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A non-cooperative interpretation of bargaining sets

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Abstract. This paper provides a non-cooperative interpretation for bargaining sets concepts in economic environments. We investigate the implementability of the Aumann-Maschler and Mas-Colell bargaining sets, and provide mechanisms whose subgame perfect equilibrium outcomes realize these sets. These mechanisms, in contrast to general mechanisms suggested in the implementation literature, have a natural structure closely related to that of the rationale underlying the bargaining sets. Furthermore, the strategy sets consist mainly of allocations and coalitions (thus avoiding any reference to preference parameters) and are finite dimensional.

JEL classification: C72, C78

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1 Introduction

In a pure exchange economy, individuals start out with a given configuration of initial endowments, and may interact among themselves to reach a final allocation of resources. Cooperative solution concepts such as bargaining sets, the Shapley value and the core try to narrow down the set of outcomes consistent with such an interaction.

A common criticism of the cooperative approach is the lack of a non- cooperative foundation that would justify the described behavior patterns. This criticism can be addressed by considering non- cooperative environments where the re-

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sulting equilibria yield the cooperative outcomes. Recent works by Bergin and Duggan (1995), Lagunoff (1994), Moldovanu and Winter (1994) Perry and Reny (1994), Serrano (1995) and Serrano and Vohra (1997) address this issue with regard to the core, Gul (1989) considers the Shapley value; Hart and Mas-Colell (1996) consider a model of non-cooperative bargaining among n participants and extend the Shapley value to non- transferable-utility games; Serrano (1993) considers the implementation of the nucleolus.

A cooperative solution concept that has been rarely treated from the implementation point of view is the bargaining set. Due to the recursive nature of its definition it has been difficult to suggest natural ways of implementing it. Perez-Castrillo and Wettstein (1998) have provided one such implementation treatment for super-additive environments by introducing a set of auxiliary individuals competing over the agents. We address this problem for pure exchange economies and provide a non-cooperative game form whose equilibria yield the Mas-Colell bargaining set. A slightly modified game form is shown to yield the Aumann-Maschler bargaining set.

Because we use the subgame perfect equilibrium our findings should be contrasted with the very general results obtained in Moore and Repullo (1988) and Abreu and Sen (1990). They provided sufficient conditions for the realization of a given behavior standard (Social Choice Correspondence) as the subgame perfect equilibria of a game form. Due, however, to the very general nature of their results and the environments they considered, the suggested game forms were quite involved. Furthermore, given the generality of the above-described results the structure of the game form was very much detached from any rationale underlying the Social Choice Correspondence under question.

In our much more specialized setting we construct game forms that do not require the transmission of any personal preference parameters. Thus, our game forms use a much smaller message space. The game forms themselves are very natural and closely related to the reasoning underlying the various bargaining and cooperation notions.

It should be noted that one shortcoming of the mechanisms constructed is their non-feasibility for some out-of-equilibrium strategy choices. This results in part from our attempt to replicate the logic inherent to the definitions of the various bargaining sets. Our game forms are also open to the objections raised by Jackson (1992) with respect to the use of non-bounded mechanisms. We view the constructs in this paper as a first step towards providing a non- cooperative foundation for cooperative solution concepts given by bargaining sets.

In Sect. 2 we present the basic model. Section 3 discusses the Mas-Colell bargaining set and presents a game form that realizes it in subgame perfect equilibria. In Sect. 4 we show the modifications necessary to realize the Aumann-Maschler bargaining set. In Sect. 5 we conclude with suggestions for future research.

2 The model: Notation and definitions

We consider a pure exchange economy E with n individuals and k commodities. Each individual is characterized by its initial endowment w_i (an element of R^k_{++}) and utility function u_i which is assumed to be continuous and strictly increasing.

An allocation x is an n-tuple of vectors in R^k_+, $(x_1, \ldots, x_n)$ such that:

$$\sum_{i=1}^{n} x_i \le \sum_{i=1}^{n} w_i \ .$$

An objection to x is a pair (S^0, x^0) where $S^0 \subseteq N$ and $x^0 \in R^{nk}_+$, such that:

(i) $x_i^0 \succeq_i x_i$ for all i in S^0 and $x_j^0 \succ_j xj$ for some j in S^0.

