The Nonsense of Bitcoin 1n Portfolio Analysis

Haim Shalit

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Monaster Center for
Economic Research
Ben-Gurion University of the Negev
P.O. Box 653
Beer Sheva, Israel

Fax: 972-8-6472941
Tel: 972-8-6472286

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Ben-Gurion University of the Negev, Beer-Sheva, Israel. shalit @bgu . a c . il

Abstract

The paper demonst rates the nonsense of using Bitcoin in financial invest ment s. By llsing mean-var·iance financ:ial analysis, st◇chast ic: dc>minance, CVaR, an<i th≤ Shapley value theory as analyt ical statistical models, I show how Bitcoin perf'orms poorly by comparing it against other traded assets. The conclusion is reached by ana lyzing daily freely available market data for the period 2018-2023.

1 Introduction

The purpose of this paper is to inquire why Bitcoin is being considered as a legitimat e investment instrument despite the fact that, in reality, it is not a valid medium of exchange nor a relia ble store of value. Bitcoin proponents see it at par with other financial assets traded in organized and regulated markets. There are a large number of well respected research academ icians who regard Bitcoin as a valid financial instrument and would like to have it regulated. This is of course a blasphemy for Bitcoin purists who claim that its main attraction is in its lack of regulation. Some professional ana lysts claim that Bitcoin should be recognized as a medium of exchange in order to allow it to legally register real-estate transactions. Renowned economists have been insulted in the media when they point out the nonsense of using Bitcoin both as an investment and as a medium of exchange.

By reading some the social networks we observe that investors have not been deterred by the larg e number of financial scandals and scams involved in Bitc·oin and other crypto currency. It is not the purpose of the present paper to address this issue here. It is remarkab le that serious investors and important investment bankers have diverted effort, money, and manpower to generate short-term revenues and invest into crypto currency markets. I am not checking their profits and losses; so I may err by writing this paper. However, I believe the remarks on Bitcoin written by Paul Krugman (2022) in the New York Times are true and well founded.

Some well known political analyst conveyed to me that terrorist organizations such as Hezbollah and others do not fund their activities with Bitcoin since block-chain records all transactions. Like other traffickers, terrorists prefer the use of gold and fiat currency to facilitate their illicit transactions. From the intense literat ure on Bitcoin and crypto currency I cite three papers: one by Akhtaruzzaman et al (2020) who used Bitcoin to build commodity portfolios. The second reference is Huberman et al (2021) who analyzed the Bitcoin payment systems although I am convinced that most Bitcoin transactions are for the purpose of hoarding value and not for the payment of goods and services. The third reference is by Eisl, Gasser, and Weinmayer (2015) who analyze Bitcoin in portfolios and use condit ional- Value-at-Risk to assess Bitcoin usefullness in diversification. In what follows I reach the opposite conclusion

In the present study I consider three classical methods to compare Bitcoin in the world of financial investments. The first model is the standard Markowitz (1952) portfolio theory conceived in the mean-variance space. The second model, being derived from the von Neuman - Morgenstern expected utility theory, uses stochastic dominance to compare Bitcoin to other assets with the help of the absolute Lorenz curve and cummulative value-at-risk (CVaR). The third approach is to use the Shapley value from cooperative game theory to establish the most valuable investment when Bitcoin is included in building efficient frontier portfolios.

2 Bitcoin and Portfolio Management

In this section I compare the ex-post performance of Bitcoin against the alternative of holding a portfolio of the most valued shares traded on Wall street. For the classic Markowitz (1952) mean-variance (MV) paradigm I use 108 shares that are mostly components of the S&PlOO index from January 2018 to May 2023. Stock rreturns are daily stock market data provided freely by Yahoo Finance. For Bitcoin the quotes are also downloaded from Yahoo Finance. Equity and bonds markets are closed on the weekends and holidays, leading to around 250 quotes per year. However, since Bitcoin is being traded 24/7, its quotes for weekends and holidays are removed, amounting to a difference of 100 data poi11ts per year. This manipulation may increase the volatility of Bitcoin as the returns move from Friday closing to Monday open. The 108 stocks statistics for the period are provided in the Appendix. The statistics for Bitcoin for the same period (January 2018 to May 2023) are: the mean return equals 0.148% and its standard deviation 4.497%. With the sample at hand we observe that the daily Bitcoin return has the largest standard deviation but not the largest mean.

We develop the two-time period MV model with Bitcoin and 108 traded securities where investors minimize the portfolio variance subject to a mean return. We construct a portfolio frontier in the MV space with N risky assets whose returns **r** that are linearly independent. This ensures the non-singularity of the variance-covariance matrix of asset returns ***E*** . We also assume that at least two risky assets have different means. We denote by ***JJ,*** the vector of assets mean returns, and by **w** the vector of portfolio weights, such that l:t $_1$ wi = 1. We assume **w** 0 hereby allowing for short sales. A frontier portfolio is obtained by minimizing the variance portfolio ½O"; = ½**w'** ***Ew*** s ubject to a required mean *f-Lv* = **w'*JJ,*** and the portfolio constraint 1 = **w'l,** where **1** is an N-vector of ones. As shown by Huang and Litzenberger (1988), the solution is obtained by minimizing the Lagrangian that includes the two constraints and deriving the first-order conditions (FOC) for a minimum, with the second-order conditions being satisfied by the 11on-singularity of' ***E.***

For the ease of presentation, we define the quadratic forms: A = **l'E** ^{1}*JJ,,* B = *JJ,*^{1}**E** ^{1}*JJ,*, C = **l'E-** ^{1}**1,** and D = BC - A^2 . The scalars B and C are positive since the matrix ***E*** is positive-definite and so is its inverse. As shown by Huang and Litzenberger (1988) sca lar D is also positive . From the FOC for a minimum variance, the optimal portfolio weights for a given mean μ_p are obtained as:

