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Discussion Paper No. 25-10

July 10, 2025

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Abstract

We study admission policies in parallel contests, in which agents choose which contest to enter and then compete for a prize based on both a nominal value and the competing agents' reputations. Agents may have either a low reputation (low type) or a high reputation (high type). We first study a single contest with an organizer who wishes to maximize total effort, and then provide general conditions for the organizer to restrict or admit low-type agents to the contest. Later, we study the admission policies in parallel contests. We investigate how a combination of admission policies influences agents' contest decisions and how this affects overall equilibrium strategies on both sides. We demonstrate, in particular, the existence of a paradoxical equilibrium in which the less prestigious contest sets higher admission limits than the more prestigious contest for the participation of low-type agents in their contests.

Keywords: Admission policy; parallel contests; reputation.

JEL classification: D44, D72, D82, J31

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1 Introduction

In our world, institutions are often distinguished by their reputations, which influence individuals' choices to join them. These more prestigious institutions usually provide greater benefits - like access to more resources and building professional networks. Admission to these institutions is usually determined by proven skills or previous accomplishments. Within these institutions, individuals compete for varying opportunities, with outcomes significantly impacting their future prospects. For an agent, finding the best institution to join entails selecting the right institution with a balance of opportunities and fierce competition for them within the institution.

In several professional sports tournaments, such as tennis tournaments, players want to compete in the most prestigious sports tournament, which not only provides a larger monetary reward but also allows them to compete against the highest-ranked players, with defeating them improving the winner's ranking as well. However, winning the most prestigious sports tournament is clearly more difficult than winning other sports tournaments. Additionally, in the world of chess, to compete in the World Chess Championship (WCC), one of the game's highest honors, players must have a FIDE rating that places them in the top tiers of the global rankings. The FIDE ratings serve as a standard measure of a player's skill, and, correspondingly, their reputation in the chess community. Players who consistently perform well in high-level events earn elite ratings, which guarantee them entry into prestigious tournaments. Winning such a contest is valued not only by the prize money, but also by the prestige of defeating other top-ranked players. When well-known figures in the chess world compete, the contest's visibility grows, as does the significance of the event and the winner. However, winning the World Chess Championship against the best chess players is not an easy task. Similarly, every young lawyer aspires to work for a highly prestigious law firm, which is likely to offer higher salaries and more complex and interesting cases to handle. However, almost every lawyer's ambition is to become a name partner in his firm, and competition for partnership in a highly prestigious law firm is unquestionably more difficult than in less prestigious firms. In all of the situations described above, not all agents are given the option of which institute or contest to participate in, whether more or less prestigious, and this decision is made based on the agents' reputations (skills) and the contest organizers' admission policies.

This paper investigates the conflict between agents who can compete in more or less prestigious contests, with more prestigious contests offering higher rewards but a lower chance of winning. Similarly, the paper investigates the conflict between contest organizers who want to maximize agents' efforts (output) in their contests by selecting the appropriate admission policy. On the one hand, they want more agents who put in more effort, but agents with very low skills may damage the contest's reputation, specifically the value of winning, which reduces the efforts of competing agents. Formally, we study a model of parallel contests with admission policies. There is a finite number of agents with varying reputation types (high and low), which are determined by their previous achievements prior to the start of the model, just as in the previous chess example, but also in almost any competitive environment. Agents can choose which contest to enter, and those who choose the same contest will compete against each other. The winner of each contest receives a prize based on both the fixed monetary award (nominal prize) and the reputations of the agents in the contest. Each contest has an organizer who wants to maximize his payoff, which is equal to the total effort put into the contest. A contest organizer must carefully consider which admission policy will result in the most expected total effort from the contest's agents. On one hand, a permissive admission policy will attract more agents, both high and low-type agents, intensifying competition and potentially increasing total effort. On the other hand, the increased presence of low-type agents reduces the participants' perceived value of winning, resulting in less effort from the agents.

In the context of parallel contests, choosing an admission policy becomes even more complex. This is because each organizer must also carefully consider the cascading effects of a change in admission policy, accounting for not only the effects of allowing lower-reputation agents to enter (in terms of the reduced value of winning in their own contest) but also the possibility that high-reputation agents will leave their contest to join others. This change in agent allocation can then influence the policies of other organizers, altering the distribution of agents across all contests and, as a result, those organizers' admission strategies.

We begin by examining the admission policies in a single contest with agents of either high or low reputation. The contest organizer first chooses an admission policy, determining whether low-type agents will be permitted to participate. The eligible agents then compete, with the winner decided by a Tullock CSF (Tullock, 1980). The value for winning each contest is given by a winning-value function that combines a fixed nominal prize with the reputational values of the competing agents. We show that adding another low-type agent to the contest has a marginal effect on whether participants' effort increases or decreases. Specifically, we demonstrate that, depending on the model's parameters, such as the number of agents of each type and their reputation types, the organizer should select an admission policy that allows low-type agents to participate up to a certain upper bound, and if their number exceeds this limit, the low-type agents are barred from participating in the contest. In the remaining cases, the admission policy permits low-type agents to enter the contest only if their number exceeds a certain threshold, at which point all low-type agents participate.

Later, we analyze parallel contests in which each organizer determines his admission policy. The agents then select a contest in which they wish to compete from those that allow them to participate. We show that, depending on the model's parameters, all combinations of the two organizers' admission policies that allow or restrict low-type agents from the contests can exist as an equilibrium. In particular, we demonstrate the existence of a non-intuitive equilibrium in which the organizer of the lower prestige contest limits the participation of low-type agents while the organizer of the higher prestige contest chooses a policy that allows low-type agents to participate. This equilibrium is attained because the more prestigious contest has a sufficient number of high-reputation agents but not too many, so that the presence of new low-reputation agents results in a limited transfer of high-reputation types to the less prestigious contest. In this case, the low-reputation agents in the more prestigious contest have no significant negative impact on the winning value, whereas their extra effort is significant. As a result, the more prestigious contest accepts low-reputation types, whereas the less prestigious contest does not.

To summarize, our findings shed light on how contest organizers determine admission policies. We determine whether and when admitting low-type agents to a contest has a negative or positive impact on the total effort. In particular, we show that in parallel contests with admission policies, the most prestigious contests may have both a higher and lower admission threshold than less prestigious contests.

1.1 Related literature

Avery et al. (2003) and Avery and Levin (2010) study the role of early admissions in determining which students are admitted. According to Avery et al. (2003), applying early can significantly improve an applicant's chances of admission to a top-tier college, potentially doubling or tripling their chances of acceptance. Furthermore, Avery and Levin (2010) argue that early admissions frequently attract highly qualified students to less prestigious colleges, as these students may be unsure of their academic standing when they apply. Chade and Lewis (2014) observe that as the capacity of lower-ranked colleges declines, they tend to raise their admission standards, sometimes exceeding the standards of higher-ranked colleges. Che and Koh (2016) show that when colleges are unsure about student preferences, they are more likely to admit students who would otherwise be passed over, resulting in fewer admissions for high-ranking students compared to those applying to lower-ranked colleges. According to this literature, the presence of a multi-stage admissions process, capacity constraints, and, in particular, incomplete information can explain why prestigious institutions admit underqualified students. In this paper, we show that in our model of parallel contests even without special admission processes like early admission or capacity constraints, and especially in a model with complete information, there are scenarios in which more prestigious institutions would benefit from setting a lower admission threshold than less prestigious ones.

Competitions between organizers who provide similar objects and compete against one another to attract agents to participate in their organization occur in a variety of settings in the economics literature. For example, in competing parallel auctions, each auction organizer sells some items, and buyers choose which auction to bid on (see, for example, Peters and Severinov 1997, Ellison et al. 2004, and Moldovanu et al. 2008). Similarly to our model, in parallel competing contests, each contest organizer awards prizes to the winners of his contest, and the agents choose which contest to compete in. Parallel contests with homogeneous agents choosing which contests to compete in have been studied, for example, by Konrad and Kovenock (2012), Juang et al. (2020), and Azmat and Möller (2009). Azmat and Möller (2018) study parallel all-pay contests with a continuum of heterogeneous agents, whereas Morgan et al. (2018) study two parallel contests with a continuum of heterogeneous agents and a deterministic contest success function with some noise.

The authors of the preceding models of parallel contests assume that the winners will receive nominal fixed prizes. Damiano et al. (2010), on the other hand, consider a continuum of agents of various types (ability or reputation) who must choose between two organizations, with an agent's payoff determined by both their average reputation and their position within the organization they joined. This implies that the prizes for winning are not fixed and are determined by the agents' allocations across contests. Cohen et al. (2024) investigate parallel contests with a finite number of agents in which the winner's prize is determined by both a nominal prize and the types (reputations) of the competing agents, and the agents' payoffs are determined following agent competition in their contests. In the models presented above, agents are active, whereas contest organizers are passive and uninvolved in the competition. Damiano et al. (2012) expand on their previous model by including a competition between the two organizations, with each organization having the authority to decide how to award prizes. In this paper, we build on our previous work by involving contest organizers who make admissions decisions, specifically which types of agents are allowed to enter their contests. The assumption that both agents and organizers are active complicates and challenges the equilibrium analysis because the agents' and organizers' payoffs are dependent on each other's strategies, despite having completely different objectives.

While we consider the reputation effect in parallel contests, there is theoretical economic literature (e.g., Mailath and Samuelson 2015) that focuses on reputation effects in repeated games in which each player builds his own reputation based on previous actions. Some of these models, for example, assume that a long-term player playing against a series of short-lived opponents can develop a reputation for playing in a specific way and thus benefit from commitment power (see Kreps and Wilson, 1982; Kreps et al., 1982; Milgrom and Roberts, 1982; Cripps et al., 2004; Ely et al., 2008). In contrast, in our parallel contests, an agent's reputation is fixed and established prior to the beginning of the competition, so it is unaffected by the agents' actions during the competition.

We assume that the agents' winning value in each contest is a combination of a fixed nominal prize and their respective reputation values. In other words, the higher the agents' reputation values in their competition, the greater their winning value. Our assumption about the winning value is similar to those in the literature on status in contests, such as Moldovanu et al. (2007) and Dubey and Geanakoplos (2010), who assume that agents receive a positive utility proportional to the number of agents in lower status categories and a negative utility proportional to the number of agents in higher status categories. As a result, the number of agents in lower-reputed categories who do not belong to their category increases the winning value of the agents in their own category.

Endogenous agent entry is a well-known method for increasing agent effort (see, for example, Fu and Lu, 2010, Cason et al. 2010, Megidish and Sela, 2013, Fu et al., 2015, and Stouras et al., 2022). In such cases, an agent's participation in a contest is defined endogenously, as agents are required to produce a minimum quality and quantity of output in order to participate in the contest. A minimum effort constraint eliminates weak players (low-type agents), and as a result, competition intensifies, causing players to increase their efforts. Indeed, Myerson (1981) found that the all-pay contest under incomplete information with the optimal participation constraint maximizes the contestants expected total effort. Furthermore, Laffont and Robert (1996) demonstrated that an all-pay contest with a reserve price is a revenue-maximizing mechanism for selling an object to agents who have linear costs and a common-knowledge budget constraint. In our model, we assume that the agent types are commonly known, and that each contest organizer decides on his admission policy, whether or not to allow agents to participate based on their types rather than their efforts or, alternatively, their outputs.

In the literature on contest theory, contest organizers' goals are commonly stated to maximize the agents' total effort (see, for example, Moldovanu and Sela, 2001, Moldovanu et al. 2012, Franke et al. 2013, Olszewski and Siegel 2020, and Zhang 2024). When the number of agents in a contest is determined exogenously, maximization of total effort and average effort are equivalent. However, in our parallel contests, agents can choose which contest to compete in, and the contest organizers have admission policies, so the goals of maximizing total effort and average effort differ. We focus on maximizing total effort; otherwise, if the contest organizer wants to maximize average effort, the analysis of our model becomes trivial because each of the organizers would choose an admission policy that restricts participation of low-type agents.

