STATUS CLASSIFICATION BY LOTTERY CONTESTS

Aner Sela

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Monaster Center for
Economic Research
Ben-Gurion University of the Negev
P.O. Box 653
Beer Sheva, Israel

Fax: 972-8-6472941
Tel: 972-8-6472286

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Abstract

We study the optimal design of status classifications in organizational structures under the assumption that agents in a lottery (Tullock) contest care about their relative position. We assume that there are two status categories and a designer who determines their sizes in order to maximize the agents' total performance. We prove that the optimal partition contains more than one agent in each status category if the number of agents is larger than three, and that the top status category contains more agents than the bottom one. This result demonstrates that in order to maximize the agents' total output the top status categories should not be exclusive to a small number of agents.

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*Department of Economics, Ben Gurion University of the Negev, 84105 Beer Sheva, Israel, anersela@bgu.ac.il

1 Introduction

It is obvious that people care about status and status concerns directly enter into their utility functions and influence their social interactions (see Frank 1985, and Frank and Cook 1995). Overwhelming experimental evidence confirms that status plays a key role in our lives.[1] Indeed, we can find numerous real-life examples of status categories since ancient times when kings and queens of feudal states awarded titles of nobility such as duke, marquis, baron, baronet, etc., until the present time, where large corporations grant status rewards such as president, vice president, senior manager, manager, etc., or academic ranks such as full professor, associate professor, and assistant professor.

We study the optimal design of status categories in organizational structures under the assumption that agents in a lottery contest care about their relative position. In our model, several homogeneous agents engage in a lottery contest (Tullock 1980) where each agent exerts an effort.[2] Then, the agents are ranked by the stochastic lottery success function according to which the agent with the highest effort is not necessarily ranked at the top, but has the highest probability to belong to the top ranking.[3] According to their ranking, the agents are partitioned into status categories. The top status category consists of those individuals with the highest ranking, the second category of those individuals with the next highest ranking, and so on. We borrow from Moldovanu et al. (2007) and Dubey and Geanakoplos (2010) the present specification of utility functions and assume that each agent cares about the number of agents in categories above and below him. In particular, agents get a positive utility that is proportional to the number of agents in lower status categories and a negative utility that is proportional to the number of agents in higher status categories. Then, the agents exert efforts according to the number of status categories and their size which are determined by a designer who wishes to maximize the total effort (output). Since the contest equilibrium depends only on the structure of the status categories and not directly on the designer's goal, our type of analysis can, in

[1] See, among others, Charness et al. 2014; Dugar et al. 2012; Tran and Zeckhauser 2012; Bhattacharya and Dugar, S. 2014, Kosfeld and Neckermann 2011; and Eckel et al. 2010.

[2] This kind of contest is under some conditions equivalentto a variety of rent-seeking contests and innovation tournaments (see Baye and Hoppe 2003).

[3] Several studies have provided axiomatic justification for this contest form (see Skaperdas 1966 and Fu and Lu 2012). For the existence of equilibriun in Tullock contests, see Szidarovszky and Okuguchi (1997), and Einy et al. (2015).

principle, be performed for a variety of other goals.

We first analyze the symmetric equilibrium effort in our lottery contest with n homogenous agents and $k \leq n$ status categories. Then, we prove that for a designer who wishes to maximize the agents' total effort the optimal number of status categories is the same as the number of agents. This result seems to be in contrast to the findings of Sela (2020) who showed that the optimal allocation of prizes and punishments in lottery contests is one prize and one punishment. In that case, there are actually three status categories, the one who won the prize, the other agents who did not win the prize and were not punished, and the one who was punished. Furthermore, if the agents have to participate whether they have a positive or a negative expected payoff, then the optimal allocation of prizes and punishments would be only one punishment which is equivalent to only two status categories: the one for the punished agent and the other for all the rest. It is worth noting that a similar conflict occurs in all-pay auctions under incomplete information between the optimal number of prizes (Moldovanu and Sela 2001) and the optimal number of status categories (Moldovanu et al. 2007).

In contrast to our result about the optimal number of status categories, in real-life situations, the number of status categories is limited. For example, there are only three academic ranks (full professor, associate professor, and assistant professor). Thus, for simplicity, we consider the case of only two categories of status A and B where r agents are allocated in the lower category B and $n - r$ agents are allocated in the higher category A. We show that if there are at least four agents, it is optimal to allocate more than one agent into the low category B in order to maximize the agents' equilibrium total effort. Similarly, if there are at least three agents, it is optimal to allocate more than one agent into the high category A in order to maximize the agents' equilibrium total effort. Furthermore, we show that in the optimal allocation of agents that maximizes the agents' equilibrium total effort, the number of agents in category B is smaller than or equal to the number of agents in category A. In other words, the number of agents in the high category should be larger than or equal to their number in the low one. Our finding that in order to maximize the agents' total output, the top status categories should not be exclusive to a small number of agents is the main message of the paper. We also prove that there is a unique optimal allocation of agents that maximizes the total effort with $n - r^*$ agents in category A and r^* in category B where $r^* < \frac{n}{2}$ can be explicitly calculated.

Afterwards, we examine the effect of additional agents on the total effort. In the standard lottery (Tullock) contest with homogenous agents the average agent's effort decreases, but the agents' total effort increases in the number of agents (see, for example, Clark and Riis 1996). In our model, on the other hand, agents can influence the size of the prizes, which may decrease the total effort (see Sela 2020). Thus, the effect of additional agents on the total effort is not clear. Nevertheless, we show that an additional agent in the low category B always increases the total effort.

