Passive Force Closure and its Computation in Compliant-Rigid Grasps

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Abstract *The classical notion of* force closure *was originally formulated for multi-fingered robot hands, where the grasping fingers* actively *apply any desired force consistent with friction constraints at the contacts. Active force closure requires sophisticated contact-force sensors and agile feedback controllers for its implementation. This paper considers a simpler notion of* passive force closure *recently introduced by Yoshikawa [17]. In passive force closure the fingers apply initial grasping forces, and the grasped object is passively stabilized against external disturbances by the frictional contacts. After motivating the usefulness of passive force closure, we characterize the conditions for its existence. Then we introduce the* passive stability set*, defined as the collection of external wrenches that can be passively resisted by a given grasp. We introduce a class of grasp arrangements where the grasping mechanism is compliant while the grasped object is rigid. Such compliant-rigid systems are common, and for these systems the passive closure set can be computed in closed form. Simulation results demonstrate the computation of the passive closure set for two and three-finger planar grasps.*

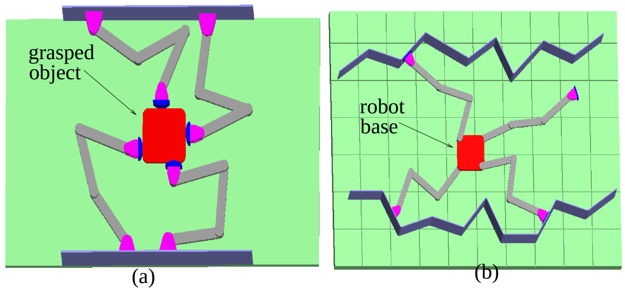

Figure 1: (a) A multi-fingered hand grasping an object. (b) A multi-limbed robot bracing against a tunnel environment.

1 Introduction

The classical notion of *force closure* was originally defined for multi-fingered robot hands [4, 14]. This notion should be called *active* force closure, since it requires that the fingers be able to actively balance any disturbing wrench (i.e. force and torque) acting on the grasped object. Active force closure requires sophisticated contact-force sensors and agile feedback controllers for its implementation, bringing the cost of such systems to prohibitive levels. However, in applications such as fixturing the grasping elements are simple devices that are preloaded against an object with initial grasping forces [8]. Physical processes at the contacts, such as friction and compliance, provide passive stabilization of the object against external disturbances. Another important application concerns articulated mechanisms that establish an initial grasp of an object using simple position-based controllers. In this case the effective compliance of the grasping mechanism together with friction at the contacts provide passive stabilization of the grasped object (Figure 1(a)). Another application is a multi-limbed robot bracing against a tunnel-like environment in static equilibrium (Figure 1(b)). Here the tunnel walls play the role of the grasped object, and the robot passively stabilizes itself by pushing against the walls using position-based controllers. In all of these examples stabilization is achieved by passive means, without active control of the contact forces.

The notion of passive force closure recently introduced by Yoshikawa [17] provides a framework for investigating passively stable grasps. By definition, a grasp is *passive force closure* if for suitably selected initial grasping forces, the contacts can passively balance any external wrench in a neighborhood about the origin. (A formal definition appears below.) This definition includes the fixturing and position-servoed grasps mentioned above. The definition also applies to objects held by multiple contacts against gravity. Such arrangements were originally considered by Reuleaux [13], and in the context of robotic manipulation by [1, 11, 15]. In this paper we consider passive stabilization by a combination of friction and compliance effects, and regard gravity as an external disturbing force. It should be emphasized at this stage that *active force closure is necessary but not sufficient for passive force closure.* The literature on active force closure is therefore only partially useful for establishing passive force closure grasps[1]. Examples of works on friction-based active force closure are

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[1] *Form closure* is another means for establishing stable grasps [16]. Form closure can be interpreted as passive force closure specialized to frictionless contacts.

[7, 12, 16], and examples of works that additionally consider the structure of the grasping mechanism are [2, 3, 6, 9].

In contrast with active force closure, a systematic study of passive force closure has only recently begun [17]. A fundamental challenge in the study of passive force closure is to characterize the conditions for its existence. Another challenge is to characterize the set of external wrenches that can be passively resisted by a given grasp. This set, called the *passive closure set*, depends on the grasp geometry, the amount of friction at the contacts, the kinematics and dynamics of the grasping mechanism, as well as the forces that establish the initial grasp. This paper describes a technique for computing the passive closure set under certain assumptions. The paper begins with a definition and characterization of *active* force closure. Then *passive* force closure is defined, and necessary and sufficient conditions for its existence are considered. Next the problem of computing the passive closure set is described. This computation involves a recursive relation which is connected to the static indeterminacy of multiply-contacted objects. The paper considers a class of grasp arrangements where a compliant mechanism holds a rigid object. Such compliant-rigid systems arise in multi-fingered hands and multi-limbed robots that interact with rigid objects using simple position-based controllers. We show that for compliant-rigid grasps the passive closure set can be computed in closed form. Then we demonstrate the computation of the passive closure set for two and three-fingered planar grasps. Experimental results are currently being collected, and these results will be reported in the final version of this paper.

2 Definition of Passive Force Closure

In this section we introduce terminology for frictional grasps and review the notion of active force closure. Then we define passive force closure and describe necessary and sufficient conditions for its existence.