The rest of the paper is organized as follows: Section 2 describes a single contest with an admissions policy. Subsection 2.1 analyzes the second stage's equilibrium efforts, while subsection 2.2 analyzes the single organizer's admission policy. In Section 3, we study the model of parallel contests with admission policies, in subsections 3.1-3.3, we examine all possible admission policy combinations for the organizers, and subsection 3.4 depicts the distribution of all these admission policy combinations. Section 4 concludes. The appendix contains the proofs.

2 A single contest

Consider a set of n asymmetric agents, $N = \{1, 2, \ldots, n\}$. Each agent $i \in N$ is either a high-type (h-type) agent with a reputation of α_h or a low-type (l-type) agent with a reputation of α_l where $\alpha_h \geq \alpha_l$. In the first stage, a contest organizer can decide whether or not to allow l-type agents from participating in the contest. If the contest organizer allows the l-type agents to participate, it will be all or nothing; he cannot allow only a subset of the l-type agents to participate. Denote the set of agents that are admitted into the contest as $\hat{N} \subseteq N$, where the number of agents that are admitted into the contest is $\hat{n}$, $\hat{n} \leq n$. In the second stage, the agents who are allowed to compete in the contest compete against one another to win the contest. The agents simultaneously exert their efforts. Let x_i denote the effort of agent $i \in \hat{N}$. The probability that agent i wins the contest is $p_i = \frac{x_i}{\sum_{j \in \hat{N}} x_j}$. When agent i exerts effort x_i, he incurs a cost $c_i(x_i) = c_i x_i$, which he bears regardless of the contest outcome, The winning value for the contest is a function $f(v, \bar{\alpha})$, where v is a nominal prize and $\bar{\alpha} = (\alpha_i)_{i \in \hat{N}}$ is a vector of all reputation types of the agents admitted to the contest. The winning value function $f(v, \bar{\alpha})$ is assumed to increase with both the nominal prize v and each of the agents' reputation types $\alpha_i \in \bar{\alpha}$. It is also assumed that adding a probability that agent i wins the contest is $p_i = \frac{x_i}{\sum_{j \in \hat{N}} x_j}$. When agent i exerts effort x_i, he incurs a cost $c_i(x_i) = c_i x_i$, which he bears regardless of the contest outcome, The winning value for the contest is a function $f(v,\bar{\alpha})$, where v is a nominal prize and $\bar{\alpha}=(\alpha_i)_{i\in\hat{N}}$ is a vector of all reputation types of the agents admitted to the contest. The winning value function $f(v,\bar{\alpha})$ is assumed to increase with both the nominal prize v and each of the agents' reputation types $\alpha_i \in \bar{\alpha}$. It is also assumed that adding a low-type agent to the contest does not increase the value function, while adding a high-type agent does not decrease it. Let $f(v,\bar{\alpha})$ denote the winning value for agents with a vector of reputation types $\bar{\alpha}=(\alpha_i)$, and $f(v,\bar{\alpha},\alpha_\gamma)$ denotes the value function of the same group of agents with an additional agent γ, whose reputation type is α_γ. Then we have

$$\text{If } \alpha_\gamma \geq \max(\bar{\alpha}), \text{ then } f(v,\bar{\alpha}) \leq f(v,\bar{\alpha},\alpha_\gamma),$$

and

$$\text{If } \alpha_\gamma \leq \min(\bar{\alpha}), \text{ then } f(v,\bar{\alpha}) \geq f(v,\bar{\alpha},\alpha_\gamma).$$

The contest organizer's payoff is the sum of all agents' efforts in the second stage. This will be referred to as a contest with an admission policy.

To analyze the contest dynamics, we begin with the second stage, in which the organizer has already determined the admission policy, specifically whether or not l-type agents will be admitted to the contest. In this stage, each agent chooses an effort level to compete against the other agents. Based on this analysis of the second stage, we examine the organizer's decision in the first stage, when he decides whether or not to allow participation of l-type agents.

2.1 The second stage

The maximization problem of agent $i \in \hat{N}$ in the second stage of the contest is

$$\max_{x_i} f(v,\bar{\alpha})\frac{x_i}{\sum_{j\in\hat{N}} x_j} - c_i x_i. \tag{1}$$

The well-known solution of the maximization problem (1) yields,

Proposition 1 *In the second stage of the contest with admission policy, for every $i \in \hat{N}$, the equilibrium effort is*

$$x_i = p_i \sum_{j \in \hat{N}} x_j,$$

where p_i is agent i's winning probability and is

$$p_i = 1 - \frac{(n-1)c_i}{\sum_{j \in \hat{N}} c_j}. \tag{2}$$

Agent i's expected payoff is

$$u_i = f(v, \bar{\alpha})(p_i)^2, \tag{3}$$

and the total effort is

$$TE = f(v, \bar{\alpha}) \frac{(n-1)}{\sum_{i \in \hat{N}} c_i}. \tag{4}$$

So far, we have analyzed the competition in the second stage. In the following section, we examine the first stage of this competition, with a focus on the impact of agents' reputations and abilities on the organizer's decision to allow these agents to compete in the second stage.

2.2 The first stage

In the first stage, the contest organizer has two options: allow or restrict low-type agents from entering the contest. When low-type agents enter the contest, there are two opposite effects on the agents' total effort. On the one hand, assuming the low-type agents are active, they contribute to the contest, increasing the total effort. On the other hand, because their reputation is lower than that of high-type agents, the value of winning is reduced when they enter the contest, reducing the agents' efforts. Thus, in the initial stages, the organizer decides on his admission policy by determining which effect is stronger. It is well known that when new agents enter a Tullock contest with a winning value based on a nominal value rather than the agents' reputation, the total effort increases despite the fact that each agent's effort falls. As a result, the organizer should only consider excluding low-type agents if their reputation reduces their winning value. Otherwise, we get the following immediate result:

Proposition 2 *In a contest with an admissions policy, if the value of winning does not decrease when l-type agents enter the contest, the organizer who wishes to maximize the agents' total effort will select an admission policy that does not exclude l-type agents from participating.*

If, on the other hand, the introduction of low-type agents to the contest, has a large negative effect on the value for winning, the organizer might consider a policy that restricts them from participating in the contest.

Proposition 3 *In a contest with an admissions policy, if there are a sufficient number of h-type agents and there exists:*

$$\frac{\partial}{\partial h}\left[\frac{f(v,\bar{\alpha}_h,\bar{\alpha}_l)}{f(v,\bar{\alpha}_h)}\right] < \frac{l(2hc_l + lc_l - (c_l - c_h)h^2 - c_l)}{h^2 c_h (h+l-1)^2}, \tag{5}$$

then, the contest organizer will choose an admission policy that restricts the l-type agents from participating in the contest.

According to Proposition 3, If there are a sufficient number of h-type agents in a contest with an admissions policy, and the inequality (5) holds, then, by (4), admission of l-type agents into the contest decreases the total effort, since we have

$$f(v,\bar{\alpha}_h,\bar{\alpha}_l)\frac{(h+l-1)}{hc_h + lc_l} - f(v,\bar{\alpha}_h)\frac{h-1}{hc_h} < 0.$$

According to Proposition 3, if an elite educational program has a sufficient number of high-qualified agents, adding some low-qualified agents may have a significant negative effect on the program's reputation, significantly decreasing the output of highly-qualified agents, while these additional new low-qualified agents' contribution is insufficient, resulting in a reduction of the total output.

The number of high-type agents is not the only factor influencing l-type agents' entry into the contest. The proposition below shows that if high-type agents are strong enough (they have low enough cost of effort), the reputation effect on the value of winning outweighs the effect of increasing the number of agents on total effort.

Proposition 4 *In a contest with an admission policy, if the h-type agents have a sufficiently low marginal cost, the organizer will choose to exclude the l-type agents from the contest.*

So far, we have shown that the number and abilities of h-type agents are critical in determining the organizer's policy; however, the characteristics of the winning value function are also important in determining which admission policy is more profitable.

Proposition 5 *In a contest with an admission policy, if the value of winning by entering l-type agents is less than or equal to half of the value of winning with only h-type agents, the organizer will always choose a policy that restricts the participation of l-type agents in the contest.*

2.2.1 The weighted average value function

In the following, we consider a set of multiplicative winning value functions based on the nominal value and the weighted average of the agents' reputations. This allows us to obtain critical conditions for determining which of the effects, reputation, or the number of agents has a greater impact on the total effort in the contest.

Definition 1 *Denote the weighted average winning value function*

$$f^A(v, \bar{\alpha}) = v \frac{\sum_{i \in \hat{N}} \alpha_i w_i}{\sum_{i \in \hat{N}} w_i},$$

where the weight of agent $i \in \hat{N}$ is w_i and the vector of weights is $\bar{w} = (w_1, w_2, ..., w_n)$.

The definition above describes an extensive group of winning value functions that depend on the values of the vector of weights $\bar{w} = (w_1, w_2, ..., w_n)$. This group includes, for example, the winning value function $f^A(v, \bar{\alpha}) = v \cdot \max\{\alpha_i \mid i \in \hat{N}\}$ and $f^A(v, \bar{\alpha}) = v \cdot \min\{\alpha_i \mid i \in \hat{N}\}$. This type of multiplicative winning value function allows the winning value to be adjusted based on the composition of the agents' reputation types and various exogenous weights. The weighted average winning value function reflects the fact that having more agents of a particular type increases their influence on the value of winning the contest. Substituting the weighted average winning value function into (4), gives the total effort of the agents in the second stage.

$$TE = v \frac{\sum_{i \in \hat{N}} \alpha_i w_i}{\sum_{i \in \hat{N}} w_i} \frac{(n-1)}{\sum_{i \in \hat{N}} c_i}. \tag{6}$$

As previously stated, multiple parameters influence the organizer's decision to allow or restrict low-type agents from participating in the contest. When dealing with a specific type of winning value function, we can quantify the effects of each parameter on the admissions policy. The number of l-type agents is a parameter that, of course, influences admission policy; on the one hand, having more l-type agents can result in a lower value for winning and thus a lower total effort. On the other hand, more l-type agents imply more agents competing and exerting more effort, which may be sufficient to compensate for the decrease in effort caused by the lower value of winning. The following proposition shed some light on which of the aforementioned effects may have a greater impact on the overall effort, and thus on the admission policy.

Proposition 6 *In a contest with an admission policy and a weighted average winning value function, let*

$$l^* = \max\left(0, h \frac{1 - \frac{\alpha_l}{\alpha_h} - \left(\frac{h}{h-1} - \frac{c_l}{c_h}\right)\frac{w_h}{w_l}}{\frac{\alpha_l}{\alpha_h}\frac{h}{h-1} - \frac{c_l}{c_h}}\right). \tag{7}$$

Then, we have the following two cases:

Case A: if,

$$h < \frac{\alpha_h c_l}{\alpha_h c_l - \alpha_l c_h}, \tag{8}$$

then the contest organizer will allow the l-type agents to participate iff $l > l^*$.

Case B: if,

$$h > \frac{\alpha_h c_l}{\alpha_h c_l - \alpha_l c_h}, \tag{9}$$

then the contest organizer will allow the l-type agents to participate iff $l < l^*$.

The result of Proposition 6 shows how complex the effect of the number of l-type agents on admission policy is. If the number of h-type agents is large enough (case B), the effect of the reduction in the value for winning on the total effort outweighs the effect caused by the additional efforts of the new l-type agents. On the other hand, if the number of h-type agents is not too large (case A), the effect of the reduction in the value for winning on the total effort can be offset if enough l-type agents enter the contest. In other words, if the number of high-type agents is large enough, low-type agents may be inefficient because their effect on total effort is negative; on the other hand, if the number of high-type agents is small enough, low-type agents are efficient because their effect on total effort is positive. It is important to note that the nature of this criterion, that is, whether we require the minimum or maximum number of l-type agents, is independent of the weights of the agents w_h and w_l. The weights, however, affect the value of the critical number l^*.

