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Abstract

We introduce heterogeneous worker preferences into a Ricardian framework of task allocation. Preference misalignment creates strategic interactions that affect teamwork and efficiency. We find that efficient outcomes may emerge as Nash equilibria only when all workers are deferential. Notably, all Nash equilibria can be classified by a *chemistry* index capturing the aggregate nature of the team. When workers as a group are self-centered, the equilibrium is inefficient because there is incomplete division of labor and workers specialize in some tasks that they should not be performing. When workers as a group are deferential, multiple Nash equilibria exist, some of which achieve full division of labor. However, workers tend to specialize in tasks that they should not be performing, and their effort allocation may run counter to their comparative advantages. These results extend to the case of teams with n members, provided that individual goals do not diverge excessively from the firm's objective. The analysis highlights limits to the scope of the role of authority in organizations.

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"The greatest improvement in the productive powers of labor, and the greater part of the skill, dexterity, and judgment with which it is any where directed, or applied, seem to have been the effects of the division of labor"

–Adam Smith, *The Wealth of Nations*, I:I, p.13

1 Introduction

Over the last few decades, a rich literature in economics has studied the returns to human capital and the distribution of earnings in the US and other countries.[1] A prominent development in modern contributions is a focus on the distinction between workers' skills and job tasks, with the assignment of skills to tasks determined by task demands, labor supplies, and technologies. In particular, recent research considers a tractable task-based Ricardian framework where workers are endowed with skills, have different comparative advantages, and where the allocation of skills to tasks is endogenous (see, for example, Acemoglu and Autor (2011), Acemoglu, Kong and Restrepo (2024)). Models that apply this framework are important in that they provide an effective account for some of the rich relationships that are observed in modern labor markets between skills, tasks, technologies, and trade.[2]

Costinot and Vogel (2015) provide a detailed review of what are termed "assignment models" or Ricardo-Roy models. This is an extensive literature that has introduced the term "task trade" to describe the international division of labor whereby different countries add value to global supply chains, and to distinguish it from goods trade. They show that these models have also contributed to a revival of international trade research.[3]

[1]The literature is extensive, but see Katz and Murphy (1992), Autor, Katz and Krueger (1998), Acemoglu (2002), Autor, Katz and Kearney (2008), and Goldin and Katz (2008).

[2]These include Deming (2017), which shows that this framework is also useful to explain the growing importance of social skills in labor market, in which workers "trade tasks" within a team as a means to exploit their comparative advantages and reduce coordination costs. The interplay between coordination gains and communication costs was already stressed in Becker and Murphy (1992), Radner (1993), Bolton and Dewatripont (1994), and Garicano (2000). See Freund (2023) for a study of the micro origins of coworker complementarities (specialization) and their impact.

[3]See Costinot and Donaldson (2012), Costinot, Donaldson, and Komunjer (2012), Costinot et al (2015), Grossman and Rossi-Hansberg (2008, 2012), and Rossi-Hansberg (2017). Much of the explosion of frameworks that have at their core a Ricardian theory is due to the influential Eaton and Kortum (2002).

Closely related, the literature on organizations and trade conveys the idea that organizational decisions provide valuable insights into the aggregate workings of the world economy, and stresses the importance of microfounding the origin and properties of production functions (Antràs and Rossi-Hansberg (2009)). From this viewpoint, Garicano and Rossi-Hansberg (2015) highlight that most human endeavors require team collaboration, and that mainstream economic models often overlook the organizational challenges inherent in production processes. They then survey the literature on knowledge-based hierarchies and offer an organizational-based explanation of a number of empirical facts as a "distinct specification of the task-based approach in which tasks are hierarchical and the production function that links the different tasks is based on an explicit organizational problem" (p.7). The current paper shares these perspectives, specifically the need to microfound team production functions in task-based organizational problems.

A common aspect of all the contributions that have emerged in a Ricardian team production framework in the literature is that they involve no strategic interactions. Essentially, the different models deal with either a profit-maximizing firm, which yields efficient outcomes or, as in the classic Dornbusch, Fisher and Samuelson (1977) and Eaton and Kortum (2002) analyses, with the competitive equilibrium concept which by the First Welfare Theorem also leads to efficient outcomes.

This absence is rather surprising because strategic interactions occupy a central position in the literature on the economics of the firm and organizations. For example, a lack of congruence between workers' and the firm's goals induces a wide range of strategic behavior, and plays a main role in the research on authority in organizations (Aghion and Tirole (1997)).[4] Similarly, there is a literature on the role of *identity* in firms. Akerlof and Kranton (2005), for instance, argue for an expanded economic model of work incentives and organizations to include the concept of identity, as "the identities of employees, who may (more or less) identify with their firms, workgroups or jobs, are central to the study of work in sociology, psychology, anthropology and management." Heterogeneous identities and personality traits among workers naturally generate, again, strategic interactions.[5]

[4]The entire principal-agent literature is based on the lack of congruence between the principal's and agent's preferences. See, e.g., Prendergast (1999), and Laffont and Martimort (2009).

[5]Antràs, Garicano and Rossi-Hansberg (2006) address the problems associated with the formation of

In this paper, we are motivated by these theoretical and empirical literatures to open up the Ricardian framework to include strategic interactions between workers. Specifically, individuals within a team may not identify *exactly* with the firm's objective function and their respective goals may not be congruent with that of the team and with each other. We study how this heterogeneity in preferences has an impact on the endogenous assignment of skills to tasks and hence on production processes. A worker may ascribe a personal worth to his own contributions to the team that diverges from the objective value he brings. We define *self-centered* workers as those who ascribe a subjective value that is greater than their objective contributions (this would be akin to home bias in international trade), and define *deferential* workers as those who undervalue their contributions. We study the implications of these teamwork frictions on the allocation of skills to tasks, specialization, and efficiency in an otherwise standard Ricardian framework with two agents. In principle, the outcome of this collective strategic choice problem would not seem apparent. Interestingly enough, we find the analysis to be tractable and that the outcome ultimately depends *only* on the aggregate nature of the team. We obtain the following results:

First, the efficient outcome in the frictionless framework emerges as a Nash equilibrium under teamwork frictions only if every worker in the team is deferential. Otherwise, strategic interactions generate departures from efficiency.

Second, when teamwork frictions are present, the Nash equilibria can be classified by a *chemistry index* that captures the aggregate nature of the group of workers. This index, which is defined as the extent to which workers *as a group* are either self-centered or deferential, is sufficient. Specifically:

(1) When workers as a group are self-centered, the Nash equilibrium is inefficient because, contrary to the frictionless framework, there is a set of tasks that *both* players perform.

(2) When workers as a group are deferential, there is a continuum of Nash equilibria. Interestingly, this multiplicity includes equilibria in which individuals do not share tasks. However, in many equilibria workers specialize in tasks that according to efficiency they should not be performing, and fail to perform tasks that efficiency dictates they should

international production teams. See also Grossman (2013) on worker heterogeneity.

specialize in.

These results have the potential to explain (a) why a complete division of labor may not always be observed empirically, even when tasks can be traded costlessly, (b) why, when there is complete division of labor, we may still observe individuals performing tasks that should be performed by other individuals from an efficiency perspective, and (c) why we may observe workers' specialization that moves *against* their comparative advantages. None of these patterns, if empirically observed, are predicted by standard Ricardian models.

After studying the role of frictions in the standard Ricardian model, we extend the analysis to the general case of teams with n members. We find that when workers ascribe a personal worth to their contributions to the team that does not diverge *excessively* from the objective value they bring, the equilibrium looks exactly as a succession, in the space of tasks, of pairs of individuals which, remarkably, behave as in the equilibria in the standard framework. Put differently, we find that the main results in the basic model are at the heart of the equilibria in teams with n members.[6] We also provide the intuition for this result and illustrate it in an example where we provide the full characterization of all Nash equilibria.

In terms of additional implications, we also view the analysis as shedding new light on the role of leadership and managers, in that departures from efficiency generate a new organizational problem in the literature (see Garicano and Rayo (2016) for a thorough review). Traditionally, managers play no part in conventional Ricardian frameworks. However, managers may now assume a pivotal role in addressing teamwork frictions. Yet, perhaps counterintuitively, this role may arise *only* when *every* worker is deferential, since it is only in this case that the efficient outcome is a Nash equilibrium that could be reachable by a talented manager. Put differently, while managers could always strive for efficiency under frictions, there are limits to the scope of their role unless every worker is deferential.

[6] As we discuss in some detail in Section 4, it turns out that our main results also generalize to other Ricardian models. When we include frictions in Deming's (2017) model of social skills, for example, all the Nash equilibria continue to be classified by the *same* chemistry index and in the same manner as in the Ricardian framework we study. The intuition is that social skills basically operate as a technology that is independent of preferences.

This offers a fresh perspective on the notion of “smart management,” one that we find to be connected solely to the *individual* character of workers (see, e.g., Adhvaryu, Kala, and Nyshadham (2023)).

Related Literature. In addition to the literatures on the Ricardian framework and organizations just noted, there is a rich body of work that documents important aspects related to teams and teamwork.

Recent empirical evidence shows how organizational decisions are taken as a reaction to productivity shocks, demand shocks, the institutional environment, and other factors (Caliendo and Rossi-Hansberg (2012), Caliendo, Monte, and Rossi-Hansberg (2015), Caliendo et al. (2020)). From this perspective, we study the microfoundations of team production functions in terms of worker heterogeneity and strategic interactions. Much like an important organizational decision of firms concerns the allocation of decision rights among employees (Aghion and Tirole (1987), Kala (2024)), interactions between heterogeneous workers in task-based Ricardian team production deliver an explicit organization problem. We discuss in some detail in a later section what variables would need to be observed (e.g., tasks, productivity, preferences) to evaluate empirically the hypotheses that emerge.

Teamwork has been broadly studied outside assignment frameworks as well. Aspects such as the determinants of group formation and participation (Alesina and La Ferrara (2000)), team incentives (Hamilton, Nickerson and Owan (2003)), team-specific capital (Jaravel, Petkova and Bell (2018)), the role of peer pressure and complementarities (Kandel and Lazear (1992), Friebel et al. (2017)), the dependence of individual productivity on the human capital of coworkers (Herkenhoff et al. (2024)), and others have been shown to be empirically important.[7] Our analysis and results bear on the effects of team composition, complementarities, and participation from the perspective of strategic interactions within teams. Also, a body of research focuses on productivity spillovers across workers, and how important it is to identify and recruit “team players,” and to monitor and incentivize teams.[8] From this standpoint, we study the role of team frictions when players have

[7]On the theoretical side, see Bonatti and Hörner (2011) on the effect of moral hazard on the amount and timing on effort, and Che and Yoo (2001) on incentives in repeated interactions within teams.

[8]See, e.g., Weidmann and Deming (2021), Neffke (2019), Mas and Moretti (2009), Arcidiacono, Kinsler,

an excessive degree of self-regarding- or other-regarding-preferences relative to what is warranted by the objective function of the firm.

Our results also relate to a literature in business, management, and sports that strongly stresses the importance of teamwork in many different contexts, and how critical but difficult it is to reach the ideal *chemistry*. In fact, few concepts are as strongly emphasized by business leaders, and historical and sports figures, as is that of a "team's chemistry." Patrick Lencioni, best-selling author of *The Five Dysfunctions of a Team* (2002), summarizes the principle as: "[I]t is teamwork that remains the ultimate competitive advantage, both because it is so powerful and rare."

Finally, as noted earlier, our analysis is motivated by, and relates to, work on heterogeneous preferences within firms (Bénabou and Tirole (2003, 2006, 2011)), including the extensive research on employee and individual "social preferences" (e.g., Fehr, Goette, and Zehnder (2009)) and on the labor market returns to different personality traits (Flinn, Todd and Wang (2024)). For instance, firms make efforts to hire applicants with certain social profiles but face trade-offs: they may attract talent at the expense of prosocial motivation (Ashraf et al., 2020). In our case, sociability manifests itself in differences between the subjective and objective value of the contributions made to the team of co-workers and thus to the firm.

The rest of the paper is structured as follows. Section 2 introduces frictions in a general Ricardian team production framework. Section 3 studies the standard framework with two agents, which we then extend to teams of n agents. Section 4 connects the theoretical findings to the empirical literature with a discussion that focuses on the measurement of the different variables that would be needed in empirical work. Section 5 concludes. An appendix collects the proofs.

and Price (2017), Halac and Prat (2016), and Halac, Kremer and Winter (2024). A modern literature also documents how teamwork is a non-cognitive "soft" skill often critical for labor market outcomes and social behaviors (e.g., Heckman, Stixrud and Urzua (2006) and Cunha, Heckman and Schennach (2010)).