(ii) $\sum_{i \in S^0} x_i^0 \le \sum_{i \in S^0} w_i$.

The core of E is defined as the set of allocations against which there exist no objections. These allocations certainly enjoy several desirable properties. They are Pareto Optimal and individually rational (each individual (weakly) prefers the core allocation to the initial allocation of endowments). If indeed reached, an allocation in the core seems to be fairly immune to recontracting among any subset of agents.

A weakness in the definition of the core is that some objections may not be reasonable. An objection may set in motion a chain of events not beneficial to all members of the objecting coalition. This avenue of thought has been pursued by Aumann and Maschler (1964), Dutta et al. (1989), Mas-Colell (1988), Maschler (1976) and Peleg (1963) and resulted in several related bargaining sets. In addition to objections, the notion of counterobjections and justified objections forms the basis for the definition of bargaining sets.

The Aumann-Maschler bargaining set is defined as follows:

An objection of i against j in x is a pair (S^0, x^0) where $S^0 \subset N$, $i \in S^0$, $j \notin S^0$ and $x^0 \in R^{nk}_+$, such that:

(i) $x_k^0 \succeq_k x_k$ for all k in S^0 and $x_i^0 \succ_i x_i$.

(ii) $\sum_{k \in S^0} x_k^0 \le \sum_{k \in S^0} w_k$.

Given an objection (S^0, x^0) of i against j a counterobjection of j against i is a pair (S^1, x^1) where $S^1 \subset N$, $i \notin S^1$, $j \in S^1$ and $x^1 \in R^{nk}_+$ such that:

(i) $x_k^1 \succeq_k x_k^0$ for all k in $S^0 \cap S^1$ and
$x_k^1 \succeq_k x_k$ for all k in $S^1 \setminus S^0$

(ii) $\sum_{k \in S^1} x_k^1 \le \sum_{k \in S^1} w_k$.

An objection is justified in the Aumann-Maschler setting if there is no counterobjection. The Aumann-Maschler bargaining set is defined as the set of allocations against which there is no justified objection.

This definition does not include efficiency and individual rationality present in the original definition of the Aumann-Maschler bargaining set. We chose this modified definition in order to present a unified treatment of the implementation of the Aumann-Maschler and the Mas-Colell bargaining sets.

Mas-Colell's bargaining set is defined slightly differently. Objections and counterobjections are defined without any reference to particular agents i and j. Counterobjections are required to strictly improve at least one participating individual's position.

An objection (S^0, x^0) to x is defined as in the core setting. A counterobjection to (S^0, x^0) is a pair (S^1, x^1) where $S^1 \subset N$ and $x^1 \in R_+^{nk}$ such that:

(i) $x_i^1 \succeq_i x_i^0$ for all i in $S^0 \cap S^1$ and
$x_i^1 \succeq_i x_i$ for all i in $S^1 \setminus S^0$
with at least one of the above preferences being strict.

(ii) $\sum_{i \in S^1} x_i^1 \leq \sum_{i \in S^1} w_i$.

An objection is justified if there is no Mas-Colell counterobjection. The Mas-Colell bargaining set is defined as the set of allocations against which there is no Mas-Colell justified objection.

Like the core, these bargaining sets enjoy a certain degree of stability against recontracting. They contain the core whenever it is non-empty and offer "reasonable" solutions even in cases where the core might be empty. We note there is an inherent non-feasibility in the way these sets are defined. The counterobjection is required to improve upon an allocation, which may not be feasible. When examining the definition of counterobjections we see they improve upon an imaginary allocation yielding the x^0 suggestion to one subset of the agents and the x suggestion to another subset. The concatenation of these two suggestions may violate the aggregate resource constraint the economy is faced with. This is a well-known characteristic of the bargaining set literature.