According to the literature on tournaments, prizes based on rank-orders of performance can be effectively used to provide incentives.[4] Thus, we assume that in addition to the status prizes, agents are awarded monetary prizes such that in each status category a different monetary prize is allocated. Accordingly, that the higher the status category is, the higher is the monetary prize. These monetary prizes change the distribution of the agents between the two status categories such that the optimal number of agents in the high category A is always larger than or equal to the optimal number of agents in this category without monetary prizes. In other words, the monetary prizes increase the relative number of agents in the high status category.

1.1 Related literature

Probably the most well-known status categories can be found in schools or similar organizations in which there are either coarse (A,B, C,..) or fine (100,99,98,....) grading. An important reason for coarse grading is that finer grading usually costs more than coarse grading (see Farhi et al. 2013). However, Harbaugh and Rasmussen (2018) assume that coarse and fine grading are equally costly and find that a certifier who is trying to maximize information to the public should paradoxically coarsen his information before reporting it. An explanation for the superiority of coarse grading is also given by Boleslavsky and Cotton (2015) who analyze competition between schools and find that schools do not give out information on students and therefore will exert more effort to increase the quality of both good and bad students. Ostrovsky and Schwarz (2010) similarly find that the average quality of students at top universities might be so high that if grades will not be uninformed, they will all get jobs, while if the grades will be informed, weaker students

[4]See Edward Lazear and Sherwin Rosen, 1981, Jerry Green and Nancy Stokey, 1983, Barry Nalebuff and Joseph Stiglitz, 1983, and Akerlof and Holden 2012).

might not get jobs. A different explanation for coarse grading is given by Lerner and Tirole (2006) who show that if there are a large number of certifiers with different objectives, grades can be coarse because each certifier optimally chooses a different pass-fail standard. In the present paper, we show that fine grades can induce more competition than coarse grades which results in higher total effort.

Other works take a different perspective on status in organizations. For instance, Fershtman and Weiss (1993) construct a general equilibrium model in which both status and wealth are determined endogenously. Hopkins and Kornienko (2004) study the effect of an exogenous change of income distribution in a model where agents care about their rank in the distribution of consumption. Becker et al. (2005) study a model where status is bought in a market by assuming that there are at least as many status classes as individuals, and that status is a complement to other consumption goods.

A related paper is Dubey and Geanakoplos (2010) who focus on absolute grading and assume (like us) complete information and that the relation between effort and output is stochastic. However, they do not assume a specific contest success function as we do. Rather they assume that the effort choice is binary, while in our model the number of effort levels is not limited. Moreover, the designer's goal in their model is to have all students choose the higher effort level, while in our model the designer's goal is to maximize the agents' total effort.

The paper most related to the current one is Moldovanu et al. (2007) who study the optimal allocation of status classes in a model under incomplete information and assume that the relation between effort and output is deterministic as in the all-pay auction.[5] They also assume as we do that the designer wishes to maximize the agents' total effort, and show that for distributions of abilities that have an increasing failure rate, a proliferation of status classes is optimal. We prove the same result but when the relation between effort and output is stochastic as in the lottery contest. However, when there are less status categories than the number of agents our results do not coincide with theirs. For example, while they show that the top status category always contains a unique agent, in our model with two status categories the top status category includes the majority of the agents. Our work demonstrates that the main result of Moldovanu et al. (2007) according to which a designer who wishes to maximize the agents' total effort would prefer a large number

[5] For all-pay auctions under incomplete information see Aman and Leininger (1996), Krishna and Morgan (1997), Modovanu and Sela (2001), (2006), Moldovanu et al. (2012), and Siegel (2014).

of status categories holds in different contests with either stochastic or deterministic success functions. On the other hand, for a limited number of status categories as in many real life cases, the optimal allocation of agents among the status categories are quite different due to the different properties of the contest success functions.

The rest of the paper is organized as follow: Section 2 presents our lottery contest for status. In Section 3, we analyze the agents' equilibrium strategies. In Section 4 we prove that the optimal number of status categories is the same as the number of agents. In Section 5, we prove some results about the allocation of agents between the two status categories. In Section 6, we show that adding new entrants to any status category increases the agents' total effort. In Section 7, we modify the model to allow monetary prizes in addition to status prizes, and then show that the ratio of the number of agents in the top category increases. Section 8 concludes. Several proofs are relegated to an Appendix.

2 The model

Consider a contest with n agents where each agent i makes an effort x_i. For simplicity, we postulate a deterministic relation between effort and output, and assume them to be equal. Efforts are submitted simultaneously and the cost of effort is $c(x_i) = x_i$. The agents are ranked according to their efforts as follows: agent i, $i = 1, ..., n$ wins the first place with probability $\frac{x_i}{\sum_{s=1}^{n} x_k}$, where x_s is agent s's effort, $s = 1, ..., n$. Then, the second place is determined by the probability success function which is based on the efforts of all the agents excluding the effort of the first winner. Thus, agent i, $i = 1, ..., n$ wins the second place with probability $\sum_{\substack{s=1 \\ s\neq i}}^{n} \frac{x_s}{\sum_{j=1}^{n} x_j} \frac{x_i}{\sum_{\substack{j=1 \\ j\neq s}}^{n} x_j}$, and so on until all the agents are ranked. Before the agents exert their efforts, the designer chooses a partition $\{(0, r_1], (r_1, r_2], ...(r_{i-1}, r_i], ..., (r_{k-1}, n]\ \}$ of the interval $(0, n]$ by the integers $r_i, i = 0, ..., n$ where $r_{i-1} < r_i$. For convenience, we define $r_0 \equiv 0$ and $r_k \equiv n$. This partition divides the agents into $k \geq 1$ status categories whereby an agent is included in category i if the place he won is between the r_{i-1}-th and the r_i-th highest ones. Each agent cares about the number of agents in categories both below and above him, and we assume that the status prize of being in category i is

