

Problem Set #3: The Current Account and Investment

Consider a small open economy. The economy consists of identical households that own capital and have access to a production technology and to the world financial markets.

Households live for two periods, and maximize utility:

$$U(C_1, C_2) = \log(C_1) + \beta \log(C_2)$$

Where C_1 and C_2 are consumption in period 1 and 2 respectively, and β is the subjective discount factor, $0 < \beta < 1$.

Households are endowed with some capital in period 1, denoted by K_1 . In each period households use capital to produce the consumption good with a production technology that is given by:

$$Q_t = A_t \sqrt{K_t} \quad t = 1, 2$$

Where Q is total production (GDP) and A is a technology parameter. An increase in A reflects technological progress as each unit of capital becomes more productive.

Assume that capital depreciates completely after it is being used in production. As a result, in order to be able to produce in the second period households must invest in period 1. Capital accumulation is therefore given by:

$$K_2 = I_1$$

Where I is investment. That is, capital in the second period is determined by investment in the first period.

Finally, residents of the economy have full access to the world financial market, and they can borrow or lend any amount at the world real interest rate, r .¹ A standard assumption for small open economy is that:

$$\beta(1+r) = 1$$

We will denote by B_1^* the amount of foreign assets households buy in the first period. In the second period they receive $(1+r)B_1^*$. Note that B_1^* may be negative.

Given this environment the flow resource constraints are given by:

$$\begin{aligned} C_1 + K_2 + B_1^* &= A_1\sqrt{K_1} \\ C_2 &= A_2\sqrt{K_2} + (1+r)B_1^* \end{aligned}$$

Notice that in the second period households neither buy new assets nor they invest (that is $I_2 = B_2^* = 0$). This is because there is no third period; therefore the households consume all remaining resources in period 2.

1. The lifetime resource constraint

Write down the lifetime (a.k.a. intertemporal) resource constraint. That is, combine the two periodical constraints into one by substituting for B_1^* . You should get an expression that equates the present value of uses (consumption and investment) to the present value of output.

2. The households' problem

- (a) Write down the households' problem. That is, specify the objective function, the choice variables, and the resource constraint (use the constraint from question 1).
- (b) Write down the Lagrangian, and take first order conditions with respect to your choice variables. Interpret your results (substitute for the Lagrange multiplier before you attempt to interpret the first order conditions).
- (c) Solve for K_2 , C_1 , and C_2 . That is, express these variables as a function of the parameters of the problem (A_1 , A_2 , K_1 , and r).

¹ The economy cannot affect the world interest rate because we assume that it is small relative to the rest of the world.

3. Saving and investment

Saving is the difference between income and consumption. In this case, income in the first period is the same as output since there is no interest income in this period (from assets that were bought before the world started, $B_0^* = 0$).

Given your solution to question 2, write down expressions for the optimal levels of period 1 saving (S_1) and investment (I_1). How do these variables depend on A_2 ? Explain.

4. The current account

- (a) Given your solution to question 3, solve for the current account in period 1.
- (b) Countries in early stages of development often run current account deficits. This is because they are expected to grow fast in the future and therefore provide attractive investment opportunities. As a result capital flows into the country; which, in turn, induces current account deficits.

Is your result in part (a) consistent with this observation? Specifically, how does the current account imbalance depend on A_2 ? Explain.