2 Ricardian Team Production with Frictions

In a general team production Ricardian model there is a team $N = \{1, \dots, n\}$ consisting of n individuals, each of whom has one unit of effort that can be spent on any one of a continuum of tasks. The assumption that an individual has one unit of effort is made without loss of generality; we can always choose the units of measurement so that it is satisfied. The productivity of a unit of effort devoted by individual $i \in N$ to task $t \in [0,1]$ is given by $\alpha_i(t)$. We assume that for each i, his productivity function $\alpha_i(t)$ is positive and continuous. We denote the comparative advantage schedule of individual i relative to individual j by $\gamma_{ij}(t) := \frac{\alpha_i(t)}{\alpha_j(t)}$, and in the case of two individuals we simply let $\gamma_{12} = \gamma$. We adopt throughout the following assumption, which was used in Wilson's (1980) analysis of Ricardian models.

Assumption 1 *For any two agents, i, j, there is no interval $[a, b]$, with $a < b$, at which $\frac{\alpha_i(t)}{\alpha_j(t)}$ is constant; i.e., there is no λ such that $\frac{\alpha_i(t)}{\alpha_j(t)} = \lambda$ for all $t \in [a, b]$.*

The team is interested in allocating the agents' efforts across tasks so as to maximize the "probability of winning," which is a monotonically increasing function of $\int u(\sum_{i=1}^{n} \alpha_i(t) l_i(t))$, where $l_i(t)$ is the effort devoted by agent $i \in N$ to task t, and $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing, continuously differentiable and strictly concave, with $\lim_{x \to 0} u'(x) = \infty$.[9] Specifically, the team is interested in effort functions $\hat{l}_i : [0,1] \to \mathbb{R}_+$, for $i \in N$, that solve

$$\max \int u\Big(\sum_{i \in N} \alpha_i(t) l_i(t)\Big) \tag{1}$$

$$\text{subject to} \int l_i(t) = 1, \qquad i \in N \tag{2}$$

The solution to this problem is well known. Each individual is assigned a set $\hat{I}_i$ of tasks such that he devotes his time exclusively to the tasks in it. This is summarized in the following proposition, the proof of which is in the appendix.

Proposition 1 *A team-optimal allocation of tasks exists and is characterized by*

$$\hat{l}_i(t) = \begin{cases} u'^{-1}\Big(\frac{\hat{\lambda}_i}{\alpha_i(t)}\Big)\frac{1}{\alpha_i(t)} & t \in \hat{I}_i \\ 0 & t \notin \hat{I}_i \end{cases} \qquad i \in N$$

[9]For any function f, we write $\int f$ or $\int f(t)$ instead of $\int_0^1 f(t)dt$.

where u'^{-1} is the inverse of u',

$$\hat{I}_i = \left\{ t \in [0,1] : \frac{\alpha_i(t)}{\alpha_j(t)} \geq \frac{\hat{\lambda}_i}{\hat{\lambda}_i} \qquad \text{for all } j \in N \right\} \tag{3}$$

and $\left(\hat{\lambda}_1, \ldots, \hat{\lambda}_n\right) \gg \mathbf{0}$ satisfy

$$\int \hat{l}_i(t) = \int_{\hat{I}_i} u'^{-1}\left(\frac{\hat{\lambda}_i}{\alpha_i(t)}\right)\frac{1}{\alpha_i(t)} = 1 \qquad i \in N. \tag{4}$$

The sets $\hat{I}_i$, $i \in N$, cover $[0,1]$, and consist of at most a countable union of intervals. Furthermore, for any two agents, i, j, the intersection $\hat{I}_i \cap \hat{I}_j$ contains at most a countable number of tasks.Two team-optimal allocations differ only on a set of tasks of measure zero.

As we will see in the next section, the standard framework in the literature assumes logarithmic preferences. In that case, the optimal allocation adopts a simple form.[10] Denoting by μ the Lebesgue measure on $[0,1]$, we have the following:

Proposition 2 *With logarithmic preferences, the team-optimal allocation is characterized by*

$$\hat{l}_i(t) = \begin{cases} \frac{1}{\mu(I_i)} & t \in \hat{I}_i \\ 0 & t \notin \hat{I}_i \end{cases} \qquad i \in N$$

where

$$\hat{I}_i = \left\{ t \in [0,1] : \frac{\alpha_i(t)}{\alpha_j(t)} \geq \frac{\mu(\hat{I}_i)}{\mu(\hat{I}_j)} \qquad \text{for all } j \in N \right\} \tag{5}$$

Proof. Since $u = \ln$, we have that $u'(t) = 1/t$ and, therefore, since in this case $u'^{-1} = u'$, we have that $u'^{-1}\left(\frac{\hat{\lambda}_i}{\alpha_i(t)}\right)\frac{1}{\alpha_i(t)} = \frac{1}{\hat{\lambda}_i}$. By (4) we have

$$\int_{\hat{I}_i} \frac{1}{\hat{\lambda}_i} = \frac{\mu(\hat{I}_i)}{\hat{\lambda}_i} = 1$$

which implies that $\hat{\lambda}_i = \mu(\hat{I}_i)$. ■

Further, it is well known that in the cases of $n = 2$ and $n = 3$, and under the assumption that the comparative advantage schedules $\gamma_{12}(t) = \alpha_1(t)/\alpha_2(t)$ and $\gamma_{23}(t) = \alpha_2(t)/\alpha_3(t)$

[10] We specify one team-optimal allocation. Recall that any two optimal allocations are equal almost everywhere.

are strictly decreasing, the team-optimal allocation can be more precisely characterized (see, for instance, Acemoglu and Autor (2011)). This is done in the following corollaries:

Corollary 1 *With logarithmic preferences, if* $n = 2$ *and* $\gamma(t) = \frac{\alpha_1(t)}{\alpha_2(t)}$ *is strictly decreasing, the team-optimal allocation is given by*

$$\left(\hat{l}_1(t), \hat{l}_2(t)\right) = \begin{cases} (\frac{1}{\hat{t}}, 0) & t < \hat{t} \\ \textit{any} & t = \hat{t} \\ \left(0, \frac{1}{1-\hat{t}}\right) & t > \hat{t} \end{cases}$$

where $\hat{t}$ *is implicitly defined by*

$$\gamma(\hat{t}) = \frac{\hat{t}}{1-\hat{t}}.$$

The proof follows directly from Proposition 2 after noticing that the sets $\hat{I}_1 = [0, \hat{t}]$ and $\hat{I}_2 = [\hat{t}, 1]$ satisfy equations (5).

Corollary 2 *With logarithmic preferences, if* $n = 3$ *and* $\frac{\alpha_1(t)}{\alpha_2(t)}$ *and* $\frac{\alpha_2(t)}{\alpha_3(t)}$ *are strictly decreasing functions, then there exist critical tasks,* $t_{12}, t_{23} \in [0, 1]$*, such that the team-optimal allocation is given by*

$$\left(\hat{l}_1(t), \hat{l}_2(t), \hat{l}_3(t)\right) = \begin{cases} (\frac{1}{t_{12}}, 0, 0) & t < t_{12} \\ (0, \frac{1}{t_{23}-t_{12}}, 0) & t_{12} \le t \le t_{23} \\ (0, 0, \frac{1}{1-t_{23}}) & t > t_{23} \end{cases} \tag{6}$$

where the critical tasks t_{12}, t_{23}*, are implicitly defined by*

$$\frac{\alpha_1(t_{12})}{\alpha_2(t_{12})} = \frac{t_{12}}{t_{23}-t_{12}} \quad \textit{and} \quad \frac{\alpha_2(t_{23})}{\alpha_3(t_{23})} = \frac{t_{23}-t_{12}}{1-t_{23}}.$$

For the reader's convenience, we offer a proof in the appendix.

Frictions. We next introduce frictions that create strategic interactions in teamwork. We assume that workers have different preferences or personality traits and, consequently, may not all share the firm's goal. Specifically, individual i assigns a subjective value $\rho_i\alpha_i$ to his productivity instead of α_i. The parameters ρ_i are similar to the congruence parameters in Aghion and Tirole (1997) in that they capture the agents' agreement with the firm's objective. The closer ρ_i is to 1, the closer are individual i's preferences to the firm's goal. Although one can think of various methods whereby a "principal" might affect the congruence of his subordinates, consistent with the literature we take these parameters as given.[11]

There are many reasons why one's subjective value could be different from one's objective contribution to the team. Individuals may have different identities, personality traits, biases or "sense of mission" (Akerloff and Kranton (2005), Bénabou and Tirole (2011), Dewatripont, Jewitt and Tirole (1999)). They may also have different intrinsic and extrinsic incentives (Bénabou and Tirole (2003, 2006)), or simply different outside options associated with their comparative advantages that create different interests in showcasing their abilities (Jäger et al, 2024). As much as firms may devote effort to identifying and selecting workers that share *exactly* the firm's objective function (and to incentivize them that way), we can expect that ρ_i is not always 1.

When $\rho_i > 1$, we may say that agent i has an excessive preference for "his own individual achievements" relative to the accomplishments of the team. Put differently, he has a greater preference to showcase his abilities relative to what is warranted from the firm's perspective. In this sense, he is *self-centered*. When $\rho_i < 1$, player i undervalues his own achievements and ability; in this sense we may say that he is *deferential*. When $\rho_i = 1$, player i's objective coincides with the goal of the firm, and we say that the player is *neutral*. Again, while we emphasize hubris or lack thereof in our description, there are many reasons why employees might intrude on each other's activities. For example, an individual may have career or other concerns and, in order to be promoted, may want to pursue more or fewer tasks than is optimal for the organization so as to signal that he is an specialist or a generalist. Simply put, the pattern of private benefits need not coincide with that of comparative advantage.

Going back to the formal model, the individuals' payoffs are given by

$$U_i(l_1, \ldots, l_n) = \int u\Big(\rho_i \alpha_i(t) l_i(t) + \sum_{j \neq i} \alpha_j(t) l_j(t)\Big) \qquad i \in N.$$

[11] For example, Aghion and Tirole (1997, p. 6) suggests "investments in the recruiting and training of new employees, design of career profiles, or enforcement of (contractual) rules restricting the subordinates' set of possible actions." In our case, in addition to vertical congruence (authority), we also have horizontal congruence (within the team), and so other factors may be relevant as well (see Kandel and Lazear (1992)).

Since their preferences are not perfectly aligned, individuals play a strategic game where their common set of strategies is the set of functions $l : [0,1] \to \mathbb{R}_+$ such that $\int l(t) = 1$, and their payoffs are given by $(U_i)_{i\in N}$.

A *Nash equilibrium* is a profile of effort functions $(l_1^*, \ldots, l_n^*)$ such that

$$U_i(l_1^*, \ldots, l_n^*) \geq U_i(l_i, l_{-i}^*) \qquad \text{for all } l_i \text{ such that } \int l_i(t) = 1.$$

Since the set of strategies is compact in the weak-* topology and the payoffs functions are continuous, the set of Nash equilibria is not empty.[12]

Assume that $(l_1^*(t), \cdots, l_n^*(t))$ is a Nash equilibrium. Then, $l_i^*(t)$ is a solution to individual i's problem:

$$\begin{aligned} &\max_{l_i} \int u\big(\rho_i \alpha_i(t) l_i(t) + \sum_{j\neq i} \alpha_j(t) l_j^*(t)\big) \\ &\text{s.t.} \int l_i(t) = 1 \end{aligned}$$

The associated Hamiltonian is

$$H(l_i(t), \lambda_i(t)) = u\Big(\rho_i \alpha_i(t) l_i(t) + \sum_{j\neq i} \alpha_j(t) l_j^*(t)\Big) - \lambda_i(\ t) l_i(t)$$

and the necessary conditions for maximization are

$$\begin{aligned} \frac{\partial H}{\partial l_i(t)} &= u'\big(\rho_i \alpha_i(t) l_i(t) + \sum_{j\neq i} \alpha_j(t) l_j^*(t)\big)\rho_i \alpha_i(t) - \lambda_i(t) \leq 0, \qquad l_i(t)\frac{\partial H}{\partial l_i(t)} = 0 \\ 0 &= \lambda_i'(t) \end{aligned}$$

These conditions are also sufficient since the integrand is differentiable and concave in l_i. Since the above conditions dictate that $\lambda_i'(t) = 0$, we have that $\lambda_i(t) = \lambda_i$ for some positive constants λ_i, for $i \in N$. Therefore, the equilibrium $(l_1^*, \cdots, l_n^*)$ satisfies, for some $(\lambda_1, \ldots, \lambda_n) \gg 0$,

$$u'\big(\rho_i \alpha_i(t) l_i^*(t) + \sum_{j\neq i} \alpha_j(\ t) l_j^*(t)\big)\rho_i \alpha_i(t) \leq \lambda_i, \quad \text{with equality if } l_i^*(t) > 0 \quad i \in N \tag{7}$$

$$\int l_i(t) = 1 \qquad i \in N \tag{8}$$

Based on this, we can make the following observation:

[12] We thank Eilon Solan for illuminating us on this point.

