THE BINARY QUALITY PRICE FUNCTION: THEORY, EMPIRICAL TESTING, AND APPLICATION TO ISRAELI TOMATO EXPORT

Oded Hochman
Department of Economics
Ben-Gurion University
of the Negev
Beersheva, Israel

Uri Regev
Monaster Economic Research Center
Ben-Gurion University
of the Negev
Beersheva, Israel

Abstract

When the quality traits of a quality-differentiated product are defined in a binary form, the resulting binary quality price function (BQPF) is shown to be convex. This implies that when many units of the good are sold in a single transaction, marketing it in packages of homogeneous product is more profitable, than in heterogeneous ones. Therefore, if sorting is possible, and relatively inexpensive, it should be undertaken. Another implication of the convexity of the BQPF concerns the case when quality uncertainty of each unit of the product exists at the time of sale. Providing quality guarantee in this case is equivalent to sorting in the former case.

The convexity of the BQPF is tested empirically using data of Israeli export glass-house tomatoes. An estimate of this BQPF has been used to determine actual quality-differentiated transfer prices paid by the single exporting agency to the various growers of this produce. A method of estimating an autoregressive model of the first order with missing observations was developed and utilized in this study and is reported in the Appendix.

1. General

Two types of quality-differentiated products are recognized in the economic literature. The first, initially recognized by Hotelling (1929), deals with a set of products with equal costs of production which are substitutes for one another. Hotelling assumed that a vector of quality characteristics typifies the quality differentiation of products, the possible values of each characteristic can be mapped on the real line, and the consumer has his own preference ordering over these values. He further assumed that the distance on the real line between two values of a given characteristic determines their level of substitutability, and every individual has a single value which is most preferred. Consequently, for each characteristic there exists a given and known distribution of its most preferred values in the population. Examples of goods with such quality characteristics are items of clothing (differentiated by color, size, etc.), and similar food items such as cereals with different flavors (Schmalensee, 1978). This kind of quality differentiated products has been investigated by Dixit and Stiglitz (1977), Spence (1976), Lancaster (1979), and others.

A second type of quality differentiated products are those in which preference ordering by individuals over different values of a given characteristic are the same, but the costs of production are higher for a more preferred quality (see Levhari and Peles (1973), Lancaster (1971), Leland (1977)). Examples of such quality traits are the life span of an electric bulb, and vitamin content in a food product.

Product quality measurements and tests performed by various producers and marketing agencies are often of a binary nature, applicable to both types of quality discussed above. In this paper it will be proved that, in competitive markets where transactions include many units of a given not perfectly homogeneous good, the binary-quality-price-function (BQPF) is a convex function of the binary quality traits (BQT). This function is also convex when the true quality of a single unit of the good is uncertain at the time of purchase.

This paper will show that the convexity of the price function implies that mixing batches of product of different quality levels prior to marketing is never profitable, while sorting at negligible cost, if possible, and sale of the various qualities under separate packaging will generally yield profits. For example, it pays to sell separately, rather than mixed, small and large tomatoes, more reliable and less reliable cars, or short and long screws. Our study also provides an explanation of the phenomena of firms guaranteeing the quality of their goods. The guarantee enables them to sell more homogeneous goods by sorting out faulty items at relatively low costs.

We will also prove empirically the hypothesis of convexity of the BQPF, using data of Israeli export glass-house tomatoes marketed in West Germany. Finally, the paper reports on the application of the estimated BQPF in the construction of quality-differentiated transfer prices, designed to mimic the expected market prices paid to the tomato growers who export their produce jointly (due to scale economies in marketing) through a single marketing agent. The agent markets the produce under a single brand name and pays the growers the transfer prices, out of which expenses are deducted.

The empirical results indicate that market price is highly sensitive to quality variations. The coefficients of quality variables are highly significant and a high coefficient of determination (R^2=0.94) has been obtained. Problems of autocorrelation with missing information were resolved by using an appropriate econometric procedure developed for this purpose (see Appendix).

The initiation of this study has been induced by rapid deterioration in the quality and prices of the Israeli glass-house tomatoes in European markets. There exists a short term (let alone a long term) tradeoff in production between quality and quantity. For example, increasing irrigation and nitrogen fertilization raises the yield while reducing quality. Since this study was started and quality-differentiated transfer payments to the growers implemented, the quality of the Israeli tomatoes has improved steadily, and have obtained the highest prices in Europe's recent marketing seasons. The implication is that the quality-differentiated transfer prices provide the growers with sufficient incentives to produce the quality levels which market prices would have induced, while the earlier practice of pooling price with export qualifying tests coupled with other administrative measures had been ineffective.

The characterization of the BQPF and resulting marketing policy implications is described below, and the empirical analysis presented. An Appendix describes the econometric methodology.

2. The Binary-Quality Price Function (BQPF) Characterization and its Practical Implications

Consider a characteristic of a quality differentiated product of any type, e.g. the color of a shirt, or the expected life span of an electric light bulb. First, divide the domain of the characteristic into two mutually exclusive subsets exhausting the whole domain. Then, define one of the subsets as the acceptance zone, and the other as the rejection zone. For example, dark red and purple shirts would be in the acceptance zone and all other colors in the rejection zone. In the light bulb example, the acceptance region might be bulbs with an expected duration of a least five hundred hours of life.