2.1 Frictional Grasps Terminology

We study 2D or 3D grasps, where a rigid object $\mathcal{B}$ is held in frictional point contact by k rigid bodies $\mathcal{A}_1,\ldots,\mathcal{A}_k$. The bodies $\mathcal{A}_1,\ldots,\mathcal{A}_k$ represent fixturing elements or the fingertips of a multi-fingered hand. Although we use the language of grasping, these bodies can also represent the footpads of a multi-limbed robot. The contact point between $\mathcal{A}_i$ and $\mathcal{B}$ is denoted r_i when expressed in $\mathcal{B}$'s body frame, and x_i when expressed in a fixed world frame (Figure 2). The two representations of the i^{th} contact point are related by the rigid-body transformation: $x_i = X\big(r_i,(d,R)\big) \triangleq Rr_i + d$, where d and R are the position and orientation of $\mathcal{B}$ with respect to the world frame. The orientation matrices R are parametrized by the exponential map, $R(\theta) = \exp(\theta)$, where $\theta \in I\!R$ in 2D and $\theta \in I\!R^3$ in 3D. The object configuration is parametrized by $q = (d,\theta) \in I\!R^m$, where $m = 3$ in 2D and $m = 6$ in 3D. The rigid-body transformation is consequently written as $X(r_i,q)$. When a force F_i acts on $\mathcal{B}$ at the i^{th} contact, it gives rise to a wrench (force and torque) denoted $\boldsymbol{w}_i$. The wrench $\boldsymbol{w}_i$ is determined by the formula: $\boldsymbol{w}_i = (DX_{r_i})^T F_i$, where DX_{r_i} is the derivative of $X(r_i,q)$ with respect to q, such that r_i is held fixed. Evaluation of this derivative gives the familiar wrench formula:

$$\boldsymbol{w}_i = \begin{pmatrix} F_i \\ \boldsymbol{\rho}_i \times F_i \end{pmatrix} \quad \text{where } \boldsymbol{\rho}_i = R(\theta)r_i.$$

In this formula $\boldsymbol{\rho}_i$ is a vector from $\mathcal{B}$'s origin to the point x_i, described in world coordinates. The torque in this formula, $\boldsymbol{\rho}_i \times F_i$, becomes in 2D a scalar times a vector perpendicular to the 2D plane. The collection of wrenches that act on $\mathcal{B}$ at a particular configuration is called *wrench space.* This space can be identified with $I\!R^m$.

We assume the standard Coulomb friction model: $|F_i^t| \le \mu |F_i^n|$, where F_i^t and F_i^n are the tangent and inward normal components of F_i, and μ is the coefficient of Coulomb friction[2]. The force F_i can only push on the object, and this constraint is described by the inequality $F_i^n \ge 0$. The friction cone at the i^{th} contact, denoted FC_i, is the collection of all frictional forces that can be applied to $\mathcal{B}$ at x_i, and it is given by

$$FC_i = \left\{F_i\colon\ F_i^n \ge 0 \text{ and } -\mu F_i^n \le F_i^t \le \mu F_i^n \right\}.$$

The set of wrenches generated by all forces in FC_i forms a cone in wrench space. The i^{th} wrench cone is denoted $\mathcal{W}_i$ and is given by

$$\mathcal{W}_i = \{\boldsymbol{w}_i\colon\ \boldsymbol{w}_i = [DX_{r_i}]^T F_i,\ \forall F_i \in FC_i\}.$$

When $\mathcal{B}$ is held by k fingers, we say that $\mathcal{B}$ is in *frictional equilibrium* if in the absence of any external wrench there exist wrenches $\boldsymbol{w}_i \in \mathcal{W}_i$ for $i = 1,\ldots,k$ such that $\sum_{i=1}^k \boldsymbol{w}_i = \vec{0}$.

2.2 Active Force Closure

Active force closure is the standard notion of force closure [12, 16]. The collection of wrenches that can

[2] In 3D there is also frictional torque about the contact normal. However, the generation of this torque requires surface contact rather than point contact between $\mathcal{A}_i$ and $\mathcal{B}$.

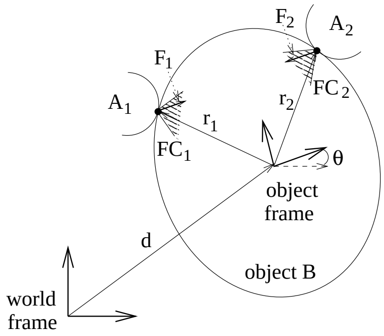

Figure 2: Basic notation for frictional grasps.

be generated by k frictional contacts is given by the set sum: $\mathcal{W}_1 + \cdots + \mathcal{W}_k = \{\boldsymbol{w}_1 + \cdots + \boldsymbol{w}_k : \boldsymbol{w}_i \in \mathcal{W}_i$ for $i = 1, ..., k\}$. This notation is used in the following definition.

Definition 1. *Let an object $\mathcal{B}$ be held in equilibrium grasp by k frictional point contacts. Let $\mathcal{W}_i$ be the wrench cone of the i^{th} contact. Then the grasp is* **active force closure** *if the sum of the wrench cones $\mathcal{W}_1 + \cdots + \mathcal{W}_k$ spans the entire wrench space $I\!R^m$, where $m=3$ in 2D and $m=6$ in 3D.*

The active aspect of the grasp lies in the assumption that the grasping bodies can generate any contact force within the respective friction cones. In practice, even a fully active grasping system can generate only bounded contact forces. However, in contrast with passive closure discussed below, knowledge of the active contact force bounds allows an immediate characterization of the external wrenches that can be actively resisted by the grasp. Next we give a simple rule for determining active force closure. In the following, a grasp is *non-marginal* when the contact forces are non-zero and lie in the interior of their respective friction cones.