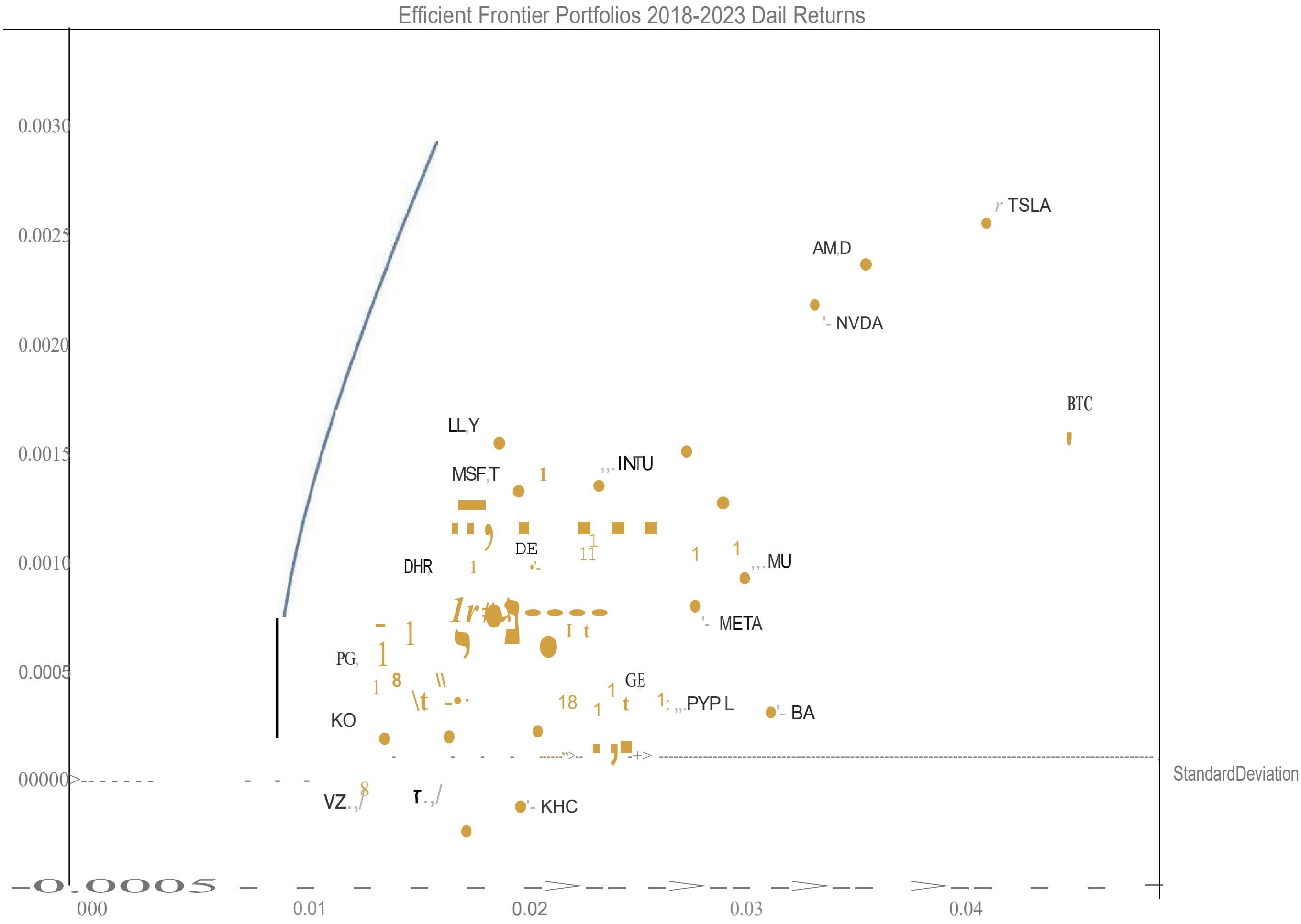
$$w_p = \frac{1}{D}\left[B \cdot \Sigma^{-1}\mathbf{1} - A \cdot \Sigma^{-1}\mu\right] + \frac{1}{D}\left[C \cdot \Sigma^{-1}\mu - A \cdot \Sigma^{-1}\mathbf{1}\right]\mu_p \tag{1}$$

The frontier portfolios delineate a hyperbola in the mean-standard deviation space leading to the portfolio variance formula for a given μ_p :

$$\sigma_p^2 = w_p' \Sigma w_p = \frac{C}{D}\left(\mu_p - \frac{A}{C}\right)^2 + \frac{1}{C} \tag{2}$$

Equation (2) is the basic formula for computing the frontier of optimal MV portfolios that is drawn as a solid line in Figure 1. From the statistics shown in the Appendix we have plotted the securities in Figure 1. First we observe clearly that three stocks (AMD, NVDA, TSLA) outperformed Bitcoin, having a smaller standard deviation and a higher mean return. Furthermore, the effic:ient frontier constructed based on the 108 stocks clearly dominated the statistics of Bitcoin for the period 2018-2023. This concludes the first examination based on the simple MV model.

Figurel: Efficient Frontier, Stocks, and Bitcoin



3 Bitcoin, the Lorenz, and CVaR

In this section[1], the concepts of stochastic dominance are presented to demonstrate the irrationality of Bitcoin in financial analysis. For that purpose I use the absolute Lorenz curve as the main analytical tool to address second-degree stochastic dominance (SSD) and cumulative value-at-risk (CVaR) to value risky assets. Stochastic dominance theory developed by Hanoch and Levy (1969), Hadar and Russell (1969), and Rothschild and Stiglitz (1970) provides economic efficiency under expected utility maximization without specifying utility functions. This results in rules that compare cumulative probability distributions (CDF) of asset returns.

Second-degree stochastic dominance (SSD), being mainly for risk-averse investors, is the most common model used in portfolio selection. By comparing the areas under the cumulative probabilities, SSD rules establish the necessary and sufficient conditions under which risky assets are preferred by all risk-averse expected utility maximizers. Computing the areas under the CDFs is not straightforward as it must be done for all the probabilities. Then, the analyst is required to compare between the various areas. One more evident alternative was developed by Shorrocks (1983) who used the absolute Lorenz curve (named here the Lorenz) to establish SSD rules. The Lorenz expresses the cumulative return on the portfolio as a function of the cumulative probability distribution. Instead of comparing conditional areas under the CDFs the following SSD rules are easier to visualize: For all risk-averse investors to prefer one asset over another, the Lorenz of one asset must lie entirely above the Lorenz of the other. In other words asset A is preferred to asset B by all risk averters if and only if

$$L_A(p) \geq L_B(p) \quad \text{for } 0 \leq p \leq 1,$$

where $L(p)$ is the Lorenz that, given the cumulative distribution $F(x)$ for asset x, is defined as:

[1] The exposition follows the presentation developed by Shalit (2014) and Shalit and Yitzhaki (2010).

$$L(p) = \int_{-\infty}^{x_p} x f(x)dx \text{ for } -\infty < x < \infty; \text{ where } x_p \text{ is defined by } p = \int_{-\infty}^{x_p} f(x)dx \quad (3)$$