$$v_i = r_{i-1} - (n - r_i).$$

Thus, the prize of being in category i is the difference between the number of agents in the lower categories (category j, $j = 1, ..., i-1$) and the number of agents in the higher categories (category j, $j = i+1, ..., k$). Note that this formulation captures the zero-sum nature of status: for any partition of status categories, the total value derived from status is

$$\sum_{i=1}^{k}(r_i - r_{i-1})v_i = \sum_{i=1}^{k}(r_i - r_{i-1})(r_i + r_{i-1} - n) = 0.$$

We refer to the above contest as a lottery contest for status.

In the Tullock contest with n symmetric agents and k prizes, $w_1 \geq w_2 \geq .... \geq w_k$, the maximization problem of an agent is

$$\max_x w_i \sum_{i=1}^{k} \frac{y^{i-1}x\left(\frac{n-1)!}{(n-i)!}\right)}{\Pi_{j=1}^{i}\left((n-j)y + x\right)} \tag{1}$$

where x is the agent's strategy and y is the symmetric strategy of all the other agents. In our model with n symmetric agents and k categories of status with $r_i - r_{i-1}$ agents in category $i, i = 1, ..., k$, the prizes are $v_i = r_{i-1} + r_i - n$ where $v_1 \leq v_2 \leq, ... \leq v_k$. Then, using the solution of (1) we obtain the symmetric equilibrium effort as follows:

Proposition 1 *The agents' symmetric equilibrium effort in the lottery contest for status with k categories of status and $r_i - r_{i-1}$ agents in category $i, i = 1, ..., k$ is*

$$x = \sum_{i=1}^{k} \sum_{j=r_{i-1}}^{r_i - 1} (r_i + r_{i-1} - n)\frac{1 - (H_n - H_j)}{n}, \tag{2}$$

where H_n is defined by

$$\begin{aligned} H_n &= \sum_{i=1}^{n} \frac{1}{i},\ n = 1, 2, ... \\ H_0 &\equiv 0. \end{aligned}$$

Proof. See Appendix. ■

The following example illustrates the equilibrium effort in a lottery contest with three status categories and six agents.

Example 1 *Consider a lottery contest for status with three status categories $A, B,$ and C, each of which* *includes two agents. Then,* $r_0 = 0, r_1 = 2, r_2 = 4,$ *and* $r_3 = 6$. *By (2), the symmetric equilibrium effort is*

$$\begin{aligned}
x &= (r_1 + r_0 - 6)(\frac{1-(H_6 - H_1)}{6} + \frac{1-(H_6 - H_0)}{6}) \\
&\quad +(r_2 + r_1 - 6)(\frac{1-(H_6 - H_3)}{6} + \frac{1-(H_6 - H_2)}{6}) \\
&\quad +(r_3 + r_2 - 6)(\frac{1-(H_6 - H_5)}{6} + \frac{1-(H_6 - H_4)}{6}) \\
&= -4(\frac{1-(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6})}{6} + \frac{1-(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6})}{6}) \\
&\quad +0(\frac{1-(\frac{1}{4}+\frac{1}{5}+\frac{1}{6})}{6} + \frac{1-(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6})}{6}) \\
&\quad +4(\frac{1-(\frac{1}{6})}{6} + \frac{1-(\frac{1}{5}+\frac{1}{6})}{6}) \\
&= 2.2444
\end{aligned}$$

3 The optimal number of status categories

So far we assumed that there are only two status categories. In this section we analyze the optimal number of status categories for a designer who wishes to maximize the agents' total effort and show that the optimal number of status categories is equal to the number of agents, namely, $k = n$, such that in each status category there is only one agent.

Proposition 2 *In the lottery contest for status, if the designer wishes to maximize the agents' total effort, the optimal number of status categories is equal to the number of agents..*

Proof. See Appendix. ■

The following example illustrates the result of Proposition 2.

Example 2 *Consider a lottery contest for status with three agents. By Example 1, if there are two status categories A and B where in the high category A there is one agent, and in the low category B there are two agents, the symmetric equilibrium effort is* $x = \frac{2}{3}$. *On the other hand, if there are two status categories A and B where in the high category A there are two agents and in category B there is one agent, the symmetric equilibrium effort is* $x = \frac{5}{6}$. *If, however, there are three status categories A, B and C, each of which includes* *one agent, then $r_0 = 0, r_1 = 1, r_2 = 2$, and $r_3 = 3$. By (2), the symmetric equilibrium effort is*

$$\begin{aligned} x &= (r_1 - r_0 - 3)(\frac{1-(H_3-H_0)}{3}) \\ &\quad +(r_2 - r_3 - 1)(\frac{1-(H_3-H_1)}{3}) \\ &\quad +(r_3 + r_2 - 3)(\frac{1-(H_3-H_2)}{3}) \\ &= (-2)(\frac{1-(\frac{1}{3}+\frac{1}{2})}{3}) \\ &\quad +(0)(\frac{1-(\frac{1}{3}+\frac{1}{2}+1)}{3}) \\ &\quad +(4)(\frac{1-\frac{1}{3}}{3}) = \frac{7}{9} \end{aligned}$$

Then, we can see that the highest symmetric equilibrium effort is obtained when there are three status categories, each category contains one agent only.