Theorem 1 (Active force closure). *Let a 2D or 3D object $\mathcal{B}$ be grasped by k frictional contacts, such that the contacts do not lie along the same spatial line when the grasp is 3D. Then the grasp is* **active force closure** *iff it is possible to establish a non-marginal equilibrium grasp of $\mathcal{B}$.*

This result appears for 2D grasps in Ref. [12].

Proof: The *grasp matrix* $G = [DX_{r_1}^T \cdots DX_{r_k}^T]$ maps the contact forces F_1,...,F_k to the net wrench acting on $\mathcal{B}$. This matrix is $3 \times 2k$ for 2D grasps and $6 \times 3k$ for 3D grasps. According to [10, Proposition 5.2], the following two conditions are necessary and sufficient for active force closure. The first is the existence of a non-marginal equilibrium grasp as stated in the proposition. The second is that G be full rank. To show the latter condition, let (v, ω) denote the instantaneous velocity of $\mathcal{B}$, where v and ω are $\mathcal{B}$'s linear and angular velocities. (In the 2D case ω is an instantaneous rotation of $\mathcal{B}$ about an axis perpendicular to the 2D plane.) Then the derivative DX_{r_i} satisfies $\frac{d}{dt}X_{r_i}(d(t), R(t)) = DX_{r_i}\begin{pmatrix} v \\ \omega \end{pmatrix} = v+\omega\times\boldsymbol{\rho}_i$. If the matrix G is not full rank, there must exist a non-zero velocity (v, w) satisfying:

$$G^T \begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v + \omega \times \boldsymbol{\rho}_1 \\ \cdots \\ v + \omega \times \boldsymbol{\rho}_k \end{pmatrix} = \begin{pmatrix} 0 \\ \cdots \\ 0 \end{pmatrix}. \quad (1)$$

In this equation $\omega \neq 0$, otherwise (1) would imply that $v = 0$. By moving v to the right in each row of (1), we obtain that $\omega \times \boldsymbol{\rho}_1 = \cdots = \omega \times \boldsymbol{\rho}_k$. It follows that $\omega \times (\boldsymbol{\rho}_i - \boldsymbol{\rho}_j) = 0$ for $1 \leq i, j \leq k$ such that $i \neq j$. In the 2D case the vectors $\boldsymbol{\rho}_i - \boldsymbol{\rho}_j$ lie in the 2D plane while ω is perpendicular to this plane. Hence $\omega \times (\boldsymbol{\rho}_i - \boldsymbol{\rho}_j) \neq 0$ and G is full rank. In the 3D case the vanishing of the cross-products $\omega \times (\boldsymbol{\rho}_i - \boldsymbol{\rho}_j)$ implies that the vectors $\boldsymbol{\rho}_1$,...,$\boldsymbol{\rho}_k$ are all parallel to ω. However, each $\boldsymbol{\rho}_i$ is a vector from $\mathcal{B}$'s origin to x_i. Hence if there are $k > 2$ contacts, these contacts must lie along the same spatial line, in contradiction with the proposition's hypothesis. Hence G is full rank in the 3D case too. □

Two comments are in order here. First, the theorem asserts that a 2-fingered frictional grasp of a 3D object is not active force closure. Indeed, the fingers cannot generate torque about the line connecting the two contacts. To achieve force closure in this case, the fingers must establish surface contact with the object, so that frictional torques about the contact normals can be generated. Second, the proof is based on the fact that the velocities $\frac{d}{dt}X_{r_i}$ cannot simultaneously vanish at the k contacts. The condition $\frac{d}{dt}X_{r_i} = 0$ means that $\mathcal{B}$ instantaneously rolls about the stationary point x_i. Indeed, a 2D object cannot simultaneously roll about two or more stationary points, while a 3D object can execute such a simultaneous rolling only when the contact points are arranged along a common line.

2.3 Passive Force Closure

Active force closure is determined solely by the arrangement of the frictional contacts on the boundary of the grasped object $\mathcal{B}$. The notion of passive force closure additionally depends on the grasping mechanism. To formalize this notion, we define three types of contacts that encapsulate three common types of restraining mechanisms.

Definition 2. *A* **passive rigid-body contact** *is a stationary rigid body that passively interacts with*

$\mathcal{B}$ *through a frictional contact. An* **active fixed-force contact** *is a frictional point contact that applies a specific force at the contact point. An* **active-compliance contact** *is a frictional contact that applies force according to a specific force-displacement relationship of the contact point.*

Let us give examples of these types of contacts. Passive rigid-body contacts are commonly used in fixturing applications to restrict the motions of a workpiece. Note that a passive rigid-body contact generates force only when $\mathcal{B}$ presses against it. Active fixed-force contacts are generated by mechanisms such as force-screw fixturing elements and force-controlled robot grippers. Active compliance contacts are generated by finger and limb mechanisms whose joints are controlled by position-servoed controllers. While the above three types of contacts are the most common, another important type of contact arise naturally in high-load fixturing applications. In these applications small elastic deformations of the contacting bodies change the idealized passive rigid-body contacts into passive compliant contacts. However, we do not consider such high-load applications in this paper. Another type of contact occurs when several active contacts are coupled together by the grasping mechanism. In order to avoid such coupled contacts, we assume that *each contact is generated by its own independent mechanism.* We are now ready to define passive force closure.

