

# **CHOOSING WISELY: A MULTI-BIDDING APPROACH**

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## **Abstract**

We analyze situations in which a group of agents (and possibly a designer) have to reach a decision that will affect all the agents. Examples of such scenarios include the selection of a candidate, the location of a nuclear reactor or the siting of a major sports event. To address the problem of reaching a decision, we propose a one-stage multi-bidding mechanism in which agents submit bids to determine the winning project. All Nash equilibria of this mechanism are efficient and immune to any coalitional deviations. Moreover, the payoffs attained in equilibrium by the agents satisfy intuitively appealing lower bounds. (JEL D78, D62)

Reaching decisions about the location of noxious facilities, such as dump-sites, environmentally hazardous plants, nuclear power generators and the like, is a highly contentious issue. For instance, in February 2000, the US Senate decided that nationwide nuclear waste would be shipped to the Yucca mountain site in Nevada (conditional to it being approved as a high-level nuclear waste repository). Despite the attractive compensation package, the State of Nevada voiced vehement opposition. President Clinton vetoed the bill, and in May 2000 the Senate failed to overturn it. As such, the major problem facing the US nuclear industry on where to site a high-level radioactive waste repository remains virtually unresolved.

Neither are decisions about the location of desirable events easily solvable. Every two years, the International Olympic Committee selects hosts for the summer and winter games, a procedure that is constantly under review and revision in order to bring about the “best possible” outcome. Until about a year ago, the decision was made by a vote based on bids and plans submitted by candidate cities. However, the recent controversy surrounding the choice of Salt Lake City to host the 2002 winter games culminated in the design of a new procedure for choosing the host of the 2006 winter games. This procedure called for the selection of two finalist cities (in this case, Turin and Sion), immediately after which the host was selected by secret ballot (in this case, Turin).

Another similar issue is currently the subject of heated debate in Spain. The Spanish government, both on the national and local level, has recently been discussing various possible projects for transferring water from rivers in the north of Spain to the arid southern regions. The parties involved have conflicting interests with regard to several aspects of the projects proposed. Particularly, the water-rich regions are hesitant to give

up parts of their water resources. The parties involved have still not agreed upon the choice of a transport mode or the size of the transfer payments that would compensate the localities that are relinquishing part of the water supply.

These and several other cases belong to a class of problems in which a group of agents has to choose one out of several projects. A project is efficient if it maximizes the aggregate welfare of the group members. Reaching an efficient outcome is trivial when the designer has all the relevant information. However, it is often the case that the parties concerned possess much more information than the designer. Hence, the designer faces a non-trivial problem if she wishes to optimally choose one of the projects. Moreover, from a normative point of view, even if the designer has all the information, it is easier to justify the use of a fixed procedure to reach a decision than to modify the procedure for various cases in light of information disclosed or already available to the designer.

Several well-known methods are available for the problem of choosing one out of a set of alternatives. One class of methods consists of various voting schemes, such as majority voting or the Borda rule. The outcomes generated by these methods need not be efficient due to strategic behavior on part of the agents. Another class of methods consists of Vickrey-Clark-Groves mechanisms. For this class, “truth-telling” is a dominant strategy that always results in the choice of an efficient alternative. However, these mechanisms are not budget-balanced and the payments collected from the agents often generate a surplus that is a loss from the agents’ point of view.<sup>1</sup>

We address the problem of reaching an efficient decision by suggesting a simple mechanism that is budget-balanced and focusing on its Nash equilibria.<sup>2</sup> We construct a *multi-bidding mechanism* in which agents place bids in order to determine which project

will be chosen. The equilibrium outcomes of the mechanism not only determine the project, but also generate a system of transfer payments that serve, in part, to compensate those agents who are not pleased with the chosen project.

The one-stage mechanism proceeds as follows. Each agent submits a vector of bids, the  $q^{\text{th}}$  component of which can be interpreted as the amount of money an agent is willing to pay if project  $q$  is chosen. The only restriction imposed on the bids submitted by an agent is that they have to sum up to zero. This introduces a measure of relative worth whereby more desirable projects receive a larger bid. The agent also selects one of the projects. We define the “aggregate bid” for a project as the sum of bids made for this project. The project with the highest aggregate bid is chosen. In case there is more than one such project, the winning project is randomly chosen from those with the highest aggregate bid that have also been selected by at least one agent. If there is no such project the winning project is randomly chosen from those with the highest aggregate bid. Once the project has been selected the agents pay the promised bid corresponding to this project, it is carried out, and any surplus is shared among the agents.