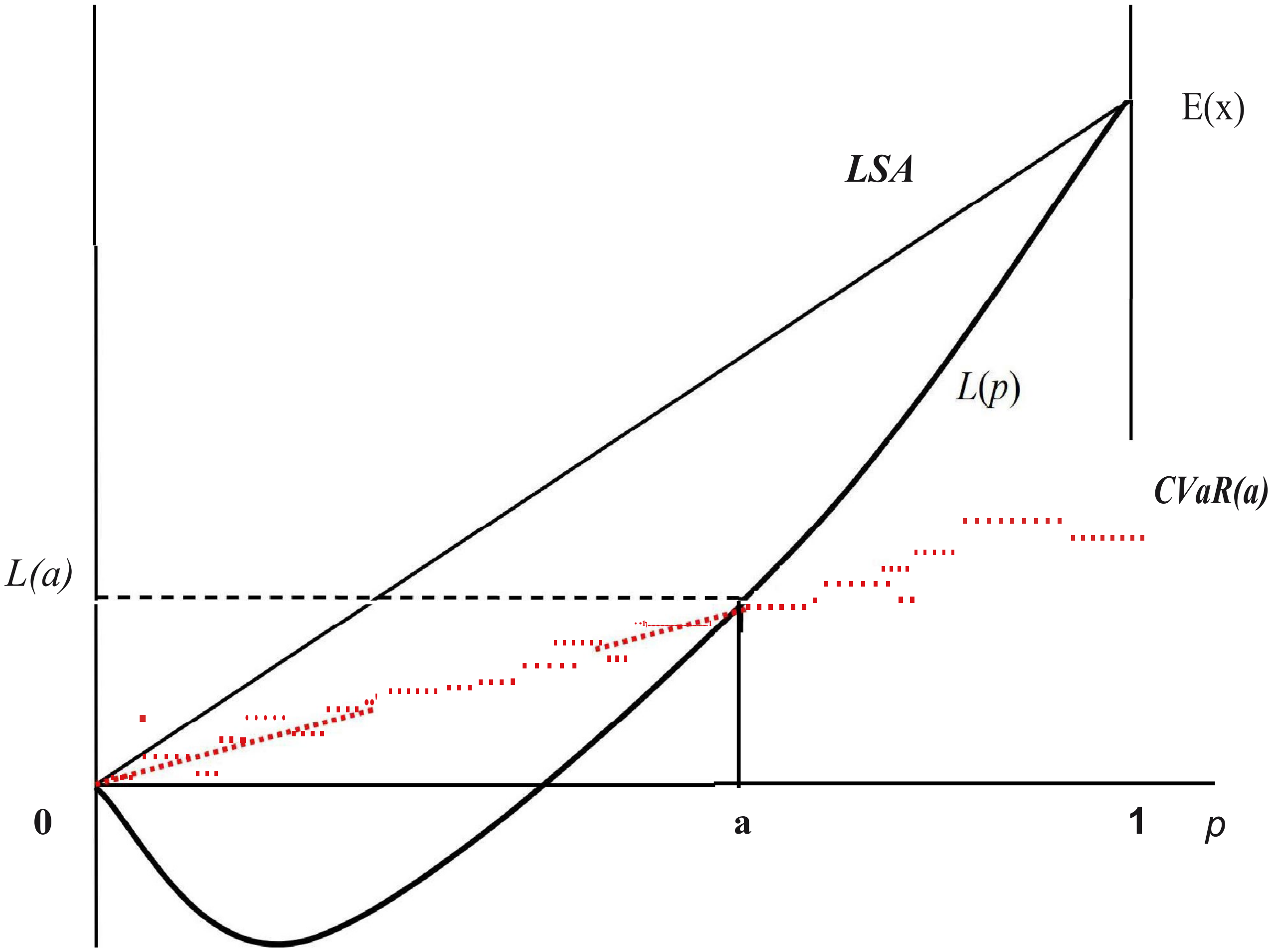
where f is the asset density function. Gastwirth (1971) proposed a simplified definition of the Lorenz curve by using the inverse of $F(x)$ designated by $F^{-1}(t) = \inf_x \{x : F(x) \geq t\}$ which is written as :

$$L(p) = \int_0^p F^{-1}(t)dt \qquad for\ 0 \le t \le 1. \quad (4)$$

Let us explore the Lorenz drawn in Figure 2. Cumulative probabilities are shown on the horizontal axis, thus returns are ranked in increasing value. The vertical axis reflects cumulative rates of returns weighted by the probabilities as formulated by Equation (3). Starting at (0,0) the Lorenz accumulates the sorted returns multiplied by their probabilities. Since the lowest returns can be losses, the Lorenz may result in negative values. The curve ends at the mean return $E(x)$ on the parallel vertical axis where all returns are used up and multiplied by their probabilities.

As explained by Shalit (2014) "... the rationale for using the Lorenz in financ ial analysis is rooted in the manner by which the Lorenz characterizes risk and mean return of investments for risk-averse investors. Such investors have concave utility functions that express declining marginal utility. The horizontal axis in Figure 2 shows the probabilities of asset returns ranked from those generating the lowest returns with the highest marginal utility to those generating the highest returns with the lowest marginal utility. The ranking of asset returns is the only information needed to sort an asset according to decreasing marginal utility. This ordering is specified by the cumulative returns multiplied by the probabilities of getting these returns. This is basically the Lorenz. The principle of distributing resources according to decreasing marginal utility or decreasing marginal product ensures that financial resources are allocated optimally. Using the Lorenz to manage portfolio risk guarantees that objective. Because the curve expresses asset behavior not as a function of returns over time but as the incidence of having lower and higher returns, it provides much more relevant information about risk and return than periodical charts..."