Definition 3. *Let an object $\mathcal{B}$ be held in equilibrium grasp by k independent frictional contacts of the types defined above. Then the grasp is* **passive force closure** *if any external wrench in a neighborhood about the origin is passively balanced by the contacts.*

Passive grasps can be implemented with controllers that simply maintain fixed joint torques or fixed joint positions, while the balancing of external wrenches is performed automatically by the contacts. Let us now discuss several properties of passive force closure grasps. First *active force closure is necessary for the existence of passive force closure*, since a neighborhood about the origin in wrench-space can be expanded to the entire space by actively increasing the contact-force magnitudes. However, *active force closure does not automatically imply passive force closure.* Figure 3(a) shows a 2-fingered frictional grasp of a rectangular object. If the two contacts are fully active the grasp is active force closure according to Theorem 1. But when the two contacts are fixed-force contacts the grasp is not passive force closure, since the contacts cannot generate a net horizontal force on $\mathcal{B}$. On the other hand, if one contact is passive while the other applies a fixed force, the grasp becomes passive force closure. Another example is the 3-fingered

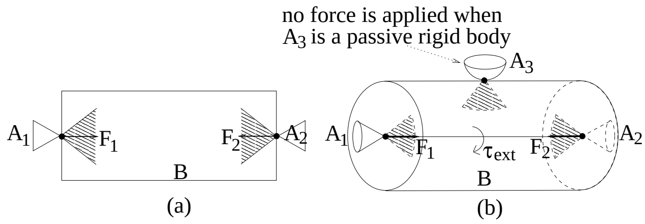

Figure 3: (a) A 2D grasp which is not passive force closure when $\mathcal{A}_1$ and $\mathcal{A}_2$ apply a fixed force. (b) A 3D grasp which is not passive force closure when $\mathcal{A}_1$ and $\mathcal{A}_2$ apply active-compliance forces while $\mathcal{A}_3$ is a passive rigid body.

frictional grasp of cylindrical object depicted in Figure 3(b). If the three contacts are fully active, Theorem 1 implies that the grasp is active force closure. But when $\mathcal{A}_1$ and $\mathcal{A}_2$ apply active-compliance forces while $\mathcal{A}_3$ is a passive rigid body, the grasp is not passive force closure since the contacts cannot generate a torque about the cylinder's axis. On the other hand, if all three fingers apply active-compliance contacts, the grasp is passive force closure.

We now give necessary and sufficient conditions for passive force closure of grasps having fixed-force or active-compliance contacts. The conditions are based on the following notion of potential energy function. The force generated by an active-compliance contact at x_i depends on the position of the contact point. The position of x_i is determined in turn by $\mathcal{B}$'s configuration q. The wrench generated by an active-compliance contact can therefore be written as $\boldsymbol{w}_i = -\nabla U_i(q)$, where $U_i(q)$ is the elastic potential energy function induced on $\mathcal{B}$ by the i^{th} compliant contact[3]. Similarly, the wrench generated by a fixed-force contact is induced by a potential function which is linear in x_i. The total potential energy of $\mathcal{B}$ is the sum $U(q) = \sum_{i=1}^{k} U_i(q)$.

Proposition 2.1. *Let a 2D or 3D object $\mathcal{B}$ be held in equilibrium grasp by k fixed-force or active-compliance contacts, such that the contacts do not lie along a common line when the grasp is 3D. Let q_0 be the object equilibrium configuration, and let $U(q)$ be the potential energy induced on $\mathcal{B}$ by the contacts. Then the following two conditions are sufficient for* **passive force closure***:*

1. *The initial equilibrium grasp is non-marginal.*
2. *The equilibrium q_0 is a non-degenerate local minimum of the elastic energy function $U(q)$ (i.e. $D^2U(q_0) > 0$).*

Moreover, in all generic grasps conditions (1) and (2) are necessary for passive force closure.

[3]The potential function $U_i(q)$ is identically zero when the i^{th} contact is broken.

The first condition of the proposition simply states the necessary condition that the grasp must be active force closure if the contacts are made fully active. The second condition is a key to understanding the difference between active and passive closure. This condition ensures that when an external wrench acts on $\mathcal{B}$, the object would automatically settle at a new configuration in the vicinity of q_0 where the contact forces balance the external wrench.

The proposition admits two generalizations. First, the proposition also applies to *frictionless* passive-rigid-body contacts. (In this case the object and finger-tips must be modeled as quasi-rigid.) Second, the proposition can be generalized to include *frictional* passive-rigid-body contacts, provided that these contacts kinematically constrain the object to move along a submanifold of its configuration space. The two conditions of the proposition apply in that case to the submanifold of allowed motions of $\mathcal{B}$. The characterization of passive force closure under general frictional passive-rigid-body contacts is the topic of current research.

3 Computation of the Passive Stability Set

Given a passive force closure grasp, the *passive stability set* is the collection of external wrenches which are automatically balanced by the contacts of the grasp. In this section we derive a closed-form formula for the passive stability set of compliant-rigid grasps. Before describing this class of grasps, let us depict a fundamental difficulty in computing the passive stability set. The Coulomb friction model allows generation of tangential forces at the contacts up to a limit determined by μ times the normal component of the contact forces. In passive grasps the normal component of the contact forces is determined by the initial grasp, and can change only in response to an external wrench $\boldsymbol{w}_{ext}$ acting on $\mathcal{B}$. In other words, the normal loadings at the contacts cannot "spontaneously" change as they do in fully active contacts. We consequently write the feasible wrench cones as $\mathcal{W}_i(\boldsymbol{w}_{ext})$. An external wrench can be possibly balanced by the contacts only when the recursive relation $\boldsymbol{w}_{ext} \in \mathcal{W}_1(\boldsymbol{w}_{ext}) + \cdots + \mathcal{W}_k(\boldsymbol{w}_{ext})$ holds true. The solution of this recursive relation is a key step in computing the passive stability set.