In the following example, we show the dynamics of Proposition 6, how, in one case, l-type agents are allowed to admit the contest, but in another case, these agents are restricted from participating in the contest.

Example 1 *Consider a contest with an admissions policy involving $h \geq 1$ high-type agents and $l \geq 1$ low-type agents. The contest has a weighted average winning value function $f^A(v, \bar{\alpha}) = v\frac{w_h h\alpha_h + w_l l\alpha_l}{w_h h + w_l l}$, where $v = 1$, and w_h and w_l are the weights of the high-type and low-type agents, respectively. Assume also that $\alpha_l = c_l = w_l = 1$.*

If the policy is to restrict participation of the l-type agents, by (6), the total effort is

$$\alpha_h \frac{h-1}{hc_h},$$

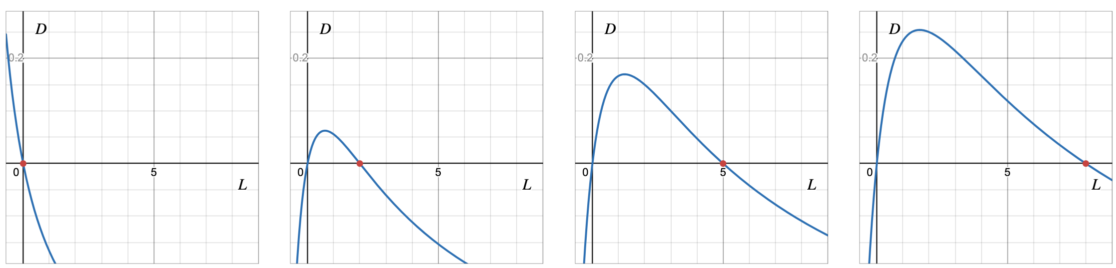
while, if the policy is to allow participation of the l-type agents, the total effort is

$$\frac{w_h h\alpha_h + w_l l\alpha_l}{w_h h + w_l l}\frac{(h+l-1)}{hc_h + lc_l}.$$

The difference of the total effort in these two cases is

$$D(h, l, \alpha_h, \alpha_l, c_h, c_l, w_h, w_l) = \frac{w_h h\alpha_h + w_l l\alpha_l}{w_h h + w_l l}\frac{(h+l-1)}{hc_h + lc_l} - \alpha_h \frac{h-1}{hc_h}. \qquad (10)$$

The contest organizer will allow l-type agents to participate if $D(\cdot) > 0$, otherwise they will be restricted.

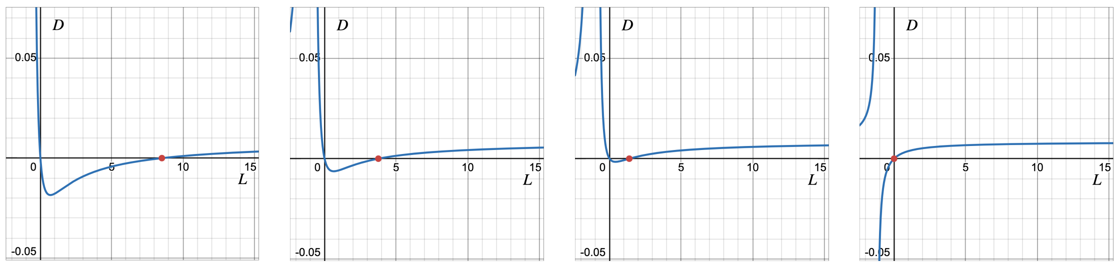


(a) $w_h = \frac{1}{2}$ (b) $w_h = 1$ (c) $w_h = 1\frac{1}{2}$ (d) $w_h = 2$

Figure 1: $D(\cdot)$ as a function of l when $h = 2, \alpha_h = 3, c_h = 1$ and varying w_h.

Figure **??** *illustrates a scenario where case B of Proposition 6 holds, meaning that, the critical number of the l-type agents, l^* given by (7), is the maximum number of l-type agents that the contest organizer will allow to participate in the contest. We can see that in figure* 1a*, the organizer does not want any l-type agents, while, in figure* 1b *the maximum number of l-type agents that are allowed to participate is $l^* = 2$, and in figure* 1d*, the criterion changes to $l^* = 8$.*

Figure **??** *shows a scenario where case A of Proposition 6 holds, meaning that the critical number of the l-type agents, l^* given by (7), is the minimum number of l-type*



(a) $w_h = 0.3$ (b) $w_h = 0.4$ (c) $w_h = 0.45$ (d) $w_h = 0.5$

Figure 2: $D(\cdot)$ as a function of l when $h = 2, \alpha_h = 1.19, c_h = 0.6$ and varying w_h.

agents that the contest organizer will allow to participate in the contest. We can see that in figure 2d, the organizer allows any number of l-type agents, while, in figure 2a, the minimum number of l-type agents that are allowed to participate is $l^* = 8.52$ meaning that at least nine l-type agents have to join the contest and less than that it is not profitable.

Example 1 shows that the contest organizer's policy is determined by the agents' reputations, abilities (costs), and the number of h-type agents in the contest. The example emphasizes that, depending on the competition's parameters, adding low-type agents to the contest may have a negative impact on the total effort, while in other cases, the effect of the additional agents may be positive.

So far, we have examined a single contest with an admission policy. The next section examines parallel contests with admission policies. In such a case, two organizers must consider their opponent's policy, namely, whether an organizer's policy allows or restricts the participation of l-type agents; this may be determined by the other organizer's policy regarding the participation of these agents. Furthermore, the admission policy of the organizers of each of the two contests has a significant impact on the agents' choice of which contest to compete in, particularly for h-type agents who have no restrictions on contest selection.

3 Parallel contests

Consider a set of n asymmetric agents, $N = \{1, 2, \ldots, n\}$, and a set of two asymmetric contests, $M = \{1, 2\}$. Similarly to the model of a single contest, each agent i in the set N is either a high-type (h-type) or a low-type (l-type) agent where the h-type agents have a reputation of α_h and the l-type agents have a reputation of α_l where $\alpha_h \geq \alpha_l$. Each of the two contests $j \in M$ has a nominal prize v_j where $v_1 \geq v_2$. In the first stage, the organizers of both contests decide on their admission policy at the same time, namely whether low-type agents can participate in the contest or not. In the second stage, given the participation constraints imposed by the contest organizers, each agent chooses a contest to compete in. The l-type agents choose only from the contests that allow their participation. The set of agents who choose to compete in contest k is denoted by N_k, and n_k is the number of agents who choose to compete in contest k. In the third stage, after all of the agents have selected a contest to compete in, agents who have chosen the same contest compete against one another to win the contest. The agents in contest k simultaneously exert their efforts. Let x_i denote the effort of agent $i \in N_k$. The probability that agent i wins contest k is $p_i = \frac{x_i}{\sum_{j \in N_k} x_j}$. Agent i who exerts an effort of x_i bears the cost of his effort $c_i(x_i) = c_i x_i$ regardless of whether he wins or loses. In addition, the winning value for the winner of contest k is the weighted average winning value function $f^A(v_k, \bar{\alpha}_k) = v_k \frac{\sum_{i \in N_k} \alpha_i w_i}{\sum_{i \in N_k} w_i}$ where v_k is the nominal prize of contest k, $\bar{\alpha_k} = (\alpha_i)_{i \in N_k}$ is the vector of all reputation types of the agents who have chosen contest k and $\bar{w}_k = (w_1, w_2, ..., w_{n_k})$ is the weights vector. This competition will now be referred to as parallel contests with admission policies.

In the first stage, both contest organizers simultaneously decide on their admissions policy, which determines whether or not l-type agents can participate in their contest. A 2x2 game in which the contest organizers can restrict or allow the entry of l-type agents exemplifies the interaction between the two contest organizers. This results in a matrix presented in table **??**, which contains four possible cases A-D.

When contest organizers consider their admissions policies, specifically whether

		Contest 2	
		Restrict Low-Type	Allow Low-Type
Contest 1	Restrict Low-Type	A	C
	Allow Low-Type	D	B

Table 1: The possible types of equilibriums for interaction of two organizers

to allow l-type agents to compete, they must consider several potential consequences. First, they should consider the effects discussed in the preceding section: the reputation effect, which lowers the value of winning and thus the total effort, and the effect of additional agents, which raises the total effort. However, unlike in the previous section, the agents are now competing in a multi-contest environment, which may provide an incentive to choose one contest over the other based on their expected payoffs in each contest. As a result, when deciding on an admission policy, a contest organizer must also consider the movement of agents into and out of the contest.

To begin the analysis and characterize the sub-game perfect equilibrium, we should start with the third stage, in which all agents who choose the same contest compete against one another by determining how much effort they exert to win the contest. However, because the analysis of the third stage is identical to that of the second stage presented in the previous section of a single contest, we can skip this stage and begin our analysis with the second stage, in which the agents choose which of the contests they will participate in.

Denote the decision of the organizer of contest $k, k = 1, 2$ in the first stage as

$$D_i = \begin{cases} 1, & l\text{-type agents allowed to participate in contest } i. \\ 0, & \text{Else.} \end{cases}$$

In contrast to the l-type agents, the h-type agents are allowed to compete in each contest regardless of the decision of the organizers. The number of l-type agents who choose to compete in each contest is determined by the number of h-type agents in that contest, as well as the admission policies of the organizers. Given an allocation of l-type agents in the contests (l_1, l_2), the number of h-type agents in each contest

(h_1, h_2) is the positive integers that satisfy,

$$u_1^h(h_1, l_1) > u_2^h(h_2 + 1, l_2) \quad \text{and} \quad u_2^h(h_2, l_2) > u_1^h(h_1 + 1, l_1) \tag{11}$$

That is, moving one h-type from contest i to contest j does not result in a higher expected payoff. The l-type agents, on the other hand, are mostly bound by the organizers' decisions in three out of four cases. Only if $(D_1, D_2) = (1, 1)$, namely, case B, the l-type agents are given the option to choose which contest they wish to participate in. In this scenario, the numbers of l-type agents in each contest (l_1, l_2) are integers satisfying:

$$u_1^l(h_1, l_1) > u_2^l(h_2, l_2 + 1) \quad \text{and} \quad u_2^l(h_2, l_2) > u_1^l(h_1, l_1 + 1) \tag{12}$$

That is, moving one l-type from contest i to contest j does not result in a higher expected payoff. We denote the solutions of (**??**) and (**??**) as (h_1^*, h_2^*) and (l_1^*, l_2^*), respectively.

	$D_2 = 0$	$D_2 = 1$
$D_1 = 0$	$(h_1, h_2, l_1, l_2) = (h_1^*, h_2^*, 0, 0)$	$(h_1, h_2, l_1, l_2) = (h_1^*, h_2^*, 0, l)$
$D_1 = 1$	$(h_1, h_2, l_1, l_2) = (h_1^*, h_2^*, l, 0)$	$(h_1, h_2, l_1, l_2) = (h_1^*, h_2^*, l_1^*, l_2^*)$

Table 2: The allocation of the agents for every organizers' decision

In the first stage, the contest organizers can consider which policy will result in the highest total effort given the agent distribution between the contests, as shown in Table **??** This is a 2x2 game where each organizer's payoff is the agents' total efforts in subsequent stages. In the following, we examine each of the four cases A-D and demonstrate under what conditions each of them can be implemented.