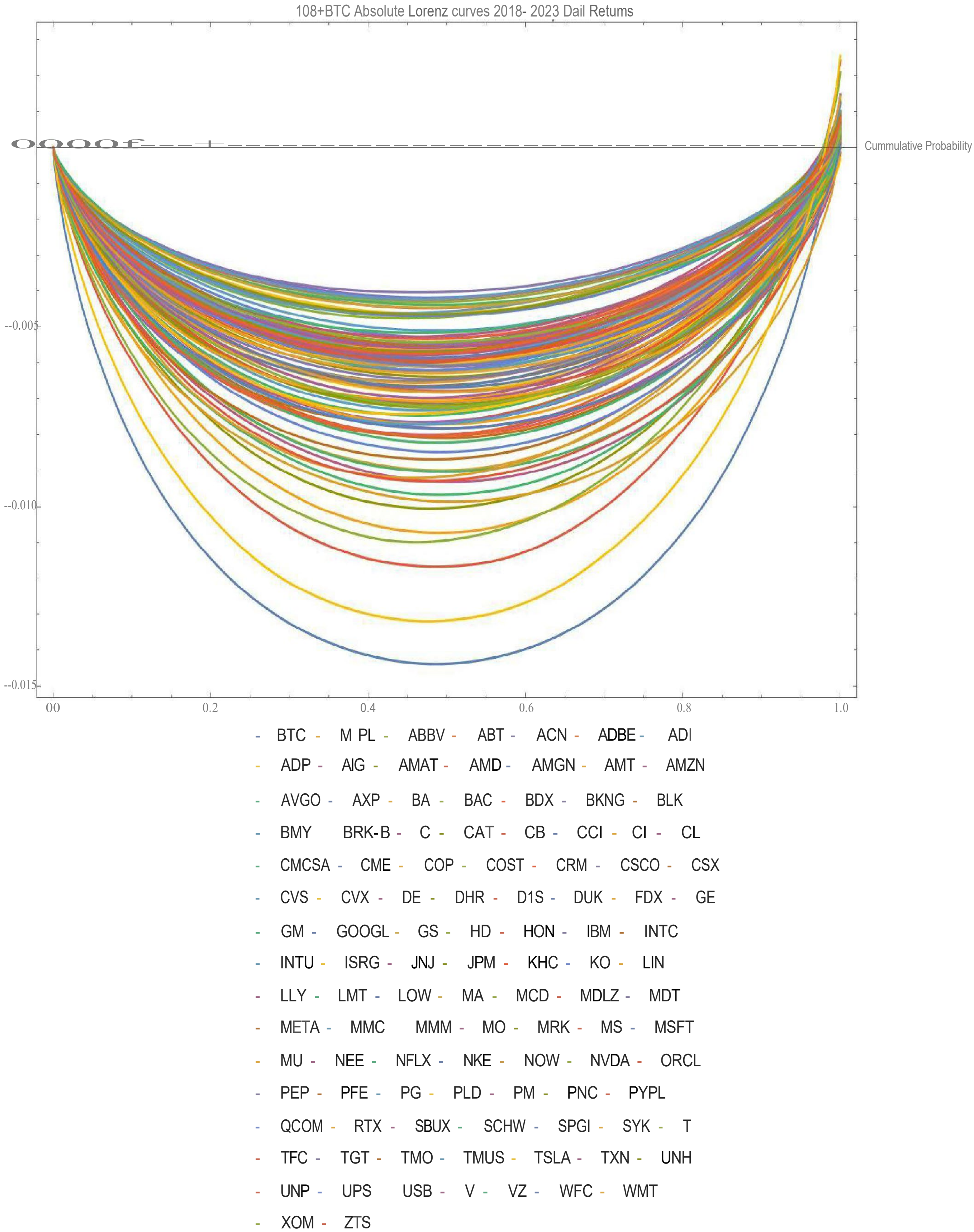
Figure 2 The Lorenz Curve



2

We now compare the Lorenz of Bitcoin against the Lorenz curves of the other securities to check whether it can be preferred by all risk averse investors according to the stochastic dominance rules. The Lorenz curves of 108 shares and the Bitcoin are computed for the period 2018-2023. This is presented in Figure 3.

Figure 3 Absolute Lorenz Curves of all Assets 2018-2023 Daily Returns



The results are convincing for the period at hand. Bitcoin is SSD dominated by most 108 securities. Although in general, Bitcoin reveals a higher mean return, its risk as expressed by a series of larger negative returns produces a Lorenz that is well below most of the other securities. It is true that the Lorenz curves intersect at some point so the the results are not overwhelming. Hence one can look at the necessary conditions for stochastic dominance tl1at use the mean and Gini's mean difference as statistics. Following Shalit and Yitzhaki (2010) we obtain from the Lorenz the two statistics that express risk and expected return. The latter is located at the terminal point of the Lorenz on the parallel vertical axis at $p = 1$. The risk underlying the Lorenz is obtained by computing the vertical differences between the Lorenz and a virtual riskless asset with the same expected return as the asset labeled the line of safe asset (LSA). In figure 2 the LSA is a straight line drawn from the origin (0, 0) to the mean *(f.Lx,* 1). The LSA expresses the expected return *f.Lx* multiplied by the probability p as plotted in Figure 2. The area between the LSA and the Lorenz is Gini's mean difference (GJVID). In portfolio management it is convenient to use one half of GMD that we label here the Gini:

$$GMD/2 \qquad rx = 2\text{cov}[x, F(x)]$$

As seen in Figure 2 and explained above, the Gini is easily obtained by computing the area between the LSA and the Lorenz as follows:

$$tr = \int_0^1 [f.LxP - L(p)]dp = \text{cov}[x, F(x)].$$

Now we can address the necessary conditions for SSD derived from the Lorenz. The mean being tl1e terminal point on the Lorenz is the first condition. A dominating asset should have a greater expected ret urn. The second condition looks at the area under the Lorenz that is the the entire area below the LSA and the area above the Lorenz which is the Gini. The other necessary condition for SSD is that the area below the Lorenz of the dominating asset be greater than the area below the Lorenz of the dominated asset. This area is one-half the mean return subtracted by the Gini. We can call this area as the Gini adjusted mean return. These requirements explain the necessary conditions for SSD by using the mean and the Gini first enunciated by Yitzhaki (1982) as the MG necessary conditions:

/J,x f.1,y

/J,x - rx f.1,y - ry

for asset *x* to SSD dominate asset *y*. To assert dominance I use these conditions to rank the shares and Bitcoin as it appears in Table 1. The ranking with respect to the mean and the Gini adjusted mean return show without any doubt the inferiority of Bitcoin according to SSD rules. Although it has a larger mean Bitcoin is MG dominated by all other assets.