A binary quality trail (BQT) is a variable which assumes the value of one, when the value of the characteristic of a unit product is in the acceptance zone, and the value of zero otherwise. For example, if a shirt is dark red or purple, the value of the associated BQT is 1, otherwise it equals 0. Likewise, if the life span of an electric light bulb is five hundred hours or more, the value of its respective BQT is 1, and otherwise it is 0. It should be noted that for each quality characteristic several independent BQTs can be defined. There are no mathematical restrictions on the acceptance zone and it could be connected or unconnected, closed or open. Thus, an acceptance zone for a characteristic 'size of shirts' could be defined by the set: $\{X: N-\alpha < X < N+\alpha$ $N=10,11,..,25;\ 0 < \alpha < 0.5\}$, where α is the allowed deviation from the marked size, X is the actual shirt size, and N is an integer marked on the shirt designating its size. Consequently, the acceptance zone contains shirts, the actual size of which deviates from the designated size by less than α. Thus BQTs are useful when questions of quality control and quality testing are involved. In this case, the set of minimum quality standards is defined as the acceptance zone.

In this study, binary quality price functions (BQPF) of two types of products are investigated. The first type is where many units of the product are sold in each market transaction, the quality of each item is fully known, but the items in the transaction are not necessarily all in the acceptance zone. The second type is where a single unit may be sold in each transaction, but the quality of any item in the transaction is uncertain at the time of sale and is revealed only later, during consumption. At the time of sale the information available is the product's quality probability distribution. In both cases, the BQT of a transaction is the probability of a single item being in the acceptance zone when drawn randomly out of the items in the transaction.

The first type of BQPF includes products in retail markets such as; most fruit and vegetables, headache pills, nails, matches, as well as raw materials in the production factor markets and most of the wholesale trade. The value of a BQT for a given transaction is the average of the BQTs of the units in the transaction of the first type and therefore is the proportion of the units in the acceptance zone. Thus if a transaction includes ten items, eight of which are in the acceptance zone, the BQT of the transaction is equal to 0.8.

The second type of BQPF is a product for which the BQT value of every single item in the transaction is uncertain at the time of sale. In this case, the BQT of a transaction is the probability that the BQT of an item drawn randomly out of the items in the transaction is one, and it can be different from one or zero even when the transaction involves only one item. Therefore, if the acceptance zone of the durability of a light bulb is five hundred burning hours or more and the probability that a light bulb will last that long is 0.7, then the value of the BQT of a transaction of such light bulbs is 0.7, independent of the size of the transaction. It is assumed that the probability (the BQT of the transaction) is fully known to all buyers and sellers at the time of sale, although each participant might have their own subjective probability. Examples of goods of the second type are all the durable goods such as electric appliances, cars, or buildings.

The BQTs are determined by quality tests conducted by the producers, consumers, and marketing agencies both at the production site and in the market. Since BQTs are defined in the process of quality testing, and this information is available to all market participants, it is beneficial to use them as the quality traits in the price quality function. In dealing with goods characterized by BQTs, it should be noted the possibility of mixing several batches of the product, each of a different quality level, to obtain a single batch with an average quality. Furthermore, it is assumed that mixing batches of different qualities is costless, while sorting into higher and lower values of BQT, if possible, is costly. This assumption holds in most cases, and leads to an important property, described in the proposition below.

Define $P(Z_1,Z_2,...,Z_n)$ to be the binary quality price function (BQPF), of a quality differentiated product of one of the two types above, in a perfectly competitive market. Customers are assumed to have full information of the levels of the BQTs of the market transactions, Z_i, for $i=1,...,n$, where n is the number of the BQTs of the good.

Proposition: Under conditions of competitive markets, the Binary Quality Price Function is a weakly convex function in its quality traits.

Proof: Competitive markets imply long run zero profits from arbitrage. First we must prove that the BQPF can not be strictly concave in this situation. It is assumed, as a working hypothesis, that the BQPF is strictly concave and proves a contradiction. Let A and B be two points on the BQPF (see Figure 1), where:

$$A = (Z^A, P(Z^A)),\ B = (Z^B, P(Z^B)),$$

$$Z^A = (Z_1^0, Z_2^0, \ldots, Z_{i-1}^0, Z_i^A, Z_{i+1}^0, \ldots, Z_n^0),$$

and $$Z^B = (Z_1^0, Z_2^0, \ldots, Z_{i-1}^0, Z_i^B, Z_{i+1}^0, Z_n^0).$$

Thus, Z^A and Z^B differ from one another only in the i^{th} element of the quality traits. Suppose that ACB in Figure 1 is the strictly concave market price function. Then an arbitrager could buy equal quantities of the good with qualities Z^A and Z^B, pay $P_A = P(Z^A)$ and $P_B = P(Z^B)$ respectively, and mix them together at no cost. The quality of the resulting product will change only in the i^{th} element, so that:

$$Z^E = [Z_1^0, \ldots, Z_{i-1}^0, 0.5(Z_i^A + Z_i^B), Z_{i+1}^0, \ldots, Z_n^0].$$

Thus the market price of the product will be Pc, while the costs of the arbitrage will be $P_D = 0.5(P_A + P_B) < P_C$. This implies positive arbitrage gains, contradicting the assumption of competitive market, and thus the BQPF cannot be strictly concave.

