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International Finance (Econ 421)

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Handout #6: Purchasing Power Parity

1. Introduction

This handout explores the relationship between prices of goods and services and the exchange rate. The main idea is that in a frictionless environment the purchasing power of a dollar in the US should be the same as in, say, Europe.¹ Under this approach the exchange rate is pinned down by the ratio of the price levels in the two locations.

It should be stressed that since we attempt to determine the exchange rate through prices, we must either assume flexible prices or expect our results to hold only in the long run.

After exploring the relationship between prices and exchange rate, we take a monetary approach to exchange rate determination. In particular we link prices to the money market and see how developments there affect the exchange rate. We will see that an increase in the domestic interest rate is associated with depreciation of the domestic currency. This result is at odds with our conclusions from the asset pricing approach driven by the interest rate parity.² We reconcile the contradiction through our assumptions regarding the behavior of prices; sticky in the short run but flexible in the long run.

2. Purchasing Power Parity (PPP)

The idea behind the PPP is that in a frictionless environment the purchasing power of a dollar in the US should be the same anywhere in the world. That is, if a basket of goods costs \$100 in the US then, after converting to foreign currency, \$100 should be able to buy us exactly the same basket anywhere in the world.

¹ By frictionless environment we mean a world with no trade costs, no tariffs, full information, etc.

² Recall that under the asset pricing approach an increase in the interest rate leads to an appreciation of the currency.

2.1 The Law of One Price (LOP)

A useful starting point for understanding the PPP is thinking of individual commodities instead of basket of goods. The LOP does just that; it is a simpler version of PPP applied to a single good. Suppose, for example, that a can of coke costs \$1 in the US and the exchange rate is \$1.25 per euro, then the LOP suggests that in Europe a can of coke should cost €0.8 (\$1 / 1.25 \$/€).

More generally, let P_i denote the *domestic* price of good i , let P_i^* denote the *foreign* price of the same good, and E is the exchange rate (domestic currency per unit of foreign currency). Using this notation the LOP states:

$$P_i = E \times P_i^* \quad (1)$$

2.2. Absolute PPP

Absolute PPP is simply the LOP applied to a basket of goods instead of a single commodity. Therefore, if we let P and P^* denote the home and foreign price levels of the basket respectively, then absolute PPP states:

$$P = E \times P^* \quad (2)$$

This relationship is called *absolute* PPP since it relates the absolute level of prices across countries to the level of the exchange rate. *Relative* PPP (discussed below) relates inflation rates to the rate of depreciation.

Note that PPP implies that the exchange rate can be pinned down by the ratio of price levels. Since we already know that exchange rates are much more volatile than prices, this relationship cannot hold in the short run. It should, however, be considered as a force that affects the convergence of prices and the exchange rate towards their long run equilibrium.

Although intuitive and appealing, unfortunately neither LOP nor absolute PPP hold in reality. There are many reasons why this is the case; we list some of them below:

- **Trade costs:** Transporting goods across borders often involves various costs such as transportation cost, tariffs, costs due to transactions in foreign currencies, costs due to

information and legal barriers, etc. Such costs affect the price in the destination market, and derive a wedge between the price there and the price where the goods are actually produced.

- **Non-tradables (individual level):** Non-tradables are those goods and services whose trade cost is so high relative to their production cost that agents everywhere choose to produce them domestically rather than trade with other countries. Typically, many non-tradables are services, for example, haircuts, movie theater tickets, bus fares, and video rentals are all non-traded services. Other examples for non-tradables that are not services are land and houses. For non-tradables there is no reason for the LOP to hold. Furthermore, any *tradable* good has some non-tradable components such as distribution cost, rent, and electricity. Such costs may derive a wedge in the price of the same good in different locations.
- **Non-tradables (aggregate level):** As in the case of individual goods, aggregate price indices include non-tradable goods and therefore absolute PPP does not hold even if LOP holds for each individual tradable good.
- **Different consumption baskets:** Aggregate price indices such as CPI are based on the typical consumption basket in each country. If countries have different consumption baskets then absolute PPP does not hold even if LOP does.
- **Imperfect competition:** Producers with some monopolistic power may price discriminate across markets, as a result the LOP would not hold.

Evidently, absolute PPP does not hold in the data. So why study it? The reason is that absolute PPP is a useful benchmark, and the data do support, at least in the long run, a variant of this relationship called **relative PPP**.