The multi-bidding mechanism is simple and straightforward. Moreover, it satisfies the appealing property that an agent is at least as well off when playing the mechanism as he is when staying out of the process (an agent can always choose a bids vector equal to the zero vector). We show that all Nash equilibrium outcomes of this mechanism are efficient. That is, the project chosen in equilibrium is efficient. Furthermore, we provide a detailed analysis of the structure of the equilibria outcomes and their ranking by the participating agents. In particular, we show that, at equilibrium, each agent’s payoff is at

least the expected payoff he would obtain in a situation where all the projects have the same probability of being developed.

We also show that the Nash equilibria of the multi-bidding mechanism are strong Nash equilibria. This makes the mechanism even more attractive since it will achieve efficient outcomes also in environments where agents might collude and coordinate their bids, which may often be the case in real-world situations.

The mechanism we propose is partly based on Lester E. Dubins' work (1977). We adopt the same environment that Dubins used to analyze the properties of decision devices and we construct a bidding mechanism that shares some common characteristics with his. His analysis mainly focused on the outcomes generated by the choice of max-min strategies, leaving open the issues related to agents' strategic behavior.

William Samuelson (1980) studied a class of procedures suggested by H. Steinhaus (1949) designed to divide indivisible objects. However, the analysis concentrated only on normative issues and ignored problems caused by strategic behavior.

Hervé Moulin (1984) proposed a two-stage game in which each agent first bids for the right to choose a project, states his utility function, and chooses a number. The agent with the highest bid (where ties are broken by the numbers chosen) selects the project to be carried out in the second stage. Then transfer payments, depending on the project chosen, are made. This mechanism implements in sub-game perfect equilibrium the equal sharing of the surplus above the "average" utility level. In a more specific environment, Matthew Jackson and Hervé Moulin (1992) considered a set of agents who decide whether to carry out a particular public project that yields a benefit to each agent. At every undominated Nash equilibrium of the mechanism they design, the public project is

undertaken exactly when its total benefit outweighs its cost. This mechanism also allows the designer to realize a variety of cost distribution rules.<sup>3</sup> Parimal Kanti Bag (1997) adapts the previous mechanism to a framework with divisible public goods when agents' benefits from public good consumption are linear.

Our mechanism would also realize efficient outcomes in Nash equilibria for discrete economic environments with public goods and externalities, continuous versions of which were considered by Hal R. Varian (1994a,b). Varian (1994a) analyzed the equilibrium outcomes of several games entailing private contributions to a public good. In one of the games he had the agents bid for the right to move first. This is a special case of our general setup where there are two agents and the two alternatives are which one of them will move first. This game did not realize Pareto efficient outcomes when taking into account all possible levels of public good provision. Another game, where agents "subsidized" the contributions made by other agents did realize Pareto efficient outcomes in subgame perfect equilibria. In another paper, Varian (1994b) considered complete information economic environments with externalities, and constructed two-stage mechanisms that realize efficient allocations in subgame perfect equilibria.

Preston R. McAfee (1992) considered the problem of dissolving a partnership, and analyzed the equilibrium properties of four simple mechanisms in environments with risk-averse agents and asymmetric information. He showed that bidding mechanisms similar to first-price and second-price sealed bid auctions attain ex post efficient outcomes in suitably restricted environments. These bidding mechanisms bear some similarities to our multi-bidding mechanism for two agents. The mechanism constructed

in our paper realizes efficient outcomes for complete information equivalents of the environments in McAfee.

An interesting subclass of the environments covered by our analysis is one in which a set of agents has to choose who from among them will develop a project. In this environment, the set of projects corresponds to the set of agents. Several papers have addressed strategic considerations for this class of problems including Howard Kunreuther and Paul R. Kleindorfer (1986), Rafael Rob (1989), Jacob Glazer and Ching-to A. Ma (1989), Arthur O'Sullivan (1993), Daniel E. Ingberman (1995), Philippe Jehiel et al. (1996) and Motty Perry and Philip J. Reny (1999). We discuss their findings and our own results regarding this environment in Section 4.

The paper proceeds as follows: In Section I, we present the basic environment and the multi-bidding mechanism, and in Section II we study the outcomes generated by the mechanism and provide a full characterization of all the equilibrium payoffs. Section III studies the environment in which a set of agents decides who from among them will develop a project that generates (positive or negative) externalities. The last Section presents conclusions and suggests further directions of research.