The *compliant-rigid grasps* are defined as grasps where a rigid object $\mathcal{B}$ is held by compliant finger mechanisms. This class of grasps also includes multi-limbed robots bracing against a rigid environment. The rigidity of $\mathcal{B}$ is an excellent approximation even though all objects exhibit some degree of compliance at the contacts. This natural compliance is negligible relative to the compliance induced by the joints of the grasping mechanism. Consider for example our experimental multi-limbed robot which resembles the one depicted in Figure 1(b). Each limb of this robot has four joints actuated by Maxon motors that generate a stiffness of 2 N/mm at the footpads. In contrast, the stiffness of objects made of Aluminum is $4.5 \cdot 10^3$ N/mm. However, not all grasps fall into the category of compliant-rigid grasps. For example, fixturing arrangements are designed to exhibit high stiffness at the contacts, and cannot be considered as compliant-rigid grasps.

We make the following simplifying assumptions. First, the finger mechanisms are assumed to interact with $\mathcal{B}$ through sharp or pointed finger-tips. This assumption implies that when a finger-tip rolls on the surface of $\mathcal{B}$, the location of the contact point remains fixed. A second assumption is that each finger generates the linear active-compliance law[4]:

$$F_i = F_i^0 - K_i(x_i - x_i^0), \tag{2}$$

where F_i^0 and x_i^0 are the contact forces and contact points at the initial equilibrium grasp, and K_i is an $n \times n$ positive semi-definite matrix ($n = 2$ in 2D and $n = 3$ in 3D). The fingers should also apply damping forces in order to ensure stability of the grasp. These damping forces are not listed in (2), as these forces vanish in the static analysis performed here.

Our first step is to express the contact forces as a function of the object configuration $q = (d, \theta)$. The i^{th} contact point is given by $x_i = R(\theta)r_i + d$, where r_i is the description of x_i in $\mathcal{B}$'s body coordinates. Let r_i^0 denote the coordinates of r_i at the initial grasp. Let $\mathcal{F}Q$ denote the collection of $\mathcal{B}$'s configurations where the contact forces lie in their respective friction cones. (The set $\mathcal{F}Q$ is considered below.) Then the pointed-finger assumption together with the rigidity of $\mathcal{B}$ guarantee that *the points r_i remain fixed in the object frame,* for all configurations $q \in \mathcal{F}Q$. Thus we may write $x_i = R(\theta)r_i^0 + d$ for $i = 1, ..., k$. Substituting for the x_i's in (2) gives the desired expression for the contact forces:

$$F_i(d, \theta) = F_i^0 - K_i\big((R(\theta)r_i^0 + d) - x_i^0\big) \qquad i = 1, ..., k. \tag{3}$$

Next we write an expression for the set of feasible configurations $\mathcal{F}Q$. This set is given by the intersection $\mathcal{F}Q = \cap_{i=1}^k \mathcal{F}Q_i$, where $\mathcal{F}Q_i$ denotes the collection of $\mathcal{B}$'s configurations where the i^{th} contact force $F_i(q)$ lies in the friction cone FC_i. Let n_i denote the inward normal to the boundary of $\mathcal{B}$ at r_i, written

[4] The analysis generalizes to any compliance law of the form: $F_i = F_i^0 + \boldsymbol{f}_i(x_i)$, such that $\boldsymbol{f}_i$ vanishes when $x_i = x_i^0$.

in $\mathcal{B}$'s body coordinates. And let N_i be the inward unit normal to the boundary of $\mathcal{B}$ at x_i, expressed in world coordinates. Then $N_i = R(\theta)n_i$, and the normal component of the i^{th} contact force is: $F_i^n = F_i \cdot N_i = F_i \cdot (R(\theta)n_i)$. The tangential component of F_i is: $F_i^t = \|[I - N_i N_i^T]F_i\| = \|[I - n_i n_i^T]R(\theta)^T F_i\|$. Substituting for F_i^n and F_i^t in the inequalities that define FC_i gives:

$$\mathcal{F}Q_i = \; \{q = (d,\theta):\; F_i \cdot (R(\theta)n_i) \geq 0 \quad \text{and} \\ \|[I - n_i n_i^T]R(\theta)^T F_i\| \leq \mu F_i \cdot (R(\theta)n_i)\} \,,$$

where μ is the coefficient of friction. Substituting for the forces F_i according to (3) gives:

$$\mathcal{F}Q_i = \{q = (d,\theta):\; F_i(d,\theta) \cdot (R(\theta)n_i) \geq 0 \quad \text{and} \\ \|[I - n_i n_i^T]R(\theta)^T F_i(d,\theta)\| \leq \mu F_i(d,\theta) \cdot (R(\theta)n_i)\}.$$

The desired set $\mathcal{F}Q$ is the intersection of the sets $\mathcal{F}Q_i$. Our final step is to identify the configurations that guarantee stable convergence of $\mathcal{B}$ to the equilibrium induced by an external wrench. This condition is captured by the requirement that the second-derivative matrix of the grasp potential energy function, $D^2U(q)$, be positive definite. A formula for $D^2U(q)$ is listed in the following lemma. A proof of the formula appears in the appendix. Given a vector $u \in I\!\!R^3$, $[u\times]$ denotes the 3×3 skew-symmetric matrix satisfying $[u\times]v = u \times v$ for all $v \in I\!\!R^3$.