Next, it is demonstrated that convexity cannot be excluded by using similar arguments. Let AEB in Figure 1 be a strictly convex BQPF. If a given quantity with quality Z^E could always be costlessly sorted into qualities Z^A and Z^B (so that Z^E is a convex combination of Z^A and Z^B), then the hypothesis of strictly convex BQPF (at E) could be rejected under competition. This is so because an arbitrager could buy goods of quality Z^E, sort them into two qualities Z^A and Z^B, sell them for an average price of P_D, to make consistent arbitrage gains contrary to the assumption of competitive market. However, sorting even if possible, is costly (in accordance with the law of entropy), so that this argument fails, resulting in the conclusion that the BQPF can indeed be strictly convex. So far, however, the difference $P_D - P_E$ is bounded from above by sorting costs.

Sorting into two predetermined quality classes, so that the original quality is a convex combination of the two, is often impossible due to interdependence between different quality traits. Suppose, for example, that the i^{th} quality trait is negatively related to the j^{th} quality trait. In this case, sorting by the i^{th} quality trait will change the level of the j^{th} quality trait as well, in such a way that the "higher" grade goods in the i^{th}

quality trait will necessarily be with a lower quality in the jth component. This implies that Z^E (in Figure 1) cannot be sorted <u>at any cost</u> into Z^A and Z^B while keeping all other traits constant. In this case, the BQPF will be determined solely by demand and supply conditions, and not by market arbitrage, and P_E may even be zero while P_A and P_B are positive.

Furthermore, the nature of the interdependence between quality traits may be different when different brands are considered, or even within the same brand between different shipments. Thus, in some instances sorting is possible and may be profitable, and in other cases it is not. Indeed, in practice sorting is often performed, e.g. with agricultural produce, which by itself implies strict convexity of the BQPF. Actually, in agricultural produce, quality deteriorates quite rapidly due to passage of time, handling, and weather conditions. Quality changes are not necessarily homogeneous, and sorting into higher and lower grades may be feasible. It was found that sorting for arbitrage gains takes place in different stages of the marketing process. This implies strict convexity of the BQPF in the relevant range, or sorting would not have been worthwhile (arbitrage gains must be at least as high as sorting costs). This completes the proof of the proposition.

Convexity of the BQPF has an important implication for marketing policy, as stated in the following corollary:

<u>Corollary</u>: marketing two quality levels separately will often yield higher profits, never lower than those obtained by marketing the product at average quality.

This corollary results from the fact that any line connecting two points of a strictly convex function will lie entirely above the function. Thus, the average price of two qualities marketed separately is given by a point on a straight line connecting their prices on the BQPF, and convexity of the BQPF implies that it is higher than the price of the mixed average quality given by the BQPF. While sorting may be costlier than the additional benefits, mixing different fruit qualities is never profitable.

Worth mentioning is an interesting type of sorting which takes place in low-quality-low-price retail markets of fresh agricultural produce, especially open markets. Shoppers in these markets are characterized by having low value of time, due to low earning power and for lack of alternative use for their shopping time. In these markets, cheap sorting performed by the customers themselves, who select the better looking units from low quality produce stands, is witnessed. Clearly, the price charged in such a stand must be higher than that charged for a product of the same quality, where no self sorting process is allowed, since the remaining lower quality merchandise must be sold at a lower price. Such a practice is worthwhile only if the BQPF is strictly convex, and the difference between the prices in the two situations is higher than the value of shoppers' time spent on sorting.

Another practical implication of the convexity of the BQPF is the following: consider a product which is sold with a single unit per transaction (e.g. appliances) whose quality is not directly observable in the market at the time of its sale, but the subjective probability of satisfying a predetermined quality level (which is the value of the BQTs) is known from quality tests, past experience, rumors, and/or advertising. When the BQPF is convex, the interest of the producer is to sort the product into high and low BQT levels, however, the information required for this task may not be available at the time of sale. Providing a guarantee to customers to replace or repair the product in case of a future failure can be considered as a form of selecting and selling with certainty the units having the desired quality (BQT=1). It is quite clear that if the price function were concave, providing a guarantee is at least not profitable. Thus, the actual prevalence of guarantees in markets with an uncertain quality of goods indicates that consumers and producers face strictly convex price functions.

3. Empirical Analysis

3.1 Approach and Purpose

The purpose of the empirical analysis carried out in this section is twofold: first, to support the proposition of convexity of a BQPF by proving it empirically for the case of Israeli export tomatoes sold in West German markets. Second, to estimate the price function in a form easily applicable as a base for quality-differentiated transfer prices paid by the exporting agency to the individual tomato growers utilizing its services.

One property of the tomatoes' BQPF, typical for perishable agricultural goods, stems from the fact that the quality of the produce deteriorates rapidly. Thus, sales at different times or locations imply different markets due to short shelf life of the produce, direct transport cost, time delays, and deterioration in the produce quality when shipped and handled. All these properties make it prohibitively expensive for arbitrage between different markets in space and time. This implies different BQPFs in different markets in time, because of change in supply (of close or perfect substitutes) or in space, due to differences in transportation costs or shipping damages to the produce quality. In addition, random effects in both demand and supply will obviously contribute to price variation between markets. Changes in the demand conditions due to differences in weather conditions, in time and space, random supply of close substitutes, and variability in the composition of the population in different locations, are also recognized. It has been assumed that the differences in the BQPFs of the different markets can be represented, by changes in the maximum price of the most abundant tomatoes in the German markets (in the relevant season), which are the Canary Island tomatoes.

Due to stable weather conditions in their growing area, the Canary Island tomatoes are of relatively stable quality with little fluctuation throughout the season. This is especially true with respect to their top quality fruit which obtains the maximum price, even if its market share could change. Thus the BQPF of the Israeli tomatoes in the German markets throughout the season has been written as P(Z,K) with a positive partial derivative with respect to K, where K denotes the maximum price of the Canary Island tomatoes in the specific market, and $Z = (Z_1,...,Z_n)$ is a vector of binary quality traits of the transactions.