What is lacking in the above definitions is the process by which individuals arrive at a point in one of these sets. Individuals usually have only their best interests in mind, and cannot be relied upon to act cooperatively. We look for a non-cooperative mode of behavior that would realize those sets or, in more technical terms, we would like to implement these sets. Returning to Hurwicz's (1960) seminal contribution, we would like to design a mechanism whose equilibria yield these sets.

These sets can be viewed as Social Choice Correspondences (SCC's). An SCC associates a set of allocations with each economy. We look for a game form whose equilibria set for any economy would coincide with the set prescribed for it by the SCC.

We specify an extensive form game tree for the game form and use the subgame perfect equilibrium concept. This construct enables us to build game trees closely related to the actual bargaining that might take place in the economy. This naturally leads to game trees which may yield non-feasible outcomes off the equilibrium path mimicking the non-feasibility which appears in the definition of the bargaining sets.

There is by now a wealth of solution concepts that the implementation literature has dealt with – starting with Nash in Maskin (1977) and proceeding to subgame perfect equilibria in Moore and Repullo (1988) and Abreu and Sen (1990), undominated Nash equilibria in Palfrey and Srivastava (1991), virtual implementation in Matsushima (1988), Abreu and Sen (1991), and finally virtual implementation in undominated strategies in Abreu and Matsushima (1992). It remains to be seen whether the sets we consider could be realized through other solution concepts as well, and whether utilizing these alternative solution concepts yields a more reasonable game form.

3 Implementing the Mas-Colell bargaining set

In the definition of the Mas-Colell bargaining set the two coalitions S^0 and S^1 can stand in any relation whatsoever. For instance, it might be the case that $S^0 \subset S^1$. This phenomenon might be counter-intuitive. We show that in the economic environments we consider, it is always possible (given an x in the Mas-Colell bargaining set) to find counterobjections not containing all the members of the original objection. This would prove useful in the construction of the game where certain fines would be levied against some of the individuals who object and do not take part in the counterobjection. Prior to proving this result we introduce the notion of a Pareto Optimal objection for a coalition S against an allocation x.

Given an allocation x and a coalition $S \subset N$, an objection (S^*, x^*) to x with $S \subset S^*$ is Pareto Optimal for S if there does not exist any other objection (S', x') with $S \subset S'$ where all the members of S are not worse off than in (S^*, x^*) with one of them being strictly better off.

The existence of Pareto Optimal objections (provided there are any objections at all) is obvious in economic environments, since they are solutions to well-defined problems of the form:

$$\begin{aligned} \max_{i \in S,\, \hat{S} \supset S} \quad & u_i(\hat{x}_i) \\ & u_j(\hat{x}_j) \geq u_j(x_j) \quad j \in \hat{S} \\ & \sum_{\hat{S}} \hat{x}_j \leq \sum_{\hat{S}} w_j \ . \end{aligned}$$

Now we can prove:

Lemma: *If* $x = (x_1, x_2, \ldots, x_n)$ *is in the Mas-Colell bargaining set then given any objection* (S^0, x^0) *there exists a counterobjection* (S^1, x^1) *where* $S^0 \not\subset S^1$.

Proof: Since x was in the Mas-Colell bargaining set there must exist a counterobjection to (S^0, x^0). If (S^0, x^0) was a Pareto Optimal objection for S^0 we have finished since $S^0 \subset S^1$ contradicts the Pareto Optimality of (S^0, x^0).

If (S^0, x^0) is not a Pareto Optimal objection, then there exists a Pareto Optimal objection for S^0 against x, (S', y') with $S^0 \subset S'$ which dominates it.

So $y'_i \succeq_i x_i^0$ i in S^0 (with at least one of the preferences being strict)
$y'_i \geq_i x_i$ in $S' \setminus S^0$.

This objection against x must also have counterobjections. Let (S^1, x^1) be such a counterobjection.

Therefore $x_i^1 \succeq_i y'_i$ i in $S^1 \cap S'$
$x_i^1 \succeq_i x_i$ i in $S^1 \setminus S'$

with at least one of the above preferences being strict (hence by continuity of preferences all the above weakly preferred than relations can be taken as strictly preferred).