Moldovanu et al. (2007) show that if the agents compete in the all-pay auction under incomplete information, and the designer wishes to maximize the agents' expected total effort, then the optimal number of status categories might be equal to the number of agents. Proposition 2 demonstrate that even when the contest success function is stochastic as in a lottery contest, the designer who wishes to maximize the agents' total effort wants to maximize the number of status categories subject to the constraint that each status category contains at least one agent. However, the real-life examples show that the number of status categories is limited and usually much smaller than the optimal one. Thus, for simplicity, in the following we focus on the case of only two status categories.

4 The optimal allocation of agents in contests with two status categories

We consider a lottery contest with n risk-neutral agents who are divided into two status categories A and B, where r agents are allocated in the lower category B and $n-r$ are allocated in the higher category A. Then, the maximization problem of an agent is

$$\max_x r \sum_{i=1}^{n-r} \frac{y^{i-1} x \left(\frac{n-1)!}{(n-i)!}\right)}{\Pi_{j=1}^{i}\left((n-j)y+x\right)} - (n-r)\sum_{i=1}^{r} \frac{y^{n-i} x (\frac{(n-1)!}{(i-1)!})}{\Pi_{j=i}^{n-1}(jy+x)} - x, \qquad (3)$$

where x is this agent's effort, and y is the symmetric effort of all his opponents. By Proposition 1 we have

Proposition 3 *In the lottery contest for status with n symmetric agents in which category A includes $n-r$ agents and category B includes r agents, the symmetric equilibrium effort is*

$$x = r\sum_{i=1}^{n-r} \frac{1-(H_n - H_{n-i})}{n} - (n-r)\sum_{i=n-r+1}^{n} \frac{1-(H_n - H_{n-i})}{n}. \tag{4}$$

The following example illustrates the effect of the allocation of agents on their total effort.

Example 3 *Consider a lottery contest for status with three agents and two status categories A and B. Then we have the following two options:*

1) In category A there is one agent and in category B there are two agents. Then, by (4), the symmetric equilibrium effort is

$$\begin{aligned} x &= 2\frac{1-(H_3-H_2)}{3} - \left(\frac{1-(H_3-H_1)+1-(H_3-H_0)}{3}\right) \\ &= 2\frac{1-\frac{1}{3}}{3} - (\frac{1-(\frac{1}{2}+\frac{1}{3})}{3} + \frac{1-(1+\frac{1}{2}+\frac{1}{3})}{3} = \frac{2}{3}. \end{aligned}$$

2) In category A there are two agents and in category B there is one agent. Then, by (4), the symmetric equilibrium effort is

$$\begin{aligned} x &= \frac{1-(H_3-H_2)}{3} + \frac{1-(H_3-H_1)}{3} - 2\left(\frac{1-(H_3-H_0)}{3}\right) \\ &= \frac{1-\frac{1}{3}}{3} + \frac{1-(\frac{1}{3}+\frac{1}{2})}{3} - 2(\frac{1-(1+\frac{1}{3}+\frac{1}{2}}{3}) = \frac{5}{6}. \end{aligned}$$

Below, we examine whether the findings of the above example can be generalized for contests with $n > 3$ agents. For this purpose, let x_r be the symmetric equilibrium effort when there are r agents in category B and $(n-r)$ agents in category A. Then, we have

Lemma 1 *In the lottery contest for status with two categories, the difference between the agents' symmetric equilibrium efforts when there are r agents and $r+1$ agents in category B is*

$$\Delta x_r = x_r - x_{r+1} = H_r - \sum_{i=1}^{n} \frac{H_{n-i}}{n} \tag{5}$$

Proof. See Appendix. ■

It is obvious that in order to maximize the agents' equilibrium efforts, the designer should allocate agents in both status categories since otherwise the agents do not have an incentive to exert positive efforts. Using (5), we show that in contrast to Example 3, $\Delta x_1 = x_1 - x_2 < 0$ for all $n \geq 4$, namely, if the number of agents is larger than three then category B should include more than one agent.

Proposition 4 *In the lottery contest for status with two status categories, if there are at least four agents, it is optimal to allocate more than one agent in the lower category B in order to maximize the agents' total effort.*

Proof. See Appendix. ■

Likewise, by (5), we show that $\Delta x_{n-2} = x_{n-2} - x_{n-1} > 0$ for all $n \geq 3$, namely, if the number of agents is larger than two, then the higher category B should include less than $n-1$ agents, namely, category A should include more than one agent.

Proposition 5 *In the lottery contest with two status categories, if there are at least three agents, it is optimal to allocate more than one agent in category A in order to maximize the agents' total effort.*

Proof. See Appendix. ■

Note that Moldovanu et al. (2007) show that when agents compete in an all-pay auction under incomplete information, the highest status category should include one agent, while by Proposition 5, in our model, the highest status category includes more than one agent. In addition, by Propositions 4 and 5, we know that there is at least one local maximal allocation of agents since Δx_r changes its sign at least once. The following result shows that Δx_r is monotonically increasing in r and therefore there is only one optimal allocation of agents between the two status categories.