3.1 Equilibrium of type A

Equilibrium of type A occurs when both contest organizers adopt a policy that restricts l-type agents' participation in their contests. The equilibrium analysis of such a case requires determining the agents' allocations for each of cases A-D and then demonstrating under what conditions the total effort exerted in both contests causes both organizers to restrict the participation of low-type agents in their contests. However, in order to find sufficient conditions for the existence of equilibrium of type A, the analysis can be simplified, and we begin by demonstrating how h-type agents select contests in the second stage, assuming that l-type agents are not allowed to participate in both contests.[1]

Lemma 1 *In parallel contests with admission policies, if, in the first stage, both organizers choose a policy that restricts the participation of l-type agents in their contests, the number of h-type agents in each contest in the second stage is given by* $(h_1^*, h_2^*) \in \{(\left\lfloor \tilde{h_1} \right\rfloor, \left\lceil \tilde{h_2} \right\rceil), (\left\lceil \tilde{h_1} \right\rceil, \left\lfloor \tilde{h_2} \right\rfloor)\}$ *where $\tilde{h_1}, \tilde{h_2}$ are given by:*

$$\begin{aligned} \tilde{h_1} &= \frac{h\sqrt{v_1}}{\sqrt{v_1}+\sqrt{v_2}} \\ \tilde{h_2} &= \frac{h\sqrt{v_2}}{\sqrt{v_1}+\sqrt{v_2}}. \end{aligned} \tag{13}$$

According to Proposition 3, if there are enough h-type agents participating in a contest and the condition (5) is satisfied, the organizer of this contest will prefer to restrict the participation of l-type agents. Furthermore, according to Lemma **??**, if both contest organizers choose a policy that restricts l-type agents from participating, the number of h-type agents who choose each contest increases with the total number of h-type agents. This means that if the total number of h-type agents is large enough, at some point in the parallel contest, the number of h-type agents in both contests given by (**??**) will be large enough for the organizers of both contests to prefer to restrict l-type agents from their contests. This argument implies the following sufficient condition for the existence of equilibrium of type A.

[1] The proof of Lemma is a modification of the Cohen et al. (2024) proof for our model.

Proposition 7 *In parallel contests with admission policies, if there are enough h-type agents and the condition (5) is satisfied, there is an equilibrium in which both organizers select an admission policy that restrict l-type agents from participating.*

According to Proposition **??**, an equilibrium of type A exists if the total number of h-type agents is large enough. However, the following example demonstrates that the critical number of h-type agents for the existence of an equilibrium of type A does not necessarily need to be very large.

Example 2 *Consider parallel contests with admissions policies. The nominal prizes are $v_1 = 4$ and $v_2 = 1$ for contests 1 and 2, respectively. The types of agents are specified in the following table:*

	h-type	**l-type**
Amount	6	2
c_i	0.9	1
α_i	3	1
w_i	1	1

Table 3: The parameters of the h-type and the l-type agents

*According to Lemma **??**, if the organizers choose policies that restrict the l-type agents from participating in the contests, the h-type agents will be distributed between the two contests as follows:*

$$h_1 = \frac{h\sqrt{\alpha_h v_1}}{\sqrt{\alpha_h v_1} + \sqrt{\alpha_h v_2}} = \frac{6\sqrt{4}}{\sqrt{4} + \sqrt{1}} = 4 \tag{14}$$

$$h_2 = \frac{h\sqrt{\alpha_h v_2}}{\sqrt{\alpha_h v_1} + \sqrt{\alpha_h v_2}} = \frac{6\sqrt{1}}{\sqrt{4} + \sqrt{1}} = 2. \tag{15}$$

According to Proposition 6 a contest organizer wants to restrict l-type agents from his contests if the number h of high-type agents in his contest is equal to or larger than,

$$\overline{h} = \frac{\alpha_h c_l}{\alpha_h c_l - \alpha_l c_h} = \frac{3 \cdot 1}{3 \cdot 1 - 1 \cdot 0.9} = 1.4. \tag{16}$$

Because that $\overline{h}$ is smaller than h_1 and h_2, both organizers will restrict participation of low-type agents in their contests to less than $l_i^*, i = 1, 2$, as given by (7). In our case,

$$l_i^* = h_i \frac{1 - \frac{\alpha_l}{\alpha_h} - (\frac{h_i}{h_i - 1} - \frac{c_l}{c_h})\frac{w_h}{w_l}}{\frac{\alpha_l}{\alpha_h}\frac{h_i}{h_i - 1} - \frac{c_l}{c_h}} = h_i \frac{1 - \frac{1}{3} - (\frac{h_i}{h_i - 1} - \frac{1}{0.9})\frac{1}{1}}{\frac{1}{3}\frac{h_i}{h_i - 1} - \frac{1}{0.9}} = -h_i \frac{7h_i - 16}{7h_i - 10}.$$

This implies that,

$$l_1^* = -4\frac{7 \cdot 4 - 16}{7 \cdot 4 - 10} = -2\frac{2}{3}$$
$$l_2^* = -2\frac{7 \cdot 2 - 16}{7 \cdot 2 - 10} = 1.$$

As a result, both contest organizers will restrict l-type agents' participation, despite the fact that there are only 6 h-type agents in both contests.

3.2 Equilibrium of type B

In this case, the organizers of both contests adopt an admission policy that allows l-type agents to compete. Such an equilibrium can be found in a variety of simple scenarios. Below we provide such a condition for the existence of an equilibrium of type B. It is well known that in the standard Tullock contest with homogeneous agents, the total effort increases with the number of agents, which is also true in our model when all agents have the same reputation type. Thus, by the continuation of the agents' equilibrium effort in the agents' reputation types, we obtain

Proposition 8 *In parallel contests with admission policies, if the reputations of the h-type and l-type agents are sufficiently close, there exists an equilibrium of type B in which both organizers select an admission policy that allows l-type agents to participate in their contests.*

3.3 Equilibria of types C and D

Type C and D equilibria are asymmetric equilibria in which one organizer selects a policy that restricts the participation of low-type agents in his contest while the other organizer allows them to participate. The more intuitive equilibrium is type C, which means that the organizer of the contest with the lower nominal prize allows low-type agents to participate, whereas the more prestige contest with the higher nominal prize restricts their participation due to their negative impact on the total effort in his contest. However, as we will see later in this section, both types C and D can exist in equilibrium. Before delving into the specific conditions under which each type of equilibrium exists, consider one sufficient condition for which organizers prefer to allow (restrict) the participation of low-type agents.

Lemma 2 *In parallel contests with admissions policies, if contest j restricts l-type agents from participating, the organizer of contest $i, i, j \in \{1, 2\}, i \neq j$ will choose to allow their participation if*

$$\left\lceil \frac{h\sqrt{v_i}}{\sqrt{v_1} + \sqrt{v_2}} \right\rceil < \frac{1}{1 - \frac{\alpha_l}{\alpha_h}\frac{c_h}{c_l}\frac{l-1}{l}}.$$

Lemma **??** provides the first result that can sustain a scenario where an organizer would prefer to let l-type agents participate in the contest, even if it means losing all h-type agents to the other contest. The condition for such a scenario, as shown in Lemma **??**, depends on the agents' parameters, weights, and the proportion of h-type to l-type agents. The primary complexity of analyzing equilibria of types C and D stems from agents' allocations in the second stage between contests.

Assume a contest organizer chooses a policy that allows l-type agents to participate. It is highly likely that some or all of the l-type agents will participate in his contest. In response to the new l-type agents, h-type agents in this contest may now consider leaving and joining the other contest, as the value of winning decreases and competition intensifies. The departure of h-type agents reduces the winning value in this contest, so l-type agents may change their preferences and follow the h-type agents to the other contest. Nonetheless, the following propositions show that such agent movements in loops can stop at some points, implying the existence of equilibria C and D. We begin with the more intuitive equilibrium of type C.

Proposition 9 *In parallel contests with admissions policies, there may be an equilibrium of type C in which the organizer of contest 1 with the higher nominal prize chooses to restrict the participation of l-type agents from his contest while the organizer of contest 2 with the lower nominal prize allows their participation.*

Proof. Proposition **??** can be proven by finding two parallel contests with specific parameters that show the existence of equilibrium of type C. For this purpose, consider two parallel contests with admissions policies, with prizes of $v_1 = 10$ and $v_2 = 1$ in contests 1 and 2. The rest of the parameters are specified in Table **??**.

	h-type	**l-type**
Amount	10	15
c_i	0.95	1
α_i	1.1	1
w_i	1	1

Table 4: The parameters of the h-type and the l-type agents

Table **??** shows the payoffs of the contest organizers for each of the four cases A-D.

		Contest 2	
		Restrict Low-Type	Allow Low-Type
Contest 1	Restrict Low-Type	$A(10.131, 0.578)$	$C(10.421, 0.933)$
	Allow Low-Type	$D(9.937, 0.868)$	$B(10.282, 0.888)$

Table 5: The expected payoffs of the organizers

We can see that the organizer of contest 2 always wants to allow l-type agents to participate in his contest. The organizer of contest 1 prefers a policy that prevents them from participating. This means that the only possible equilibrium is of type C.[2] ■

Proposition **??** demonstrates the existence of an equilibrium of type C, where l-type agents can only participate in the contest with the lower nominal prize and not in the contest with the higher nominal prize. The following proposition demonstrates that, in some cases, the opposite occurs: the contest with the higher nominal prize is the only contest that admits l-type agents, whereas the contest with the lower nominal prize restricts their participation.

Proposition 10 *In parallel contests with admissions policies, there may be an equilibrium of type D in which the organizer of contest 1 with the higher nominal prize allows l-type agents to participate in his contest while the organizer of contest 2 with the lower nominal prizes restricts their participation.*

Proof. Proposition **??** can be proven by finding two parallel contests with specific parameters that show the existence of equilibrium of type D. For this purpose, consider two parallel contests with admissions policies, with prizes $v_1 = 10$ and $v_2 = 1$ in contests 1 and 2. The rest of the parameters are specified in Table **??**.

	h-type	**l-type**
Amount	10	15
c_i	0.99	1
α_i	1.1	1
w_i	1	1

Table 6: The parameters of the h-type and the l-type agents

Table **??** shows the payoffs of the contest organizers for each of the four cases A-D.

We can see that the contest 1 organizer is always willing to allow l-type agents to participate in his contest. The organizer of contest 2, however, does not wish to allow their participation. This means that the only possible equilibrium is of type D.[3] ■

[2]The Appendix contains the complete proof, including all of the calculations.
[3]The Appendix contains the complete proof, including all of the calculations.

		Contest 2	
		Restrict Low-Type	Allow Low-Type
Contest 1	Restrict Low-Type	$A(9.722, 0.555)$	$C(10, 0.933)$
	Allow Low-Type	$D(9.761, 0.888)$	$B(10.025, 0.833)$

Table 7: The expected payoffs of the organizers

3.4 A distribution of equilibrium types

So far, we have shown that there is an equilibrium of all types, A-D. However, the existence of a subgame-perfect equilibrium based on pure strategies is not always guaranteed. For example, this may occur when organizer 1 always wants to choose the opposite policy than organizer 2, while organizer 2 always wants the same policy as organizer 1. Given that there are five possible outcomes (cases A-D and no equilibrium) in parallel contests with admission policies, it is unclear how common each of these outcomes is. Below, we describe and illustrate the distribution of each type of outcome.

As previously stated, determining the outcome of simultaneous contests with admission policies, even for a specific case with numerical parameters, requires numerous calculations. This is primarily because the allocations of agents across contests in the second stage can change dramatically if an organizer changes his admission policy. Consequently, we have implemented a computer simulation that simulates the results of parallel contests with admission policies. The following example shows the distribution of possible outcomes.