Observation 1 *If $\rho_i > 1$ for some $i \in N$, then the team-optimal strategies do not constitute a Nash equilibrium.*

Proof. Let $(\hat{l}_1, \cdots, \hat{l}_n)$ be the team-optimal strategies and let $(\hat{I}_k)_{k \in N}$ and $(\hat{\lambda}_k)_{k \in N}$ be the associated sets of tasks and multipliers defined in Proposition 1. We will show that if $(\hat{l}_1, \cdots, \hat{l}_n)$ constitutes an equilibrium, then $\rho_i \leq 1$. Since $(\hat{I}_k)_{k \in N}$ are nonempty compact sets that cover $[0, 1]$, there must be a player $j \neq i$ such that $\hat{I}_i \cap \hat{I}_j \neq \emptyset$. Then, for $t_{ij} \in \hat{I}_i \cap \hat{I}_j$, we have that

$$\frac{\alpha_i(t_{ij})}{\alpha_j(t_{ij})} = \frac{\hat{\lambda}_i}{\hat{\lambda}_j}. \tag{9}$$

If the team-optimal strategies constituted a Nash equilibrium, then from (7–8), there would be positive numbers λ_i and λ_j such that

$$u'\Big(\rho_i \alpha_i(t) u'^{-1}\big(\frac{\hat{\lambda}_i}{\alpha_i(t)}\big)\frac{1}{\alpha_i(t)}\Big)\rho_i\alpha_i(t) \quad = \quad \lambda_i \quad \text{if } t \in int\, \hat{I}_i \tag{10a}$$

$$u'\Big(\alpha_j(t) u'^{-1}\big(\frac{\hat{\lambda}_j}{\alpha_j(t)}\big)\frac{1}{\alpha_j(t)}\Big)\rho_i\alpha_i(t) \quad \leq \quad \lambda_i \quad \text{if } t \in int\, \hat{I}_j \tag{10b}$$

Manipulating, we obtain

$$\rho_i u'^{-1}\big(\frac{\hat{\lambda}_i}{\alpha_i(t)}\big) \quad = \quad u'^{-1}\big(\frac{\lambda_i}{\rho_i \alpha_i(t)}\big) \quad \text{if } t \in int\, \hat{I}_i \tag{11a}$$

$$\frac{\hat{\lambda}_j}{\alpha_j(t)}\rho_i\alpha_i(t) \quad \leq \quad \lambda_i \quad \text{if } t \in int\, \hat{I}_j \tag{11b}$$

By continuity of α_i/α_j,

$$\frac{\hat{\lambda}_j}{\alpha_j(t_{ij})}\rho_i\alpha_i(t_{ij}) \leq \lambda_i$$

and by (9) we have that

$$\hat{\lambda}_i \rho_i \leq \lambda_i$$

Substituting in (11a), and taking into account that u^{-1} is a convex function,

$$\begin{aligned} \rho_i u'^{-1}\Big(\frac{\hat{\lambda}_i}{\alpha_i(t_{ij})}\Big) \quad &= \quad u'^{-1}\Big(\frac{\lambda_i}{\rho_i \alpha_i(t_{ij})}\Big) \\ &\leq \quad u'^{-1}\Big(\frac{\hat{\lambda}_i}{\alpha_i(t_{ij})}\Big) \end{aligned}$$

which implies that $\rho_i \leq 1$. ■

This observation says that it is enough for a single individual to be self-centered for the team-optimal strategies to not be an equilibrium. The intuition is that self-centered individuals tend to overstep the boundaries set by efficient behavior, and as a result full division of labor does not take place. What if none of the individuals is self-centered and all are deferential? In that case the team-optimal allocation may be but it is not necessarily a Nash equilibrium. However, for the standard framework with logarithm utility that we study in the next section, we can state the following:

Observation 2 *Under logarithmic utility, if $\rho_i \leq 1$ for all $i \in N$, then the team-optimal strategies constitute a Nash equilibrium.*

Proof. The proof consists in verifying that there are λ_i, for $i = 1, \ldots, n$ such that the team-optimal strategies identified in Proposition 1, satisfy condition (7). Substituting the team-optimal strategies into these conditions we obtain that they must satisfy

$$\begin{aligned} \rho_i u'^{-1}\Big(\frac{\hat{\lambda}_i}{\alpha_i(t)}\Big) &= u'^{-1}\Big(\frac{\lambda_i}{\rho_i \alpha_i(t)}\Big) \quad \text{if } t \in int\, \hat{I}_i \\ \frac{\hat{\lambda}_j}{\alpha_j(t)}\rho_i \alpha_i(t) &\leq \lambda_i \quad \text{if } t \in int\, \hat{I}_j \end{aligned}$$

Since $u'^{-1}(y) = 1/y$, and taking into account that $\frac{\hat{\lambda}_j}{\alpha_j(t)} \leq \frac{\hat{\lambda}_i}{\alpha_i(t)}$ for $t \in int\, \hat{I}_j$, it can be checked that for $\lambda_i = \hat{\lambda}_i$, $i = 1, \ldots, n$, the above conditions are satisfied. ■

3 Equilibria in Standard Ricardian Framework

The assumption of a Cobb-Douglas function is standard in the literature; see, for example, Dornbusch, Fischer, and Samuelson (1977), Acemoglu and Autor (2011), and Deming (2017), among others. In this section, we study this framework, which we refer to as the standard Ricardian framework for the case of two agents, which itself represents the cornerstone case in the literature. We then extend our analysis to the more general case with $n > 2$ agents.

3.1 Characterization of Equilibria

Let (l_1^*, l_2^*) be an equilibrium. Then, it satisfies the equilibrium conditions (7–8) for some $\lambda_1, \lambda_2 > 0$, which in this case become

$$\frac{\rho_i \alpha_i(t)}{\rho_i \alpha_i(t) l_i(t) + \alpha_j(t) l_j^*(t)} \leq \lambda_i, \quad \text{with equality if } l_i^*(t) > 0 \qquad i = 1, 2 \tag{12}$$

$$\int l_i(t) = 1 \qquad i = 1, 2 \tag{13}$$

Define the following sets: $A_1 = \{t : l_1^*(t) > 0, l_2^*(t) = 0\}$, $A_2 = \{t : l_2^*(t) > 0, l_1^*(t) = 0\}$, and $B = \{t : l_2^*(t) > 0, l_1^*(t) > 0\}$. The sets A_1 and A_2 contain the tasks exclusively performed by player 1 and player 2, respectively. The set B is the set of task that are shared by both players. The set $\{t : l_2^*(t) = 0, l_1^*(t) = 0\}$ of tasks performed by neither player has measure zero (and can be assumed, without loss of generality, to be empty) because otherwise individuals' utilities would each be $-\infty$ and, therefore, each player would have a profitable deviation. It follows from (12–13) that the equilibrium has the following form:[13]

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t \in A_1 \\ \frac{\rho_1 \rho_2}{\rho_1 \rho_2 - 1} \left(\frac{\rho_1 \gamma(t) - \frac{\lambda_1}{\lambda_2}}{\lambda_1 \rho_1 \gamma(t)}, \frac{\frac{\lambda_1}{\lambda_2} - \frac{\gamma(t)}{\rho_2}}{\lambda_1} \right) & t \in B \\ \left(0, \frac{1}{\lambda_2}\right) & t \in A_2 \end{cases} \tag{14}$$

In order to fully characterize the equilibria, it is necessary to identify the sets A_1, B and A_2, and the coefficients λ_1 and λ_2. This is what the next three propositions, whose proofs are in the appendix, will do. As we shall see, we find that there are three types of equilibria and that these types depend *only* on whether the product of the congruence parameters $\rho_1 \rho_2$ is greater than, equal to, or smaller than one. We refer to $\rho_1 \rho_2$ as the team's *chemistry index*. Following the same terminology we have used for individuals, when this index is larger than one, we say that the team's chemistry is self-centered. When this index equals one, we say that the team's chemistry is neutral, and when it is less than one we say that the team's chemistry is deferential.

[13] As we will show later, if $\rho_1 \rho_2 = 1$, the set B will be empty.

Proposition 3 *(Team Chemistry: Neutral) When* $\rho_1\rho_2 = 1$, *the equilibrium consists of the following strategies:*

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{t^*}, 0) & t < t^* \\ any & t = t^* \\ \left(0, \frac{1}{1-t^*}\right) & t > t^* \end{cases}$$

where the critical task t^* *is the only one that satisfies*

$$\rho_1\gamma(t^*) = \frac{t^*}{1-t^*}.$$

Note that, since γ is decreasing, the critical task t^* is unique. Also note that this equilibrium consist of the efficient strategies for the case where the comparative advantage is $\rho_1\gamma$ instead of γ, and furthermore that $\rho_1 > 1$ if and only if $t^* > \hat{t}$. Namely, the self-centered player takes upon himself *additional* tasks compared to those that he would take at the team-optimal outcome. Further, the greater the congruence parameter of the self-centered individual, the farther the equilibrium critical task is from the team-optimal one. Nevertheless, we should emphasize that, though not team-optimal, the equilibrium is efficient because the resulting division of labor does respect the comparative advantages of the agents.

Proposition 4 *(Team Chemistry: Self-Centered) When* $\rho_1\rho_2 > 1$, *the equilibrium consists of the following strategies:*

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t < t_1 \\ \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\left(\frac{1-\frac{\gamma(\ t_2)}{\gamma(t)}}{\lambda_1}, \frac{1-\frac{\gamma(t)}{\gamma(t_1)}}{\lambda_2}\right) & t \in [t_1, t_2] \\ \left(0, \frac{1}{\lambda_2}\right) & t > t_2 \end{cases}$$

where the tasks t_1, t_2 *and the multipliers* λ_1, λ_2 *are the unique solutions to*

$$\begin{aligned} \int_0^1 l_1^*(t) &= \int_0^1 l_2^*(t) = 1 \\ \frac{\gamma(t_1)}{\rho_2} &= \frac{\lambda_1}{\lambda_2} = \rho_1\gamma(t_2). \end{aligned}$$

When the team chemistry is self-centered, the equilibrium effort functions partition the set of tasks into three intervals. There are two tasks, $t_1 < t_2$, such that:

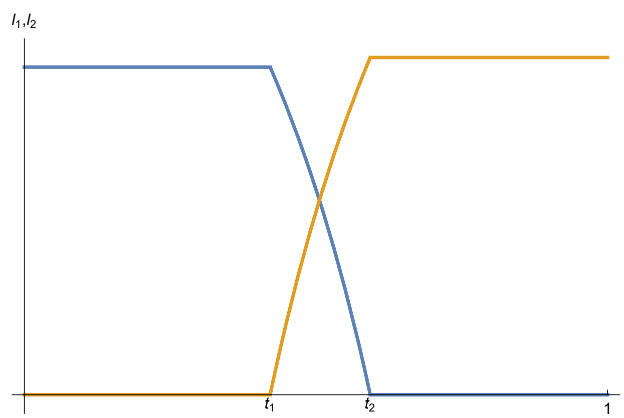
- tasks in the left interval $[0, t_1)$ are performed exclusively by player 1;

- tasks in the right interval $(t_2, 1]$ are performed exclusively by player 2, and

- tasks in the middle interval $[t_1, t_2]$ are performed by both players.

It can be checked that the equilibrium strategies are continuous and weakly monotonic. Since both players are self-centered and want to perform more tasks than they should, the result is that there are some tasks which are performed by *both* of them. This is precisely the source of the equilibrium inefficiency. Finally, note that as the product $\rho_1\rho_2$ tends to 1, the equilibrium converges to the one when $\rho_1\rho_2 = 1$. Figure 1 depicts the equilibrium strategies when $\rho_1\rho_2 > 1$.

Figure 1: Example of an equilibrium where $\rho_1\rho_2 > 1$.