2.3. Relative PPP

Relative PPP relates prices in two countries and the exchange rate through their rate of change. To derive relative PPP take absolute PPP in time t and divide it by the absolute PPP of time $t-1$ and get:

$$\frac{P_t}{P_{t-1}} = \frac{P_t^* E_t}{P_{t-1}^* E_{t-1}}$$

Next take logs from both sides to get:

$$\pi_t = \pi_t^* + \varepsilon_t \quad (3)$$

Where $\pi_t \equiv \log(P_t/P_{t-1})$, $\pi_t^* \equiv \log(P_t^*/P_{t-1}^*)$, and $\varepsilon_t \equiv \log(E_t/E_{t-1})$. π , π^* , and ε are domestic inflation rate, foreign inflation rate, and rate of depreciation, respectively.³ That is, relative PPP predicts that the depreciation rate is equal to the difference in inflations.

Relative PPP is a useful concept since it may hold even when the LOP and absolute PPP do not. As mentioned earlier the data provide evidence in favor of relative PPP in the long run, although the convergence toward this long run equilibrium is slow.

In our model of exchange rate determination we will use relative PPP as a long run relationship that will help us to pin down the effect of economic developments on exchange rate expectations. It should be noted that although relative PPP, as expressed in equation (3), relates inflation differentials to the depreciation rate; it can also be expressed in terms of absolute levels. Specifically, letting q denote an arbitrary positive constant, relative PPP can be expressed as:

$$qP_t = E_t \times P_t^* \quad (4)$$

Notice that for $q = 1$ equation (4) is simply absolute PPP. To show that equation (4) is equivalent to (3) we need to show that one implies the other. Clearly, following the same derivation as before (4) implies equation (3). We will now show that the opposite is also true. Equation (3) implies that:

$$\frac{P_t}{P_{t-1}} = \frac{P_t^* E_t}{P_{t-1}^* E_{t-1}} \quad \text{for all } t$$

Rearrange:

$$\frac{P_t^* E_t}{P_t} = \frac{P_{t-1}^* E_{t-1}}{P_{t-1}} \quad \text{for all } t$$

The ratio P^*E/P is therefore constant over time. Letting q denote this ratio, we get:

$$qP_t = E_t \times P_t^*$$

³ For any variable x : $\log(x_t/x_{t-1}) \approx (x_t - x_{t-1})/x_{t-1}$. This is a good approximation if x_t is not too far from x_{t-1} .

which is the same as equation (4). We have therefore shown that (3) and (4) are in fact the same statement. Hence, we can express relative PPP using either of these expressions.

3. Prices and Exchange Rates in the Long Run

3.1. The Monetary Approach

In this section we take a monetary approach to exchange rate determination as we make the connection between PPP and the money market. We will see how developments in the money market affect the exchange rate in the long run.

Recall that absolute PPP pins down the exchange by the price ratio in two countries:

$$E = P/P^*$$

From our discussion on the money market in handout 5, the price levels are given by:

$$P = M^s/L(R, Y) \quad \text{and} \quad P^* = M^{s^*}/L(R^*, Y^*)$$

Therefore, the long run exchange rate is:

$$E = \frac{M^s}{M^{s^*}} \cdot \frac{L(R^*, Y^*)}{L(R, Y)} \quad (5)$$

In this approach the exchange rate is completely determined by the demand and supply of money. As in any demand and supply system, the price of foreign currency in terms of domestic currency, i.e. the exchange rate, increases with its (relative) demand and falls with its (relative) supply. That is, as the demand for domestic currency increases the currency appreciates (E falls), and as domestic money supply increases the currency depreciates (E increases).

Factors that affect the demand for money also affect the exchange rate. An increase in domestic output, for example, increases money demand and therefore the currency appreciates. An increase in the domestic interest rate reduces money demand and therefore the currency depreciates. Notice that this result contradicts the prediction of the interest parity condition, where we have concluded that an increase in the domestic interest rate leads to an immediate appreciation.

It should be stressed that this is a long run analysis, and therefore the changes we have just discussed should be considered as permanent developments that affect the long

run position of the economy. The interest parity condition, on the other hand, was used in a short run analysis where we held prices fixed.

3.2. Inflation, Interest Parity and the Exchange Rate

Recall that in order to equate returns the interest parity condition suggests that the expected depreciation of a currency is equal to the interest rate differential:

$$\varepsilon^e = R - R^*$$

If relative PPP holds then market participants will also expect it to hold, and therefore:

$$\pi^e = \pi^{*e} + \varepsilon^e$$

Substitute for ε^e from the interest parity and we get:

$$R - R^* = \pi^e - \pi^{*e} \quad (6)$$

Equation (6) helps us understand why the domestic currency depreciates (in the long run) when the nominal interest rate, R , increases. Higher nominal interest rate implies higher inflation;⁴ which, in turn, reduces the purchasing power of the domestic currency. As a result, the currency loses value and depreciates.