Example 3 *Consider two parallel contests with admissions policies, with nominal prizes of $v_1 = 10$ and $v_2 = 1$. There are ten h-type and ten l-type agents. The parameters for l-type agents are $c_l = \alpha_l = w_l = 1$. The simulation involves varying the values of the parameters w_h and α_h for the h-type agents, with $c_h = 0.98$. Figure* **??** *shows the equilibrium outcomes.*

Example **??** illustrates that, given the reputation and weight of h-type agents, all possible outcomes are achievable. When h-type agents' reputations are sufficiently

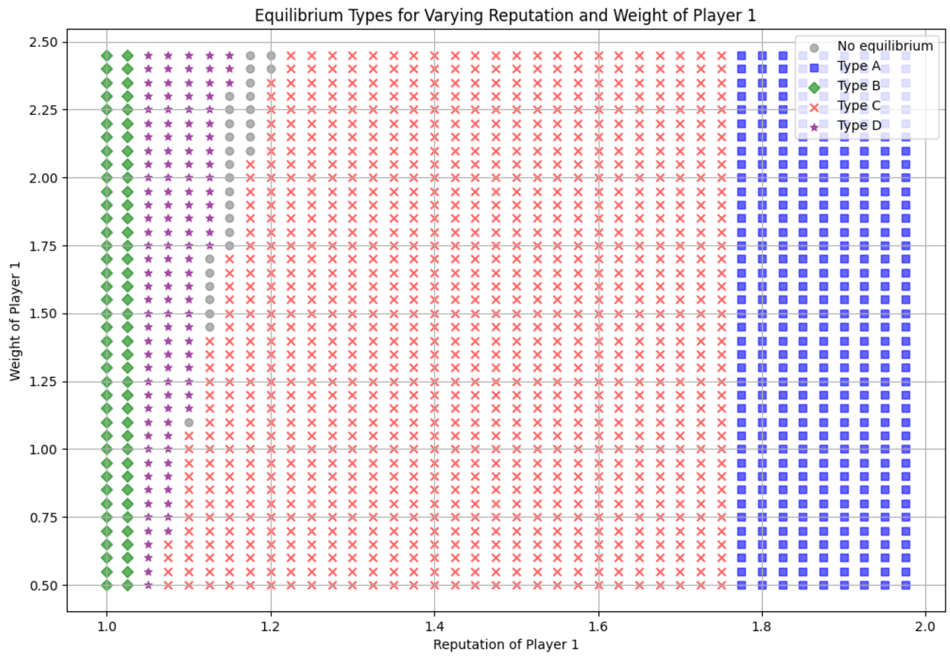


Figure 3: Equilibrium outcomes with varying w_h and α_h.

high in comparison to those of l-type agents, both organizers choose to exclude l-type agents, resulting in an equilibrium of type A. As the reputation of h-type agents declines but remains relatively high, admitting l-type agents becomes appealing to contest 2's organizer because their inclusion does not significantly reduce the contest's perceived prestige. In contrast, for the organizer of contest 1, the loss of prestige from admitting l-type agents outweighs the benefit of increased participation, resulting in an equilibrium of type C.

As the reputation of h-type agents declines further, the opposite dynamic emerges: contest 1, the more prestigious contest, maintains its perceived prestige by ensuring a sufficient number of high-reputation participants, even when low-reputation agents are included. In this case, the participation of l-type agents benefits organizer 1 more than it harms the contest's prestige. In contrast, for contest 2, the less prestigious contest, organizer 2 finds it untenable to include l-type agents which results in equilibrium of type D. Finally, when the reputations of h-type and l-type agents are nearly equal, the equilibrium shifts to type B, in which both organizers accept l-type agents. In this case, low-reputation agents have little impact on the prestige levels of the contests, so their inclusion is acceptable to both organizers.

In Example **??**, equilibrium types are distributed for two parallel contests with specific parameters. However, we can learn that the equilibrium types of A and B occur for extreme reputations of the high-type agents, either large (type A) or small (type B).ining range of reputations is dominated by type C equilibrium, with type D, the paradoxical equilibrium, being the least common.

Despite the fact that in Example **??** the distribution of equilibrium types has been done for two parallel contests with specific parameters we can learn that the equilibrium types of A and B occur for extreme reputation of the h-type agents either large (type A) or small (type B). The remaining range of reputations is dominated by equilibrium of type C, with type D, the paradoxical equilibrium, being the least common.

4 Conclusion

We investigate how institutions determine their admission policies. We study a model in which an agent's payoff is determined by the composition of a monetary prize and the reputation of the agents in his contest. The admission policy is determined by the organizers and can be permissive or restrictive. A permissive policy, that is, one that allows lower-type agents to participate in the contest, may increase the contest's intensity and thus total effort; however, it may also reduce the value of winning, lowering total effort in the contest.

In a single contest with an admission policy, we show that adding another low-type agent can either increase or decrease the contestants' total effort. We specifically provide the conditions under which the organizer may choose a restrictive admission policy in which only a few low-type agents participate up to an upper bound, as well as the conditions under which a permissive policy in which all low-type agents participate only if their number exceeds a certain lower bound.

In parallel contests in which each organizer selects his own admission policy, we show that any combination of admission policies that allow or exclude low-type agents from the contests can be an equilibrium. In particular, we demonstrate the existence of an equilibrium in which the organizer of the lower prestige contest restricts the participation of low-type agents while the organizer of the higher prestige contest chooses a policy that allows low-type agents to participate.

Our findings answer the question why do some of the most prestigious institutions admit participants who appear to be underqualified? We show that if there are enough high-reputation agents, the common policy is likely to be to restrict admission of low-reputation agents; however, if the total number of high-reputation agents is insufficient, any combination of admission policies is possible, including one in which the higher prestige institutes adopt a permissive admission policy that allows participation of low-reputation (underqualified) agents.

We make some assumptions in our parallel contests, such as the fact that each contest has only one winner and that the second stage competition is the well-known Tullock contest. However, our analysis clearly shows that our results are quite robust, and, for example, multiple winners in each contest or a more general Tullock contest success function will not significantly alter our quality findings in this paper.

5 Appendix

5.1 Proof of Proposition 3

Without loss of generality, we assume that all of the l-type agents are active. In such a case, the contest organizer prefers to allow the participation of the l-type agents if

$$f(v, \bar{\alpha}_h, \bar{\alpha}_l)\frac{(h+l-1)}{hc_h+lc_l} > f(v, \bar{\alpha}_h)\frac{h-1}{hc_h}.$$

The LHS represents the total effort with all l-type agents, while the RHS represents the total effort without l-type agents. The last inequality can be written as follows:

$$\frac{f(v, \bar{\alpha}_h, \bar{\alpha}_l)}{f(v, \bar{\alpha}_h)} > \frac{(h-1)(hc_h+lc_l)}{hc_h(h+l-1)}. \tag{17}$$

We know that the winning value function does not increase as the number of low-type agents increases. Thus, for every v and $\bar{\alpha}$, if $\alpha_\gamma \geq \max(\bar{\alpha})$, then $f(v_k, \bar{\alpha}) \leq f_k(v, \bar{\alpha}, \alpha_\gamma)$, and if $\alpha_\gamma \leq \min(\bar{\alpha})$, then $f(v, \bar{\alpha}) \geq f(v, \bar{\alpha}, \alpha_\gamma)$. This implies that $\frac{f(v,\bar{\alpha}_h,\bar{\alpha}_l)}{f(v,\bar{\alpha}_h)} \leq 1$ and.

$$\lim_{h\to\infty}\frac{f(v, \bar{\alpha}_h, \bar{\alpha}_l)}{f(v, \bar{\alpha}_h)} = 1$$

On the other hand, we have

$$\lim_{h\to\infty}\frac{(h-1)(hc_h+lc_l)}{hc_h(h+l-1)} = \lim_{h\to\infty}\frac{h^2c_h+hlc_l-hc_h-lc_l}{h^2c_h+hc_hl-hc_h} = 1.$$

According to our condition (**??**),

$$\frac{\partial}{\partial h}\left[\frac{f(v, \bar{\alpha}_h, \bar{\alpha}_l)}{f(v, \bar{\alpha}_h)}\right] < \frac{\partial}{\partial h}\left[\frac{(h-1)(hc_h+lc_l)}{hc_h(h+l-1)}\right] = \frac{l(2hc_l+lc_l-(c_l-c_h)h^2-c_l)}{h^2c_h(h+l-1)^2}. \tag{18}$$

Therefore, we obtain that

$$\lim_{h\to\infty} \frac{f(v,\bar{\alpha}_h,\bar{\alpha}_l)}{f(v,\bar{\alpha}_h)} > \lim_{h\to\infty} \frac{(h-1)(hc_h+lc_l)}{hc_h(h+l-1)}.$$

This means that if the condition (**??**) is satisfied and there are enough h-type agents in the contest, the contest organizer will choose to restrict the l-type agents. *Q.E.D.*

5.2 Proof of Proposition 4

We assume, without loss of generality, that all of the l-type agents are active. In such a case, by (**??**), the contest organizer prefers to allow the l-type agents' participation if

$$\frac{f(v,\bar{\alpha}_h,\bar{\alpha}_l)}{f(v,\bar{\alpha}_h)} > \frac{(h-1)(hc_h+lc_l)}{hc_h(h+l-1)}.$$

As the cost of the h-type agents decreases, the RHS increases, resulting in

$$\lim_{c_h\to 0^+} \frac{(h-1)(hc_h+lc_l)}{hc_h(h+l-1)} = \lim_{c_h\to 0^+} \frac{(h-1)lc_l}{hc_h(h+l-1)} = \infty.$$

Furthermore, because the l-type agents do not increase the value of winning, we get that

$$\frac{f(v,\bar{\alpha}_h,\bar{\alpha}_l)}{f(v,\bar{\alpha}_h)} \leq 1.$$

As such, for any values of v, h, l, c_h, α_h and α_l, there is a marginal cost c_h^* such that for every $c_h' \leq c_h^*$,

$$\frac{f(v,\bar{\alpha}_h,\bar{\alpha}_l)}{f(v,\bar{\alpha}_h)} < \frac{(h-1)(hc_h'+lc_l)}{hc_h'(h+l-1)}.$$

Thus, if h-type agents have a low enough marginal cost, the organizer will choose a policy that restricts l-type agents' participation in the contest. *Q.E.D.*

5.3 Proof of Proposition 5

In a contest with an admission policy, if the value of winning by entering l-type agents is less than or equal to half of the value of winning with only h-type agents, the organizer will always choose a policy that restricts l-type agents' participation in the contest.

If the value for winning satisfies

$$\frac{f(v, \bar{\alpha}_h, \bar{\alpha}_l)}{f(v, \bar{\alpha}_h)} \leq \frac{1}{2},$$

by Proposition 3, in order to show that the organizer wishes to restrict l-type agents' participation in the contest, it is sufficient to show that,

$$Q = \frac{(h-1)(hc_h + lc_l)}{hc_h(h+l-1)} > \frac{1}{2}. \tag{19}$$

Below we show that it is impossible that

$$\frac{(h-1)(hc_h + lc_l)}{hc_h(h+l-1)} < \frac{1}{2}. \tag{20}$$

1. The effect of the number of h-type agents (h) on $Q:$ Differentiating Q by h results in,

$$\frac{\partial}{\partial h}\left[\frac{(h-1)(hc_h + lc_l)}{hc_h(h+l-1)}\right] = -\frac{l((c_l - c_h)h^2 - 2c_l h - c_l l + c_l)}{c_h h^2 (h+l-1)^2},$$

which is positive if,

$$(c_l - c_h)h^2 - 2c_l h - c_l l + c_l < 0,$$

or, alternatively, if,

$$\frac{h^2}{h+l-1} < \frac{c_l}{c_l - c_h}.$$

The l-type agents are active when, $h < \frac{c_l}{c_l - c_h}$, and since $\frac{h^2}{h+l-1} < h$, we obtain that the last inequality $\frac{h^2}{h+l-1} < \frac{c_l}{c_l - c_h}$ holds. This implies that as the number of h-type agents increases, so does Q.