We adopt here Griliches' (1971) and Rosen's (1974) hedonic price estimation method of identifying the contribution of individual quality components to the price of a product. The general theory does not preclude any functional form for the hedonic price function. By Taylor theorem, any analytical function can be approximated by a finite degree polynomial, and indeed in several recent studies a polynomial has been adopted as a functional form of a hedonic function (Brown and Rosen (1982), and Brown and Mendelson (1984)).

When testing for convexity of the BQPF we have chosen a quadratic function, so that the alternative hypotheses to be tested becomes simple and straightforward: the estimated function is either convex or not over the entire domain. In higher degree polynomials or in other functional forms, the estimate may be convex in one domain and concave in another. Thus, when using a quadratic function, the question of its convexity can be uniquely resolved. Another, more down to earth reason, is that it allows all quality traits to be included as the statistical theory dictates. This is not possible in higher degree polynomials due to the large number of regressors involved.

Israeli tomatoes grown by individual growers have been, and still are, exported and sold collectively by a single exporting agency under one or two brand names. Exporting agricultural produce is subject to economies of scale, and therefore marketing it by several exporting agencies is more expensive than through a single one. In the past, all growers received the same "pooling" price, which caused a lack of incentive for growing quality fruit by the individual grower. This led the growers to reduce the quality of the fruit, which in turn led to a deterioration in the market price, and threatened the existence of the whole industry. Consequently, in order to establish incentives for producing high quality tomatoes, it was decided to perform quality tests in the packing house which would determine the produce quality of each individual grower, according to which they would be paid. The differentiated quality transfer pricing system constructed for this purpose, required the estimation of the market BQPF. Data associating the quality of tomatoes at the packing house and its foreign market price, was collected and recorded.

An estimate of the BQPF, useful as a pricing base, is one in which the number of BQTs are minimal. Quality testing in the packing house should be simple and cheap. Since fewer traits involve less quality measurements, reducing the number of traits is desirable. Correlations between different quality traits may exist but not necessarily in a linear form, thus it may be possible to increase the effective correlation between quality traits by raising the polynomial degree and making more variables redundant in the estimate. A third degree polynomial with a reduced number of traits was derived as the best estimate for the BQPF for the practical purpose of obtaining actual quality-differentiated transfer prices paid to the glasshouse tomato growers in Israel.

3.2 The Data

The data includes information about the wholesale prices (in German Marks/kg) of two brands of Israeli tomatoes in Germany throughout one growing season (winter 1979). These prices were obtained in wholesale German markets a few days after harvest and packing. The two brands differ in their growing technology. The quality traits of the Israeli tomatoes were estimated by samples taken at the gate of the packing houses on harvest dates, before sorting and packing had taken place. The traits that were measured include percentage of firm, pink, red, small and medium sizes, and discolored fruit. In the packing house, the fruit were sorted by color intensity and by size, so that only sales of relatively homogeneous fruit with these latter traits took place. Those, more or less, exhaust the quantitative quality traits which can be readily measured without destroying the product. Chemical content or taste of the product were not measured. The process of obtaining and linking together the different pieces of the data was quite complex and involved matching of three data sets:

(1) BQT levels at harvest date, by packing house, half week, and brand. Tomatoes were sampled at random from each of five packing houses, every half week to determine their quality levels.

(2) Daily sales records of wholesalers (panelists) in Germany. These data included daily information about quantities received, quantities sold, and quantities in stock, as well as gross revenue of daily sales (yielding average daily prices). These records supplied the information separately for the two Israeli brands in our sample.[1]

(3) Linkage information; All Israeli tomatoes were sent to West Germany by air through Koln. A team was placed at the Koln airport to record the identification of every delivery from Koln to all destinations in Germany.

The details reported include the harvest date, packing house identification, brand and size of each delivery, as well as its destination (wholesalers) and delivery date. The matching of these three sets produced the basic data for this analysis, which included the average quality vector of the tomatoes for each sale, as well as its price. In addition, data was collected on the competing tomato prices from official publications (ZMP) which recorded maximum and minimum prices every two days in each city for all types of tomatoes sold there. The final data set included 510 observations out of a total of over 2,000 actual transactions which took place during that season. The actual transactions were made by about twenty-five wholesalers in nine regions of West Germany from January through May, 1979. The transactions of any given wholesaler were not necessarily made every working day, mainly due to lack of supply. Therefore, the data consist of incomplete time series information.

3.3 Autocorrelation

As with most time series data involving market prices, autocorrelation must be taken into consideration in the analysis, however, the conventional treatment of autocorrelation cannot be applied directly to this problem because of missing observations due to different time intervals between consecutive transactions in the same market. A method to resolve this problem and treat autocorrelation with missing observations has been developed. The accepted model of 1st order autocorrelation (AR1) states:

$$Y_t = \sum b_j X_{jt} + U_t, \text{ for } t=1,\dots,T$$

and $$U_t = \rho\, U_{t-1} + e_t$$

where U_t is the unobserved random disturbance, ρ is the autocorrelation coefficient, and e_t is distributed $N(0,\sigma^2)$, with $Ee_te_s = 0$ for $s = t$. The Generalized Least Squares (GLS) estimation procedure has been extended to the case of randomly missing observations, where time intervals between consecutive observations are not fixed. The GLS estimators were obtained by applying ordinary least square technique to the following transformations of the original data (see Appendix):