Lemma 3.1. *Let a rigid object $\mathcal{B}$ be grasped by k active-compliance contacts each satisfying the linear compliance law (2). Then the formula for $D^2U(q)$ in the 3D case is:*

$$D^2U(q) = \\ \sum_{i=1}^{k} \begin{bmatrix} K_i & K_i[\boldsymbol{\rho}_i\times \\ [\boldsymbol{\rho}_i\times]^T K_i & [\boldsymbol{\rho}_i\times]^T K_i\,[\boldsymbol{\rho}_i\times] + ([F_i\times][\boldsymbol{\rho}_i\times])_s \end{bmatrix} \quad (4)$$

where for a given matrix A, $A_s = \frac{1}{2}(A + A^T)$. The formula for $D^2U(q)$ in the 2D case is:

$$D^2U(q) = \sum_{i=1}^{k} \begin{bmatrix} K_i & K_i J\boldsymbol{\rho}_i \\ (J\boldsymbol{\rho}_i)^T K_i & (J\boldsymbol{\rho}_i)^T K_i J\boldsymbol{\rho}_i - F_i \cdot \boldsymbol{\rho}_i \end{bmatrix}, \quad (5)$$

where $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

The set of configurations that satisfy the stability condition, denoted $\mathcal{P}$, is given by

$$\mathcal{P} = \{q = (d,\theta):\; \lambda_{min}(D^2U(q)) > 0\} \,,$$

where λ_{min} denotes the minimal eigenvalue of a matrix. The net wrench generated on $\mathcal{B}$ by the contact forces is $\boldsymbol{w} = \sum_{i=1}^{k}(F_i, \boldsymbol{\rho}_i \times F_i)$. Since F_i and $\boldsymbol{\rho}_i$ are functions of q, $\boldsymbol{w}$ can be interpreted as a mapping from configuration space to wrench space. The passive stability set, denoted $\mathcal{W}_{passive}$, is the image in wrench-space of the configurations q in $\mathcal{F}Q \cap \mathcal{P}$ under the mapping $\boldsymbol{w}(q)$:

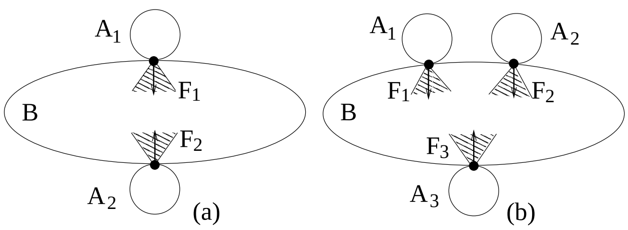

Figure 4: Passive force closure grasps of an elliptical object using (a) tow and (b) three fingers.

$$\mathcal{W}_{passive} = \left\{ \boldsymbol{w} = \sum_{i=1}^{k} \begin{pmatrix} F_i(q) \\ \boldsymbol{\rho}_i(q) \times F_i(q) \end{pmatrix} \;:\; q \in \mathcal{F}Q \cap \mathcal{P} \right\}.$$

Any wrench $\boldsymbol{w}_{ext}$ in $\mathcal{W}_{passive}$ would be automatically balanced by the contacts of the grasp.

4 Simulation Results

In this section we compute the passive stability set of the 2-finger and 3-finger passive force closure grasps depicted in Figure 4. In both grasps the object is an ellipse with major and minor axes whose length is four and two length-units. In both examples the contacts are frictional with a coefficient of friction $\mu = 0.3$. In both examples the fingers apply active-compliance forces, with a stiffness matrix of $K_i = I$. That is, each finger applies a uniform one-unit force per one-unit of deflection of the respective contact point. In the 2-finger grasp the magnitudes of the initial forces are set to $\|F_1^0\| = \|F_2^0\| = 50$ force units. Figure 5(a) shows the collection of feasible configurations $\mathcal{F}Q \cap \mathcal{P}$, where $\mathcal{F}Q = \mathcal{F}Q_1 \cap \mathcal{F}Q_2$. The coordinates in the figure are (d_x, d_y, θ), where (d_x, d_y) are in length-units and θ in radians. Note that the d_y-coordinate of $\mathcal{F}Q \cap \mathcal{P}$ varies in the interval $[-10, 10]$, while the d_x-coordinate of this set varies in the interval $[-20, 20]$. This difference can be explained by the fact the deflection of the ellipse along the y-axis generates pure tangential forces which are bounded by μ times the normal forces generated by deflection of the ellipse along the x-axis. The passive stability set $\mathcal{W}_{passive}$ of the 2-finger grasp is depicted in Figure 5(b). The coordinates in this figure are (F_x, F_y, τ). Note that the asymmetry of $\mathcal{F}Q \cap \mathcal{P}$ now appears as an asymmetry of $\mathcal{W}_{passive}$ along the F_x and F_y axes. Finally, the magnitudes of the initial forces in the 3-finger grasp are set to $\|F_1^0\| = \|F_2^0\| = 25$ and $\|F_3^0\| = 50$ force units. Figure 6(a) depicts the set of feasible configurations $\mathcal{F}Q \cap \mathcal{P}$,

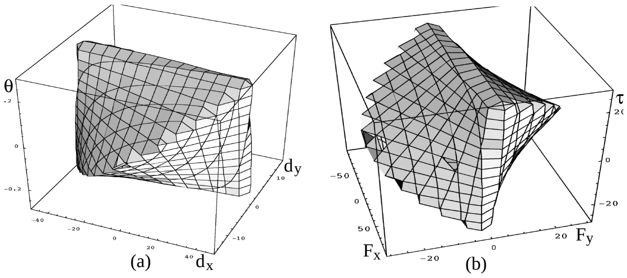

Figure 5: The feasible configurations set and the passive stability set of the 2-finger grasp.