2. The effect of the marginal effort cost of the h-type agents (c_h) on Q: Differen-

tiating Q by c_h results in,

$$\frac{\partial}{\partial c_h}\left[\frac{(h-1)(hc_h+lc_l)}{hc_h(h+l-1)}\right]=-\frac{(h-1)lc_l}{(l+h-1)hc_h^2},$$

The last term is always negative. Thus, the value of Q decreases as c_h increases.

Based on the analysis above, To minimize Q, we must minimize h while maximizing c_h, as shown in the analysis above. Thus, to obtain the minimal value of Q, we need to set $h = 2$, and $c_h = c_l$. Substituting $h = 2, c_h = c_l$ into (**??**) yields:

$$\frac{(2-1)(2c_h+lc_h)}{2c_h(2+l-1)}=\frac{l+2}{2(l+1)}.$$

We can see that the LHS is greater than $\frac{1}{2}$ for all $l > 0$. Thus, as demonstrated, the inequality of (**??**) is valid. *Q.E.D.*

5.4 Proof of Proposition 6

We consider two scenarios: one in which l-type agents are allowed to enter the contest, and another in which they are restricted from participating and only h-type agents do. In each of these scenarios, we calculate and compare the total effort given by (6). By this comparison we can find the point in which the organizer is indifferent between the two alternatives. Let h and l be the number of h-type and l-type agents, respectively. If the contest organizer allows l-type agents to participate, the total effort in the contest in the second stage is

$$TE = v\frac{w_h h\alpha_h + w_l l\alpha_l}{w_h h + w_l l}\frac{(h+l-1)}{hc_h+lc_l}.$$

However, if the organizer decides to exclude the l-type agents from the contest, the total effort in the second stage is

$$TE = v\alpha_h\frac{h-1}{hc_h}.$$

Thus, the contest organizer prefers to allow the participation of l-type agents in the contest if

$$v\frac{w_h h\alpha_h + w_l l\alpha_l}{w_h h + w_l l}\frac{(h+l-1)}{hc_h + lc_l} - v\alpha_h\frac{h-1}{hc_h} > 0.$$

Multiplying both sides by $\frac{1}{v}(w_h h + w_l l)(hc_h + lc_l)$ yields

$$(w_h h\alpha_h + w_l l\alpha_l)(h+l-1) - \alpha_h\frac{h-1}{hc_h}(w_h h + w_l l)(hc_h + lc_l) > 0,$$

which is equivalent to

$$Q = \frac{h-1}{h}\left(\frac{\alpha_l}{\alpha_h}\frac{h}{h-1} - \frac{c_l}{c_h}\right)l^2 + (h-1)\left(\frac{\alpha_l}{\alpha_h} + \left(\frac{h}{h-1} - \frac{c_l}{c_h}\right)\frac{w_h}{w_l} - 1\right)l > 0 \quad (21)$$

When we equalize Q to zero, the two roots are

$$l_1^* = 0, l_2^* = h\frac{1 - \frac{\alpha_l}{\alpha_h} - (\frac{h}{h-1} - \frac{c_l}{c_h})\frac{w_h}{w_l}}{\frac{\alpha_l}{\alpha_h}\frac{h}{h-1} - \frac{c_l}{c_h}}.$$

Note that l_2^* might be either positive or negative, thus we can denote $l^* = \max(0, l_2^*)$. It is clear that Q has a minimum if

$$\frac{\alpha_l}{\alpha_h}\frac{h}{h-1} - \frac{c_l}{c_h} > 0,$$

and a maximum if

$$\frac{\alpha_l}{\alpha_h}\frac{h}{h-1} - \frac{c_l}{c_h} < 0.$$

In other words, Q as a minimum if

$$h < \frac{\alpha_h c_l}{\alpha_h c_l - \alpha_l c_h}, \quad (22)$$

and a maximum if

$$h > \frac{\alpha_h c_l}{\alpha_h c_l - \alpha_l c_h}. \quad (23)$$

If (**??**) holds, and the number of l-type agents l is less than l^*, it means that the total effort that includes the efforts of the l-type agents is less than if the l-type agents do not participate. However, If the number of l-type agents is larger than l^*, allowing them to participate in the contest increases the total effort. Similarly, if (**??**) holds, the contest organizer will choose a policy that allows the l-type agents to participate only if the number of l-type agents is less than l^*. *Q.E.D.*

5.5 Proof of Lemma ??

We slightly modify Cohen et al.'s (2024) proof for our model. According to Proposition 1, the expected payoff of an h-type agent in contest $k, k = 1, 2$ is given by

$$u_k^h = f(v_k, \bar{\alpha}_k)\left(1 - \frac{(n_k - 1)c_i}{\sum_{j \in N_k} c_j}\right)^2.$$

Substituting the winning value function $f(v_k, \bar{\alpha}_k) = v_k \frac{\sum_{i \in N_k} \alpha_i w_i}{\sum_{i \in N_k} w_i}$ yields

$$u_k^h = v_k \frac{\sum_{i \in N_k} \alpha_i w_i}{\sum_{i \in N_k} w_i}\left(1 - \frac{(n_k - 1)c_i}{\sum_{j \in N_k} c_j}\right)^2.$$

Since there are no l-type agents in the contests, we can simplify the expected payoff and obtain

$$u_k^h = v_k \alpha_h \left(1 - \frac{(h_k - 1)c_h}{h_k c_h}\right)^2 = \frac{v_k \alpha_h}{h_k^2}.$$

If we compare the expected payoff of an h-type agent in each of the contests we obtain

$$u_1^h = \frac{v_1 \alpha_h}{h_1^2} = \frac{v_2 \alpha_h}{h_2^2} = u_2^h.$$

By substitute $h_2 = h - h_1$ we have,

$$\tilde{h_1} = \frac{h\sqrt{v_1}}{\sqrt{v_1} + \sqrt{v_2}},$$
$$\tilde{h_2} = \frac{h\sqrt{v_2}}{\sqrt{v_1} + \sqrt{v_2}}.$$

Thus, $u_1^h = u_2^h$ when exactly $\widetilde{h}_1$ agents choose to compete in contest 1 and $\widetilde{h}_2$ choose to compete in contest 2. However, $\widetilde{h}_i, i = 1, 2$ are not necessarily integers. In that case, let us assume that in equilibrium, $\tilde{h_1} - 2$ agents choose to compete in contest 1 and $\tilde{h_2} + 2$ agents choose to compete in contest 2 . Then, there exists that $u_1^h > u_2^h$, namely, the agents' expected payoff in contest 1 is larger than in contest 2. But, then, an agent from contest 2 could move to contest 1 and he will have a larger expected payoff than an agent in contest 2. This is because that if the allocation of agents is $\tilde{h_1} - 1$ in contest 1 and $\tilde{h_2} + 1$ agents in contest 2, there exists that $u_1^h > u_2^h$. This argument reduces the number of possible equilibrium allocations to only two combinations of $\left\lfloor \tilde{h_k} \right\rfloor$-the largest integer that is smaller than $\tilde{h_k}$, and $\left\lceil \tilde{h_k} \right\rceil$ the smallest integer that is larger than $\tilde{h_k}$ $k = 1, 2$. Hence, if the l-type agents aren't allowed to enter the contests, h_k^* agents choose to compete in contest $k = 1, 2$ where $(h_1^*, h_2^*) \in \{(\left\lfloor \tilde{h_1} \right\rfloor, \left\lceil \tilde{h_2} \right\rceil), (\left\lceil \tilde{h_1} \right\rceil, \left\lfloor \tilde{h_2} \right\rfloor)\}$.

5.6 Proof of Proposition ??

Proposition 6 states that given the number of h-type agents in the contest, the contest organizer selects his admission policy based on the number of l-type agents in relation to their critical number, which is given by

$$l^* = h\frac{1 - \frac{\alpha_l}{\alpha_h} - (\frac{h}{h-1} - \frac{c_l}{c_h})\frac{w_h}{w_l}}{\frac{\alpha_l}{\alpha_h}\frac{h}{h-1} - \frac{c_l}{c_h}}.$$

If the agents' reputation types are close enough, we get that

$$\lim_{\alpha_l \to \alpha_h} h \frac{1 - \frac{\alpha_l}{\alpha_h} - (\frac{h}{h-1} - \frac{c_l}{c_h})\frac{w_h}{w_l}}{\frac{\alpha_l}{\alpha_h}\frac{h}{h-1} - \frac{c_l}{c_h}} = \lim_{\alpha_l \to \alpha_h} h \frac{-(\frac{h}{h-1} - \frac{c_l}{c_h})\frac{w_h}{w_l}}{\frac{h}{h-1} - \frac{c_l}{c_h}} = \lim_{\alpha_l \to \alpha_h} -h\frac{w_h}{w_l} < 0.$$

Thus, the critical value l^* is negative when α_l is close enough to α_h. According to Proposition 6, this means that either the organizers choose to allow or restrict the participation of all the l-type agents, regardless of their number. Furthermore, according to Proposition 6, if $h < \frac{\alpha_h c_l}{\alpha_h c_l - \alpha_l c_h}$, the contest organizer will allow l-type agents to participate if $l > l^*$. However, when α_l is close enough to α_h, this condition is equivalent to $h < \frac{c_l}{c_l - c_h}$ indicating that the l-type agents are active. Thus, if the l-type agents are active, the organizer should allow them to participate. To summarize, if the agents' reputation types are close enough, the organizers allow all agents to participate in both contests. *Q.E.D.*

5.7 Proof of Lemma ??

Assume, without loss of generality, that $i = 1$ and $j = 2$, that is, we assume that the organizer of contest 2 restricts l-type agents from participating and we want the find the condition under which the organizer of contest 1 allow their participation. Because the organizer of contest 2 does not allow l-type agents to participate in his contest, if the organizer of contest 1 allows them to do so, they will all enter contest 1. Adding l-type agents to contest 1 causes some h-type agents to leave for contest 2. Consider the worst-case scenario in which all h-type agents choose to leave contest 1, but contest 1's organizer still wants the $l-$type agents to participate. According to (4), the total effort in contest i is

$$f(v_i, \bar{\alpha}_h, \bar{\alpha}_l)\frac{(h_i + l_i - 1)}{h_i c_h + l_i c_l} \tag{24}$$

Thus, assuming that no h-type agents are present in contest 1 yields that the total effort in this contest is

$$f(v_1, \bar{\alpha}_l)\frac{l-1}{lc_l}$$

If the total effort (**??**) without h-type agent is greater than the total effort without l-type agents, the organizer will undoubtedly choose a policy that allows l-type agents to participate. Formally, the organizer will allow l-type agents if,

$$f(v_1, \bar{\alpha}_l)\frac{l-1}{lc_l} > f(v_1, \bar{\alpha}_h)\frac{h_1-1}{h_1 c_h}.$$

Multiply both sides by $\frac{c_h h_1}{f(v_1, \bar{\alpha}_h)}$ yields

$$\frac{f(v_1, \bar{\alpha}_l)}{f(v_1, \bar{\alpha}_h)}\frac{c_h}{c_l}\frac{l-1}{l}h_1 > h_1 - 1,$$

which is equivalent to

$$\left(1 - \frac{f(v_1, \bar{\alpha}_l)}{f(v_1, \bar{\alpha}_h)}\frac{c_h}{c_l}\frac{l-1}{l}\right)h_1 < 1.$$