$$y_i^* = (Y_i - \rho^{Di} Y_{i-1})/\sqrt{S_i}$$

$$(1)\quad X_{ij}^* = (X_{ij} - \rho^{Di} X_{i-1,j})\,/\sqrt{S_i};\ j=1,\dots J;\ i=2,\dots,I$$

$$Y_1^* = Y_1\sqrt{(1-\rho^2)}, \text{ and } X_1^* = X_1\sqrt{(1-\rho^2)},$$

where the index i counts the order of the observation, and Di denotes the time interval between the i^{th} and its preceding observation, and

$$(2)\quad S_i = (1-\rho^{2Di})/(1-\rho^2).$$

Since in this case (as is usual) ρ is unknown, ρ and the regression coefficients vector b have been estimated simultaneously using the maximum likelihood estimation method. Ignoring constants, the log likelihood function adjusted to the missing information case, can be written (see Appendix):

$$(3)\quad \ln(L) = -I.\ln[\sum_{i=1}^{I} (Y_i^* - \sum_j b_j X_{ij}^*)^2/I] + \ln(1-\rho^2) - \sum_{i=1}^{I} \ln(S_i),$$

where Y_i^* and X_{ij}^* are defined in equation (1), S_i is defined in (2), and I is the number of observations. The search method suggested by Hildreth and Dent (1974) for simultaneous estimation of ρ and b, where the b's are estimated for grid values of ρ, and the maximum likelihood criterion is used to select the estimates, has been used.

In this case, the dependent variable Y was the price of the Israeli tomatoes and the X_j's were the quality traits and the Canary Island price. Data were collected from nine different regions in West Germany, and the ρ's estimated separately for each region. The ρ's were found to be significantly different from one another, and their estimates appear in Table 1.

3.4 Estimation Procedure and Results

The Functional Form: The outcome of section 3.1 is that the hedonic binary quality price function (BQPF) to be estimated is $P(Z_1,...,Z_n, K)$. The dependent variable is the price of the Israeli tomatoes, and the independent variables are BQTs, Z_i, and the Canary Island tomatoes' maximum price, K.

In order to test the convexity of the BQPF, it is assumed that the function is quadratic in quality traits, and seven measured BQTs are applied: percentages of firm (F), discolored (D), small size (S), medium size (M), pink (P), and red (R) tomatoes. In addition, the price of the Canary Island tomatoes (K) has been included as a regressor, including its interactions with the quality traits and its square term. The results appear in Table 2.

The results of Table 2 were used to test statistically the convexity of the BQPF with respect to the quality traits only (but not the price of the Canary Island tomatoes). Namely, these results were used to test whether the matrix of second derivatives with respect to the quality traits is negative semi-definite or not. The values of all interaction parameters (such as AD, etc.) compose the off diagonal elements of this matrix, and the diagonal terms are twice the parameters of the square terms (FF, RR, etc.).

The Wald test for non-linear statistical hypotheses [Judge et. al. (1985 pp. 215-16)] has been used for every principal minor of the matrix of second partial derivatives of the BQPF[2]. The results of the Wald test show that all minors of order two or higher are not significantly different from zero, and in Table 2 it can be seen directly that one of the square terms (DD) is positive and significantly different from zero, which implies strict convexity. Thus, it can be concluded from these tests that the hypothesis of convexity is accepted. However, the convexity of the function is rather weak.

For the determination of quality-differentiated tomato transfer prices, the smallest number of variables to represent the quality of the fruit as it affects price was sought, in order to save on quality tests costs. It was also assumed that the BQPF was linear in K, an hypothesis confirmed by the previous analysis of quadratic function. The Canary Island price variable was allowed to affect the intersect and the coefficients (slopes) of all quality variables. The assumption of linearity of the BQPF in the random variable K is also convenient, since it means that the expected BQPF is simply the estimated function, with the expected price of the Canary Island tomatoes substituted for K.

In the estimation aimed at finding a basis for quality pricing a two stage stepwise procedure was used: beginning with a third degree polynomial, and in the first stage at each step BQT was selected to be added to the already existing BQT's, or raised the degree of the polynomial. Each BQT was represented by all its possible combinations (for the given degree) with the BQTs already included in the model. For example, if trait A is the only trait currently in the model of a third degree polynomial, and trait B is the new one considered for adding, then the following combinations are considered for adding simultaneously: B, B^2, B^3, B^2A, BA^2. As shown in the example, each trait was introduced along with its interaction with the Canary Island tomatoe price K, and the trait's contribution to the F-statistic was then calculated. This procedure was repeated for all possible BQTs. An alternative to adding a new trait was to raise the polynomial degree. Thus, if the current polynomial is of third degree and A and B are traits already in the model, then the following combinations of variables are considered to be added simultaneously, A^4, B^4, AB^3, A^2B^2, A^3B, as well as the interaction terms with the Canary Island price (K). Out of all these possible alternatives, the one selected is the one with the highest F-statistic.

In the second stage, out of the full new set of combinations that represent the new trait thus chosen, selection was again by stepwise regression procedure, for the combinations that contributed significantly to the model. This two stage procedure was then repeated until a new set of combinations representing a new trait or degree with significant contribution could not be found.

The reason for using this procedure, as noted before, was to minimize the number of traits in the model and to find the most important ones. This would later lead to a reduction in the cost of quality testing in the packing house, requiring the measurement of a smaller number of traits. It is quite possible that a different function, using different traits, could be found as well, but that would be the case in general, where one must choose between many correlated variables to be included in a model.