Proposition 6 *In the lottery contest for status with two status categories, there is a unique optimal allocation of agents with $n - r^*$ agents in category A and r^* agents in category B where*

$$r^* = \min\{r : \Delta x_r > 0\}.$$

Proof. See Appendix. ■

The following example illustrates this proposition.

Example 4 *Consider a lottery contest for status with four agents and two categories A and B. By (5), we have*

$$\begin{aligned}\Delta x_1 &= x_1 - x_2 = \frac{(4-1)}{4}H_1 - \frac{1}{4}\sum_{\substack{i=1\\ i\neq 2}}^{4} H_{i-1}\\ &= \frac{3}{4} - \frac{1}{4}((1+\frac{1}{2}) + (1+\frac{1}{2}+\frac{1}{3})) = -0.0833,\end{aligned}$$

$$\begin{aligned}\Delta x_2 &= x_2 - x_3 = \frac{(4-1)}{4}H_2 - \frac{1}{4}\sum_{\substack{i=1\\ i\neq 3}}^{4} H_{i-1}\\ &= \frac{3}{4}(1+\frac{1}{2}) - \frac{1}{4}(1+(1+\frac{1}{2}+\frac{1}{3})) = 0.416\,67,\end{aligned}$$

and

$$\begin{aligned}\Delta x_3 &= \frac{(4-1)}{4}H_3 - \frac{1}{4}\sum_{\substack{i=1\\ i\neq 4}}^{4} H_{i-1}\\ &= \frac{3}{4}(1+\frac{1}{2}+\frac{1}{3}) - \frac{1}{4}(1+(1+\frac{1}{2})) = 0.75.\end{aligned}$$

We can see that Δx_r increases in r. Since $\Delta x_1 < 0$ and $\Delta x_2 > 0$, we obtain that $x_2 > x_1$ and $x_2 > x_3$. Therefore, the optimal allocation of agents is two in each of the two categories.

By Propositions 4 and 5, we know that if the number of agents is sufficiently large (more than three) each of the categories should include more than one agent. Using (5), we show that $\Delta x_{\frac{n}{2}} > 0$ for an even n. Thus, since Δx_r is monotonically increasing in r, we obtain the following result.

Proposition 7 *In the lottery contest for status with two status categories, if the designer maximizes the agents' equilibrium effort, the number of agents in the low category B is smaller than or equal to the number of agents in the high category A.*

Proof. See Appendix. ■

5 Additional agents

In the standard lottery contest with homogenous agents, the agents' average effort decreases in the number of agents while the agents' total effort increases in the number of agents (see, Tullock 1980). In our model, agents affect the size and the number of prizes, and since additional prizes may decrease the total effort, it is not clear whether an additional agent has the same effect on the equilibrium effort as in the standard lottery contest. Below we investigate this question.

The total effort in a contest with n agents where r of them are in the low category B is:

$$TE(n,r) = r\left(\sum_{i=1}^{n-r} 1 - (H_n - H_{n-i})\right) - (n-r)\left(\sum_{i=n-r+1}^{n} 1 - (H_n - H_{n-i})\right). \tag{6}$$

when we add one agent to the low category B, the total effort is

$$TE(n+1,r+1) = (r+1)\left(\sum_{i=1}^{n-r} 1 - (H_{n+1} - H_{n+1-i})\right) - (n-r)\left(\sum_{i=n-r+1}^{n+1} 1 - (H_{n+1} - H_{n+1-i})\right). \tag{7}$$

A comparison of (6) and (7) yields

Proposition 8 *In the lottery contest for status, an additional agent in category B always increases the agents' total effort.*

Proof. See Appendix. ■

The intuitive explanation for the result given by Proposition 8 is as follows: When one agent is added to category B, the value of each of the $n-r$ agents in category A increases from r to $r+1$, while the value of each of the r agents in category B is not changed and remains $-(n-r)$. By this argument, it is clear that their total effort increases as well.

6 Monetary prizes

We now assume that in addition to the status prizes according to which agents get positive utility proportional to the number of agents in lower status categories and negative utility proportional to the number of agents in higher status categories, agents are awarded monetary prizes such that the higher the status category is, the higher is the monetary prize. Formally, consider a set of k monetary prizes $w_k \geq w_{k-1} \geq ... \geq w_1$ where $\sum_{j=1}^{k} w_j = m$ and a family of division points $\{r_i\}_{i=0}^{k}$ where $r_0 = 0$ and $r_k = n$ determines a partition with k categories. A contestant ranked in the top category k (i.e., a contestant whose effort is among the top $r_k - r_{k-1}$) receives a monetary prize of w_k, a contestant in the second highest category receives a prize of

w_{k-1}, and so on. Thus, an agent who is in the status category $i, 1 \leq i \leq k$ receives a total prize (monetary + status) of

$$v_i = w_i + r_{i-1} - (n - r_i).$$

In a case of two status categories, the maximization problem of an agent who exerts an effort of x is given by

$$\max_x (r + w_A) \sum_{i=1}^{n-r} \frac{y^{i-1} x \left(\frac{n-1)!}{(n-i)!}\right)}{\Pi_{j=1}^{i} ((n-j)y + x)} - (n - r + w_B) \sum_{i=1}^{r} \frac{y^{n-i} x (\frac{(n-1)!}{(i-1)!})}{\Pi_{j=i}^{n-1} (jy + x)} - x,$$

where y is the symmetric effort of all the other agents. By symmetry, $x = y$ and the equilibrium effort is

$$x = (w_A + r) \left(\sum_{i=1}^{n-r} \frac{1 - (H_n - H_{n-i})}{n} \right) + (n - r + w_B) \left(\sum_{j=1}^{r} \frac{(H_n - H_{j-1} - 1)}{n} \right). \quad (8)$$