Since $\frac{c_h}{c_l} < 1$, $\frac{l-1}{l} < 1$, and $\frac{f(v_1,\bar{\alpha}_l)}{f(v_1,\bar{\alpha}_h)} < 1$, we obtain that $\frac{f(v_1,\bar{\alpha}_l)}{f(v_1,\bar{\alpha}_h)}\frac{c_h}{c_l}\frac{l-1}{l} < 1$. This implies that, $1 - \frac{f(v_1,\bar{\alpha}_l)}{f(v_1,\bar{\alpha}_h)}\frac{c_h}{c_l}\frac{l-1}{l} > 0$, or that,

$$h_1 < \frac{1}{1 - \frac{f(v_1,\bar{\alpha}_l)}{f(v_1,\bar{\alpha}_h)}\frac{c_h}{c_l}\frac{l-1}{l}}. \tag{25}$$

If contest 1 has a small number of h-type agents (h_1), the organizer of this contest will choose to allow l-type agents to participate, even if it means all h-type agents will leave the contest. Furthermore, according to Lemma **??**, before the addition of l-type agents to contest 1, the number of h-type agents in contest 1 was at most

$$\left\lceil \tilde{h_1} \right\rceil = \left\lceil \frac{h\sqrt{v_1}}{\sqrt{v_1} + \sqrt{v_2}} \right\rceil.$$

In a weighted average winning value function, when there are only l-type or h-type agents, the ratio between the winning value functions in these cases is

$$\frac{f(v_1,\bar{\alpha}_l)}{f(v_1,\bar{\alpha}_h)} = \frac{v_1 \frac{w_l l \alpha_l}{w_l l}}{v_1 \frac{w_h h \alpha_h}{w_h h}} = \frac{\alpha_l}{\alpha_h}.$$

Thus, we can replace $\frac{f(v_1,\bar{\alpha}_l)}{f(v_1,\bar{\alpha}_h)}$ with $\frac{\alpha_l}{\alpha_h}$ and rewriting (**??**) as

$$\left\lceil \frac{h\sqrt{v_1}}{\sqrt{v_1}+\sqrt{v_2}} \right\rceil < \frac{1}{1-\frac{\alpha_l}{\alpha_h}\frac{c_h}{c_l}\frac{l-1}{l}}. \tag{26}$$

In other words, if (**??**) holds and the organizer of contest 2 does not allow l-type agents in his contest, the organizer of contest 1 will allow them. This is because, even if all h-type agents leave the contest, the overall effort in contest 1 increases.

5.8 Proof of Proposition ??

Consider two parallel contests with admissions policies and nominal prizes of $v_1 = 10$ and $v_2 = 1$. The rest of the parameters are specified in Table **??**. To analyze the

	h-type	**l-type**
Amount	10	15
c_i	0.95	1
α_i	1.1	1
w_i	1	1

Table 8: The parameters of the h-type and the l-type agents for C equilibrium

equilibrium in the first stage, we first calculate the equilibrium values in the second stage for each combination of admission policies, namely cases A-D, and then, based on the expected total effort in each contest for each combination of admission policies, we calculate the first stage admission policy equilibrium.

5.8.1 The analysis of case A

In case A, both contest organizers decide to restrict the participation of l-type agents. Lemma **??** states that if the organizers restrict the participation of l-type agents, the h-type agents are allocated as follows:

$$\begin{aligned} h_1 =& \frac{h\sqrt{\alpha_h v_1}}{\sqrt{\alpha_h v_1} + \sqrt{\alpha_h v_2}} = \frac{h\sqrt{v_1}}{\sqrt{v_1} + \sqrt{v_2}} = \frac{10\sqrt{10}}{\sqrt{10} + \sqrt{1}} = 7.586 \\ h_2 =& \frac{h\sqrt{\alpha_h v_2}}{\sqrt{\alpha_h v_1} + \sqrt{\alpha_h v_2}} = \frac{h\sqrt{v_2}}{\sqrt{v_1} + \sqrt{v_2}} = \frac{10\sqrt{1}}{\sqrt{10} + \sqrt{1}} = 2.402. \end{aligned} \tag{27}$$

It can be verified that the allocation of h-type agents in the contest is therefore $(h_1, h_2) = (8, 2)$. Using (4), the total effort in each contest is

$$\begin{aligned} TE_1^A &= v_1\alpha_h \frac{h_1 - 1}{h_1 c_h} = 10 \cdot 1.1 \frac{8 - 1}{8 \cdot 0.95} \approx 10.131 \\ TE_2^A &= v_2\alpha_h \frac{h_2 - 1}{h_2 c_h} = 1 \cdot 1.1 \frac{2 - 1}{2 \cdot 0.95} \approx 0.578. \end{aligned}$$

5.8.2 The analysis of case D

In case D, the contest organizer of contest 1 allows the participation of l-type agents, whereas the organizer of contest 2 does not. In this case, l-type agents cannot move between contests, whereas h-type agents can. Thus, we need to analyze how the h-type agents are distributed between the two contests. According to Proposition 1, the expected payoff of h-type agents in contest $i, i = 1, 2$ is given by

$$u_h^i = v_i \frac{w_h h_i \alpha_h + w_l l_i \alpha_l}{w_h h_i + w_l l_i} \left(1 - \frac{(h_i + l_i - 1)c_h}{h_i c_h + l_i c_l}\right)^2.$$

Given the parameters α, w, c, $l_1 = 15, l_2 = 0$, we obtain the expected payoffs of the h-type agents in both contests

$$u_h^1 = 10\frac{1.1h_1 + 15}{h_1 + 15}\left(1 - \frac{(h_1 + 15 - 1) \cdot 0.95}{0.95h_1 + 15}\right)^2$$
$$u_h^2 = \frac{1.1h_2}{h_2}\left(1 - \frac{(h_2 - 1) \cdot 0.95}{0.95h_2}\right)^2.$$

When we compare the expected payoffs, we can determine the distribution of h-type agents in contests 1 and 2, such that that no h-type agent wishes to move to another contest. When we compare the expected payoffs, we are able to obtain the distribution of h-type agents in contests 1 and 2, such that there is no an h-type agent who want to move to another contest. It can be shown that, in this case, the allocation of the h-type agents is $(h_1, h_2) = (6, 4)$. Then, by (4) the total effort in each contest is

$$TE_1^D = 10\frac{6 \cdot 1.1 + 15}{6 + 15}\frac{(6 + 15 - 1)}{6 \cdot 0.95 + 15} = 9.937$$
$$TE_2^D = 1.1\frac{(4 - 1)}{4 \cdot 0.95} = 0.868.$$

5.8.3 The analysis of case C

In case C, the organizer of contest 2 allows the l-type agents to participate, whereas the organizer of contest 1 does not. To find the allocation of the h-type agents across the contests, we substitute the parameters α, w, c, $l_1 = 0, l_2 = 15$ into the expected payoff of the h-type agents in each of the contests and obtain that

$$u_h^1 = 10 \cdot 1.1\left(1 - \frac{(h_1 - 1) \cdot 0.99}{0.99h_1}\right)^2$$
$$u_h^2 = \frac{1.1h_2 + 15}{h_2 + 15}\left(1 - \frac{(h_2 + 15 - 1) \cdot 0.99}{0.99h_2 + 15}\right)^2.$$

When we compare the expected payoffs, we can find the distribution of h-type agents in contests 1 and 2, ensuring that no h-type agent wishes to move to another contest.

For the given parameters, the h-type agent allocation is $(h_1, h_2) = (10, 0)$. Then, by (4), the total effort in each contest is

$$TE_1^C = 10\frac{10 \cdot 1.1}{10}\frac{10-1}{10 \cdot 0.95} = 10.421$$
$$TE_2^C = \frac{15}{15}\frac{15-1}{15} = 0.933.$$

5.8.4 The analysis of case B

In case B, l-type agents can participate in both contests 1 and 2. Thus, the analysis of case B must take into account both agents' endogenous allocations, as they can be allocated in both contests. Based on our analysis of case C, we conclude that if all l-type agents are allocated in contest 2, all h-type agents prefer to be allocated in contest 1. Thus, we begin by looking at how the l-type agents are distributed under the assumption that all h-type agents remain in contest 1. Later, after determining the allocation of the l-type agents, we can validate how the h-type agents are allocated based on the allocation of the l-type agents and continue recursively until we reach the final allocation of both types of agents across contests. In the case that all h-type agents are allocated to contest 1, the expected payoffs of the l-type agents in contests 1 and 2 are

$$u_l^1 = 10\frac{1.1 \cdot 10 + l_1}{10 + l_1}\left(1 - \frac{10 + l_1 - 1}{0.95 \cdot 10 + l_1}\right)^2$$
$$u_l^2 = \left(1 - \frac{l_2 - 1}{l_2}\right)^2.$$

When we compare the expected payoffs, we can determine how many l-type agents are allocated in contests 1 and 2, respectively. It can be shown that, in this case, the allocation of the l-type agents is $(l_1, l_2) = (6, 9)$. Given that some l-type agents are now allocated in contest 1, we must reconsider the distribution of h-type agents across contests. The expected payoffs of the h-type agents in contests 1 and 2, given the allocation of the l-type agents, are

$$u_h^1 = 10\frac{1.1h_1+6}{h_1+6}\left(1-\frac{0.95(h_1+6-1)}{0.95h_1+6}\right)^2$$
$$u_h^2 = \frac{1.1h_2+9}{h_2+9}\left(1-\frac{0.95(h_2+9-1)}{0.95h_2+9}\right)^2.$$

When we compare the expected payoffs, we obtain that all h-type agents want to remain in contest 1. Thus, in case B, the allocation of agents across the contests is $(h_1, h_2, l_1, l_2) = (10, 0, 6, 9)$. Then, the total efforts in both contests are

$$TE_1^D = 10\frac{10\cdot 1.1+6}{10+6}\frac{(10+6-1)}{10\cdot 0.95+6} = 10.282$$
$$TE_2^D = \frac{(9-1)}{9} = 0.888.$$

5.8.5 Summary of the analyses of cases A-D

Because the organizers' expected payoffs are the total efforts exerted in their contests, we can summarize their expected payoffs in each of the four A-D cases into Table ??.

		Contest 2	
		Restrict Low-Type	Allow Low-Type
Contest 1	Restrict Low-Type	$A(10.131, 0.578)$	$C(10.421, 0.933)$
	Allow Low-Type	$D(9.937, 0.868)$	$B(10.282, 0.888)$

Table 9: The expected payoffs of the organizers

In this 2x2 game, it is clear that contest 2's organizer always wants l-type agents to participate in his contest. In response, the organizer of contest 1 prefers to restrict their participation. This means that we have an equilibrium of type C. $Q.E.D.$

5.9 Proof of Proposition ??

Consider two parallel contests with admissions policies, with nominal prizes of $v_1 = 10$ and $v_2 = 1$. The rest of the parameters are specified in Table ??.

	h-type	**l-type**
Amount	10	15
c_i	0.99	1
α_i	1.1	1
w_i	1	1

Table 10: The parameters of the h-type and the l-type agents for D equilibrium

Similarly to Proposition **??**, we first calculate the equilibrium in the second stage for each combination of admission policies, namely cases A-D, and then, based on the expected total effort in each contest, we calculate the equilibrium in the first stage.