Proposition 5 *(Team Chemistry: Deferential) When $\rho_1\rho_2 < 1$, the equilibrium consists of the following strategies:*

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t \in A_1 \\ \frac{\rho_1\rho_2}{1-\rho_1\rho_2}\left(\frac{\frac{\gamma(t_2)}{\gamma(t)}-1}{\lambda_1}, \frac{\frac{\gamma(t)}{\gamma(t_1)}-1}{\lambda_2}\right) & t \in B \\ (0, \frac{1}{\lambda_2}) & t \in A_2 \end{cases}$$

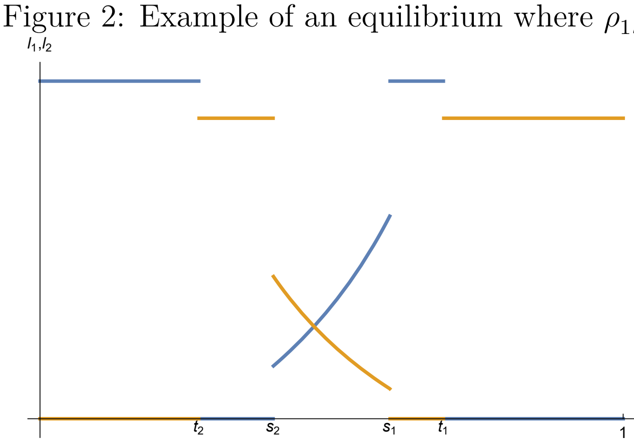
where $A_1 = [0, t_2] \cup B_1$, $A_2 = [t_1, 1] \cup B_2$, and $B = (t_2, t_1) \setminus (B_1 \cup B_2)$, for some $B_1, B_2 \subset (t_2, t_1)$, and where the tasks t_1, t_2 and the multipliers λ_1, λ_2 solve

$$\int_0^1 l_1^*(t) = \int_0^1 l_2^*(t) = 1 \tag{15}$$

$$\frac{\gamma(t_1)}{\rho_2} = \frac{\lambda_1}{\lambda_2} = \rho_1\gamma(t_2). \tag{16}$$

As in the previous case, there are two critical tasks, t_2 and t_1, this time with $t_2 < t_1$ such that player 1 is the sole performer of tasks in $[0, t_2]$ and player 2 is the sole performer of tasks in $[t_1, 1]$. However, players now *do not necessarily* make simultaneous efforts in all the tasks in the remaining interval (t_2, t_1). This interval is partitioned into three sets, $\{B_1, B_2, B\}$, which need not be intervals themselves, and only in one of them, B, tasks are shared. In the other two sets, B_1 and B_2, strict division of labor takes place. Importantly, as long as the strategies and the critical tasks satisfy (15–16), these three intervals are arbitrary, which means that the equilibrium need not be continuous or monotonic. Note that even if there is full division of labor, i.e., if $B = \emptyset$, there might be large inefficiencies. Indeed, if the set B_1 contains a non-negligible set of tasks located to the right of some tasks in B_2, the outcome will be inefficient; by switching some tasks, individuals could achieve a better outcome for everyone. Figure 2 illustrates an example of an equilibrium in which $B_1 = (s_1, t_1)$ and $B_2 = (t_2, s_2)$. Note that the equilibrium is neither continuous nor monotonic. Although each individual is the sole performer of tasks in which he has a strong comparative advantage, we can see that in the interval of tasks $(t_{2,}t_1)$ in which the comparative advantage is weaker, the allocation of tasks may be chaotic. Indeed, as we move from t_2 to t_1, player 1 makes no effort in $[t_2, s_2]$, then he shares tasks with player 2 in $[s_2, s_1]$, then he is the sole performer of tasks in $[s_1, t_1]$ and, finally, from t_1 onwards, he defers to player 2. Player 2 has, mutatis mutandi, a similar pattern of behavior.

Figure 2: Example of an equilibrium where $\rho_1\rho_2 < 1$.

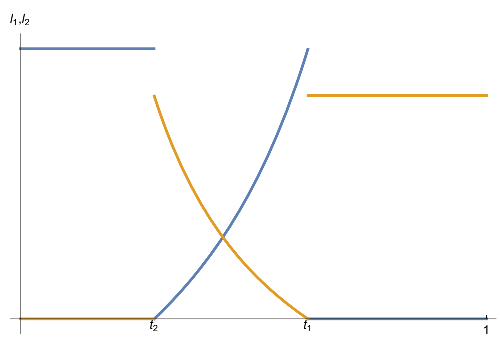


This case shows that when the aggregate nature of the team is deferential, many types of equilibria are possible: We may not only observe tasks being performed by both players, but also patterns of effort that are *not* comonotonic with the workers' comparative advantages.

Finally, among the plethora of possible equilibria patterns, we can identify two special cases that lead to simple equilibria. Their characterization is in the appendix.

Special Case A: In this case, both players invest positive effort in every task $t \in (t_2, t_1)$. Formally, $A_1 = [0, t_2]$, $B = (t_2, t_1)$, and $A_2 = [t_1, 1]$. Figure 3 illustrates this kind of equilibrium. Note however that the equilibrium strategies are not continuous and that, contrary to the $\rho_1 \rho_2 > 1$ case, they are not monotone either. That is, within the interval (t_2, t_1), players' effort levels move *against* their comparative advantage.

Figure 3: Example of an equilibrium where $\rho_1 \rho_2 < 1$.



Special Case B: In this case, there is full division of labor. Formally, $A_1 = [0, t_2] \cup B_1$ and $A_2 = B_2 \cup (t_1, 1)$ where $B_1 \cup B_2 = (t_2, t_1)$. As long as the strategies and the critical tasks satisfy (15–16), the partition $\{B_1, B_2\}$ is arbitrary. Thus, a particularly simple case is one in which there is a critical task $\tilde{t} \in (t_2, t_1)$ such that $A_1 = [0, \tilde{t})$ and $A_2 = (\tilde{t}, 1]$. Task $\tilde{t}$ is not unique, and according to Observation 2, can be chosen to be the team-optimal cutoff $\hat{t}$. Figure 4 illustrates this kind of equilibrium. Here the behavior is similar to the $\rho_1 \rho_2 = 1$ case, and it tends to it as the product $\rho_1 \rho_2$ tends to 1.

Figure 5 summarizes the Nash equilibria that obtain in the Ricardian framework under teamwork frictions.

Figure 4: Example of an equilibrium where $\rho_1\rho_2 < 1$.

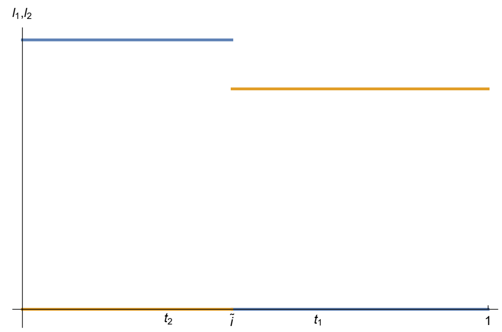
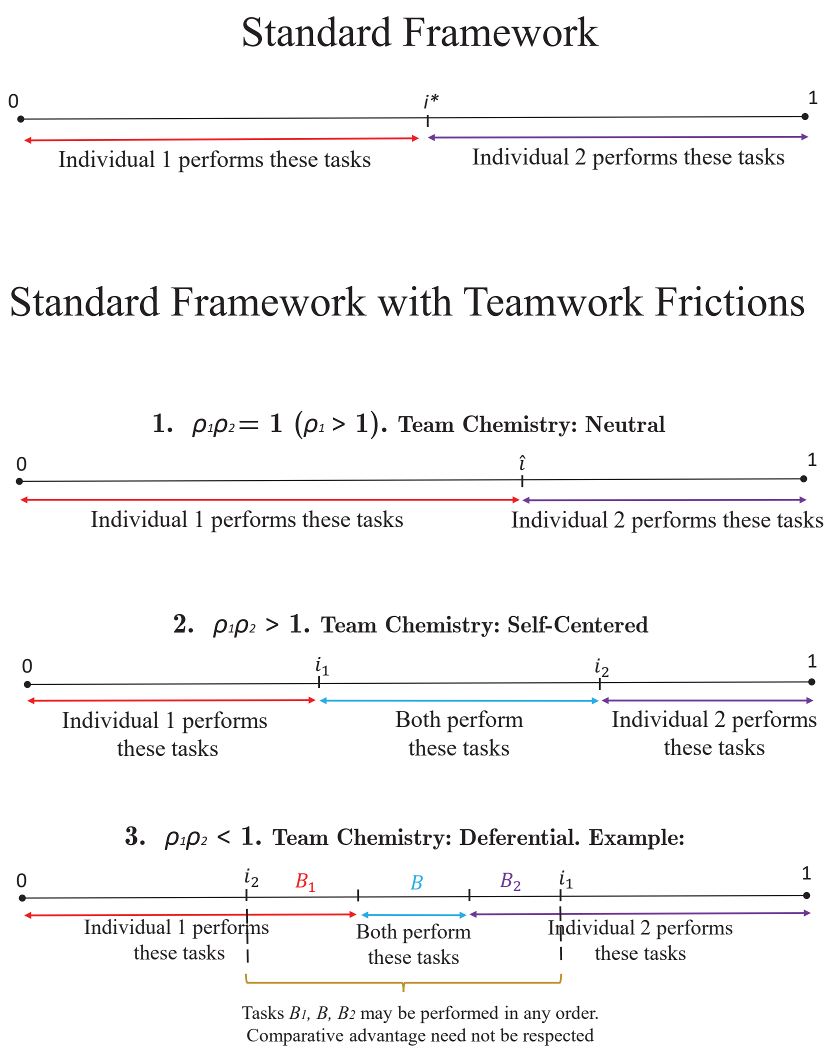


Figure 5: Equilibria in standard Ricardian framework



Summing up, we have found that in the standard Ricardian framework, frictions typically generate a range of tasks which can be, and sometimes are, performed by both players. The closer a firm gets to hire workers that share its objective, the smaller such range. When firms do not succeed in their hiring process we will often find deviations from the firm-optimal outcome. As for the sources of these deviations, when the team chemistry is neutral, deviations arise only because workers are specializing in certain tasks that they should not be performing. When the team chemistry is self-centered, there is a range of tasks that are performed by both players; moreover, efficiency cannot be reached as an equilibrium by even the most talented manager. However, when the team chemistry is deferential, many different equilibria are possible. These include cases in which even when players do not share tasks, their effort levels do not respect their comparative advantages. Put differently, in this case inefficiency manifests itself as a lack of division of labor and in patterns of specialization that are not comonotonic with individuals' comparative advantages. Perhaps surprisingly, however, in this scenario, the efficient outcome is an equilibrium *if both* players are deferential. This means that if a talented manager could identify the efficient allocation of tasks and impose them to his workers before they interact, then his workers will choose to perform those tasks in their Nash equilibrium.

We discuss in some detail in the discussion section how future research may be able to test the empirical implications of these results. We just note now that, at a broader level, these findings offer new insights that could help explain why prosocial motivation tends to be associated with positive labor market outcomes (e.g., Ashraf et al. (2020), Kosse et al. (2020)), why these motivations are often sought after, and why a body of organizational behavior research finds that employees' social preferences may have significant effects on firm performance (Penner et al. (2005), Meier (2007)). Authority then plays a critical role to move from possibly chaotic patterns to efficient outcomes in these cases. Interestingly, chaotic patterns are actually observed some times. For example, Bassi et al. (2024) study manufacturing firms in Uganda and find that they resemble more a collection of self-employed individuals sharing a production space than a firm with specialized labor.

We next study the extent to which the analysis remains tractable in a framework with n members.

3.2 Equilibrium in Framework with n Agents

It should be readily intuitive that when there are more than two agents, equilibria may dictate that more than two individuals share the same tasks for some values of the congruence parameters. For this reason, a full description of all the equilibria will be cumbersome. However, by the continuity properties of Nash equilibrium, if the congruence parameters are close enough to one, then the equilibria will be close to the team-optimal allocation and, as a result, no more than two individuals will perform each task. It turns out that, in this case, it is possible to provide a simple description of the equilibria. As anticipation of the main results, we find that the equilibria obtained in the standard framework, along with the corresponding pairwise chemistry indices $\rho_i\rho_j$, play an essential role. In what follows we first describe the general form of the equilibrium with n agents, and then provide the intuition and full characterization in a specific example with three agents. The reader is invited, if he so wishes to skip the next sub-subsection and come back to it after the three-agent case is understood.

3.2.1 Form of Equilibrium

Recall that the team-optimal allocation partitions the set of tasks into intervals and assigns a subset of them to each player. Consider a pair of players j and k who at the team-optimal allocation are assigned adjacent intervals. We call them *adjacent agents*. Then, if their chemistry index is close enough to one, in equilibrium there will be an interval of tasks $[a, b]$ exclusively performed by j, by k, or by both, but not by any other individual. Then, the equilibrium behavior of j and k, restricted to the tasks in $[a, b]$ will mimic their equilibrium behavior in a two-player game in which they allocate efforts to $[a, b]$. To see this, let $(l_i^*)_{i \in N}$ be an equilibrium of the n-player game. Then, by the consistency property of Nash equilibrium (Peleg and Tijs (1996)), the restriction of (l_j^*, l_k^*) to the tasks in $[a, b]$ must be an equilibrium of the two-player game in which they have $\int_a^b l_j^*$ and $\int_a^b l_k^*$ units of effort, respectively, available for allocation to tasks in $[a, b]$, and in which their objective functions are $\ln\left(\rho_j \alpha_j(t) l_j(t) + \alpha_k(t) l_k(t)\right)$ and $\ln\left(\alpha_j(t) l_j(t) + \rho_k \alpha_k(t) l_k(t)\right)$, respectively (recall that $l_i^*(t) = 0$ for all $t \in [a, b]$ and $i \neq j, k$).