This interaction between PPP and the interest parity condition highlights the role of our assumptions on the behavior of prices in determining the exchange rate. In the short run prices are fixed; therefore, higher domestic interest rate implies that saving in domestic currency is more attractive and hence the currency appreciates. In the long run prices adjust, and higher interest rate leads to higher inflation, and since the domestic currency loses purchasing power relative to its foreign counterpart, it depreciates.

3.3. The Fisher Effect

As we have just seen, in the long run, a higher interest rate leads to higher inflation. The main force behind this result is money neutrality, that is, in the long run nominal changes cannot have real effects. This result, i.e. higher nominal interest rate leads to higher inflation, is known as the *Fisher effect*.

⁴ In our model there is no uncertainty and since we assume rational expectations we do not distinguish between expected inflation and its actual level.

In this case inflation is higher in order to keep the **real interest rate** constant and to equate *real returns* across countries. To see that (6) equate real returns recall that the real interest rate, denoted by r , is the difference between the nominal interest rate and expected inflation, $R - \pi^e$. Therefore, after rearranging (6) we get:

$$r = r^* \quad (7)$$

3.4. The Price Level in the Long Run

In Handout 5 we made a case for long run money neutrality, and showed how the price level is pinned down using this assumption. However, the discussion there implicitly assumed that money does not grow over time and consequentially there was no inflation. After discussing relative PPP and the Fisher effect, we now have a richer set of tools to analyze how prices are determined in the long run. We therefore turn to generalize our results to the case where money is constantly growing.

Recall that we assume that money supply and output are exogenous;⁵ hence, after taking these factors as given, equilibrium in the money market gives us a relationship between the price level and nominal interest rate:

$$P = M^s / L(R, Y) \quad (8)$$

In the long run prices and money must grow at the same rate, otherwise real money balances will either shrink to zero (and the nominal interest rate explodes) or real money balances go to infinity (and the interest rate goes to zero). Hence, the growth rate of prices, i.e. inflation, is pinned down by the growth rate of money supply.

This, however, does NOT pin down the price *level* – to that end we use our insight from the Fisher effect. The force behind the Fisher effect is that in the long run the real interest rate is fixed and cannot be affected by nominal changes. Therefore, given expected inflation (which is the same as the growth rate of money), π^e , and the real interest rate, r , we can pin down the nominal interest rate by $R = \pi^e + r$.

⁵ So far we have assumed that output is exogenous in both the short run and the long run. In Handout 8 we will endogenize the *short run* behavior of output, while maintaining the assumption of long run exogeneity. We will assume that in the long run output is determined by the supply of production factors. Therefore, the discussion here about long run determination of prices will carry over to our general model where output is endogenized.

Given R (together with M^s and Y), we have pinned down the right hand side of (8) and therefore pinned down the price level as well.

4. Monetary Policy and the Exchange Rate in the Long Run

We are now ready to analyze the long run effect of changes in the monetary policy on the exchange rate. So far we have analyzed changes in the level of the money supply; however, monetary policy is often conducted by controlling the *growth rate* of money, not its absolute level. Therefore, in this section we analyze the effect of a permanent increase in the *growth rate* of money. Note that since this is a long run analysis we will assume flexible prices.

Assume that initially the economies are in their long run equilibrium, and consider a permanent unanticipated increase in the growth rate of the money supply from μ_0 to μ_1 . Panel A in Figure 1 illustrates this change. Notice that the money supply does NOT increase on impact, only its growth rate increases.

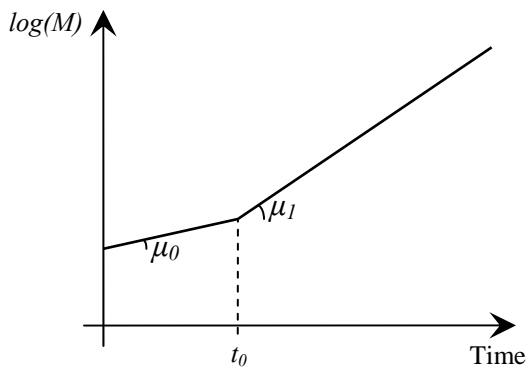
In the new long run equilibrium prices must grow at the same rate as money; therefore, the inflation rate increases as well from π_0 to π_1 , where $\pi_0 = \mu_0$ and $\pi_1 = \mu_1$. Panel C in Figure 1 illustrates this change by a higher slope in $\log(P)$ after the shock.