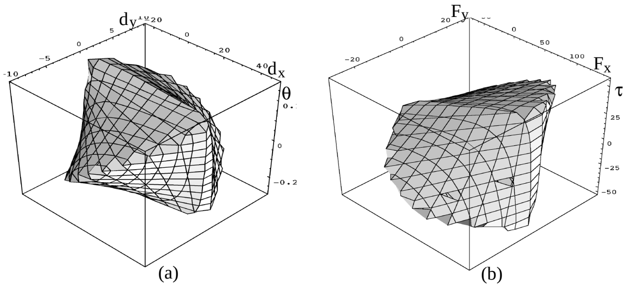

Figure 6: The feasible configurations set and the passive stability set of the 3-finger grasp.

where $\mathcal{F}Q = \mathcal{F}Q_1 \cap \mathcal{F}Q_2 \cap \mathcal{F}Q_3$. Figure 6(b) shows the stability set $\mathcal{W}_{passive}$ for the 3-finger grasp.

5 Conclusion

In active force closure grasps the fingers resist external wrenches by actively applying the required forces at the contacts. Active grasping systems require sophisticated contact-force sensors and agile feedback controllers for their implementation. In passive force closure grasps the contacts establish initial grasping forces, and the balancing of external wrenches is performed automatically by the contacts. Passive grasping systems can be implemented with controllers that simply maintain fixed joint torques or fixed joint positions. Using the framework established by Yoshikawa [17], we defined three types of contacts that commonly occur in passive grasps: active-compliance contacts, fixed-force contacts, and passive-rigid-body contacts. Then we formally defined *passive force closure*, and provided necessary and sufficient conditions for generic passive force closure grasps. These conditions imply that active force closure is necessary but not sufficient for passive force closure. To guarantee passive force closure, the grasped object must automatically converge to a nearby equilibrium where the contact forces balance the external wrench. It should be noted that the necessary and sufficient conditions admit active-compliance and fixed-force contacts, but only special cases of passive-rigid-body contacts.

Next we computed the *passive stability set* of compliant-rigid grasps. In this class of grasps a rigid object $\mathcal{B}$ is held by compliant grasping mechanisms. Compliant-rigid grasps provide an excellent approximation for common scenarios where a multi-fingered hand grasps hard objects, or a multi-limbed robot braces against a rigid environment. Assuming that the mechanisms generate active-compliance forces, we derived a closed-form expression for the passive stability set. A key component in this derivation has been the ability to express the contact forces as a function of the object configuration, thereby avoiding the static indeterminacy problem. Simulation results depict the passive stability set for 2-finger and 3-finger grasps. A thorough investigation of the passive stability set is currently in progress. In particular, we are studying the effect of contact points location, amount of friction, and magnitude of grasping forces on the size and shape of the passive stability set. Finally, we are in the process of constructing an experiment that would allow us to measure the passive stability set and compare this data with our analytical results.

Future extensions of this work will include the following two topics. First, we wish to obtain necessary and sufficient conditions for passive grasps having frictional rigid-body contacts. The main challenge in this work is the need to account for complex phenomena such as micro-slip, wedging, and hysteresis. A second future research topic is the development of planning tools which are based on the passive stability set. These tools would be useful for object manipulation by passive grasps, as well as for locomotion of multi-limbed robots.

A Conditions for Passive Force Closure

Proof sketch of Proposition 2.1: Let $\mathcal{N}$ be a small neighborhood of configurations about q_0. As $\mathcal{B}$'s configuration varies in $\mathcal{N}$, the contact forces vary in a neighborhood about the contact forces of the initial grasp. Since the initial grasp is non-marginal, by a continuity argument all contact forces generated by varying $\mathcal{B}$'s configuration in $\mathcal{N}$ still lie in their respective friction cones. (This statement holds true even when the location of some contact points changes due to local rolling of $\mathcal{B}$.)

Next we establish that any external wrench in a neighborhood about the origin can be balanced by feasible contact forces. When $\mathcal{B}$ is at a configuration $q \in \mathcal{N}$, the net wrench generated by the contacts is given by the negated gradient $-\nabla U(q)$. Consider now the gradient $\nabla U(q)$ as a mapping from configuration space to wrench space. By assumption $\nabla U(q_0) = 0$. According to the Inverse Function Theorem, ∇U maps an open neighborhood about q_0 to an open neighborhood about the zero wrench if the derivative of ∇U at q_0, $D^2U(q_0)$, is non-singular. Since q_0 is a non-degenerate local minimum of U, $D^2U(q_0)$ is non-singular as required.

Finally we establish that $\mathcal{B}$ would automatically settle at a configuration where the contact forces balance the external wrench acting on $\mathcal{B}$. Let $\boldsymbol{w}_{ext}$ denote a fixed external wrench acting on $\mathcal{B}$. The dynamics of $\mathcal{B}$ is governed by the equation: $M(q)\ddot{q} + B(q,\dot{q}) = -\nabla U(q) + \boldsymbol{w}_{ext}$. (The contacts also apply damping forces which we ignore for simplicity.) The external influences on $\mathcal{B}$ can be written as the negated gradient of a composite potential function: $\Phi(q) = U(q) - \boldsymbol{w}_{ext} \cdot q$. A general result concerning the dynamics of mechanical systems states that the flow of a damped mechanical system governed by a potential function Φ is attracted to the local minima of Φ [5]. We have already shown that for every $\boldsymbol{w}_{ext}$ in a neighborhood about the origin there exists a configuration q_1 such that $\nabla\Phi(q_1) = 0$. The equilibrium point q_1 is a stable attractor if it is a local minimum of Φ i.e., if $D^2\Phi(q_1) > 0$. But $D^2\Phi(q) = D^2U(q)$, and the entries of $D^2U(q)$ vary continuously with q. Since the eigenvalues of a matrix vary continuously with its entries, $D^2U(q)$ remains positive definite in a neighborhood of q_0. By shrinking $\mathcal{N}$ if necessary, we conclude that q_1 is a local minimum of Φ, and $\mathcal{B}$ would automatically settle at q_1.