Table 1: Bitcoin and Securities Ranked wrt to Gin i-Adju sted-Mean

Symbol	Mean	Gini	i-r	Symbol	Mean	Gini	i-r	Symbol	Mean	Gini	−r
PEP	0.05%	0.65%	-0.60%	ACN	0.07%	0.91%	-0.84 %	CAT	0.06%	1.11 %	-1.05 %
PG	0.05%	0.67%	-0.61%	MO	0.01%	0.85%	-0.84%	ADI	0.08%	1.14 %	-1.06%
KO	0.04%	0.66%	-0.62%	CB	0.04%	0.88%	-0.84%	AXP	0.07%	1.13%	-1.06%
MDLZ	0.06%	0.68%	-0.62%	V	0.07%	0.92%	-0.85%	AVGO	0.13%	1.19%	-1.07%
JNJ	0.03%	0.65%	-0.62%	SPGI	0.08%	0.93%	-0.85%	PNC	0.03%	1.10%	-1.07%
MCD	0.06%	0.69%	-0.63%	TMO	0.09%	0.94%	-0.85%	USB	0.00%	1.08%	-1.08%
WMT	0.05%	0.70%	-0.65%	CSCO	0.05%	0.91%	-0.86%	BAC	0.03%	1.11%	-1.08%
CL	0.02%	0.68%	-0.66%	UNP	0.05%	0.92%	-0.87%	ISRG	0.09%	1.18%	-1.08%
VZ	0.00%	0.65%	-0.66%	PLD	0.08%	0.95%	-0.88%	INTU	0.10%	1.22%	-1.11 %
MMC	0.07%	0.74%	-0.67%	AMT	0.05%	0.92%	-0.88%	AMZN	0.08%	1.20%	-1.11 %
MRK	0.08%	0.75%	-0.67%	SBUX	0.07%	0.95%	-0.88%	ADBE	0.09%	1.21 %	-1.12 %
BRK-B	0.05%	0.72%	-0.67%	CCI	0.03%	0.91%	-0.88%	BKNG	0.06%	1.18%	-1.12 %
COST	0.09%	0.77%	-0.68%	CSX	0.06%	0.95%	-0.89%	FDX	0.02%	1.17%	-1.14 %
DUK	0.03%	0.73%	-0.70%	MSFT	0.12%	1.01%	-0.89%	CRM	0.08%	1.25%	-1.17%
BMY	0.03%	0.78%	-0.75%	UPS	0.06%	0.95%	-0.89%	C	0.01%	1.18%	-1.17%
LMT	0.05%	0.82%	-0.77%	SYK	0.07%	0.96%	-0.90%	WFC	0.01%	1.19%	-1.18%
NEE	0.07%	0.84%	-0.77%	CVS	0.03%	0.93%	-0.90%	AIG	0.04%	1.24%	-1.20 %
BDX	0.03%	0.81%	-0.78%	CMCSA	0.02%	0.93%	-0.91%	INTC	0.01%	1.21%	-1.20%
AMGN	0.04%	0.82%	-0.78%	MMM	-0.03%	0.88%	-0.91%	SCHW	0.04%	1.24%	-1.20%
LLY	0.15%	0.92%	-0.78%	KHC	-0.01%	0.91%	-0.92%	TFC	0.01%	1.22%	-1.21%
ABT	0.06%	0.86%	-0.80%	JPM	0.05%	0.99%	-0.94 %	QCOM	0.09%	1.33 %	-1.24 %
HON	0.04%	0.85%	-0.80%	RTX	0.04%	0.99%	-0.95 %	META	0.07%	1.35 %	-1.28 %
ADP	0.07%	0.87%	-0.80%	LOW	0.09%	1.04 %	-0.95 %	COP	0.10%	1.40 %	-1.31 %
LIN	0.08%	0.89%	-0.81%	TGT	30 .09%	1.03%	-0.95%	NOW	0.14%	1.46%	-1.32%
CME	0.05%	0.86%	-0.81%	AAPL	0.13%	1.08%	-0.95%	GM	0.03%	1.37%	-1.34%
MDT	0.03%	0.84%	-0.81%	MA	0.09%	1.04%	-0.95%	GE	0.04%	1.41%	-1.37 %
DHR	0.08%	0.90%	-0.81%	GOOGL	0.08%	1.05%	-0.97%	PYPL	0.03%	1.40%	-1.37 %
PFE	0.04%	0.85%	-0.81%	BLK	0.05%	1.03%	-0.98%	NFLX	0.10%	1.51%	-1.41%
T	0.00%	0.81%	-0.81%	NKE	0.07%	1.05%	-0.98%	AMAT	0.12%	1.56%	-1.44 %
ORCL	0.08%	0.90%	-0.81%	GS	0.05%	1.04%	-1.00%	BA	0.03%	1.52%	-1.49 %
IBM	0.03%	0.84%	-0.82%	TXN	0.07%	1.07 %	-1.00 %	MU	0.08%	1.63%	-1.54 %
PM	0.03%	0.85%	-0.82%	CVX	0.06%	1.06%	-1.01%	NVDA	0.21%	1.77%	-1.56 %
ZTS	0.08%	0.90%	-0.82%	DIS	0.01%	1.03 %	-1.02 %	AMD	0.24%	1.90 %	-1.66 %
ABBV	0.06%	0.89%	-0.83%	CI	0.04%	1.07%	-1.03%	TSLA	0.26%	2.16 %	-1.91%
HD	0.06%	0.89%	-0.83%	DE	0.09%	1.12%	-1.03%	BTC	0.15%	2.30 %	-2.16 %
UNH	0.08%	0.91%	-0.83%	MS	0.07%	1.12%	-1.04%				
TMUS	0.07%	0.91%	-0.84%	XOM	0.06%	1.11 %	-1.05 %				

Relat ed to the Lorenz, there is an additional criterion that we like to test that is the Value-at-Risk (Va R). The measure quantifies exposure to risk as the amount needed to keep in a safe asset to overcome defa ult. VaR is a safety-first risk measure defined as the negative quantile of probability p expressed as

$$VaR(p) = -F^{-1}(p)$$

As we observe $VaR(p)$ is one element of the Lorenz obtained directly from the cumulative distribution function. However as it is well established VaR lacks the basic properties of a valid risk measure as explained by Artzner et al (1999). For coping with VaR's lack of coherence, conditional-Value-at-Risk (CVaR) was developed by Rockefa ller and Uryasev (2000). T he basic idea is to calculate $CVaR(p)$ as the mean value of all the quant iles below the original VaR in the lower tail of the cumulative distribution function which can be obtained directly from the Lorenz:

$$CVaR(p) = -\frac{1}{p}\int_0^p F^{-1}(t)dt = -\frac{L(p)}{p}$$

In Figure 2 the $CVaR(a)$ for probability a is the slope of the straight line that runs from (0,0) to $(a, L(a))$. Under these circumstances, the estimate d CVaR becomes a specific value of the Lorenz. Hence, for a given data set,CVaR is estimated by ranking and summing up the observations. The CVaR at 5% and at 10% for our data set are presented in Table 2.

Table 2: CVaR (5%) and CVaR(IO%) for all Asset s

Symbol	CVaR5%	CVaR10%	Symbol	CVaR5%	CVaR10%	Symbol	CVaR5%	CVaR10%
BTC	28.73%	14.30%	CVX	13 .59%	6.76%	NFLX	19.31%	9.61%
AAPL	13.31%	6.62%	DE	14.18%	7.05%	NKE	13.35%	6.65%
ABBV	11.38%	5.66%	DHR	11.28%	5.61%	NOW	18.39%	9.14%
ABT	11.03%	5.48%	DIS	13.54%	6.74%	NVDA	21.94%	10.90 %
ACN	11.42%	5.68%	DUK	9.38%	4.67%	ORCL	11.11%	5.53%
ADBE	15.35%	7.64%	FDX	15.40%	7.66%	PEP	8 .05%	4.01%
ADI	14.62%	7.27%	GE	18.59%	9.25%	PFE	11.06%	5.50%
ADP	11.01%	5.48%	GM	18.03%	8.97%	PG	8.42%	4.19%
AIG	15.98%	7.95%	GOOGL	13.29%	6.62%	PLD	12.07%	6.01%
AMAT	20.07%	9.98%	GS	13.61%	6.77%	PM	11.11%	5.53%
AMD	23.32%	11.61 %	HD	11.35%	5.65%	PNC	14.33%	7.13%
AMGN	10.55%	5.25%	HON	10.91%	5.43%	PYPL	18.56%	9.23%
AMT	12.00%	5.97%	IBM	10.95%	5.45%	QCOM	16.93%	8.42%
AMZN	15.26%	7.60%	INTC	16.17%	8.05%	RTX	12.55%	6.25%
AVGO	14.90 %	7.41%	INTU	15.39%	7.66%	SBUX	11.85%	5.90%
AXP	14.20%	7.06%	ISRG	14.84%	7.38%	SCHW	16.41%	8.16 %
BA	19.70%	9.81%	JNJ	8.35%	4.16%	SPGI	11.74%	5.84%
BAC	14.50%	7.21%	JPM	12.74%	6.34%	SYK	12.17%	6.05%
BDX	10.64%	5.29%	KHC	12.16%	6.05%	T	10.82%	5.38%
BKNG	15.29%	7.61%	KO	8.36%	4.16%	TFC	16.06 %	7.99%
BLK	13.25%	6.60%	LIN	11.26%	5.60%	TGT	12.89 %	6.41%
BMY	10.18%	5.06%	LLY	10.97%	5.46%	TMO	11.99%	5.95%
BRK-B	9.21%	4.58%	LMT	10.31%	5.13%	TMUS	11.44%	5.69%
C	15.60%	7.76%	LOW	13.01%	6.47%	TSLA	26.35%	13.11 %
CAT	14.41%	7.17%	MA	13.07%	6.50%	TXN	13.93%	6.93%
CB	11.32%	5.63%	MCD	8.59%	4.27%	UNH	11.36%	5.65%
CCI	11.98%	5.96%	MDLZ	8.46%	4.21%	UNP	11.89%	5.91%
CI	13.98%	6.95%	MDT	10.97%	5.46%	UPS	12.13%	6.03%
CL	8.92%	4.43%	META	17.34%	8.63%	USB	14.14%	7.04%
CMCSA	12.41%	6.18%	MMC	9.21%	4.58%	V	11.56%	5.75%
CME	11.00%	5.47%	MMM	12.08%	6.02%	VZ	8 .74%	4.35%
COP	17.96%	8.93 %	MO	11.33%	5.64%	WFC	15.63%	7.78%
COST	9.50%	4.72%	MRK	9.28%	4.62%	WMT	8.89%	4.42%
CRM	16.04%	7.98%	MS	14.37%	7.15%	XOM	14.41%	7.16%
CSCO	11 .70%	5.82%	MSFT	12.41%	6.18%	ZTS	11.46%	5.70%
CSX	1 2.18%	6.06%	MU	21.43%	10.66 %			
CVS	1 2.40%	6.17%	NEE	10.52%	5.23%			