5.9.1 The nalysis of Case A

In case A, both contest organizers decide to restrict the participation of l-type agents. In that case, according to Lemma **??**, if the organizers choose a policy that restricts the l-type agents from participation, the h-type agents are allocated as follows:

$$\begin{aligned} h_1 =& \frac{h\sqrt{\alpha_h v_1}}{\sqrt{\alpha_h v_1} + \sqrt{\alpha_h v_2}} = \frac{h\sqrt{v_1}}{\sqrt{v_1} + \sqrt{v_2}} = \frac{10\sqrt{10}}{\sqrt{10} + \sqrt{1}} = 7.586 \\ h_2 =& \frac{h\sqrt{\alpha_h v_2}}{\sqrt{\alpha_h v_1} + \sqrt{\alpha_h v_2}} = \frac{h\sqrt{v_2}}{\sqrt{v_1} + \sqrt{v_2}} = \frac{10\sqrt{1}}{\sqrt{10} + \sqrt{1}} = 2.402. \end{aligned} \tag{28}$$

It can be verified that the allocation of h-type agents in the contest is $(h_1, h_2) = (8, 2)$. Then, according to (4), the total efforts in each of the contests are

$$\begin{aligned} TE_1^A &= v_1 \alpha_h \frac{h_1 - 1}{h_1 c_h} = 10 \cdot 1.1 \frac{8 - 1}{8 \cdot 0.99} \approx 9.722 \\ TE_2^A &= v_2 \alpha_h \frac{h_2 - 1}{h_2 c_h} = 1 \cdot 1.1 \frac{2 - 1}{2 \cdot 0.99} \approx 0.555. \end{aligned}$$

5.9.2 The analysis of case D

In case D, the organizer of contest 1 allows l-type agents to participate, but the organizer of contest 2 does not. Thus, in contrast to case A, h-type agents who face more l-type agents in contest 1 may wish to move on to contest 2. First, we need to analyze how the h-type agents are allocated. According to Proposition 1, the expected payoff for h-type agents in contest $i, i = 1, 2$ is

$$u_h^i = v_i \frac{w_h h_i \alpha_h + w_l l_i \alpha_l}{w_h h_i + w_l l_i} \left(1 - \frac{(h_i + l_i - 1)c_h}{h_i c_h + l_i c_l}\right)^2.$$

Using the parameters α, w, c, $l_1 = 15, l_2 = 0$, we can calculate the expected payoffs of h-type agents in both contests.

$$u_h^1 = 10\frac{1.1h_1 + 15}{h_1 + 15}\left(1 - \frac{(h_1 + 15 - 1) \cdot 0.99}{0.99h_1 + 15}\right)^2$$
$$u_h^2 = \frac{1.1h_2}{h_2}\left(1 - \frac{(h_2 - 1) \cdot 0.99}{0.99h_2}\right)^2.$$

When we compare expected payoffs, we can determine the distribution of h-type agents in both contests. It can be shown that in this case, the distribution of the h-type agents is $(h_1, h_2) = (5, 5)$. Then, according to (4), the total effort in each of the contests is

$$TE_1^D = 10\frac{5 \cdot 1.1 + 15}{5 + 15}\frac{(5 + 15 - 1)}{5 \cdot 0.99 + 15} = 9.761$$
$$TE_2^D = 1.1\frac{(5 - 1)}{5 \cdot 0.99} = 0.888.$$

5.9.3 The analysis of case C

In case C, the organizer of contest 2 allows the l-type agents to participate, whereas the organizer of contest 1 does not. As a result, all 15 l-type agents enter contest 2. To allocate the h-type agents across the contests, we substitute the parameters α, w, c, $l_1 = 0, l_2 = 15$ into the expected payoff of the h-type agents in each of the contests, and obtain

$$u_h^1 = 10 \cdot 1.1 \left(1 - \frac{(h_1 - 1) \cdot 0.99}{0.99 h_1}\right)^2$$
$$u_h^2 = \frac{1.1 h_2 + 15}{h_2 + 15} \left(1 - \frac{(h_2 + 15 - 1) \cdot 0.99}{0.99 h_2 + 15}\right)^2.$$

When we compare expected payoffs, we can determine the distribution of h-type agents in both contests. It can be shown that, in this case, the distribution of the h-type agents is $(h_1, h_2) = (10, 0)$. Then, according to (4), the total effort in each of the contests is

$$TE_1^C = 10 \frac{10 \cdot 1.1}{10} \frac{10 - 1}{10 \cdot 0.99} = 10$$
$$TE_2^C = \frac{15}{15} \frac{15 - 1}{15} = 0.933.$$

5.9.4 The analysis of case B

In case B, l-type agents can participate in both contests. Similar to Proposition **??**, we consider how the l-type agents are allocated under the assumption that all h-type agents remain in contest 1. After determining the allocation of the l-type agents, we validate how the h-type agents are allocated based on the allocation of the l-type agents and continue recursively until we reach the final allocation of both types of agents across the contests.

If all h-type agents are allocated to contest 1, the expected payoffs of the l-type agents in both contests are

$$u_l^1 = 10 \frac{1.1 \cdot 10 + l_1}{10 + l_1} \left(1 - \frac{10 + l_1 - 1}{0.99 \cdot 10 + l_1}\right)^2$$
$$u_l^2 = \left(1 - \frac{l_2 - 1}{l_2}\right)^2.$$

When we compare expected payoffs, we can determine the distribution of l-type agents in both contests. It can be shown that, in this case, the distribution of the l-type agents is $(l_1, l_2) = (9, 6)$. Given that some l-type agents are now allocated in contest 1, we need to reconsider the distribution of h-type agents across contests. The expected payoffs of the h-type agents in both contests are

$$u_h^1 = 10\frac{1.1h_1 + 9}{h_1 + 9}\left(1 - \frac{0.99(h_1 + 9 - 1)}{0.99h_1 + 9}\right)^2$$
$$u_h^2 = \frac{1.1h_2 + 6}{h_2 + 6}\left(1 - \frac{0.99(h_2 + 6 - 1)}{0.99h_2 + 6}\right)^2.$$

When we compare the expected payoffs, we find that all h-type agents want to stay in contest 1. . Thus, in case B, the allocation of agents across the contests is $(h_1, h_2, l_1, l_2) = (10, 0, 9, 6)$. The total effort in each of the contests is

$$TE_1^D = 10\frac{10 \cdot 1.1 + 9}{10 + 9}\frac{(10 + 9 - 1)}{10 \cdot 0.99 + 9} = 10.025$$
$$TE_2^D = \frac{(6 - 1)}{6} = 0.833.$$

5.9.5 Summary of the analyses of cases A-D

Given that the organizers' expected payoff is the total effort in their contests, we can summarize their expected payoffs in each of the four cases A-D in Table **??**.

		Contest 2	
		Restrict Low-Type	Allow Low-Type
Contest 1	Restrict Low-Type	$A(9.722, 0.555)$	$C(10, 0.933)$
	Allow Low-Type	$D(9.761, 0.888)$	$B(10.025, 0.833)$

Table 11: The expected payoffs of the organizers

The organizer of contest 1 clearly wants l-type agents to participate in his contest. As a result, the organizer of contest 2 prefers to restrict their participation. This implies that we have an equilibrium of type D. $Q.E.D.$

References

[1] Avery, C., Fairbanks, A., & Zeckhauser, R. (2003). The early admissions game : Joining the elite. Harvard University Press.

[2] Avery, C., & Levin, J. (2010). Early admissions at selective colleges. *American Economic Review*, 100(5), 2125-2156.

[3] Azmat, G., & Möller, M. (2009). Competition among contests. *The RAND Journal of Economics*, 40(4), 743-768.

[4] Azmat, G., & Möller, M. (2018). The distribution of talent across contests. *The Economic Journal*, 128 (609), 471–509.

[5] Cason, T. N., Masters, W. A., & Sheremeta, R. M. (2010). Entry into winner-take-all and proportional-prize contests: An experimental study. *Journal of Public Economics*, 94(9-10), 604-611.

[6] Chade, H., Lewis, G., & Smith, L. (2014). Student portfolios and the college admissions problem. *Review of Economic Studies*, 81(3), 971-1002.

[7] Che, Y. K., & Koh, Y. (2016). Decentralized college admissions. *Journal of Political Economy*, 124(5), 1295-1338.

[8] Cohen, C., Rabi, I., & Sela, A. (2024). Reputation in Contests. Working paper, Ben-Gurion University.

[9] Cripps, M. W., Mailath, G. J., & Samuelson, L. (2004). Imperfect monitoring and impermanent reputations. *Econometrica*, 72(2), 407-432.

[10] Damiano, E., Li, H., & Suen, W. (2010). First in village or second in Rome? *International Economic Review,* 51 (1), 263–288.

[11] Damiano, E., Li, H., & Suen, W. (2012). Competing for talents. *Journal of Economic Theory,* 147 (6), 2190–2219.

[12] Dubey, P., Geanakopolos, J. (2010). Grading exams: 100, 99, 98,..., A, B, C ? *Games and Economic Behavior* 69(1), 72-94.

[13] Ellison, G., Fudenberg, D., & Möbius, M. (2004). Competing auctions. *Journal of the European Economic Association*, 2(1), 30-66.

[14] Ely, J., Fudenberg, D., Levine, D. K. (2008). When is reputation bad?. *Games and Economic Behavior*, 63(2), 498-526.

[15] Franke, J., Kanzow, C., Leininger, W., & Schwartz, A. (2013). Effort maximization in asymmetric contest games with heterogeneous contestants. *Economic Theory*, 52, 589-630.

[16] Fu, Q., & Lu, J. (2010). Contest design and optimal endogenous entry. *Economic Inquiry*, 48(1), 80-88.

[17] Fu, Q., Jiao, Q., & Lu, J. (2015). Contests with endogenous entry. *International Journal of Game Theory*, 44, 387-424.

[18] Juang, W. T., Sun, G. Z., & Yuan, K. C. (2020). A model of parallel contests. *International Journal of Game Theory*, 49 (2), 651–672.

[19] Konrad, K. A., & Kovenock, D. (2012). The lifeboat problem. *European Economic Review*, 56 (3), 552–559.

[20] Kreps, D. M., & Milgrom, P., Roberts, J., & Wilson, R. (1982). Rational cooperation in the finitely repeated prisoners' dilemma. *Journal of Economic Theory*, 27(2), 245-252.

[21] Kreps, D. M., & Wilson, R. (1982). Reputation and imperfect information. *Journal of Economic Theory*, 27(2), 253-279.

[22] Laffont, J. J., & Robert, J. (1996). Optimal auction with financially constrained buyers. *Economics Letters*, 52(2), 181-186.

[23] Mailath, G. J., & Samuelson, L. (2015). Reputations in repeated games. Handbook of Game Theory with Economic Applications, 4, 165-238.

[24] Megidish, R., & Sela, A. (2013). Allocation of prizes in contests with participation constraints. *Journal of Economics & Management Strategy*, 22(4), 713-727.

[25] Milgrom, P., & Roberts, J. (1982). Predation, reputation, and entry deterrence. *Journal of Economic Theory*, 27(2), 280-312.

[26] Moldovanu, B., & Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review,* 91, 542-558.

[27] Moldovanu B., Sela, A., Shi, X. (2007). Contests for status, *Journal of Political Economy*, 115(2), 338-363.

[28] Moldovanu, B., Sela, A., & Shi, X. (2008). Competing auctions with endogenous quantities. *Journal of Economic Theory*, 141(1), 1-27.

[29] Moldovanu, B., Sela, A., & Shi, X. (2012). Carrots and sticks: prizes and punishments in contests. *Economic Inquiry*, 50(2), 453-462.

[30] Morgan, J., Sisak, D., & Vardy, F. (2018). The ponds dilemma. *The Economic Journal,* 128 (611), 1634–1682.

[31] Myerson, R. B. (1981). Optimal auction design. *Mathematics of operations research*, 6(1), 58-73.

[32] Olszewski, W., & Siegel, R. (2020). Performance-maximizing large contests. *Theoretical Economics*, 15(1), 57-88.

[33] Peters, M., & Severinov, S. (1997). Competition among sellers who offer auctions instead of prices. *Journal of Economic Theory*, 75(1), 141-179.

[34] Stouras, K. I., Hutchison-Krupat, J., & Chao, R. O. (2022). The role of participation in innovation contests. *Management Science*, 68(6), 4135-4150.

[35] Tullock, G. (1980). Efficient Rent Seeking. In J. M. Buchanan, R. D. Tollison, & G. Tullock, Toward a theory of the rent-seeking society. Texas A & M University.

[36] Zhang, M. (2024). Optimal contests with incomplete information and convex effort costs. *Theoretical Economics*, 19(1), 95-129.