## **I. The Environment and the Multi-bidding Mechanism**

We consider a set of agents  $\mathcal{N} = \{1, \dots, n\}$  and a set of possible projects  $\mathcal{K} = \{1, \dots, k\}$ . The agents' utilities depend on the project chosen and the agents have to choose which project will be carried out. The utility (payoff) of agent  $i$  if project  $q$  is carried out is given by  $v_q^i$ .

Project  $q$  is *efficient* if:

$$\sum_{i \in \mathcal{N}} v_q^i \geq \sum_{i \in \mathcal{N}} v_p^i \quad \text{for all } p \in \mathcal{K}$$

We denote by  $\mathcal{E}$  the set of efficient projects and by  $V^e$  the maximum total value that can be attained by developing the project, that is,

$$V^e = \sum_{i \in \mathcal{N}} v_q^i \quad \text{for some } q \in \mathcal{E}.$$

Although each agent has complete information, that is, he knows all the values of  $v_q^i$ , the planner does not. Alternatively, we can assume that the planner, while possibly having some partial information, wants to design a system that will be applicable to any similar situation.

We construct a *multi-bidding mechanism* through which agents influence the choice of the project. The winning project will be the one having received the highest bid. Informally, the mechanism is described as follows: Each agent  $i$  announces  $k$  bids, one for each possible project, that are constrained to sum up to zero. Furthermore, he chooses one of the projects. The project with the highest aggregate bid is chosen as the winner. We define the aggregate bid for a project as the sum of bids made to that project. In case of a tie, the winning project is randomly chosen among those with the highest aggregate bid that have been selected by at least one agent. Otherwise, the winning project is randomly chosen among those with the highest aggregate bid. Once the winning project is identified, the bids corresponding to it are paid (or received). The surplus, if any, is shared among the agents, and the project is developed.



A key feature of our mechanism is that we allow for more than one bid by each agent. Forcing the agents to submit a single bid makes it impossible to “fine-tune” the strategy and leads to inefficient equilibria.

We now describe the multi-bidding mechanism more formally:

Each agent  $i \in \mathcal{N}$  makes bids  $b^i_q$  in  $\mathbb{R}$ , one for every  $q$  in  $\mathcal{K}$  with  $\sum_{q \in \mathcal{K}} b^i_q = 0$ , and announces  $m^i \in \mathcal{K}$ . Hence, a strategy for agent  $i$  is a vector  $((b^i_q)_{q \in \mathcal{K}}, m^i)$  in  $\mathcal{H}^k \times \mathcal{K}$  ( $m^i$  is “interpreted” to mean that the agent would like to see project  $m^i$  developed), where

$$\mathcal{H}^k = \left\{ \mathbf{z} \in \mathbb{R}^k \mid \sum_{q \in \mathcal{K}} z_q = 0 \right\}.$$

For each  $q \in \mathcal{K}$ , we let  $B_q = \sum_{i \in \mathcal{N}} b^i_q$  denote the aggregate bid for project  $q$ .<sup>4</sup> The set of projects with the highest aggregate bid is denoted by  $\Omega(b) = \{ q \in \mathcal{K} \mid B_q \geq B_p \text{ for all } p \in \mathcal{K} \}$ . Finally,  $\Omega_1(b, m) = \{ q \in \Omega(b) \mid m^i = q \text{ for some } i \in \mathcal{N} \}$  denotes the set of projects with the highest aggregate bid that have been selected by some agent. The winning project  $\alpha$  is randomly chosen among the members of  $\Omega_1(b, m)$  if  $\Omega_1(b, m)$  is not empty. Otherwise, it is randomly chosen in  $\Omega(b)$ .

The final payment to any agent  $i$  is given by  $(-b^i_\alpha + B_\alpha/n)$ .<sup>5</sup>

## II. Equilibrium Outcomes of the Multi-bidding Mechanism

We are interested in the equilibrium outcomes generated by the multi-bidding mechanism; that is, which project will be finally implemented and what will be the final

utility levels of the agents? As this is a game of complete information, we analyze the resulting Nash equilibria (NE). We start by proving several important properties satisfied by all equilibrium strategies. In Lemma 1 we show that in equilibrium the aggregate bid for any project must be zero. Using Lemma 1, we derive in Lemma 2 a set of inequalities satisfied by all equilibrium strategies.

LEMMA 1: In any NE of the multi-bidding mechanism, the aggregate bid of any project is zero, i.e.,  $B_q = 0$  for any  $q$ .