Specifically, if the congruence parameters are close enough to one, we can find a collection of intervals $([a_\ell, b_\ell])_{\ell=1}^{L}$, not necessarily disjoint, such $\cup_\ell [a_\ell, b_\ell] = [0, 1]$ and such that for each $\ell = 1, \ldots, L$ there is a unique pair of players, j and k, which depend on the interval $[a_\ell, b_\ell]$, such that

1. $l_j^*(t) + l_k^*(t) > 0$ for $t \in [a_\ell, b_\ell]$
2. $l_j^*(t) > 0$ for some $t \in [a_\ell, b_\ell]$ and $l_k^*(t) > 0$ for some $t \in [a_\ell, b_\ell]$
3. $l_i^*(t) = 0$ for all $t \in [a_\ell, b_\ell]$ and for all $i \neq j, k$.

Then, there are positive numbers λ_j and λ_k (the solution to (7–8)) such that the equilibrium strategies of these players, restricted to the tasks in $[a_\ell, b_\ell]$ can be written as

$$\left(l_j^*(t), l_k^*(t)\right) = \begin{cases} (\frac{1}{\lambda_j}, 0) & t \in A_j \\ \frac{\rho_j \rho_k}{\rho_j \rho_k - 1} \left(\frac{\rho\ _j \gamma(t) - \frac{\lambda_j}{\lambda_k}}{\lambda_j \rho_j \gamma(t)}, \frac{\frac{\lambda_j}{\lambda_k} - \frac{\gamma(t)}{\rho_k}}{\lambda_j} \right) & t \in B \\ \left(0, \frac{1}{\lambda_k}\right) & t \in A_k \end{cases} \tag{17}$$

where A_k, B and A_k are pairwise disjoint sets whose union is $[a_\ell, b_\ell]$.

Note that (17) is the general expression of the equilibrium strategies in the two-player game stated in equation (14) where individual j takes the role of player 1 and individual k takes the role of player 2. That is, the tasks in $[a_\ell, b_\ell]$ are distributed between j and k, so that the tasks in A_j are performed exclusively by the former, tasks in A_k are performed exclusively by the latter, and taks in B are performed jointly by both of them according to the equilibrium of the corresponding two-player game.

3.2.2 Characterization of Equilibria with Three Agents

The equilibria can be better illustrated in the case analyzed in Acemoglu and Autor (2011) in which the comparative advantages schedules γ_{12} and γ_{23} are strictly decreasing. In this case, the team-optimal strategies have already been described in Corollary 2. As just noted, if the congruence factors are close enough to one, it will be possible to partition the set of tasks into intervals so that each interval will be assigned to only two players who will divide it in a way that mimics the two-player equilibrium, the nature of which will depend on whether the chemistry index between these two individuals is less than, equal to, or bigger than one. In particular, the following propositions can be checked to be true by verifying that the strategy profiles satisfy conditions (7–8).

Proposition 6 *(Chemistry between Adjacent Agents: Neutral) Let the congruence parameters satisfy $\rho_1\rho_2 = \rho_2\rho_3 = 1$, and consider the critical tasks defined by*

$$\frac{\alpha_1(t_{12})}{\rho_2\alpha_2(t_{12})} = \frac{t_{12}}{t_{23} - t_{12}} \qquad \textit{and} \qquad \frac{\rho_2\alpha_2(t_{23})}{\alpha_3(t_{23})} = \frac{t_{23} - t_{12}}{1 - t_{23}}$$

If

$$\frac{\alpha_1(t_{23})}{\rho_2\alpha_3(t_{23})} \leq \frac{t_{12}}{1 - t_{23}} \leq \frac{\rho_2\alpha_1(t_{12})}{\alpha_3(t_{12})} \tag{18}$$

then, the following strategy profile constitutes an equilibrium:

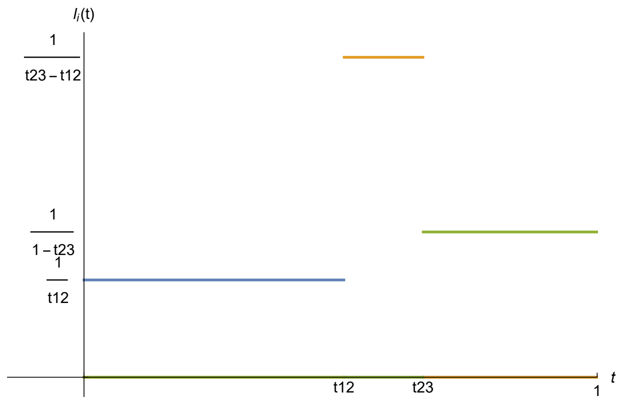
$$(l_1^*(t), l_2^*(t), l_3^*(t)) = \begin{cases} (\frac{1}{t_{12}}, 0, 0) & t < t_{12} \\ (0, \frac{1}{t_{23}-t_{12}}, 0) & t_{12} < t < t_{23} \\ (0, 0, \frac{1}{1-t_{23}}) & t_{23} < t \end{cases}$$

Proof. Note that the strategy profile (l_1^*, l_2^*, l_3^*) is precisely the efficient allocation when the productivity functions are given by α_1, $\rho_2\alpha_2$, and α_3. (See Proposition 2 and Corollary 2.) This implies that $t_{23} > t_{12}$ and that the strategies are well defined. In order to see that they constitute an equilibrium, it is enough to check that, along with $\lambda_1 = t_{12}$, $\lambda_2 = t_{23} - t_{12}$ and $\lambda_3 = 1 - t_{23}$, they satisfy conditions (7–8). We leave this mechanical task to the reader. ■

Note that condition (18) sets a limit to how far from 1 can the congruence parameters be for the above strategies to constitute an equilibrium. It also shows that the limit is not symmetric: While player 2 can be arbitrarily self-centered, he cannot be too deferential. Indeed, if ρ_2 is too small, then players 1 and 3 will be too self-centered and will want to step on each other's feet, so to speak. Figure 6 illustrates a case that meets this condition. Note that, restricted to the interval $[0, t_{23})$, the behavior of players 1 and 2 mimics the two-player equilibrium. Similarly, restricted to the interval $(t_{12}, 1]$, the behavior of players 2 and 3 also mimics the two-player equilibrium.

Next, when the chemistry indices are not neutral, the equilibrium $(l_1^*(t), l_2^*(t), l_3^*(t))$ generates two intervals of tasks that may be shared by players 1 and 2, and by players 2

Figure 6: Example of an equilibrium where $\rho_1\rho_2 = \rho_2\rho_3 = 1$.



and 3, respectively. The extreme points of these intervals are denoted t_1 and t_2, and s_2 and s_3, respectively, and satisfy the following familiar conditions:

$$\begin{aligned}
\int_0^1 l_1^*(t) &= \int_0^1 l_2^*(t) = \int_0^1 l_3^*(t) = 1 \\
\frac{\gamma_{12}(t_1)}{\rho_2} &= \frac{\lambda_1}{\lambda_2} = \rho_1\gamma_{12}(t_2) \\
\frac{\gamma_{23}(s_2)}{\rho_3} &= \frac{\lambda_2}{\lambda_3} = \rho_2\gamma_{23}(s_3).
\end{aligned} \tag{19}$$

The next proposition describes the equilibria when the congruence parameters are close to one, and when the chemistry indices between players 1 and 2 and between players 2 and 3 are bigger than one. The proof consists of verifying that the stated strategies satisfy the equilibrium conditions (7–8), and is left to the reader.

Proposition 7 *(Chemistry between Adjacent Agents: Self-Centered) Assume that* $\rho_1\rho_2 > 1$ *and* $\rho_2\rho_3 > 1$ *and consider the strategies given by*

$$(l_1^*(t), l_2^*(t), l_3^*(t)) = \begin{cases}
(\frac{1}{\lambda_1}, 0, 0) & t < t_1 \\
\frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\left(\frac{1 - \frac{\gamma_{12}(t_2)}{\gamma_{12}(t)}}{\lambda_1}, \frac{1 - \frac{\gamma_{12}(t)}{\gamma_{12}(t_1)}}{\lambda_2}, 0\right) & t \in [t_1, t_2] \\
\left(0, \frac{1}{\lambda_2}, 0\right) & t \in (t_2, s_2) \\
\frac{\rho_2\rho_3}{\rho_2\rho_3 - 1}\left(0, \frac{1 - \frac{\gamma_{23}(t_3)}{\gamma_{23}(t)}}{\lambda_2}, \frac{1 - \frac{\gamma_{23}(t)}{\gamma_{23}(t_2)}}{\lambda_3}\right) & t \in [s_2, s_3] \\
(0, 0, \frac{1}{\lambda_3}) & t > s_3
\end{cases}$$

where the tasks t_1, t_2, t_3 *and the multipliers* $\lambda_1, \lambda_2, \lambda_3$ *are the solutions to conditions (19).*

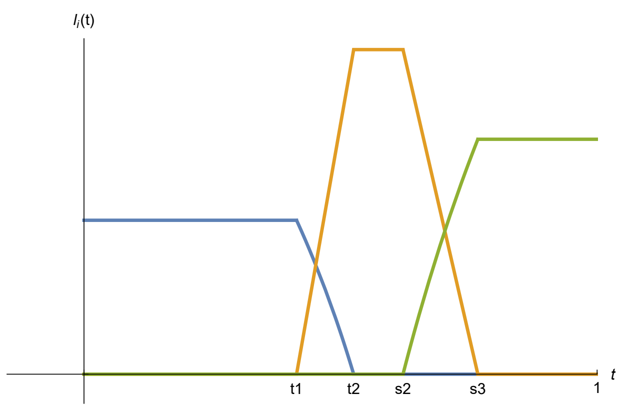
If the congruence parameters are close enough to 1 so that $t_2 < s_2$, the above strategies constitute an equilibrium.

In this case, the equilibrium effort functions partition the set of tasks into five intervals. There are four tasks, $t_1 < t_2 < s_2 < s_3$, such that:

- tasks in the interval $[0, t_1)$ are performed exclusively by player 1;
- tasks in the interval $[t_1, t_2]$ are performed by players 1 and 2, and
- tasks in the interval $(t_2, s_2]$ are performed exclusively by player 2,
- tasks in the interval $[s_2, s_3]$ are performed by players 2 and 3,
- tasks in the interval $(s_3, 1]$ are performed exclusively by player 3.

Figure 7 depicts an equilibrium when $\rho_1\rho_2 > 1$ and $\rho_2\rho_3 > 0$. Note that, restricted to the interval $[0, s_2)$, the behavior of players 1 and 2 mimics the two-player equilibrium. Similarly, restricted to the interval $(t_2, 1]$, the behavior of players 2 and 3 mimics the two-player equilibrium.

Figure 7: Example of an equilibrium where $\rho_1\rho_2 > 1$ and $\rho_2\rho_3 > 1$.



The next proposition describes equilibria when both pairwise chemistry indices are smaller than 1. The proof consists of checking that they satisfy the equilibrium conditions (7–8), and is left to the reader.

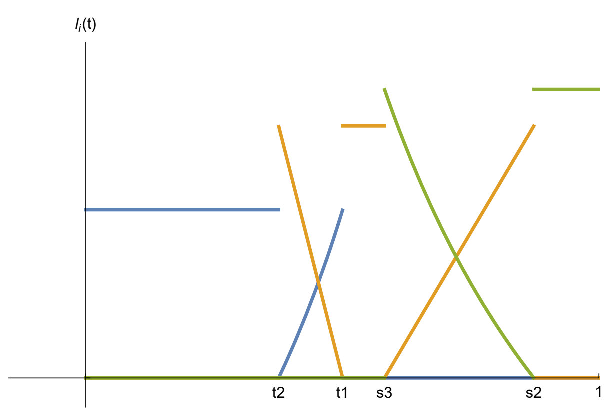
Proposition 8 *(Chemistry between Adjacent Agents: Deferential) Assume that* $\rho_1\rho_2 < 1$, *and* $\rho_2\rho_3 < 1$, *and consider the strategies given by:*

$$(l_1^*(t), l_2^*(t), l_3^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t \in A_1 \\ \frac{\rho_1\rho_2}{1-\rho_1\rho_2}\left(\frac{\frac{\mu(t_2)}{\mu(t)}-1}{\lambda_1}, \frac{\frac{\mu(t)}{\mu(t_1)}-1}{\lambda_2}\right) & t \in B \\ \left(0, \frac{1}{\lambda_2}\right) & t \in A_2 \\ \frac{\rho_3\rho_2}{1-\rho_3\rho_2}\left(\frac{\frac{\mu(s_3)}{\mu(t)}-1}{\lambda_2}, \frac{\frac{\mu(t)}{\mu(s_2)}-1}{\lambda_3}\right) & t \in C \\ \left(0, \frac{1}{\lambda_3}\right) & t \in A_3 \end{cases}$$

where $A_1 = [0, t_2] \cup B_1$, $A_2 = [t_1, s_3] \cup B_2 \cup C_2$, $A_3 = [s_2, 1] \cup C_3$, *and* $B = (t_2, t_1) \setminus (B_1 \cup B_2)$, *and where the tasks* t_1, t_2, s_2, s_3 *and the multipliers* $\lambda_1, \lambda_2, \lambda_3$ *solve conditions (19). If the congruence parameters are close enough to 1 so that* $t_1 < s_3$, *the above strategies constitute an equilibrium.*

Figure 8 depicts an equilibrium when $\rho_1\rho_2 < 1$ and $\rho_2\rho_3 < 0$. The equilibrium is an extreme one in which $B_1 = B_2 = C_2 = C_3 = \emptyset$. Note that, restricted to the interval $[0, s_3)$, the behavior of players 1 and 2 mimics the two-player equilibrium (Special Case A). Similarly, restricted to the interval $(t_1, 1]$, the behavior of players 2 and 3 mimics the two-player equilibrium (Special Case A).