Empirical estimation of the BQPF for the determination of quality differentiated transfer prices to the Israeli export tomato growers to the German market (March - May, 1979), yields the results presented in Table 3 and Figure 2. From Table 3, the matrix of second partial derivatives with respect to quality traits has been evaluated at the average sample values and found to be positive semi-definite, and strictly positive definite when the small size trait (S) was omitted. These results again corroborate the theoretical arguments concerning the shape of the BQPF. Actually, the degree of convexity is underestimated in our empirical results, since each observation represents an average of all daily sales of a wholesaler. Thus the estimated BQPF will be closer to linear than the true BQPF. To see this, consider the following argument: Suppose an observation represents the average of two sales of different quality levels; then the average price of this observation will be on a straight line connecting the two prices of the respective sales on the BQPF, which by the definition of convexity, is above the BQPF at this average quality level. Thus, the function estimated from such points will be closer to linear than the true one is, as argued above. If the BQTs of each transaction differ considerably from one another, the linearizing effect is greater. Because for some traits the fruit were sorted into extreme values (e.g. by size and color), the linearizing effect was more pronounced in those traits.

From Table 3 this model explains 95% of the price variation (R^2=0.95). The high R^2 value is partially due to the fact that the Canary Island price variable, K, captured the random effects uncorrelated with quality. The value of the Durbin-Watson statistic (2.04) means that autocorrelation was completely eliminated. In Figure 2 it is shown that in the domain of the average plus-minus two standard deviations, the function is convex, and behaves according to the theory. Firmness affects the price positively, and discoloration negatively, as expected (100 minus discoloration has a positive slope). Also, the percentage of small tomatoes affects the price negatively, which is confirmed by field operators.

The first partial derivatives of the BQPF with respect to the quality characteristics have been calculated at their average values from Table 3. These derivatives are (.032, .038, -.001) for (F, D, S) respectively. Accepting the hedonic price theory, these values are the implicit prices of the quality characteristics. For example, an increase of 1% in firm tomatoes raises the price of tomatoes by .032 GM/kg, while a reduction of 1% in discolored tomatoes (D = 100 - discolored) raises the tomato price by .037 GM/kg. The values are calculated at the average sample values, but can be calculated from the same function separately for every possible quality level.

Appendix

In this Appendix we present an autoregressive model of first order (AR1) of time series data with time gaps between successive observations which has been used in this study.

Assume the following model:

$$Y = Xb + U$$

$$U_1 = e_1$$

$$\text{(A1)} \quad U_t = \rho U_{t-1} + e_t, \; t=2,\ldots,T$$

$$e \sim N(0, \sigma^2 \ell)$$

where Y, U, and e, are each T*1 vectors, U_{t-1} and e_t are uncorrelated, b is an J*1 vector of unknown parameters, X is a T*J matrix independent of U, possibly of fixed numbers, ρ is an unknown parameter and ℓ is the identity matrix.

Suppose two consecutive observations are separated by varying time intervals (say days). Let i denote the observation index, then t(i) denotes the number of days from the first to the i^{th} observation [t(1)=1], and D^i=t(i) - t(i-1) denotes the number of time intervals between two consecutive observations in the sample. From equation (A1) it follows that:

$$\text{(A2)} \quad U_{t(i-1)} = \rho^{D^i} U_{t(i-1)} + \sum_{m=0}^{D^i-1} \rho^m e_{t(i-1)-m} = \rho^{D^{i,w}} U_w + \sum_{m=0}^{D^{i,w}-1} \rho^m e_{t(i)-m}$$

for i = 2,...,I, and i > w. For all i > w, $D^{w,i} = D^{i,w} = \sum_{m=w+1}^{i} D^m$ denotes the number of days between the w^{th} and the i^{th} observations. Heretofore, we denote $U_i = U_{t(i)}$, and thus $U_{t(i)-D^i} = U_{t(i-1)} = U_{i-1}$. Given $-1 < \rho < 1$, it follows from (A1) and (A2) that:

$$\text{(A3)} \quad Var(U_i) = \sigma^2/(1-\rho^2)$$

is constant, and:

$$\text{(A4)} \quad E\, UU^T = \phi = \varphi\, \sigma^2/(1-\rho^2)$$

where:

$$\text{(A5)} \quad \varphi = \begin{bmatrix} 1 & ,\rho^{D^{1,2}} & ,\rho^{D^{1,3}} & ,\ldots\ldots\ldots\ldots, & \rho^{D^{1,I}} \\ \rho^{D^{2,1}} & , 1 & ,\rho^{D^{2,3}} & ,\ldots\ldots\ldots\ldots, & \rho^{D^{2,I}} \\ . & & & & . \\ \rho^{D^{I,1}} & ,\rho^{D^{I,2}} & ,\rho^{D^{I,3}} & \ldots\ldots\ldots\ldots\ldots, & 1 \end{bmatrix}$$

Define:

$$\text{(A6)} \quad S_i = (1-\rho^{2D^i})/(1-\rho^2)$$

(A7) $S_{i,w} = (1 - \rho^{2(D^i+D^w)})/(1-\rho^2) = S_w + \rho^{2D^w} S_i$

then, by direct multiplication ($\varphi\ \varphi^{-1} = I$) it can be shown that:

(A8)

$$\varphi^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} 1/S_2\ , & -\rho^{D^2}/S_2\ , & 0\ , & ,..0, & 0\ , & 0 \\ -\rho^{D^2}/S_2, & S_{2,3}/S_2S_3, & -\rho^{D^3}/S_3\ , & 0\ ,..0, & 0\ , & 0 \\ 0\ -\rho^{D^3}/S_3 & , S_{3,4}/S_3S_4, & -\rho^{D^4}/S_4, & ..0, & 0\ , & 0 \\ . & & & & & . \\ . & & & & & . \\ 0\ , & 0 & , \ldots\ldots\ldots\ldots\ldots & 0, & -\rho^{D^I}/S_I, & 1/S_I \end{bmatrix}$$

Define:

(A9)

0

$$P = \frac{1}{\sqrt{(1-\rho^2)}} \begin{bmatrix} \sqrt{(-\rho^2)}\ , & 0\ , & 0\ , & \ldots\ldots\ldots\ldots & , & & 0 \\ -\rho^{D^2}/\sqrt{S_2}\ , & 1/\sqrt{S_2}\ , & 0 & , 0\ ,\ldots, & 0\ , & & 0 \\ 0 & , -\rho^{D_3}/\sqrt{S_3}, & 1/\sqrt{S_3}\ , & 0\ ,\ldots, 0\ , & 0\ , & & 0 \\ . & & & & & & . \\ . & & & & & & . \\ 0\ , & 0\ , & 0\ , & 0\ ,\ldots, 0 & ,{}^{D}-p\ I/\sqrt{S_I}, & & 1/\sqrt{S_I} \end{bmatrix}$$

This matrix satisfied $P^TP = \varphi^{-1}$, which can be verified by direct multiplication.

Given the above assumptions in (A1) of autocorrelated disturbances with missing information, and if ρ is known, the best linear unbiased estimator (BLUE) for the vector b is given by:

(A10) $\tilde{b} = (X^TP^TPX)^{-1}X^TP^TPy$,
where P is defined in equation (A9).

The proof is immediate since the matrix P is defined in (A9) so that $P^TP=\varphi^{-1}$. Therefore, the estimator defined in (A10) is the generalized least square estimator which is BLUE (Judge et al., p. 115).

For the actual computations one needs to compute $y^*=Py$ and $X^*=PX$, so that the GLS estimator of b will be the ordinary least square obtained from the transformed variables. This transformation is given by:

$$y_i^* = (y_i - \rho^{D^i} y_{i-1})/\sqrt{S_i}$$

$$X_{ij}^* = (X_{ij} - \rho^{D^i} X_{i-1,j})/\sqrt{S_i} \; ; \quad j=1,\dots K; \quad i=2,\dots,I$$

$$y_1^* = y_1\sqrt{(1-\rho^2)}, \text{ and } X_1^* = X_1\sqrt{(1-\rho^2)}$$

Estimation procedure

When ρ is unknown (as is usually the case) one can not obtain a BLUE estimator, and a non linear least square (NLS) methods has to be used to estimate ρ and b. The maximum likelihood estimators (MLE) has been used, (see e.g. Hildreth and Dent (1974)). Employing scanning procedures, for grid points of ρ we have estimated b, where the criterion of maximum likelihood determined the optimal value of ρ. For the general multivariate normal distribution, the likelihood function is (e.g. Judge et. al. 1980):

$$\text{(A12)} \quad L(b,\sigma^2,\rho \mid y) = 2\pi^{-I/2} \; |\phi|^{-1/2} \exp[(y-Xb)^T\phi^{-1}(y-Xb)]$$

where I is the number of observations, and ϕ is defined in (A4).

For our case of missing observations, using (A4) through (A8), it follows that:

$$\text{(A13)} \quad |\varphi|^{-1} = |\varphi^{-1}| = (1-\rho^2)^{1-I} \Big/ \prod_{i=1}^{I} S_i$$

then, by (A4):

$$\text{(A14)} \quad |\phi|^{-1/2} = \sigma^{-I} \; (1-\rho^2) \Big/ \prod_{i=1}^{I} S_i$$

Since $P = \varphi^{-1/2}$, using the notation $X^* = PX$, $y^* = Py$, and $U^* = y^*-X^*b$, the log of the likelihood function can be written (ignoring constants):

$$\text{(A15)} \quad \ln(L) = -I.\ln(U_T^* .U^* /I) + \ln(1-p^2) - \sum_{i=1}^{I} \ln(S_i),$$

where Si is defined in equation (A7). To estimate ρ and b, search methods suggested by Hildreth and Dent (1974) have been applied, where the b's have been estimated for grid values of ρ, and the maximum likelihood criterion has been used to select the estimate of ρ and the vector b.

Footnotes

1. The difference between the two lines 'Peretz' and 'Aviv' was entirely due to the growing technique. The 'Peretz' line, grown under special stress conditions, was obviously superior and obtained higher prices. Under the regime of quality pricing in subsequent years, Peretz growing techniques were fully or partially adopted by all growers.

2. The Wald test statistic λ_W that we have used is defined by:

$$\lambda_W = \quad I.q(b)'.\{(\partial q/\partial \beta|\beta) \tilde{\Sigma} (\partial q/\partial \beta|\beta)'\}^{-1} q(b)$$

where: I = number of observations

$q(b)$ = non-linear vector function, which in our case is a principal minor of the matrix of second derivatives of the BQPF, and $Q(b)'$ is the transpose of $q(b)$.

$\tilde{\Sigma}$ = the estimates of the variance-covariance matrix of the parameters.