Since S' was Pareto Optimal, S^0 cannot be contained in S^1. If S^0 is contained in S^1 then all members of S^0 can be made better off compared to their S' allocation, in contradiction to S' being Pareto Optimal for S^0. Hence, as before, $S^0 \not\subset S^1$. We now want to show that (S^1, x^1) is a counterobjection against (S^0, x^0).

$S^1 \cap S^0 \subset S^1 \cap S'$ so on this set $x_i^1 \succ_i x_i^0$.

$S^1 \setminus S^0 = (S^1 \setminus S') \cup (S^1 \cap (S' \setminus S^0))$ so on $S^1 \setminus S'$ we have $x_i^1 \succ_i x_i$ and on any point in S^1 and S' but not in S^0 we have $x_i^1 \succ_i y'_i \succeq_i x_i$.

Before formally specifying the multi stage game whose subgame perfect equilibria would coincide with the Mas-Colell bargaining set, we describe its main features as follows. It would be a 5-stage mechanism. At Stage 1 each individual sends a feasible allocation. These allocations are averaged to yield a final allocation with which Stage 1 ends. The particular averaging performed has the property that in the case where all individuals submit the same proposal, no single individual can change the resulting allocation by a unilateral deviation.

At Stage 2, individuals can either agree or send in a triplet (S^{0i}, x^{0i}, n^i) : S^{0i} is a subset of N, x^{0i} is an allocation and n^i is a positive number. The triplet can be interpreted to mean that coalition S^{0i} is objecting via the allocation x^{0i}. If all individuals agree, the game ends and the resulting allocation is the one derived in Stage 1. If more than one triplet is sent in, the game ends with the individual announcing the largest number receiving the aggregate endowment, whereas the rest receive the zero bundle. If there is one such triplet the game moves to Stage 3.

At Stage 3 the members of S^{0i} are approached sequentially. The first individual that does not agree ends the game and the resulting allocation is once more that of Stage 1. If they all agree, the game proceeds to Stage 4.

At this stage individuals may send in pairs (S^{1i}, x^{1i}) where $S^{0i} \not\subset S^{1i}$. If no pairs are sent, the game ends. The members of S^{0i} receive the bundles prescribed by x^{0i}, and the others receive the bundles prescribed by x from Stage 1.[1] If there

[1] This may violate aggregate feasibility, but as discussed before is inherent in the logic underlying the bargaining sets.

are objections, the game moves to Stage 5, and the prevailing objection is the one associated with the largest number from Stage 2 (ties are broken in favor of the higher indexed individual).

At Stage 5 the members of S^{1i} are approached sequentially. If a disagreement is encountered, the game ends with the allocation of x^{0i} to members of S^{0i}, x^i to the rest and a further fine imposed on the individual whose objection was selected in Stage 4. In the case where all members of S^{1i} agree, the game ends with members of S^{1i} receiving their bundles according to x^{1i}. Other individuals receive the bundles prescribed by x with a further fine for the individuals that belonged to the first objection but not to the counterobjection.

Formally, the game in extensive form consists of a 5-stage game (some of the stages may consist of several substages).

At Stage 1, each individual i sends in a feasible allocation x^i. The mechanism proceeds to construct a weighted average of the x^i's as follows:

$$\alpha^i = \sum_{j,j' \neq i} \left| x^j - x^{j'} \right|$$

$$\alpha = \sum_{i=1}^{n} \alpha^i$$

$$\beta^i = \begin{cases} \dfrac{\alpha^i}{\alpha} & \text{if} \quad \alpha > 0 \\ \dfrac{1}{n} & \text{if} \quad \alpha = 0 \end{cases}$$

$$\bar{x} = \sum_{i=1}^{n} \beta^i x^i \ .$$

At Stage 2, each individual i sends in the message YES or a triplet (S^{0i}, x^{0i}, n^i); $S^{0i} \subset N$, x^{0i} is a feasible allocation and n^i is a positive number. If all individuals send in YES, the game ends with the resulting allocation $\bar{x}$ across the individuals. If two or more individuals do not send in YES, the game ends. The individual announcing the largest number receives the aggregate endowment, and the rest receive the zero bundle. If exactly one individual does not send in YES the game proceeds to Stage 3.