The larger CVaR the riskier the stock . From Table 2 we see that BTC ranks among the riskier ones. No surprise here. This part icular true for the stocks NVDA, AMD, and TSLA vs Bitcoin. The main difference is that CVaR considers only low-ret urn risks for a given probability whereas the Lorenz and the mean-Gini conditions for SSD consider risk and mean return for the entire distribut ion.

4 The Shapley Value of Bitcoin in a Efficient Portfolio

To compute the exact value of Bitcoin in a standard investment model, I am using the Shapley value from cooperative game theory. I consider a portfolio of stocks as an n-person cooperative game where the financial assets are players in the game. The goal is to measure the contribution of each stock to the general outcome of the portfolio. The Shapley value extracts the true and exact contribution of each stock to the portfolio's total value. The presentation of this section follows Shalit (2021).

The game purpose is to minimize portfolio risk expressed by the variance for a given mean return. For a set N of n securities, the Shapley value is calculated from the contribution of each and every security in the portfolio. To capture the way each security contributes to the entire portfolio, we compute the risk v for each and every subset of stocks $S \subset N$. In total, we have 2^N portfolios or coalitions including the empty set. The marginal contribution of each security k to the subset portfolio S is given by $v(S) - v(S \setminus \{k\})$, where $v(S)$ is the risk of portfolio S, and $v(S \setminus \{k\})$ is the risk of the portfolio S without security k. Portfolios are predefined and all the orderings are equally probable. Hence, $S \setminus \{k\}$ is the portfolio that precedes k, and its contribution to coalition S is computed when all the orderings of S are accounted for. Given equally probable orderings, we compute their expected marginal contribution. Therefore, we need the probability that, for a given ordering, the subset $S \subset N$, $k \subset S$ is seen as the union of security k and all the securities that precede it. Two probabilities are used here: First, the probability that k is in s (s being the number of stocks in S) being equal to $1/n$, and second, that $S \setminus \{k\}$ arises when $s - 1$ securities are randomly chosen from $N \setminus \{k\}$, that is $(n - s)!(s - 1)!/(n - 1)!$.

The Shapley value for security k is obtained by averaging the marginal contributions to the risk of all portfolios for the set of N securities and the risk function v, which in mathematical terms is written as

$$Sh_k(N,v) = \sum_{S \subset N, k \subset S} \frac{(n-s)!(s-1)!}{n!} \left[v(S) - v(S \setminus \{k\}) \right] \quad (5)$$

or, alternatively,

$$Shk(N,v) \quad L_{ScN,kcS} \frac{8}{n.} [v(S \cup \{k\}) - v(S)] . \qquad (6)$$

Shapley value theory works best with a single attribute imputed to all game participants, thus I use optimal portfolios whose expected returns are always at their minimum risk. Consider the set of frontier portfolios generated by minimizing the portfolio variance for a given expected return. The portfolio frontier in the mean- standard deviation space was elaborated in Section 2 above. The result is the variance formulated by Equation (2) that is the set of the optimal MV portfolios. That variance is used to calculate the Shapley value of the asset son the MV efficient frontier. For the mean return *JJ,p* , the variance of Equation (2) can be written equivalently as:

$$aP^2 = \frac{1}{D}(C\mu P^2 - 2A\mu p + B). \qquad (7)$$

Then, for an arbitrary set of required mean returns *JJ,p,* we calculate with Equation (7) the frontier portfolio variance for each subset $S \cup i$ N. The Shapley value is computed following Equation (6) using the variance-covariance matrix *Es* and the quadratic forms $As = 1$ $E s^{1}$ JJ,s , $Bs = p, E s^{1}$ JJ,s, $Cs = 1$ $E s^{1}1s$, and $Ds = BsCs - A\}$ for all the 2N subsets S N. The Shapley value for each stock i in an optimal front ier portfolio subject to a given mean *JJ,p* is obtained as

$$Shi(aP; /J,p) = \sum_{s=0}^{N-1} \sum_{ScN\backslash i} \frac{(n-s-1)!s!}{n!} [a^2(\mu p, S \cup \{i\}) - a^2(\mu p, S)] \quad \forall i \in N. \qquad (8)$$

Finally, for a given return *JJ,p,* the Shapley values add up to their optimal portfolio variance at *JJ,p* as

$$\sum_{i=1}^{N} Shi(a;; JJ,p) = a;(p,p) . \qquad (9)$$

lt seems now natural to discuss the Sh3apley value as expressed by Equation (8) for an asset in an optimal portfolio. Given that efficient portfolios have the lowest variance for a given mean , the increm ental risks *a;(p,p, S* U { i}) - *a;(p,p, S)* are non-positive for any asset i and any set S that does not contain i . Indeed as assets are added to the portfolio the variance does not increase. However, Shapley value computation also includes the incremental risk of going from an empty portfolio to a portfolio of one risky asset i whose increment is usually positive. Hence, as it is shown in the empirical analysis, Shapley values of assets in optimal portfolios can be either negative or positive. Negative Shapley values imply that these assets reduce their risk contribution to the portfolio as mean return increase. Positive Shapley values imply increasing risk assets along the efficient frontier and therefore increase mean return.