Figure 8: Example of an equilibrium where $\rho_1\rho_2 < 1$ and $\rho_2\rho_3 < 1$.



Finally, the next proposition describes equilibria in which individuals 1 and 2 have a chemistry index less than one, while individuals 2 and 3 have a chemistry index bigger than one.

Proposition 9 *(Chemistry between Adjacent Agents: Mixed) Assume that* $\rho_1\rho_2 < 1$*, and* $\rho_2\rho_3 > 1$ *, and consider the strategies given by:*

$$(l_1^*(t), l_2^*(t), l_3^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t \in A_1 \\ \frac{\rho_1\rho_2}{1-\rho_1\rho_2}\left(\frac{\frac{\mu(t_2)}{\mu(t)}-1}{\lambda_1}, \frac{\frac{\mu(t)}{\mu(t_1)}-1}{\lambda_2}\right) & t \in B \\ (0, \frac{1}{\lambda_2}) & t \in A_2 \\ \frac{\rho_2\rho_3}{\rho_2\rho_3-1}\left(0, \frac{1-\frac{\mu_{23}(t_3)}{\mu_{23}(t)}}{\lambda_2}, \frac{1-\frac{\mu_{23}(t)}{\mu_{23}(t_2)}}{\lambda_3}\right) & t \in [j_2, j_3] \\ (0, \frac{1}{\lambda_3}) & t > j_3 \end{cases}$$

where $A_1 = [0, t_2] \cup B_1$*,* $A_2 = [t_1, j_3] \cup B_2$*, and* $B = (t_2, t_1) \setminus (B_1 \cup B_2)$*, and where the tasks* t_1, t_2, s_2, s_3 *and the multipliers* $\lambda_1, \lambda_2, \lambda_3$ *solve conditions (19). If the congruence parameters are close enough to 1 so that* $t_1 < s_3$*, the above strategies constitute an equilibrium.*

Summing up, the standard framework extended to n agents remains tractable when individuals' preferences do not deviate *excessively* from the value they bring to the firm, and the Nash equilibria resemble exactly that of a collection of pairs of individuals who behave as in the standard framework with two agents.

4 Discussion: Empirics

The labor and organizational economics literature offers scant insights into how employees might infringe upon the prerogatives of others and the implications this has for workforce and organizational structure. Even a minimal infringement because of preference heterogeneity or different personality traits generates a new class of organizational problems, the nature of which involves strategic interactions.

We see many interesting directions in which the analysis may be taken, and perhaps the most immediate one concerns empirics. Considering the recent empirical developments, it is not inconceivable that rich datasets will soon allow us to know the extent to which specialization is observed empirically, whether the division of labor is *always* efficient, and whether specialization *never* moves against the schedule of comparative advantages. These seem extremely strong predictions of the basic frictionless frameworks, predictions that are intuitively unlikely to be always satisfied. Viewed from this perspective, the results we have obtained have the potential to explain (a) why a full division of labor might not always occur in practice, even when tasks can be exchanged without cost (Deming (2017)), (b) why, even under complete division of labor, individuals may still carry out tasks that would be more efficiently performed by others, and (c) why workers' specialization might run *counter* to their comparative advantages. Frictions that create strategic interactions in teamwork generate these patterns, and we have found that these patterns depend only on the aggregate nature of the team. There are other predictions as well. For example, without being exhaustive, we may expect that the scope of authority will be linked to the nature of the team (and be greater in deferential teams), that when full specialization is observed, the schedule of comparative advantages will more likely be respected in self-centered teams than in deferential teams, or that self-centered teams will tend to exhibit a lower overlap of tasks the closer they are to having a neutral chemistry.

Where are we more likely to find sufficiently rich datasets that would allow us to begin implementing suitable empirical work? Predicting this in advance is challenging, but differences in preferences and personalities (as different from differences in knowledge) may perhaps be more prevalent in the formation of international production teams with individuals from different countries and cultures (see, for example, Antràs et al. (2006, 2008)). Settings in which team members are residual claimants sharing the fortunes and misfortunes of the firm (as in partnerships and corporations that make worker compensation contingent on company profits) would also seem better candidates than other settings.

In general, however, we believe the literature is approaching the point where it could provide a decisive empirical impetus for studying Ricardian frameworks with frictions in the near future. This optimism stems from its existing combination of observational data (including survey data) and experimental approaches, which together could make empirical investigations increasingly feasible. What variables would we need to observe? Here is some empirical guidance:

World Management Survey WMS (`http://worldmanagementsurvey.org`). What is often referred to as the new empirical economics of management has produced this high-quality interview-based evaluation tool. It defines and scores key management practices focusing on main areas (monitoring, targets, incentives/people management) and sectors (manufacturing, healthcare, education, and retail) with the goal of "digging deeper into the black box of productivity across industries and countries." In the case of the Manufacturing Survey Instrument (Bloom et al., 2021), the survey asks directly "How much do managers decide how tasks are allocated across workers in their teams?" We believe that it would be ideal to extend the panel dimension of the WMS to include this question for all other sectors as well. As for the available answers in the WMS, there are five options: either managers or workers may make all, about equal, or most the decisions. While the whole range of answers should be informative and empirically tractable, the cleanest empirical settings would probably be those in which workers make all the decisions.

Tasks: A key innovation in Bassi et al. (2024) is to measure *time use* within the firm, tracking how entrepreneurs and their employees allocate each hour of their workday to many pre-specified tasks, including both production tasks (e.g., specific steps of the production process) and non-production tasks (e.g., interacting with customers, supervision, input procurement). Viewed from the perspective of our analysis, the critical additional feature to add would be an assessment *individual performances* on those tasks. This would allow us to know the schedule of comparative advantages, which is of course essential to assess Ricardo-Roy models of production.

Preferences and Personality Types: As noted earlier, different settings (international teams, corporations in which worker compensation is contingent on firms' profits) may offer observable outcomes that facilitate some inference on workers' preferences. But perhaps the more direct way is simply to assess workers' personality and behavioral traits explicitly, as in the economics literature on social behavior, cognitive and non-cognitive abilities, psychology, and the Big-5 personality factors (e.g., Heckman, Stixrud and Urzua (2006), Borghans et al. (2008), Almludn et al. (2011), Flinn, Todd and Wang (2024)). This would seem readily feasible. Indeed, personality, intelligence and other psychology tests are routinely used in a variety of settings including business, education, civil service, and the military.

Social Skills: These skills can be a critical determinant of team production in many settings. They are also different from, and harder to assess than, personality measures. As Weidmann and Deming (2021) show, in order to measure these skills, absent any natural experiments, we may need to take an experimental approach. This could be implemented at the same time as the individual performance on different tasks is assessed. Specifically, their methodology randomly assigns individuals to multiple teams, and then measures each team's performance on different tasks.[14] This allows measuring social skills and workers' personalities separately, and thus also their separate impact on task allocation and team performance given the schedule of comparative advantages.

Of course, an important question is the extent to which the predictions we have obtained are robust to the inclusion social skills. To answer this question, we have considered Deming's (2017) model where the objective of the team is a monotonically increasing function of the sum of the individuals' outputs, and individual i's output is a function of the labor inputs that he and his teammate devote to the various tasks applied towards this output. Agents are characterized by their own productivity, but now this productivity is lowered when effort is devoted to the teammate's output.[15] It is well known that when all workers share the objective function of the firm, costly social skills generate a range of non-traded tasks. Now, when individuals are characterized by congruence parameters different from one so that, as in the preceding analyses, their preferences are not perfectly aligned, what are the Nash equilibria in this model? What are the testable implications? In an earlier working paper (Palacios-Huerta, Steinberg, and Volij (2024)) we find that: (1) the efficient outcome is a Nash equilibrium only if every worker in the team is deferential; otherwise, strategic interactions generate inefficient outcomes, and (2) all the Nash equilibria can be characterized precisely and their nature is the *same* as in the characterization obtained in the standard Ricardian framework that we have studied. Moreover, it involves exactly the *same* chemistry index.

The intuition for why the results are robust and the same findings obtain is that social

[14]Interestingly, in their study lab participants are assessed on three dimensions of the Big-5 personality inventory that could have been readily applied to our description of individual types—Conscientiousness, Extroversion, and Agreeableness. Agreeableness, for example, "is a personality trait referring to individuals that are perceived as kind, sympathetic, cooperative, warm, honest, and considerate [and] reflecting individual differences in cooperation and social harmony. People who score high on measures of agreeableness are empathetic and altruistic, while those with low agreeableness are prone to selfish, lack of empathy... and may show dark triad tendencies, such as narcissistic, antisocial, and manipulative behavior." (https://en.wikipedia.org/wiki/Agreeableness)

[15]This formalization follows the one of transportation costs in Dornbusch, Fisher, and Samuelson (1977).

skills basically act as a trading technology that is independent of preferences, and so it continues to be the case that the nature of the players' chemistry determines the class of Nash equilibria while the individuals identities determine the scope of authority. Obviously, what does change is, as in the case of social skills without frictions, the range of tasks in which we may observe specialization: a greater the cost of trading tasks implies a greater range of non-traded tasks.[16] In terms of empirics, these results also mean that separating social skills from personality characteristics would be necessary and sufficient to study the same implications we have obtained.

5 Concluding Remarks

Krugman (1998) concludes his essay *Ricardo's Difficult Idea* by noting that "Ricardo's idea is truly, madly, deeply difficult. But it is also utterly true, immensely sophisticated – and extremely relevant to the modern world." Over the last three decades, the emergence of new datasets has significantly enriched our understanding of key empirical developments in labor markets and firms. These advancements have highlighted the need for more comprehensive frameworks to study how changes in earnings and employment are shaped by the interplay between worker skills, job tasks, technological advancements, and evolving trading opportunities. It is, therefore, unsurprising that Ricardo's ideas have become a cornerstone not only in trade literature but also in these contemporary developments in labor economics. What we find surprising, however, is that a common feature of all contributions rooted in Ricardian team production frameworks is the absence of strategic interactions. This omission is particularly striking given the central role strategic interactions play in the extensive literature on the economics of firms and organizations. Our goal has been to provide a first step toward bridging this gap by incorporating strategic considerations into the Ricardian framework and exploring their implications for team production dynamics and organizational outcomes.

In terms of empirics, a combination and extension of the existing surveys, tests, and

[16] Needless to say, other differences have to do with the different nature of the model. For example, since this is a model in which a principal would be allowed to "delineate tasks," he could find it optimal to separate the tasks (say, physically) to prevent encroaching. We thank Jean Tirole for this point.

methods already in place in the literature offer a promising avenue to test Ricardo-Roy models with frictions arising from preferences. We conclude with three questions that in our judgement would each be a good starting point for future research:

(a) *Managers and Incentives.* Our analysis has focused on preferences, and thus deviates from standard incentive theory. However, once the framework is opened up to include authority explicitly, the analysis will readily link to this literature (see, e.g., Prendergast (2007, 2008), Halac and Prat (2016), Halac, Kremer and Winter (2024)). More generally, there is no reason to expect that the organizational problem arising from strategic considerations within teams will be independent from other problems why organizations fail (see Garicano and Rayo (2016) for a thorough survey).

(b) *Partnerships and Profit Sharing: Endogenous* ρ_i. When team members are all residual claimants, incentives take a different form: "incentives are generated when an individual empathizes with those whose income he affects" (Kandel and Lazear, 1992, p. 816). Thus, norms, mutual monitoring, corporate culture, peer pressure, shame and guilt may operate as an endogenous formation of ρ_i.