To describe Stage 3 let t denote the individual who sent in an "objection". The individuals comprising S^{0t} are approached sequentially and have to send in an element of $\{$YES,NO$\}$. The first NO that is encountered ends the game with $\bar{x}$ as the final allocation. If all members of S^{0t} send in YES, the game moves to Stage 4.

At Stage 4 individuals send in either the message YES or a pair (S^{1i}, x^{1i}); where $S^{1i} \subset N$, $S^{0t} \not\subset S^{1i}$ and x^{1i} is an allocation. If all individuals send in YES, the game ends, and the outcome allocation is described as follows: First the bundles prescribed by x^{0t} are assigned to members of S^{0t} and the $\bar{x}$ bundles are assigned to individuals outside of S^{0t}. If not all individuals send in YES the game moves to Stage 5.

To describe Stage 5, consider the set of individuals that sent in counterobjections. Let t' denote the highest indexed individual among them. t' plays the same role as t in Stage 2. The individuals comprising $S^{1t'}$ are approached sequentially and have to send in an element of {YES,NO}.The first NO that is encountered ends the game with the final allocation constructed in Stage 4 previously, except for individual t' whose bundle is multiplied by a half. If all members of $S^{1t'}$ send in YES, the bundles prescribed by $x^{1t'}$ are assigned to members of $S^{1t'}$ and the x bundles are assigned to individuals outside of $S^{1t'}$. The bundles of individuals that belonged to S^{0t} but not to $S^{1t'}$ are multiplied by a half, and the rest are left intact.

Prior to stating our theorem we have to add one technical assumption which will remain in force throughout the rest of the paper and rule out boundary points in the correspondences we want to implement. Namely we assume that an allocation in the bargaining sets never assigns the zero bundle to one or more of the individuals. This is necessary since the fines that the mechanism imposes would not affect individuals assigned the zero bundle.

Theorem 1. *The subgame perfect equilibrium outcomes of the extensive form game described above coincide with the Mas-Colell bargaining set.*

Proof: First we show that any element of the Mas-Colell bargaining set can be realized as a subgame perfect equilibrium (SPE). Let x be in the Mas-Colell bargaining set. The following n-tuple of strategies yields x and constitutes an SPE. At Stage 1, each individual sends in the message x. At Stage 2 all individuals send in YES (there is no need to specify their response in case something other than x was reached in Stage 1). In Stage 3, if ever reached, individuals agree to any objection, provided they are not worse off, and furthermore, there does not exist any counterobjection precluding them. (Note that by the lemma, if x is in the Mas-Colell bargaining set, there would always be at least one individual for which a precluding counterobjection exists.) If the objection does not satisfy these criteria, the individual sends in the message NO. At Stage 4, any individual that can find a valid counterobjection, including the individual, sends in the best such counterobjection. At Stage 5 individuals that are in the counterobjection, as well as in the objection, agree if the counterobjection makes them better off than in the objection. Individuals who are in the counterobjection but were not in the original objection agree if the counterobjection makes them better off than the allocation Stage 1 ended with.

To show that this is an SPE one has to check only deviations in Stage 2 and onwards, since any change in Stage 1's strategy would not change the final outcome. Moving to Stage 2 we see that since x was in the Mas-Colell bargaining set, no individual can come up with an objection yielding him a strictly preferred bundle that would be approved in Stage 3, so there is no profitable deviation in Stage 2. In Stage 3, an individual might consider agreeing to an objection that, if adopted, would make the individual worse off. However, that would mean that the individual anticipates that this objection would be replaced by a counterobjection later on. This can never be the case since a counterobjection that is replaced cannot be accepted in the first place. Any member of the objecting coalition that does not take part in the counterobjection is made worse off by the acceptance of the objection and hence he would refuse it.