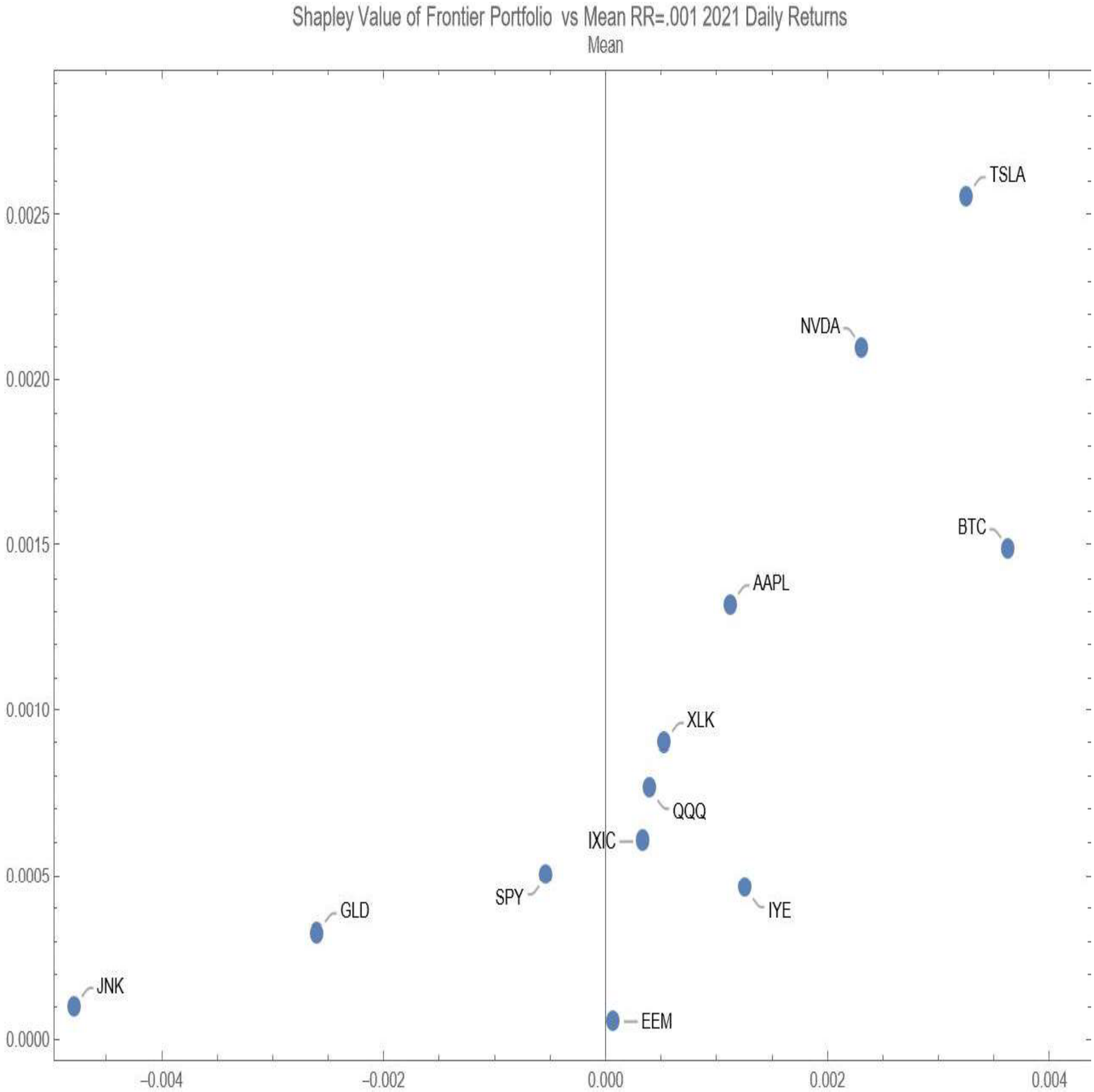
For the empirical analysis I have reduced the number of assets because of the dimensionality of calculating the Shapley value. I have chose 12 assets that include indices and single stocks. The statistics for these assets are reported in the Appendix .

The Shapley value results are presented below and plotted in Figure 4 in the space mean return-Shapley value. What we confirmed by using the Shapley value in an efficient portfolio of ma.ior financial assets and BTC is that Bitcoin is not the most valued player in terms of risk and mean return.

Table 4: Shapley Values of Assets on the Efficient Frontier

Symbol	Mean	Shapley value
AAPL	0.132%	0.113%
BTC	0.149%	0.363%
EEM	0.006%	0.007%
GLD	0.032%	-0.259%
IXIC	0.060%	0.034%
IYE	0.046%	0.126%
JNK	0.010%	-0.479%
NVDA	0.210%	0.231%
QQQ	0.076%	0.040%
SPY	0.050%	-0.053%
TSLA	0.255%	0.326%
XLK	0.090%	0.053%

Figure 4: Shapley Values and Means of Frontier Portfolio Assets



5 Conclusion

Daily financial market data from the period 2018-2023 used in various standard rational models have confirmed st at ist ically beyond any doubt that Bitcoin is an overpriced asset that does not deserve to be included in a plausible portfolio. Furthermore, because Bitcoin is unregulated asset traded on days and hours that investors play, sleep, or enjoy the Sabbat h, its movements cannot be used effectively with other financial assets . Although Bitcoin has many derivative instruments that can protect its volatility on the weekends , the only play I see (and I do not recommend it) is to short it.

In the 11 century Europ eans Crusaders fought their way to Jerusalem. On the road they encountered a beautiful wild flower, the tulip which still grow in the plains of Israel. The invaders domesticated and produced the national flower of Holland. Five centuries later, Dutch traders scuttled for the privilege to hold the precious flower, causing the Tulip mania. We now enjoy the flower in our gardens. Block chain techniques are here to stay too.

References

[1] Md Akhtaruzzaman, Ahmet Sensoyc, and Shaen Corbet. The infiuen ce of Bitcoin on portfolio diversification and design. *Finance Research Letters,* 37(101344), November 2020.

[2] Alexander Eisl, Stephan M. Gasser, and Karl Weinmayer. Caveat emptor: Does Bitc·oin improve portfolio diversification'? *SSRN,* (2408997), 2015.

[3] .J. L. Gastwirth. A genera l definition of the lorenz curve. *Econometrica,* pages 1037-1039., 1971.

[4] J. Hadar and W. R. Russell. Rules for ordering uncertain prospects? *American Economic Review,* 59:25- 34, 1969.

[5] Giora Hanoch and Haim Levy. The efficiency analys is of choice involvin g risk. *Review of Economic Studies,* pages 335-346, 1969.

[6] Chi-fu Huang and Robert H. Litzenberger. *Foundations for Financial Economics.* Elsevier Science Publishing Co, New York NY, 1988.