The new depreciation rate, ε , is pinned down by relative PPP, equation (3). Since there is no change in the foreign country the depreciation rate of the domestic currency increases one-to-one with the inflation rate. We will assume for simplicity that $\pi^* = 0$, hence the depreciation rate increases from ε_0 to ε_1 , where $\varepsilon_0 = \mu_0$ and $\varepsilon_1 = \mu_1$. Panel D in Figure 1 illustrates this change by a higher slope in $\log(E)$ after the shock.

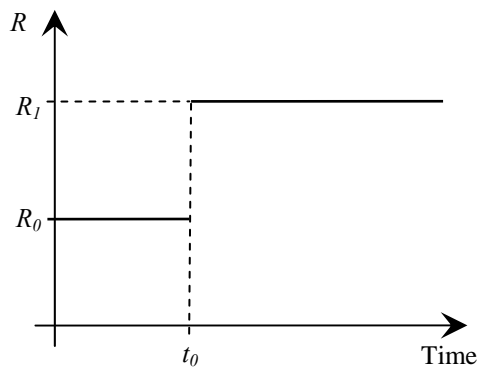
The change in the nominal interest, R , is pinned down by the interest parity condition. Again, since there is no change in the foreign country the domestic interest rate must increase one-to-one with the depreciation rate from R_0 to R_1 , where $R_1 = R_0 + \mu_1 - \mu_0$. Panel B in Figure 1 illustrates the increase in R .

Figure 1: A Permanent Increase in the Growth Rate of M^s
Time Path of Key Variables

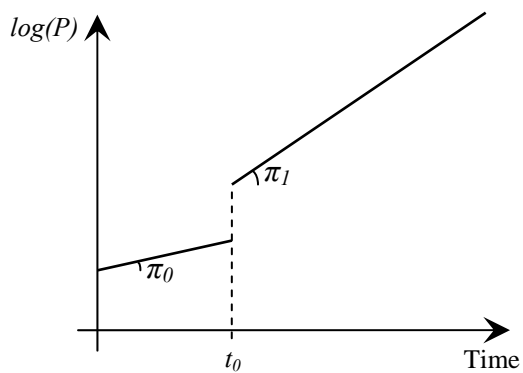
Panel A: Money



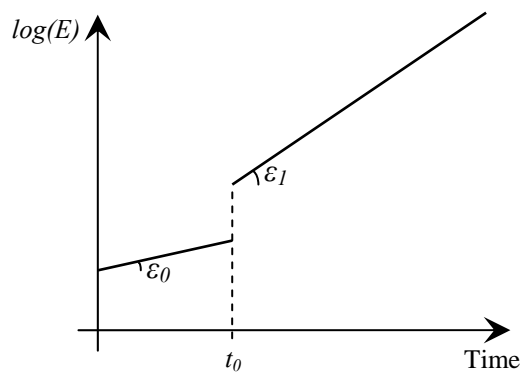
Panel B: Interest Rate



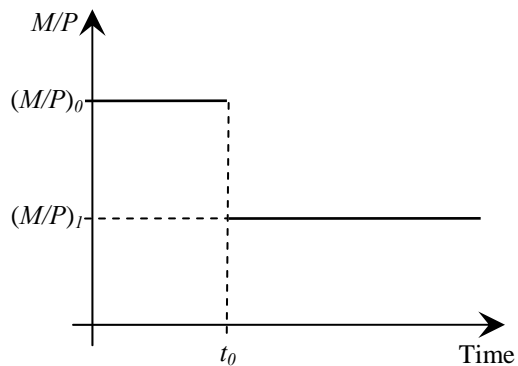
Panel C: Price level



Panel D: Exchange Rate



Panel E: Real Money Balances



Finally, notice that we still haven't pinned down the behavior of prices and the exchange rate on impact. In the money market the interest rate increases on impact while output is fixed. This, in turn, reduces *real* money demand. In order to maintain equilibrium in the money market, real money supply, M^s/P , must fall as well (the change in M/P is illustrated in Panel E). Recall that nominal money supply, M^s , does not change on impact; therefore, prices must increase in order to reduce real money supply and restore equilibrium. By relative PPP (expressed in levels), equation (4), the level of the exchange rate moves together with the price level. Therefore, on impact, the exchange rate increases as well. These changes are illustrated in panels C and D in Figure 1.

In contrast to our previous analysis of a discrete change in the level of M^s , here the exchange rate reacts immediately in anticipation of higher *future* depreciation rates. This creates momentarily excess supply of domestic currency, and hence the exchange rate depreciates on impact.

As in the case of exchange rate overshooting this result is important since it provides an explanation for the high volatility of exchange rates. In this case the exchange rate jumps immediately even though the shock itself, an increase in the growth rate of money, has no immediate effect on the level of the money supply. The discrete increase in the exchange rate is generated solely by expectations for a higher depreciation rate in the future.