two Gini indexes.

The last column shows the composition of the actual market portfolio. As expected, it does not appear in any of the efficient portfolios, but it is interesting to note that it is close to the efficiency frontier. For this reason, one cannot conclude from our findings that an investment in the market portfolio during the period investigated was not efficient.

The necessary conditions for an efficient set constructed by the SSD approach, which were described in the previous section in connection with a single-security investment, can be applied here too in order to reduce the set of efficient portfolios.

As expected, there is no efficient SSD portfolio that dominated the others. The investor who wanted a higher return and was ready to accept a higher risk would have had to put more of his money in risky stocks. But portfolio (3), whose average rate of return was 6.1 percent a month, dominated portfolio (8), and so the latter can be removed from the efficient set ($\mu_3 - \mu_8 = 0.78$; $\mu_3 - \Gamma_3 - (\mu_8 - \Gamma_8) = 0.24$).

The lower panel of Table 4 presents the efficient portfolios for various rates of return attainable on individual securities according to the mean-variance criterion. The investor who wanted the highest return, 7.65 percent a month, would have had to put his money in land, construction, and development stocks, which involved the highest risk. Combinations of commercial bank stocks, land, construction, and development stocks, and convertible bonds likewise promised a high return along with a high risk, while portfolios composed of commercial bank, investment company, and land, construction, and development stocks and indexed bonds yielded a smaller return but also had a smaller risk. An investment in a diversified portfolio reduces the risk compared with an individual security. This is exemplified by portfolio (4), which had a 7.4 percent average monthly return and a 11.1 percent monthly standard deviation. A group that gave the same rate of return — investment company stocks — had a risk index of 14.6 percent (see Table 1); i.e. the risk index of an efficient portfolio was 75 percent as high as that of the individual security.

The other conclusions emerging from the lower panel of Table 4 are essentially the same as those drawn from the upper panel. The differences in the standard deviation and the Gini obtained by the two methods are small. This, however, is not true of the differences in the composition of the portfolios: during the period investigated these ranged between 2 and 3 percent for each asset's portfolio proportion (cf., for example, column 5 in both panels of the table). Mean-variance portfolios generally consist of more securities than the corresponding mean-Gini portfolios (one security more on average). The efficient set constructed according to the mean-variance was larger than the MG efficient set after imposing on the portfolios the SSD necessary conditions.

Table 5 presents the efficient portfolios constructed by our two methods for real rates of return, which generally differ only slightly from the nominal returns. The most striking changes are that, unlike the nominal portfolio, the real portfolio includes mortgage bank stocks (2) according to both approaches, and also capital notes under the mean-variance approach; but when the SSD necessary conditions are applied by means of MG, portfolio (8) drops out of the efficient set. Since this is the only portfolio that includes mortgage bank stocks, then in the case of mean-Gini the use of real data does not alter the conclusions drawn from the nominal data. This is contrary to the MV results: here the shift from nominal to real data alters the composition of the efficient set, and this confers a certain advantage on the mean-Gini method.

The principal finding of this empirical analysis is the similarity of the efficient portfolios constructed by our two methods. But it must be emphasized that to some extent this similarity derived from the use of data on groups of securities: the distribution of the rates of return for groups of securities can be expected to more closely approximate the normal distribution than does the distribution of the returns on individual securities.

**Table 5**

**EFFICIENT PORTFOLIOS, REAL VALUES, DECEMBER 1976 TO DECEMBER 1980**[a]

(Percentages)

| Monthly rate of return | 2.373 | 2.259 | 2.085 | 1.879 | 1.249 | 1.168 | 1.061 | 0.991 | 0.90 | 0.06 | 0.80 Market |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | (7) | (11) | (4) | (1) | (12) | (5) | (6) | (2) | (3) | (8) | portfolio |
| **Mean-Gini** | | | | | | | | | | | |
| Stocks | | | | | | | | | | | |
| (1) Commercial banks | | 18.94 | 56.87 | 99.35 | 63.33 | 59.03 | 52.03 | 48.52 | 43.69 | 4.28 | 30.6 |
| (2) Mortgage banks | | | | | | | | | | 2.95 | 2.0 |
| (3) Specialized financial institutions | | | | | | | | | | 2.63 | 4.9 |
| (4) Investment companies | | | | 0.58 | 1.86 | 1.71 | 2.26 | 2.40 | 2.38 | | 5.9 |
| (7) Land, construction, development | 100.00 | 63.45 | 36.64 | | | | | | | | 1.6 |
| Bonds | | | | | | | | | | | |
| (8) Linked to consumer price index | | | | 0.07 | 34.81 | 39.25 | 45.19 | 49.08 | 53.93 | 70.47 | 47.1 |
| (9) Traded in foreign currency | | | | | | | | | | 19.67 | 3.0 |
| (11) Convertible bonds | | 17.61 | 6.49 | | | | | | | | |
| Total | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | |
| Gini index | 8.889 | 7.355 | 5.469 | 4.128 | 2.820 | 2.669 | 2.478 | 2.359 | 2.218 | 1.545 | 4.550 |
| Gini index — optimal MV portfolio | 8.889 | 7.360 | 5.475 | 4.128 | 2.821 | 2.670 | 2.481 | 2.359 | 2.222 | 1.576 | |
| **Mean-variance** | | | | | | | | | | | |
| Stocks | | | | | | | | | | | |
| (1) Commercial banks | | 17.69 | 55.93 | 99.12 | 64.75 | 60.37 | 54.58 | 50.12 | 45.96 | 10.10 | 30.6 |
| (2) Mortgage banks | | | | | | | | | | 0.01 | 2.0 |
| (3) Specialized financial institutions | | | | | | | | | | 0.23 | 4.9 |
| (4) Investment companies | | | | 0.78 | 0.49 | 0.43 | 0.37 | 0.96 | 0.28 | 0.02 | 5.9 |
| (6) Commerce and services | | | | | | | | | | 0.01 | 0.1 |
| (7) Land, construction, development | 100.00 | 59.26 | 33.47 | | 0.07 | 0.07 | 0.06 | | 0.06 | 0.01 | 1.6 |
| Bonds | | | | | | | | | | | |
| (8) Linked to consumer price index | | | | 0.09 | 34.68 | 39.13 | 44.99 | 48.92 | 53.71 | 66.56 | 47.1 |
| (9) Traded in foreign currency | | | | | | | | | | 23.27 | 3.0 |
| (11) Convertible bonds | | 23.05 | 10.61 | | | | | | | 0.01 | 0.3 |
| (12) Capital notes | | | | | | | | | | 0.00 | 0.0 |
| Total | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | **100.00** | |
| Standard deviation | 15.962 | 13.298 | 10.151 | 7.928 | 5.351 | 5.046 | 4.659 | 4.416 | 4.127 | 2.830 | 4.55 |
| Standard deviation — optimal MG portfolio | 15.962 | 13.310 | 10.160 | 7.928 | 5.352 | 5.047 | 4.663 | 4.420 | 4.132 | 2.946 | |

[a] See the notes to Table 4.

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