PROOF:

If  $\Omega(b) = \mathcal{K}$  the claim is satisfied since the sum of aggregate bids is zero. If  $\Omega(b) \neq \mathcal{K}$ , that is, the winning project(s) received a positive aggregate bid, there must be another project that received a negative aggregate bid. In this case, any agent could, by slightly reducing the bid he made for the winning projects and increasing the bids (so as to satisfy the summing up to zero) for the losing projects, keep the same set of winning projects and increase his payoff.

Lemma 1 implies that in any NE, players' utilities from a given project  $q$  are given by  $(v_q^i - b_q^i)$ , and not  $(v_q^i - b_q^i - B_q/n)$ , since in equilibrium  $B_q = 0$  for any  $q$ . Moreover, in equilibrium, the bids are such that each agent can by a slight change in his bid decide upon the identity of the winning project. Indeed, since all aggregate bids are equal, agent  $i$  can turn project  $q$  into a certain winner by slightly increasing  $b_q^i$ . Similarly, agent  $i$  could turn project  $p$  into a sure loser by slightly decreasing  $b_p^i$ . Therefore, the project

finally chosen must be the most preferred one for every agent given the equilibrium bids. This result is stated as Lemma 2.

LEMMA 2: If project  $q$  is chosen as a NE outcome of the multi-bidding mechanism, the following condition must be satisfied:

$$(1) \quad v_q^i - b_q^i \geq v_p^i - b_p^i \text{ for all } i \in \mathcal{N} \text{ and } p \in \mathcal{K},$$

The inequalities derived in Lemma 2 are now used to show that all Nash equilibrium outcomes are efficient.

THEOREM 1: In every NE of the multi-bidding mechanism, the project chosen is efficient.

PROOF:

Assume that project  $q$  was chosen. Take any project  $p$  different from  $q$ . Consider condition (1) corresponding to this particular project  $p$ . Summing up over the set of agents, we get:

$$\sum_{i \in \mathcal{N}} v_q^i - \sum_{i \in \mathcal{N}} b_q^i \geq \sum_{i \in \mathcal{N}} v_p^i - \sum_{i \in \mathcal{N}} b_p^i.$$

Since aggregate bids for projects  $q$  and  $p$  are zero, we obtain

$$\sum_{i \in \mathcal{N}} v_q^i \geq \sum_{i \in \mathcal{N}} v_p^i.$$

Therefore project  $q$  is efficient.

Hence, the multi-bidding mechanism leads to an efficient project even though the authorities in charge lack precise information on the benefits associated with each project. The intuition for this result follows from Lemma 2. If the project chosen in equilibrium was not efficient, then it would be possible to increase the aggregate utility of the agents by the choice of an alternate project. This would certainly improve the situation of at least one person and (by Lemma 2) that person could, by slightly modifying his strategy, attain a better outcome, in contradiction to the fact that the choice of the project was an equilibrium outcome.

Theorem 1 states the main property of the NE. The (constructive) proof of the existence of such NE is postponed until Theorem 3. We now provide some characteristics of the utilities obtained by the agents in equilibrium.

Denote by  $u^i$  the final level of utility that agent  $i$  obtains by participating in the multi-bidding mechanism. Also, denote by  $\underline{v}^i$  the expected utility agent  $i$  obtains if there is a probability of  $1/k$  that each project is carried out. Formally,

$$\underline{v}^i = \frac{1}{k} \sum_{q \in K} v_q^i .$$

Note that the payoff  $\underline{v}^i$  is associated with the random assignment mechanism, which is a popular benchmark for evaluating performances. The following theorem shows that the  $\underline{v}^i$  utility levels provide a lower bound on the payoffs received by the agents at the equilibria of the multi-bidding mechanism.

**THEOREM 2:** In every NE of the multi-bidding mechanism, the final utility of any agent  $i$  is greater or equal to his average valuation, that is,  $u^i \geq \underline{v}^i$  for all  $i$  in  $\mathcal{N}$ .