In Stage 4 an individual cannot get a better outcome by not sending in the counterobjection. Sending in a non-valid counterobjection would not alter the outcome, so there is no profitable deviation in Stage 4.

In Stage 5, an individual can never gain by deviating, since the individual is choosing the best course of action, given that Stage 5 has been reached.

Now we will show that any SPE of this game form yields an element of the Mas-Colell bargaining set. First note that an SPE can never involve any objections. If there is one objection, it always pays for an individual, other than the objecting one, to object as well and send in a large enough number. It is also clear that if there is more than one objection, there would be no equilibrium choice for the numbers individuals send in at Stage 1. So the outcome in any SPE must coincide with the x of Stage 1.

If the x reached in Stage 1 were not an element of the Mas-Colell bargaining set, there would have been a justified objection (S^0, x^0) and an individual j in it for which $x_j^0 \succ_j x_j$. Individual j will obtain a strictly preferred outcome by sending in this objection. The members of S^0 will agree to the objection, since it does not make them worse off. This contradicts the fact that we started from an SPE.

Thus, in any SPE the x arrived at must be an element of the Mas- Colell bargaining set.

4 Implementing the Aumann-Maschler bargaining set

To implement the Aumann-Maschler bargaining set we consider a mechanism similar to the one constructed previously, the main difference being that the mechanism will treat objections and counterobjections in terms of specific individuals i and j.

Briefly, the mechanism consists of five stages. Stage 1 is the same as Stage 1 in the mechanism implementing the Mas-Colell bargaining set. In Stage 2, each individual i can send in a YES or an objection against an individual j ($j \neq i$), together with a number affecting the fine imposed on individual j in case j does not counter object. Stage 3 is also the same as in the mechanism for the Mas-Colell bargaining set. In Stage 4 individual j may send in a YES or counter object. A YES ends the game with allocations determined basically as in the previous game, that is by the objection and the allocation from Stage 1, the only change being a fine imposed on individual j. A counterobjection moves the game to Stage 5. As before, if the counterobjection is accepted, the game ends with the allocation determined by the counterobjection and the allocation from Stage 1 and individual i is fined. If the counterobjection is rejected, the game ends with the same allocation as in Stage 4.

To describe the game formally note that Stage 1 is identical to Stage 1 in the mechanism implementing the Mas-Colell bargaining set. In Stage 2, each individual i sends in YES or a four-tuple $(S^{0i}, x^{0i}, n^i, j^i)$: $S^{0i} \subset N$, $i \in S^{0i}$, $j^i \notin S^{0i}$ and x^{0i} is a feasible allocation. If all individuals send in YES, the game ends with $\bar{x}$ being the outcome allocation. If two or more individuals do not send in YES, the game ends with the individual announcing the largest n^i receiving everything. If exactly one individual (denoted by t) does not send in YES, the game proceeds to Stage 3, which is identical to Stage 3 in the game implementing the Mas-Colell bargaining set. In Stage 4, individual j (the individual appearing in t's objection from Stage 2) sends in YES or a pair (S^{1j}, x^{1j}); $S^{1j} \subset N$, $j \in S^{1j}$, $t \notin S^{1j}$ and x^{1j} is a feasible allocation. If j sends in YES, the game ends with following outcome allocation: The x^{0t} bundles are given to the members of S^{0t} and the $\bar{x}$ bundles are given to the rest except for individual j. Individual j receives the $\bar{x}$ bundle scaled down by $\frac{n^t}{1+n^t}$. If j sends in a "counterobjection", the game moves to Stage 5.

In Stage 5, the individuals comprising S^{1j} are approached sequentially, and have to send in an element of $\{$YES,NO$\}$. The first NO that is encountered ends the game with the outcome constructed almost identically to Stage 4, the only difference being that j's bundle is scaled down by $\frac{n^t}{2(1+n^t)}$. If all members of S^{1j} send in YES, the game ends with members of S^{1j} receiving the bundles prescribed by x^{1j}, the $\bar{x}$ bundles are given to the rest except for individual t and individual t receives the $\bar{x}$ bundle scaled down by half.