(c) *Optimal Chemistry.* We have taken as given the matching of types within a team. It might be possible to derive the optimal chemistry (matching pattern) for a given (infinite) population of types. This could have implications for the optimal regulation of tasks. It would be worth investigating, under the assumption of asymmetric information when agents privately know their type, whether matching–or possibly assortative matching–will operate as an incentive device for agents to truthfully announce their type.

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A Appendix

This appendix is divided in two parts. In part A.1 we derive the efficient strategies in the general case and prove Corolary 2. In part A.2 we characterize the Nash equilibria in the standard framework with teamwork frictions.

A.1 Proof of Proposition 1 and Corollary 2

Proof of Proposition 1: A team-optimal strategy profile is the solution to the optimal control problem (1). It exists since the objective function is continuous and the set of allocations is compact in the weak-* topology. Uniqueness follows from the strict concavity of the utility function and the linearity of the technology: if there were two team-optimal allocations that differed on a set of tasks with positive measure, their convex combination would be a feasible allocation and would result in a higher team-payoff, which is impossible. The associated Hamiltonian is

$$H\Big(\big(l_i(t),\lambda_i(t)\big)_{i=1}^{n}\Big) = u\Big(\sum_{i=1}^{n}\alpha_i(t)l_i(t)\Big) - \sum_{i=1}^{n}\lambda_i(t)l_i(t)$$

and the corresponding necessary conditions for a solution are

$$\frac{\partial H}{\partial l_i(t)} = u'\Big(\sum_{i=1}^{n}\alpha_i(t)l_i(t)\Big)\alpha_i(t) - \lambda_i(t) \leq 0 \text{ with equality if } l_i(t)>0, i\in N \quad (20a)$$

$$0 = \lambda_i'(t), \qquad i \in N \quad (20b)$$

$$\int_0^1 l_i(t) = 1, \qquad i\in N \quad (20c)$$

Consider a solution $(l_i(t), \lambda_i(t))$ to the above conditions. By (20b), there is a constant λ_i such that for all i, $\lambda_i(t) = \lambda_i$. Furthermore, by (20a), $\lambda_i > 0$. Since, $\lim_{x\to 0} u'(x) = \infty$, at the solution, all tasks are performed, i.e., $\sum_{i=1}^{n}\alpha_i(t)l_i(t) > 0$. Consequently, it follows from (20a) that for the tasks performed by agent i we have that

$$\frac{\alpha_i(t)}{\alpha_j(t)} \geq \frac{\lambda_i}{\lambda_j} \qquad j\in N,\ l_i(t)>0$$

Define for each agent i,

$$I_i = \left\{t\in[0,1] : \frac{\alpha_i(t)}{\lambda_i} \geq \frac{\alpha_j(t)}{\lambda_j} \quad \text{for all } j\in N\right\}. \quad (21)$$

The set I_i is the set of tasks that are performed by agent i. By Assumption 1, it is a finite or countable union of intervals. Since for every $t\in[0,1]$ there is a player i such that $\frac{\alpha_i(t)}{\lambda_i} \geq \frac{\alpha_j(t)}{\lambda_j}$ for all $j\in N$, we have that $\cup_i I_i = [0,1]$. Also, by (20c), these sets are non-empty. As a result, since the sets I_i are compact and the interval $[0,1]$ is connected, for each i, there must be a $j \neq i$ such that $I_j \cap I_i \neq \emptyset$. Therefore, if $t_{ij} \in I_i \cap I_j$, then $\frac{\alpha_i(t_{ij})}{\alpha_j(t_{ij})} = \frac{\lambda_i}{\lambda_j}$. Therefore, for $t \in \ int\ I_j$, we have that $l_j(t) > 0$ and $l_i(t) = 0$, for $i \in N$, $i \neq j$, and consequently, by (20a)

$$u'\big(\sum_{i=1}^{n} \alpha_i(t) l_i(t)\big)\alpha_i(t) = u'(\alpha_j(t) l_j(t))\alpha_j(t) = \lambda_j \qquad t \in int\, I_j$$

which implies,

$$l_j(t) = u'^{-1}\Big(\frac{\lambda_j}{\alpha_j(t)}\Big)\frac{1}{\alpha_j(t)} \qquad t \in int\, I_j$$

Since $\int_0^1 l_j(t) = \int_{I_j} l_j(t) = 1$,

$$\int l_j(t) = \int_{I_j} u'^{-1}\Big(\frac{\lambda_j}{\alpha_j(t)}\Big)\frac{1}{\alpha_j(t)} = 1.$$

We conclude that the team's optimal strategies satisfy

$$l_i(t) = \begin{cases} u'^{-1}\left(\frac{\lambda_j}{\alpha_j(t)}\right)\frac{1}{\alpha_j(t)} & t \in I_i \\ 0 & t \notin I_i \end{cases} \qquad i \in N \tag{22}$$

where for each $i \in N$,

$$I_i = \left\{ t \in [0,1] : \frac{\alpha_i(t)}{\alpha_j(t)} \geq \frac{\mu(I_i)}{\mu(I_j)}, \qquad \text{for all } j \in N \right\}. \qquad \blacksquare$$

Proof of Corollary 2: Recall that at the optimum, the sets of tasks assigned to the agents satisfy

$$\hat{I}_1 = \left\{ t \in [0,1] : \frac{\alpha_1(t)}{\alpha_2(t)} \geq \frac{\mu(\hat{I}_1)}{\mu(\hat{I}_2)}, \frac{\alpha_1(t)}{\alpha_3(t)} \geq \frac{\mu(\hat{I}_1)}{\mu(\hat{I}_3)} \right\}$$
$$\hat{I}_2 = \left\{ t \in [0,1] : \frac{\alpha_2(t)}{\alpha_1(t)} \geq \frac{\mu(\hat{I}_2)}{\mu(\hat{I}_1)}, \frac{\alpha_2(t)}{\alpha_3(t)} \geq \frac{\mu(\hat{I}_2)}{\mu(\hat{I}_3)} \right\}$$
$$\hat{I}_3 = \left\{ t \in [0,1] : \frac{\alpha_3(t)}{\alpha_1(t)} \geq \frac{\mu(\hat{I}_3)}{\mu(\hat{I}_1)}, \frac{\alpha_3(t)}{\alpha_2(t)} \geq \frac{\mu(\hat{I}_3)}{\mu(\hat{I}_2)} \right\}$$

Since none of them is empty, there are t_1 and t_2 such that

$$\frac{\alpha_1(t_1)}{\alpha_2(t_1)} \geq \frac{\mu(\hat{I}_1)}{\mu(\hat{I}_2)} \geq \frac{\alpha_1(t_2)}{\alpha_2(t_2)}$$

By the intermediate value theorem, there is t_{12} such that

$$\frac{\alpha_1(t_{12})}{\alpha_2(t_{12})} = \frac{\mu(\hat{I}_1)}{\mu(\hat{I}_2)}$$

Since $\frac{\alpha_1(t)}{\alpha_2(t)}$ is decreasing, this means that $t \notin \hat{I}_2$ for all $t < t_{12}$, and $t \notin \hat{I}_1$ for all $t > t_{12}$. Similarly, there is t_{23} such that

$$\frac{\alpha_2(t_{23})}{\alpha_3(t_{23})} = \frac{\mu(\hat{I}_2)}{\mu(\hat{I}_3)}$$

and since $\frac{\alpha_2(t)}{\alpha_3(t)}$ is decreasing, this means that $t \notin \hat{I}_2$ for all $t > t_{23}$, and $t \notin \hat{I}_3$ for all $t < t_{23}$. Since $\hat{I}_2$ is non-empty, we conclude that $t_{12} < t_{23}$. As a result, since $\hat{I}_1 \cup \hat{I}_2 \cup \hat{I}_3 = [0,1]$, the tasks on which the players specialize are given by

$$\hat{I}_1 = [0, t_{12}], \qquad \hat{I}_2 = [t_{12}, t_{23}], \qquad \hat{I}_3 = [t_{23}, 1]. \tag{23}$$

The result then follows from Proposition 2. ■

A.2 Characterization of Equilibria under Frictions

Proof of Proposition 3: Assume that $\rho_1\rho_2 = 1$. In this case, conditions (7–8) become

$$\frac{\rho_1\alpha_1(t)}{\rho_1\alpha_1(t)l_1^*(t) + \alpha_2(t)l_2^*(t)} - \lambda_1 \leq 0 \text{ with equality if } l_1^*(t) > 0 \tag{24a}$$

$$\frac{\alpha_2(t)}{\rho_1\alpha_1(t)l_1^*(t) + \alpha_2(t)l_2^*(t)} - \lambda_2 \leq 0 \text{ with equality if } l_2^*(t) > 0 \tag{24b}$$

$$\int l_1(t) = \int l_2(t) = 1 \tag{24c}$$

Comparing these conditions to the conditions for efficiency in (20), we see that they are obtained from (20) by simply replacing $\alpha_1(t)$ by $\rho_1\alpha_1(t)$. Therefore, we can conclude that when $\rho_1\rho_2 = 1$, the equilibrium strategies are given by

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{t^*}, 0) & t < t^* \\ \text{anything} & t = t^* \\ \left(0, \frac{1}{1-t^*}\right) & t > t^* \end{cases}$$

where t^* satisfies

$$\rho_1\gamma(t^*) = \frac{t^*}{1-t^*}. \quad ■$$

Having dealt with the case where $\rho_1\rho_2 = 1$, we henceforth assume that $\rho_1\rho_2 \neq 1$. Recall that the equilibrium strategies assume the form (see equation (14))

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t \in A_1 \\ \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\left(\frac{\rho_1\gamma(t) - \frac{\lambda_1}{\lambda_2}}{\lambda_1\rho_1\gamma(t)}, \frac{\frac{\lambda_1}{\lambda_2} - \frac{\gamma(t)}{\rho_2}}{\lambda_1}\right) & t \in B \\ \left(0, \frac{1}{\lambda_2}\right) & t \in A_2 \end{cases} \tag{25}$$

Furthermore, since in equilibrium players use up all their effort, we have that

$$\int_0^1 l_1^*(t) = \int_0^1 l_2^*(t) = 1. \tag{26}$$

Since $(l_1^*(t), l_2^*(t)) \geq (0,0)$ for all $t \in B$, we must have

$$\frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\big(\rho_1\gamma(t) - \frac{\lambda_1}{\lambda_2}\big) \;\geq\; 0 \;\text{ for all } t \in B \tag{27a}$$

$$\frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\big(\frac{\lambda_1}{\lambda_2} - \frac{\gamma(t)}{\rho_2}\big) \;\geq\; 0 \;\text{ for all } t \in B \tag{27b}$$

Given that γ is decreasing, these two inequalities imply that the tasks in B will all belong to an interval whose extreme points, t_1 and t_2, satisfy

$$\frac{\gamma(t_1)}{\rho_2} = \frac{\lambda_1}{\lambda_2} \;=\; \rho_1\gamma(t_2). \tag{28}$$

Let t_1 and t_2 solve the above equations. These two critical tasks will be used in the next two proofs.