[7] Gur Huberman, Jacob D. Leshno, and Ciamac Moallemi. Monopoly without a monopolist: an economic ana lysis of the Bitcoin payment system. *Review of Economic Studies,* 88(6) :3011 - 3040, November 2021.

[8] Harry Markowitz. Portfolio selection. *Journal of Finance,* 7(1):77-91, 1952.

[9] Guilermo Owen. *Game Theory.* Emerald Group Publishing, 2013.

[10] Rockafellar R. and 8. Uryasev. Optimization of conditionalva lue-at-risk. *Journal of Risk,* 2:21-41, 2000.

[11] Alvin E. Roth. *The Shapleyv value: essays in honor of Lloyd S. Shapley,* chapter Introduction, pages 1-30. Cambridge Univers ity Press, New York, 1988.

[12] Mic·hael Rothschild and Joseph Stiglitz. Increasing risk i: A definition. *Journal of Economic Theory,* 2:66-84, 1970.

[13] Haim Shalit. Portfolio risk management using the lorenz curve. *Journal of Portfolio Management,* 40(3):152?159., Spring 2014.

[14] Haim Shalit. Using the Shapley value of stocks as systematic risk. *Joumal of Risk Finance,* 21(4):459, 468, October 2020.

[15] Haim Shalit. The Shapley value decomposition of optimal portfolios. *Annals of Finance,* 17(1):1, 25, March 2021.

[16] Haim Shalit and Shlomo Yitzhaki. The mean-Gini efficient portfolio frontier. *Joumal of Financial Research,* 28(1):59-75, Spring 2005.

[17] Haim Shalit and Shlomo Yitzhaki. How does beta explain stochastic dominance effic·iency? *Review of Quantitative Finance and Accounting,* 35(4):431-444, November 2010.

[18] Lloyd S. Shapley. A value for n-person games. In H.W. Kuhn and A.W. Tucker, editors, *Contributions to the Theory of Games, Vol II,* volume 28 of *Annals of Mathematics Studies ,* pages 307-317. Princeton Un iversity Press, Princeton , New .Jersey, 1953.

[19] Anthony F. Shorrocks. Ranking income distributions. *Economica,* 50:3- 17, 1983.

[20] Shlomo Yitzhaki. Stoc:hast ic· dominance, mean varianc:e, and Gini's mean difference. *American Economic Review,* 72(1):178-185, 1982.

Appendix

Exhibit Al: Shares statistics Daily returns 2018-2023

Symbol	Mean	Stdev	Symbol	Mean	Stdev	Symbol	Mean	Stdev
AAPL	0.132%	2.064%	CVX	0.058%	2.200%	NEE	0.070%	1.693%
ABBV	0.060%	1.775%	DE	0.092%	2.139%	NFLX	0.100%	2.971%
ABT	0.065%	1.667%	DHR	0.084%	1.709%	NKE	0.065%	2.067%
ACN	0.074%	1.783%	DIS	0.011%	2.048%	NOW	0.143%	2.731%
ADBE	0.095%	2.351%	DUK	0.033%	1.491%	NVDA	0.210%	3.320%
ADI	0.083%	2.183%	FDX	0.025%	2.340%	ORCL	0.083%	1.849%
ADP	0.068%	1.748%	GE	0.040%	2.682%	PEP	0.052%	1.364%
AIG	0.037%	2.508%	GM	0.029%	2.620%	PFE	0.036%	1.630%
AMAT	0.118%	2.894%	GOOGL	0.083%	1.994%	PG	0.054%	1.351%
AMD	0.242%	3.562%	GS	0.047%	2.050%	PLD	0.076%	1.872%
AMGN	0.042%	1.602%	**HD**	0.059%	1.793%	PM	0.026%	1.700%
AMT	0.046%	1.797%	HON	0.045%	1.686%	PNC	0.025%	2.165%
AMZN	0.081%	2.260%	IBM	0.026%	1.692%	PYPL	0.027%	2.702%
AVGO	0.126%	2.326%	INTC	0.012%	2.396%	QCOM	0.088%	2.584%
AXP	0.071%	2.339%	INTU	0.103%	2.311%	RTX	0.043%	2.026%
BA	0.028%	3.116%	ISRG	0.095%	**2.272%**	SBUX	0.067%	1.918%
BAC	0.031%	2.207%	JNJ	0.028%	1.294%	SCHW	0.038%	2.393%
BDX	0.031%	1.550%	JPM	0.052%	1.995%	SPGI	0.079%	1.859%
BKNG	0.056%	2.286%	KHC	-0.014%	1.970%	SYK	0.066%	1.923%
BLK	0.051%	2.010%	KO	0.042%	1.325%	T	-0.003%	1.624%
BMY	0.029%	1.502%	LIN	0.083%	1.700%	TFC	0.014%	2.461%
BRK-B	0.048%	1.432%	**LLY**	0.146%	1.860%	TGT	0.085%	2.152%
C	0.007%	2.402%	LMT	0.050%	1.659%	TMO	0.090%	1.767%
CAT	0.058%	2.087%	LOW	0.090%	2.109%	TMUS	0.069%	1.786%
CB	0.044%	1.764%	MA	0.089%	2.035%	TSLA	0.255%	4.107%
CCI	0.031%	1.760%	MCD	0.059%	1.474%	TXN	0.069%	2.008%
CI	0.043%	2.122%	MDLZ	0.058%	1.367%	UNH	0.083%	1.842%
CL	0.019%	1.343%	MDT	0.025%	1.645%	UNP	0.054%	1.799%
CMCSA	0.024%	1.796%	META	0.071%	**2.773%**	UPS	0.056%	1.880%
CME	0.046%	1.763%	MMC	0.074%	1.483%	USB	-0.001%	2.178%
COP	0.097%	2.778%	MMM	-0.032%	1.719%	V	0.070%	1.805%
COST	0.091%	1.518%	MO	0.008%	1.642%	VZ	-0.004%	1.258%
CRM	0.083%	2.440%	MRK	0.077%	1.451%	WFC	0.011%	2.336%
CSCO	0.048%	1.806%	MS	0.070%	2.201%	WMT	0.048%	1.444%
CSX	0.064%	1.881%	MSFT	0.124%	1.945%	XOM	0.060%	2.108%
CVS	0.025%	1.796%	MU	0.084%	2.997%	ZTS	0.082%	1.755%

Exhibit A2 Some 12 Assets for Shapley value

Symbol	Mean	St-Dev
AAPL	0.132%	2.064 %
BTC	0.149%	4.498%
EEM	0.006%	1.437%
GLD	0.032%	0.910%
IXIC	0.060%	1.591%
IYE	0.046%	2.248 %
JNK	0.010%	0.636%
NVDA	0.210%	3.320%
QQQ	0.076%	1.624%
SPY	0.050%	1.330%
TSLA	0.255%	4.107%
XLK	0.090%	1.739%