For a function to be weakly convex, every principal minor of the matrix of its second derivatives should be non negative. Thus, the statistic λ_W that was calculated for every such principal minor, is an appropriate statistical test for convexity of the BQPF.

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Table 1. Estimated Serial Correlation Coefficients for the Regions

Region	1	2	3	4	5	6	7	8	9
Rho	.99	.95	.99	.84	.11	.72	.79	.97	.83

Table 2. Estimation of a Quadratic BQPF

Variable[a]	Parameter estimate	t for HO: parameter=0[b]
Intercept	-0.01193614	-0.666
Firm (F)	-0.02133779	-1.503
Discolor (D)	-0.11041645	-2.827*
Small (S)	0.05265600	3.086*
Pink (P)	0.00537223	0.204
Red (R)	0.33247497	2.419*
Kanx (K)	2.26479451	3.211*
Medium (M)	0.07639221	4.198*
FF	0.00002727	0.494
FD	0.00029227	2.035*
FS	-0.00011066	-1.576
FP	0.00005703	0.663
FM	-0.00016128	-2.152*
FR	-0.00036271	-0.522
FK	0.00155851	0.532
DD	0.00095310	3.309*
DS	-0.00014003	-0.670
DP	-0.00016481	-0.877
DM	-0.00017762	-0.801
DR	-0.00102246	-0.772
DK	-0.00691169	-1.029
SS	-0.00003113	-0.727
SP	-.00000804	-0.045
SQ	-0.00005958	-0.792
SR	-0.00162045	-3.249*
SK	-0.00763440	-1.706
PP	0.00001662	0.201
PM	-0.00006475	-0.339
PR	-0.00129673	-1.529
PK	0.00403533	0.909
MM	-0.00002574	-0.466
MR	-0.00152792	-3.018*
MK	-0.01152729	-2.482*
RR	0.00197585	0.561
RK	0.00442226	0.163
KK	-0.16678517	-1.588

R-square = .9378; Durbin-Watson Statistic = 2.127; Number of observations = 509.

a. F = percentage of firm tomatoes.
   D = 100 - percentage of discolored tomatoes.
   S = percentage of small tomatoes.
   P = percentage of pink tomatoes.
   R = percentage of red tomatoes.
   K = price of Canary Island tomatoes
   M = percentage of medium size tomatoes.
   FF = F squared
   FD = F times D, etc.

b. An asterisk above a t-value means that the coefficient is significantly different from zero at 95% confidence level.

Table 3. Estimation of the Hedonic Price Function

Variable[a]	Parameter estimate	t for HO: parameter=0[b]
F^2	$.177*10^{-3}$	1.3
FS	$-.403*10^{-3}$	4.0*
F^3	$.282*10^{-6}$	0.3
D	.0484	4.3*
D^3	$-.255*10^{-5}$	-2.13*
S	.0155	2.5*
K	2.419	7.9*
KF^2	$-.792*10^{-5}$	-.2
KFS	$-.604*10^{-4}$	-1.73
KF^2S	$-.137*10^{-6}$	-.5
KFDS	$.235*10^{-5}$	3.35*
KF^2D	$-.524*10^{-6}$	-1.15
KD	-.04405	-7.16*
KD^3	$.243*10^{-5}$	5.78*
KD^2S	$-.749*10^{-6}$	-2.66*
Intercept	-.0307	-1.91

R-squared = 0.948; Durbin-Watson Statistic - 2.04; Number of observations = 509;

a. F = percentage of firm tomatoes (average value in the sample = 70.35).
   D = 100 - percentage of discolored tomatoes (average value in the sample = 86.39).
   S = percentage of small tomatoes (average value in the sample = 71.13).
   K = price of Canary Island tomatoes (average value in the sample = 3.13).
   FS = F times S
   KFS = K times F times S, etc.

b. An asterisk above a t-value means that the coefficient is significantly different from zero at 95% confidence level.

FIGURE 1.

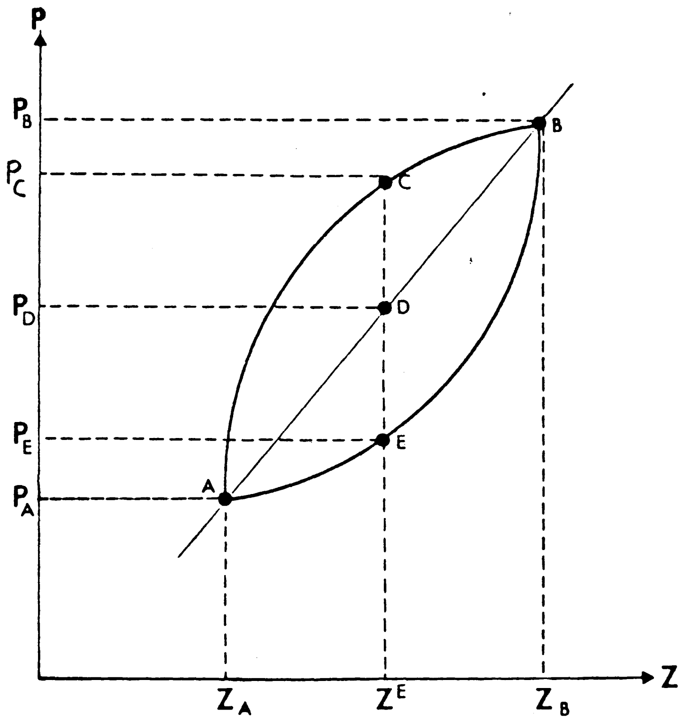

FIGURE 2.