PROOF:

Consider an equilibrium outcome in which a project  $q$  is chosen. Summing up condition (1) for all  $p$  in  $\mathcal{K}$ , we obtain:

$$k(v_q^i - b_q^i) \geq \sum_{p \in \mathcal{K}} (v_p^i - b_p^i) \quad \text{for all } i \in \mathcal{N}.$$

Given that the sum of  $b_p^i$ 's over the set  $\mathcal{K}$  equals zero by construction, we get:

$$u^i = v_q^i - b_q^i \geq \frac{1}{k} \sum_{p \in \mathcal{K}} v_p^i = \underline{v}^i \quad \text{for all } i \in \mathcal{N}.$$

The intuition behind Theorem 2 is also simple. Agent  $i$  can always choose a vector of bids so that he guarantees himself a certain final utility irrespective of the choices made by other agents. Agent  $i$  can choose bids in  $\mathbb{H}^k$  satisfying  $v_q^i - b_q^i = v_p^i - b_p^i$  for all  $q, p$  in  $\mathcal{K}$ , namely  $b_q^i = v_q^i - \underline{v}^i$  for all  $q$ . It is easy to check that by using this “safe” strategy, agent  $i$ 's payoff always equals at least  $\underline{v}^i$ . Indeed, agent  $i$  obtains precisely  $\underline{v}^i$  if  $B_q = 0$  for all  $q$  (as happens at equilibrium); he obtains strictly more than  $\underline{v}^i$  if  $B_q > 0$  for some  $q$ . Hence, the utility that agent  $i$  obtains in any NE must be, at least,  $\underline{v}^i$ .

Theorems 1 and 2 highlight the main properties of the equilibrium outcomes. All equilibrium outcomes are efficient and guarantee for each agent at least his expected utility when the probability of carrying out the project is uniformly distributed among all the projects. Hence, all equilibrium outcomes belong to the set  $\mathcal{P}$ , where

$$\mathcal{P} \equiv \left\{ \mathbf{u} \in \mathbb{R}^n \mid \sum_{i \in \mathcal{N}} u^i = V^e \text{ and } u^i \geq \underline{v}^i \text{ for all } i \in \mathcal{N} \right\}.$$

We now derive further properties of the equilibrium strategies using the efficiency of any equilibrium outcome.

LEMMA 3: At any NE of the multi-bidding mechanism,  $m^i \in \mathcal{E}$  for every agent  $i$ .

PROOF:

Since in equilibrium all aggregate bids are equal, if an agent  $i$  sent  $m^i \notin \mathcal{E}$ , there would be a positive probability that an inefficient project will be chosen at equilibrium. This is in contradiction to Theorem 1.

LEMMA 4: In any NE of the multi-bidding mechanism, all agents are indifferent to the identity of the project chosen as long as it is efficient.

PROOF:

Assume we are at a NE where an efficient project  $q$  is chosen. Furthermore, there is another efficient project  $p$ . Since

$$\sum_{i \in N} v_q^i = \sum_{i \in N} v_p^i ,$$

it is also the case that:

$$\sum_{i \in N} v_q^i - \sum_{i \in N} b_q^i = \sum_{i \in N} v_p^i - \sum_{i \in N} b_p^i .$$

Therefore, all the inequalities involving projects  $q$  and  $p$  in equation (1) must be satisfied as equalities. Hence, at this NE, any agent  $i$  gets the same payoff if project  $q$  or  $p$  is chosen.

We can now use the above properties to provide a full characterization of all the equilibrium strategies.

LEMMA 5: A set of strategies constitutes a NE of the multi-bidding mechanism if and only if the following three properties hold:

- (a) The aggregate bid for every project is equal to zero.
- (b) Given the equilibrium bids, an agent's payoff is maximal if an efficient project is chosen, and it is the same regardless of which efficient project is chosen.
- (c) The message  $m^i \in \mathcal{E}$  for every agent  $i$ .

PROOF:

The “only if” part has been shown in the previous lemmata. We now prove that the conditions are also sufficient. Suppose that the strategies of the agents satisfy conditions (a) – (c). We prove that no agent  $i$  in  $\mathcal{N}$  profits by deviating from these strategies.

Note first that conditions (a) and (c) imply that  $\Omega_1(b, m) \subseteq \mathcal{E}$  and hence by (b) every agent is indifferent to the identity of the project chosen in  $\Omega_1(b, m)$ . Therefore, given the bids, no agent  $i$  can gain by a change in his message  $m^i$  since his utility is maximized when an efficient project is chosen. If an agent decides to increase his bid for a particular project, it will be chosen with certainty (because of (a)). However, since the projects in  $\Omega_1(b, m)$  yielded a maximal payoff, his payoff will decrease.

Theorem 3 constructively shows that any vector in  $\mathcal{P}$  can indeed be realized as an equilibrium outcome of our mechanism. Thus we provide a full characterization of the set of NE utilities.