As in the previous sections, let x_r be the equilibrium effort when there are r agents in category B and $(n-r)$ agents in category A. Then, we have

Lemma 2 *In the lottery contest for status with two categories A and B and monetary prizes w_A and w_b, the difference between the agents' symmetric equilibrium efforts where there are r agents and $r+1$ agents in category B is*

$$\Delta x_r = x_r - x_{r+1} = (w_A + w_B) \frac{1 - H_n}{n} + \frac{(w_A + w_B + n)}{n} H_r - \sum_{i=1}^{n} \frac{H_{n-i}}{n}. \quad (9)$$

Proof. See Appendix. ■

By analyzing the effect of the monetary rewards on Δx_r given by (9), we obtain that

Proposition 9 *In the lottery contest for status, the optimal number of agents in category A when there are monetary prizes is always larger than or equal to the optimal number of agents in category A without monetary prizes. Furthermore, if the number of agents is larger than three, the optimal number of agents in each category is larger than one.*

Proof. See Appendix. ■

7 Conclusion

We studied homogeneous agents who are partitioned into status categories according to the stochastic lottery success function of their outputs. We described the structure of the optimal partition into two status categories from the point of view of a designer who wishes to maximize the total output. We also studied the interplay between pure status and monetary prizes.

A comparison with the work of Moldovanu et al. (2007), who studied a similar model but with a deterministic success function (all-pay auction) revealed both different and similar results. We both prove that the optimal number of status categories is the maximal one, namely, for each agent there is a different status category. On the other hand, we prove that the top status category includes the majority of the agents while they prove that it includes only one agent. Last, they show that if monetary prizes are added to the status prizes then the optimal number of status categories completely changes and becomes two and the top status category contains one agent , while we show that monetary prizes may change the distribution of the agents between the two status categories such that the relative number of agents in the top status category increases. Consequently, the main message of this paper is that the top status categories should not be exclusive, and if the goal is to maximize the agents' output, there is no reason to limit the number of agents in the top status category to only a few agents.

8 Appendix

8.1 Proof of Proposition 1

According to Sela (2020), in the Tullock contest with n symmetric players and k prizes $w_i > w_{i+1}, i = 1, ..., k$, the symmetric equilibrium effort is

$$x = \sum_{i=1}^{k} w_i \frac{1 - (H_n - H_{n-i})}{n}, \tag{10}$$

where H_n, $n \geq 1$ is given by

$$H_n = \sum_{i=1}^{n} \frac{1}{i},$$

and

$$H_0 \equiv 0.$$

Since the status prize of being in category i is

$$v_i = r_{i-1} - (n - r_i),$$

where $v_{i+1} > v_i, i = 1, ..., k$, and there are $r_i - r_{i-1}$ agents in category i who receive the same prize of v_i, we obtain that the symmetric equilibrium effort is

$$x = \sum_{i=1}^{k} \sum_{j=r_{i-1}}^{r_i - 1} (r_i + r_{i-1} - n) \frac{1 - (H_n - H_j)}{n}.$$

Q.E.D.

9 Proof of Proposition 2

Consider two adjacent categories that will be denoted by m and $m+1$, $1 \leq m < k$. By (2), the contribution of the agents in these status categories to the total effort is

$$\begin{aligned} y &= \sum_{i=m}^{m+1} \sum_{j=r_{i-1}}^{r_i - 1} (r_i + r_{i-1} - n) \frac{1 - (H_n - H_j)}{n} \\ &= \sum_{j=r_m}^{r_{m+1}-1} (r_{m+1} + r_m - n) \frac{1 - (H_n - H_j)}{n} \\ &\quad + \sum_{j=r_{m-1}}^{r_m - 1} (r_m + r_{m-1} - n) \frac{1 - (H_n - H_j)}{n}. \end{aligned}$$

If we add the agents from category m to category $m+1$, there is no effect on the agents' efforts in other categories, and the contribution of the agents in this new status category on the total effort is then

$$\widetilde{y} = \sum_{j=r_{m-1}}^{r_{m+1}-1} (r_{m+1} + r_{m-1} - n) \frac{1 - (H_n - H_j)}{n}.$$

The difference between these cases is

$$\begin{aligned} \widetilde{y} - y &= -(r_m - r_{m-1}) \sum_{j=r_m}^{r_{m+1}-1} \frac{1 - (H_n - H_j)}{n} \\ &\quad + (r_{m+1} - r_m) \sum_{j=r_{m-1}}^{r_m - 1} \frac{1 - (H_n - H_j)}{n}, \end{aligned}$$