Theorem 2. *The subgame perfect equilibrium outcomes of the extensive form game described above coincide with the Aumann-Maschler bargaining set.*

Proof: The proof proceeds along similarly to the proof of Theorem 1. Let x be in the Aumann-Maschler bargaining set. At Stage 1 each individual sends in the allocation x. At Stage 2, all individuals send in YES. In Stage 3, if ever reached, individuals agree to any objection of i against j, provided they are not worse off, and there does not exist a counterobjection by j excluding them. In Stage 4, if individual j (the one against which there was an objection) can find a counterobjection, j sends in the best such counterobjection. In Stage 5, the individual raising the counterobjection agrees, provided it is preferred to the scaled down bundle received otherwise. The other individuals behave as in the previous mechanism.

To show that this is an SPE we can, as before, start by checking deviations from Stage 2 onwards. Since x was in the Aumann-Maschler bargaining set, no individual (regardless of how large n^i is chosen) can suggest an objection that would be approved of in Stage 3. The j individual would always be able to find a valid counterobjection, which would be strictly preferred to receiving the scaled down bundle from $\bar{x}$. By continuity of preferences, the j individual would thus be able to suggest a counterobjection that would make everyone in it, other than j, better off than in $\bar{x}$, and would still leave j better off than if j were to receive the scaled down bundle. Arguments similar to those employed in Theorem 1 show that there are no profitable deviations in Stages 3, 4 and 5 as well.

Before proving that any SPE outcome is an element of the Aumann-Maschler bargaining set we make the following observation. Let $x = (x_1, \ldots, x_n)$ be an allocation which is not in the Aumann-Maschler bargaining set. Then there must exist an objection of an individual i against an individual j for which any counterobjection that leaves everyone but j at least as well off (see the definition of a counterobjection in the Aumann-Maschler set up), yields j a utility level less than $u_j(x_j) - \varepsilon$.

As before, it is obvious that no SPE can involve objections. Next we show that the allocation reached in Stage 1 must be an element of the Aumann-Maschler bargaining set. If the x reached in Stage 1 is not in the bargaining set, there must, by the observation just made, exist an objection of i against j, which together with a large enough number announced by i (to make the scaling down of j nearly insignificant), would not be opposed by j. Individual i by sending in this objection will end up with a preferred outcome in contradiction to the fact his previous strategy was part of an SPE.

5 Conclusions

We have demonstrated that certain cooperative solution concepts can be given a non-cooperative foundation by providing game forms whose subgame perfect equilibrium outcomes coincide with the solutions in question. The game forms we have constructed are closely related to the rationales underlying the various solutions, and possess intuitively appealing strategy spaces. Individuals basically send in allocations and coalitions. This is in sharp contrast to the well-known general constructions in implementation theorems requiring the transmission of utility profiles.

These findings provide a positive foundation for the study of these sets, since it is demonstrated an outside designer who has no information on the preferences of the individuals comprising the society can realize them.

There are several more cooperative solution concepts and it remains to be seen whether game forms of the type constructed here would be useful for implementing them. It is natural to ask whether by the addition of more stages the consistent bargaining set of Dutta et al. (1989) could be implemented.

It is also possible to provide alternate definitions of the bargaining sets in terms of the reference point for the counterobjection. In the bargaining sets in this paper, an individual who is in the counterobjection, but was not in the original objection, compares the bundle offered by the counterobjection to the x (original suggestion) bundle. This creates the feasibility issues mentioned before. A modified bargaining set, where the comparison is with respect to the initial endowments, could be implemented by a mechanism that would be feasible also outside of equilibrium. The properties of these modified bargaining sets could be a topic for further research. Vind (1992) raises closely related issues.

A further direction of research would be to look for game forms implementing these cooperative solutions via alternate solution concepts. The derived mechanisms should then be critically compared with the ones suggested in this paper.

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