The necessity of condition (2) is based on the following argument. Suppose that $\mathcal{B}$ settles at the equilibrium q_1 induced by $\boldsymbol{w}_{ext}$. If the equilibrium point q_1 is not a local minimum of Φ, then it must be either a degenerate local minimum or a saddle of Φ. A degenerate local minimum can occur only in non-generic grasps. A saddle is an *unstable* equilibrium that generically attracts only a thin set of initial points. If the equilibrium at q_1 is a saddle, the trajectory of $\mathcal{B}$ that starts at q_0 would generically diverge away from the saddle. □

B Computation of $D^2U(q)$

In this appendix we compute the two formulas for $D^2U(q)$ which appear in Lemma 3.1. To begin with, $U(q) = \sum_{i=1}^k U_i(q)$ where $U_i(q)$ is the elastic energy induced by the i^{th} active-compliance contact. Using the linear compliance law (2), the elastic energy induced by the i^{th} contact is:

$$\begin{aligned} U_i(q) &= \tfrac{1}{2}\big(X_i(r_i^0,q) - x_i^0\big)^T K_i\big(X_i(r_i^0,q) - x_i^0\big) \\ &\quad -F_i^0 \cdot (X_i(r_i^0,q) - x_i^0), \end{aligned}$$

where $x_i = X_i(r_i^0, q) = R(\theta)r_i^0 + d$. The first derivative of U_i is: $DU_i(q) = -DX_{r_i}(q)^T F_i(q)$, where $F_i(q) = F_i^0 - K_i(x_i(q) - x_i^0)$. The second derivative of U_i is:

$$D^2U_i(q) = DX_{r_i}^T(q) K_i DX_{r_i}(q) - D^2X_{r_i}(q)^T F_i(q). \tag{6}$$

Recall that $[u\times]$ is the 3×3 skew-symmetric matrix satisfying $[u\times]v = u\times v$ for all $v \in \mathbb{R}^3$. Then $DX_{r_i}(q) = [I, (-\boldsymbol{\rho}_i)\times]$. The second derivative, $D^2X_{r_i}(q)$, is a vector-valued symmetric bilinear function. To obtain a formula for $D^2X_{r_i}(q)$, we compute the derivative of $DX_{r_i}(q)$ along a configuration-space trajectory $q(t)$. The velocity of $\mathcal{B}$ along $q(t)$ is denoted $\dot{q} = (v, \omega)$, where v and ω are $\mathcal{B}$'s linear and angular velocities. Since $\boldsymbol{\rho}_i = R(\theta)r_i^0$, $\frac{d}{dt}DX_{r_i}(q(t)) = [0, (-\dot{R}r_i^0)\times] = [0, (\boldsymbol{\rho}_i \times \omega)\times]$. The action of this derivative on the force F_i is: $\left(\frac{d}{dt}DX_{r_i}(q(t))\right)^T F_i = \begin{pmatrix} 0 \\ (\boldsymbol{\rho}_i\times\omega)\times F_i \end{pmatrix}$. Using a triple cross-product identity, we obtain that $(\boldsymbol{\rho}_i \times \omega) \times F_i = [(\boldsymbol{\rho}_i \cdot F_i)I - \boldsymbol{\rho}_i F_i^T]\omega = -[F_i\times][\boldsymbol{\rho}_i\times]\omega$. On the other hand, the chain rule implies that $\frac{d}{dt}DX_{r_i}(q(t)) = (D^2X_{r_i}(q))\dot{q}$. Hence the action of $D^2X_{r_i}(q)$ on F_i is given by the following matrix:

$$D^2X_{r_i}(q)^T F_i = \begin{bmatrix} 0 & 0 \\ 0 & -([F_i\times][\boldsymbol{\rho}_i\times])_s \end{bmatrix}.$$

Substituting for $DX_{r_i}(q)$ and $D^2X_{r_i}(q)^T F_i$ in (6) gives:

$$D^2U_i(q) = \begin{bmatrix} K_i & K_i[\boldsymbol{\rho}_i\times] \\ [\boldsymbol{\rho}_i\times]^T K_i & [\boldsymbol{\rho}_i\times]^T K_i [\boldsymbol{\rho}_i\times] + ([F_i\times][\boldsymbol{\rho}_i\times])_s \end{bmatrix}. \tag{7}$$

The summation $D^2U(q) = \sum_{i=1}^k D^2U_i(q)$ gives formula (4) for 3D grasps. In the 2D case, we evaluate each $D^2U_i(q)$ of (7) along a velocity vector $\dot{q} = (v,\omega)$ such that $v = (v_x, v_y, 0)$ and $\omega = (0,0,\omega_z)$. In particular, $[\boldsymbol{\rho}_i\times]\omega = \omega_z J\boldsymbol{\rho}_i$ where $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $[F_i\times][\boldsymbol{\rho}_i\times]\omega = (\boldsymbol{\rho}_i \cdot F_i)\omega_z$. When these terms are substituted into (7) and the sum $D^2U(q) = \sum_{i=1}^k D^2U_i(q)$ is taken, formula (5) is obtained.

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