Proof of Proposition 4: Since $\rho_1\rho_2 > 1$ and $\gamma(t)$ is decreasing, it follows from (28) that $t_1 < t_2$. It follows from (27) that

$$\frac{\gamma(t)}{\rho_2} \leq \frac{\lambda_1}{\lambda_2} \;\leq\; \rho_1\gamma(t) \qquad \text{for all } t \in B. \tag{29}$$

Also, using (28) we have that

$$\frac{\gamma(t)}{\rho_2} \;>\; \frac{\lambda_1}{\lambda_2} \qquad \text{for all } t < t_1$$

$$\frac{\lambda_1}{\lambda_2} \;>\; \rho_1\gamma(t) \qquad \text{for all } t > t_2.$$

These two inequalities, along with (29) imply that $B \subset [t_1, t_2]$. It follows from (8) that

$$\frac{\lambda_1}{\lambda_2} \leq \frac{\gamma(t)}{\rho_2}, \text{ for all } t \in A_1.$$

Since γ is decreasing, it follows from (28) that

$$\frac{\gamma(t)}{\rho_2} < \frac{\lambda_1}{\lambda_2} \text{ for all } t > t_1.$$

These two inequalities imply that $A_1 \subset [0, t_1]$. Similarly, it follows from (7) that

$$\rho_1\gamma(t) \leq \frac{\lambda_1}{\lambda_2} \quad \text{for all } t \in A_2$$

and since γ is decreasing, it follows from (28) that

$$\frac{\lambda_1}{\lambda_2} < \rho_1\gamma(t) \quad \text{for all } t < t_2,$$

which implies that $A_2 \subset [t_2, 1]$. Since $A_1 \cup B \cup A_2 = [0,1]$, we conclude that

$$\begin{aligned} [0,t_1) &\subset A_1 \subset [0,t_1] \\ (t_1,t_2) &\subset B \subset [t_1,t_2] \\ (t_2,1] &\subset A_2 \subset [t_2,1]. \end{aligned}$$

Summarizing, the equilibrium is given by

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t < t_1 \\ \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\left(\frac{\rho_1\gamma(t) - \frac{\lambda_1}{\lambda_2}}{\lambda_1\rho_1\gamma(t)}, \frac{\frac{\lambda_1}{\lambda_2} - \frac{\gamma(t)}{\rho_2}}{\lambda_1}\right) & t \in [t_1, t_2] \\ \left(0, \frac{1}{\lambda_2}\right) & t > t_2 \end{cases}$$

where the tasks t_1, t_2 and the multipliers λ_1, λ_2 are characterized by (26) and (28). Using (28), the equilibrium strategies can be written as

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t < t_1 \\ \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\left(\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1}, \frac{1 - \frac{\gamma(t)}{\gamma(t_1)}}{\lambda_2}\right) & t \in [t_1, t_2] \\ (0, \frac{1}{\lambda_2}) & t > t_2 \end{cases}$$

It remains to show that the equilibrium critical tasks and multipliers are unique. Suppose that $(l_1(t), l_2(t))$ and $(l_1'(t), l_2'(t))$ are two equilibria with parameters $(t_1, t_2, \lambda_1, \lambda_2)$ and $(t_1', t_2', \lambda_1', \lambda_2')$, respectively. Then, the following equalities hold:

$$\frac{\gamma(t_1)}{\rho_2} = \frac{\lambda_1}{\lambda_2} = \rho_1\gamma(t_2). \tag{30a}$$

$$\frac{\gamma(t_1')}{\rho_2} = \frac{\lambda_1'}{\lambda_2'} = \rho_1\gamma(t_2'). \tag{30b}$$

Suppose without loss of generality that $t_1 \le t_1'$. Then, $\frac{\lambda_1}{\lambda_2} \ge \frac{\lambda_1'}{\lambda_2'}$ and $t_2 \le t_2'$. The uniqueness will follow from the next three claims.

Claim 1 $\lambda_1' \ge \lambda_1$.

Proof. Assume by contradiction that $\lambda_1' < \lambda_1$. There are two cases to consider:

Case 1: $t_1' \leq t_2$. Given that γ is decreasing,

$$
\begin{aligned}
\frac{1}{\lambda_1} &< \frac{1}{\lambda_1'} \\
\frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} &\leq \frac{1}{\lambda_1} < \frac{1}{\lambda_1'} \qquad t \in [t_1, t_1'] \\
\frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} &< \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2')}{\gamma(t)}}{\lambda_1'} \qquad t \in [t_1', t_2]
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
1 &= \int_0^1 l_1^*(t) = \int_0^{t_1} \frac{1}{\lambda_1} + \int_{t_1}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} \\
&= \int_0^{t_1} \frac{1}{\lambda_1} + \int_{t_1}^{t_1'} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} + \int_{t_1'}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} \\
&< \int_0^{t_1} \frac{1}{\lambda_1'} + \int_{t_1}^{t_1'} \frac{1}{\lambda_1'} + \int_{t_1'}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2')}{\gamma(t)}}{\lambda_1'} + \int_{t_2}^{t_2'} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2')}{\gamma(t)}}{\lambda_1'} \\
&= \int_0^{t_1'} \frac{1}{\lambda_1'} + \int_{t_1'}^{t_2'} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} = \int_0^1 l_1'(t) = 1
\end{aligned}
$$

which is absurd.

Case 2: $t_1' \geq t_2$. In this case we have that

$$
\begin{aligned}
\frac{1}{\lambda_1} &< \frac{1}{\lambda_1'} \\
\frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} &\leq \frac{1}{\lambda_1} < \frac{1}{\lambda_1'} \qquad t \in [t_1, t_2]
\end{aligned}
$$

Then,

$$
\begin{aligned}
1 &= \int_0^1 l_1^*(t) = \int_0^{t_1} \frac{1}{\lambda_1} + \int_{t_1}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t_2)}{\gamma(t)}}{\lambda_1} \\
&< \int_0^{t_1'} \frac{1}{\lambda_1'} < \int_0^1 l_1(t) = 1
\end{aligned}
$$

which is absurd.

We conclude that $\lambda_1' \geq \lambda_1$. ■

Claim 2 $\lambda_2' \leq \lambda_2$.

Proof. Assume by contradiction that $\lambda_2' > \lambda_2$. There are two cases to consider:

Case 1: $t_1' \leq t_2$. Given that γ is decreasing,

$$\begin{aligned}
\frac{1}{\lambda_2} &> \frac{1}{\lambda_2'} \\
\frac{1}{\lambda_2} &> \frac{1}{\lambda_2'} \geq \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1')}}{\lambda_2'} \qquad t \in [t_2, t_2'] \\
\frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1)}}{\lambda_2} &> \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1')}}{\lambda_2'} \qquad t \in [t_1', t_2]
\end{aligned}$$

Therefore,

$$\begin{aligned}
1 &= \int_0^1 l_2^*(t) = \int_{t_1}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1)}}{\lambda_2} + \int_{t_2}^1 \frac{1}{\lambda_2} \\
&= \int_{t_1}^{t_1'} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1)}}{\lambda_2} + \int_{t_1'}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1)}}{\lambda_2} + \int_{t_2}^{t_2'} \frac{1}{\lambda_2} + \int_{t_2'}^1 \frac{1}{\lambda_2} \\
&> \int_{t_1}^{t_1'} 0 + \int_{t_1'}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1')}}{\lambda_2'} + \int_{t_2}^{t_2'} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1')}}{\lambda_2'} + \int_{t_2'}^1 \frac{1}{\lambda_2'} \\
&= \int_{t_1'}^{t_2'} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1')}}{\lambda_2'} + \int_{t_2'}^1 \frac{1}{\lambda_2'} = \int_0^1 l_2'(t) = 1
\end{aligned}$$

which is absurd.

Case 2: $t_1' \geq t_2$. We have that

$$\begin{aligned}
\frac{1}{\lambda_2} &> \frac{1}{\lambda_2'} \\
\frac{1}{\lambda_2} &> \frac{1}{\lambda_2'} \geq \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1')}}{\lambda_2'} \qquad t \in [t_1', t_2']
\end{aligned}$$

Then,

$$\begin{aligned}
1 &= \int_0^1 l_2^*(t) = \int_{t_1}^{t_2} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1)}}{\lambda_2} + \int_{t_2}^1 \frac{1}{\lambda_2} \\
&> \int_{t_1}^{t_1'} 0 + \int_{t_1'}^{t_2'} \frac{\rho_1\rho_2}{\rho_1\rho_2 - 1}\frac{1 - \frac{\gamma(t)}{\gamma(t_1')}}{\lambda_2'} + \int_{t_2'}^1 \frac{1}{\lambda_2'} = \int_0^1 l_2(t) = 1
\end{aligned}$$

which is absurd.

We conclude that $\lambda_2' \leq \lambda_2$. ∎

Claim 3 *$\lambda_1' = \lambda_1$ and $\lambda_2' = \lambda_2$.*

Proof. We know by the previous claims that $\lambda_1' \geq \lambda_1$ and $\lambda_2' \leq \lambda_2$. Assume by contradiction that at least one of these inequalities is strict. In this case we have $\frac{\lambda_1}{\lambda_2} < \frac{\lambda_1'}{\lambda_2'}$, which we know not to be true. ■

Since $\lambda_1' = \lambda_1$ and $\lambda_2' = \lambda_2$, it follows from equations (30) that $(t_1, t_2, \lambda_1, \lambda_2) = (t_1', t_2', \lambda_1', \lambda_2')$. ■

Proof of Proposition 5: Since $\rho_1\rho_2 < 1$, it follows from (28) that $t_2 < t_1$. Since γ is decreasing, it follows from (27) that

$$\rho_1\gamma(t) \leq \frac{\lambda_1}{\lambda_2} \leq \frac{\gamma(t)}{\rho_2} \qquad \text{for all } t \in B. \tag{31}$$

Also, by (28) we have that

$$\begin{aligned} \frac{\gamma(t)}{\rho_2} &< \frac{\lambda_1}{\lambda_2} && \text{for all } t > t_1 \\ \frac{\lambda_1}{\lambda_2} &< \rho_1\gamma(t) && \text{for all } t < t_2. \end{aligned}$$

These two inequalities, along with (31) imply that $B \subset [t_2, t_1]$. It follows from (7–8) that

$$\begin{aligned} \frac{\rho_2}{\gamma(t)/\lambda_1} - \lambda_2 &\leq 0 && \text{for all } t \in A_1 \\ \frac{\rho_1\gamma(t)}{1/\lambda_2} - \lambda_1 &\leq 0 && \text{for all } t \in A_2. \end{aligned}$$

or, rearranging,

$$\begin{aligned} \frac{\lambda_1}{\lambda_2} &\leq \frac{\gamma(t)}{\rho_2} && \text{for all } t \in A_1 \\ \rho_1\gamma(t) &\leq \frac{\lambda_1}{\lambda_2} && \text{for all } t \in A_2. \end{aligned}$$

It follows from (28) that

$$\begin{aligned} \frac{\lambda_1}{\lambda_2} &> \frac{\gamma(t)}{\rho_2} && \text{for all } t > t_1 \\ \rho_1\gamma(t) &> \frac{\lambda_1}{\lambda_2} && \text{for all } t < t_2. \end{aligned}$$

These two inequalities imply that, $A_1 \subset [0, t_1]$ and $A_2 \subset [t_2, 1]$. Therefore,

$$\begin{aligned} A_1 &= [0, t_2) \cup B_1 \\ A_2 &= B_2 \cup (t_1, 1] \end{aligned}$$

where B_1 and B_2 are two disjoint subsets such that where $\{B_1, B_2, B\}$ is a partition of $[t_2, t_1]$ into three measurable cells. The tasks in B_1 are performed exclusively by player 1. The tasks in B_2 are performed exclusively by player 2. The tasks in B are simultaneously performed by both players. Except for the requirement that they satisfy (26), the partition can be arbitrary. ■

Special Cases. There are two extreme types of equilibrium when $\rho_1\rho_2 < 1$, one in which $B = \emptyset$ and the other when $B = [t_2, t_1]$.

Special Case A: In this case, the equilibrium strategies have the following form:

$$(l_1^*(t), l_2^*(t)) = \begin{cases} (\frac{1}{\lambda_1}, 0) & t \in A_1 \\ \left(0, \frac{1}{\lambda_2}\right) & t \in A_2 \end{cases}$$

where

$$\int_{A_1} \frac{1}{\lambda_1} = \int_{A_2} \frac{1}{\lambda_2} = 1$$

This implies that

$$\lambda_1 = |A_1|\,,\ \lambda_2 = |A_2.| \tag{33}$$

We see that for some $\alpha \in [0,1]$, $\lambda(B_1) = \alpha(t_1 - t_2)$ and $\lambda(B_2) = (1-\alpha)(t_1 - t_2)$. Then, by (33)

$$\begin{aligned} \lambda_1 &= |A_1| = t_2 + \alpha(t_1 - t_2) \\ \lambda_2 &= |A_2| = (1 - t_1) + (1-\alpha)(t_1 - t_2) = 1 - \lambda_1. \end{aligned}$$

We conclude that the equilibrium must satisfy

$$\rho_1\gamma(t_2) = \frac{t_2 + \alpha(t_1 - t_2)}{1 - t_2 - \alpha(t_1 - t_2)} = \frac{\gamma(t_1)}{\rho_2}$$

for some $\alpha \in [0,1]$, t_1 and t_2.

Special Case B: In this case, in which the support of the equilibrium strategies are intervals, namely $A_1 = [0, t^*)$ and $A_2 = [t^*, 1]$, we have that

$$\rho_1\gamma(t_2) = \frac{t^*}{1 - t^*} = \frac{\gamma(t_1)}{\rho_2}$$

for some t_1 and t_2. These two equations characterize an equilibrium as long as $t^* \in [\underline{t}, \overline{t}]$, where the bounds $\underline{t}$ and $\overline{t}$ are defined by

$$\begin{aligned} \rho_1\gamma(\overline{t}) &= \frac{\overline{t}}{1 - \overline{t}} \\ \frac{\underline{t}}{1 - \underline{t}} &= \frac{\gamma(\underline{t})}{\rho_2}. \end{aligned}$$

In the equilibrium where $B = [t_2, t_1]$ the equilibrium is given by (25) where

$$\begin{aligned} [0, t_2) &\subset A_1 \subset [0, t_2] \\ (t_2, t_1) &\subset B \subset [t_2, t_1] \\ (t_1, 1] &\subset A_2 \subset [t_1, 1]. \end{aligned}$$

and the tasks t_1, t_2 and the multipliers λ_1, λ_2 are characterized by (26) and (28).