With some calculations we obtain

$$
\begin{aligned}
\widetilde{y}-y &= -(r_m-r_{m-1})\sum_{j=r_m}^{r_{m+1}-1}\frac{H_j}{n}+(r_{m+1}-r_m)\sum_{j=r_{m-1}}^{r_m-1}\frac{H_j}{n} \\
&< -(r_m-r_{m-1})(r_{m+1}-r_m)\min_j\{\frac{H_j}{n}\}_{j=r_m}^{r_{m+1}-1}+(r_{m+1}-r_m)(r_m-r_{m-1})\max_j\{\frac{H_j}{n}\}_{j=r_{m-1}}^{r_m-1} \\
&= -(r_m-r_{m-1})(r_{m+1}-r_m)\frac{H_{r_m}}{n}+(r_{m+1}-r_m)(r_m-r_{m-1})\frac{H_{r_{m-1}}}{n} \\
&= (r_{m+1}-r_m)(r_m-r_{m-1})\frac{-1}{nr_m}<0.
\end{aligned}
$$

Thus, if the goal is to maximize the agents' total effort, it is optimal to divide any category into two adjacent categories. *Q.E.D.*

9.1 Proof of Lemma 1

By (4), we have

$$
\begin{aligned}
\Delta x_r &= x_r-x_{r+1}=r\sum_{i=1}^{n-r}\frac{1-(H_n-H_{n-i})}{n}-(n-r)\sum_{i=n-r+1}^{n}\left(\frac{1-(H_n-H_{n-i})}{n}\right) \\
&\quad -(r+1)\sum_{i=1}^{n-r-1}\frac{1-(H_n-H_{n-i})}{n}+(n-r-1)\sum_{i=n-r}^{n}\left(\frac{1-(H_n-H_{n-i})}{n}\right) \\
&= -\sum_{i=n-r+1}^{n}\left(\frac{1-(H_n-H_{n-i})}{n}\right)+(n-r-1)\frac{1-(H_n-H_r)}{n} \\
&\quad -\sum_{i=1}^{n-r-1}\frac{1-(H_n-H_{n-i})}{n}+r\frac{1-(H_n-H_r)}{n} \\
&= (n-1)\frac{1-(H_n-H_r)}{n}-\sum_{\substack{i=1\\ i\neq n-r}}^{n}\left(\frac{1-(H_n-H_{n-i})}{n}\right) \\
&= H_r-\sum_{i=1}^{n}\frac{H_{n-i}}{n}.
\end{aligned}
$$

Q.E.D.

9.2 Proof of Proposition 4

By (5) we have

$$
\begin{aligned}
\Delta x_1 &= x_1-x_2=H_1-\sum_{i=1}^{n}\frac{H_{n-i}}{n} \\
&< 1-\frac{(n-1)}{n}-\frac{(n-2)}{n}\frac{1}{2}-\frac{(n-3)}{n}\frac{1}{3} \\
&= \frac{-5n+18}{6n}
\end{aligned}
$$

Thus, we obtain that $\Delta x_1 = x_1 - x_2 < 0$ for all $n \geq 4$, namely, the symmetric equilibrium effort is higher if there are two agents in category B than if there is only one agent. *Q.E.D.*

9.3 Proof of Proposition 5

By (5), we have

$$\begin{aligned}
\Delta x_{n-2} &= H_{n-2} - \sum_{i=1}^{n} \frac{H_{n-i}}{n} \\
&> 1 - \frac{n-1}{n} - \frac{\frac{1}{(n-1)}}{n} \\
&= \frac{n-2}{n(n-1)}
\end{aligned}$$

Thus, we obtain that $\Delta x_{n-2} = x_{n-2} - x_{n-1} > 0$ for all $n \geq 3$, namely, the symmetric equilibrium effort is higher if there are two agents in category A than if there is only one agent. *Q.E.D.*

9.4 Proof of Proposition 6

By (5), we have

$$\Delta x_r = x_r - x_{r+1} = H_r - \sum_{i=1}^{n} \frac{H_{n-i}}{n}$$

Since Δx_r is monotonically increasing, the optimal number of agents in category B is given by

$$r^* = \min\{r : \Delta x_r > 0\}.$$

Since by Proposition 4 $\Delta x_1 < 0$, and by Proposition 5 $\Delta x_{n-2} > 0$, the monotonicity of Δx_r implies that there is a unique optimal allocation according to which there are $n - r^*$ agents in category A and r^* agents in category B. *Q.E.D.*

9.5 Proof of Proposition 7

Without loss of generality, we assume that n is even. Then, by (5), we have

$$\begin{aligned}
\Delta x_{\frac{n}{2}} &= H_{\frac{n}{2}} - \sum_{i=1}^{n} \frac{H_{n-i}}{n} \\
&= \frac{1}{n}\Big(\sum_{j=0}^{\frac{n}{2}-1}\big(\frac{n}{2}-j\big)\frac{1}{\frac{n}{2}-j} - \sum_{j=1}^{\frac{n}{2}-1}\big(\frac{n}{2}-j\big)\frac{1}{\frac{n}{2}+j}\Big) \\
&\geq \frac{1}{n}\sum_{j=1}^{\frac{n}{2}-1}\big(\frac{n}{2}-j\big)\Big(\frac{1}{\frac{n}{2}-j} - \frac{1}{\frac{n}{2}+j}\Big)\Big) > 0.
\end{aligned}$$

Thus, if n is even allocating $\frac{n}{2}$ agents in category B yields a higher equilibrium effort than allocating $\frac{n}{2}+1$ agents.[6] Hence, since by Proposition 6, Δx_r is monotonically increasing in r, we obtain that the optimal number of agents in category B is smaller than or equal to $\frac{n}{2}$. $Q.E.D.$

9.6 Proof of Proposition 8

The difference between the total effort in a contest with $n+1$ agents when $r+1$ of them are allocated in category B (which is given by (7)) and the total effort in a contest with n agents when r of them are allocated in category B (which is given by (6)) is

$$\begin{aligned}
& TE(n+1, r+1) - TE(n,r) \\
= \; & (r+1)\left(\sum_{i=1}^{n-r} 1 - (H_{n+1} - H_{n+1-i})\right) - (n-r)\left(\sum_{i=n-r+1}^{n+1} 1 - (H_{n+1} - H_{n+1-i})\right) \\
& - r\left(\sum_{i=1}^{n-r} 1 - (H_n - H_{n-i})\right) + (n-r)\left(\sum_{i=n-r+1}^{n} 1 - (H_n - H_{n-i})\right) \\
= \; & \sum_{i=1}^{n-r} 1 - (H_{n+1} - H_{n+1-i}) + r\sum_{i=1}^{n-r}(-H_{n+1} + H_n + H_{n+1-i} - H_{n-i}) \\
& -(n-r)\sum_{i=n-r+1}^{n}(H_n - H_{n+1} + H_{n+1-i} - H_{n-i}) - (n-r)(1 - H_{n+1}) \\
= \; & \sum_{i=1}^{n-r} H_{n+1-i} + r\sum_{i=1}^{n-r}\frac{1}{n+1-i} - (n-r)\sum_{i=n-r+1}^{n}\frac{1}{n+1-i} \\
\geq \; & \sum_{i=1}^{n-r} H_{n+1-i} - (n-r)\sum_{i=n-r+1}^{n}\frac{1}{n+1-i} \\
= \; & \sum_{i=1}^{n-r} H_{n+1-i} - (n-r)H_{r+1} > 0
\end{aligned}$$

[6] If n is odd then the same argument holds for $\lfloor \frac{n}{2} \rfloor$.

Therefore, $TE(n+1, r+1) - TE(n, r) > 0$, which means that an additional agent in category B always increases the agents' total effort. $Q.E.D.$

$Q.E.D.$

9.7 Proof of Lemma 2

By (8), we have

$$\begin{aligned}
\Delta x_r &= x_r - x_{r+1} = (w_A + r)\sum_{i=1}^{n-r}\frac{1-(H_n - H_{n-i})}{n} - (w_B + n - r)\sum_{i=n-r+1}^{n}\left(\frac{1-(H_n - H_{n-i})}{n}\right) \\
&\quad -(w_A + r + 1)\sum_{i=1}^{n-r-1}\frac{1-(H_n - H_{n-i})}{n} + (w_B + n - r - 1)\sum_{i=n-r}^{n}\left(\frac{1-(H_n - H_{n-i})}{n}\right) \\
&= -\sum_{i=n-r+1}^{n}\left(\frac{1-(H_n - H_{n-i})}{n}\right) + (w_B + n - r - 1)\frac{1-(H_n - H_r)}{n} \\
&\quad -\sum_{i=1}^{n-r-1}\frac{1-(H_n - H_{n-i})}{n} + (w_A + r)\frac{1-(H_n - H_r)}{n} \\
&= (w_A + w_B + n - 1)\frac{1-(H_n - H_r)}{n} - \sum_{\substack{i=1 \\ i \neq n-r}}^{n}\left(\frac{1-(H_n - H_{n-i})}{n}\right) \\
&= (w_A + w_B)\frac{1-H_n}{n} + \frac{(w_A + w_B + n)}{n}H_r - \sum_{i=1}^{n}\frac{H_{n-i}}{n}.
\end{aligned}$$

$Q.E.D.$

9.8 Proof of Proposition 9

By (9), we have

$$\frac{d\Delta x_r}{dw_A} = \frac{d\Delta x_r}{dw_B} = \frac{1 - H_n + H_r}{n}.$$

Thus, if r is sufficiently large, Δx_r increases in the monetary prizes, and if r is sufficiently small, Δx_r decreases in the monetary prizes. Therefore we obtain that $\Delta x_1 < 0$ and $\Delta x_{n-2} > 0$ as well as in the lottery contest for status without monetary prizes. This implies that if $n \geq 4$,the optimal number of agents in category A and in category B is larger than one. In particular, we have

$$\frac{d\Delta x_r}{dw_A} = \frac{d\Delta x_r}{dw_B} > \frac{1 - \frac{n-r}{r+1}}{n} = \frac{2r + 1 - n}{n}.$$

Since for $r \geq \frac{n}{2}$, $\frac{dx_r}{dw_A} = \frac{dx_r}{dw_B} > 0$, we obtain that $\Delta x_{\frac{n}{2}} > 0$ with monetary prizes as well without them. This implies that the optimal number of agents in category A when there are monetary prizes is always larger than or equal to the optimal number of agents in category B. Since $\frac{d\Delta x_r}{dw_A} = \frac{d\Delta x_r}{dw_B} > 0$ for all $r > \frac{n}{2}$ we obtain that the optimal number of agents in category A when there are monetary prizes is always larger than or equal to the optimal number of agents in category A without monetary prizes. $Q.E.D.$

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