**THEOREM 3:** Any  $\mathbf{u} \in \mathcal{P}$  can be realized as a Nash equilibrium outcome of the multi-bidding mechanism.

**PROOF:**

See the Appendix.

The following corollary sets our results in the framework of implementation theory.

**COROLLARY 1:** The multi-bidding mechanism implements in NE the set of utility vectors given by:

$$\left\{ (u^1, \dots, u^n) \in \mathbf{R}^n \mid \sum_{i \in \mathbf{N}} u^i = V^e \text{ and } u^i \geq \underline{v}^i \text{ for all } i \in \mathbf{N} \right\} .$$

**PROOF:**

It follows directly from theorems 1, 2, and 3.

One concern regarding the realization of efficient outcomes via NE is that the results are usually not immune to coalitional manipulations. A subset of the agents can often improve their situation by a joint deviation from the equilibrium strategies. A NE is a *strong NE* if for any subset of agents the strategies played by its members are a best response to the strategies adopted by the remaining agents. We now show that the NE of



the multi-bidding mechanism are immune to coalitional deviations and constitute, in fact, strong NE.<sup>6</sup> This adds further credibility to our efficiency results in that they remain intact even when one allows for collusion among agents.

**THEOREM 4:** Any NE of the multi-bidding mechanism is a strong NE.

**PROOF:**

Take any NE of the multi-bidding mechanism and let  $(b_q^i)_{i \in \mathcal{N}, q \in \mathcal{K}}$  be the bids announced in that equilibrium. Assume by way of contradiction that there exists a subset of agents  $C \subseteq \mathcal{N}$  which could achieve a better outcome for all of its members by deviating to a new set of strategies (while agents outside of  $C$  keep their NE strategies). Let  $(d_q^i)_{i \in C, q \in \mathcal{K}}$  denote the bids used in the new set of strategies.

Let  $q$  be a project that is chosen as an outcome of the mechanism given the new bids and messages. The sum of utilities obtained by the agents in  $C$  is:

$$\sum_{i \in C} (v_q^i - d_q^i) = \sum_{i \in C} v_q^i - \sum_{i \in C} d_q^i.$$

Since the (new) aggregate bid for project  $q$  must be non-negative, and the bids by the agents outside of  $C$  did not change, it must be the case that the sum of the new bids of the agents in  $C$  is at least as large as previously, hence:

$$\sum_{i \in C} v_q^i - \sum_{i \in C} d_q^i \leq \sum_{i \in C} v_q^i - \sum_{i \in C} b_q^i = \sum_{i \in C} (v_q^i - b_q^i) \leq \sum_{i \in C} (v_p^i - b_p^i),$$

for any efficient  $p$ .

Given that the last expression is the aggregate utility of the members in  $C$  at the NE, the deviation by  $C$  could not have increased the aggregate utility of the members of  $C$ .

Hence, it is not possible for the agents in  $C$  to propose a joint set of strategies in which they will be better off than under the NE strategies.

A strong NE is immune to any coalitional deviation and as such is also a coalition proof equilibrium (coalition proofness looks only at a subset of all the possible deviations, see Douglas B. Bernheim et al. (1987) for a precise definition). Hence, the mechanism implements the efficient outcomes in coalition proof equilibria as well.

### **III. In Whose Backyard?**

One subclass of the environments we have considered in the previous sections has been extensively discussed in the literature, namely the case where each project is “associated” with one agent. Consider the case where one of  $k$  cities is to host a project. The project can either be a “good”, for example a museum or a large sports event, or it can be a “bad”, such as a dump-site or a nuclear waste disposal facility. In this setting  $v_q^i$  and  $b_q^i$  can be interpreted as follows:  $v_q^i$  is the utility of city  $i$  if city  $q$  ends up with the project;  $b_q^i$  is the amount of money city  $i$  is ready to pay in case city  $q$  gets the project. If the project is a “good”, then  $v_q^i$  is typically a negative number if  $q \neq i$ , and a positive number if  $q = i$ . The opposite holds if the project is a “bad”. Clearly, the mechanism we have constructed applies to this environment and realizes efficient outcomes in this setting with (positive or negative) externalities. In this case, the lower bound  $\underline{v}^i$  for the equilibrium payoffs has a natural interpretation. Given that the environment is one in which every agent has the same a priori rights, a natural and fair (although not efficient)

procedure for allocating the project is to randomly choose one of the agents with probability  $1/n$ . The value  $\underline{v}^i$  is the expected payoff of agent  $i$  under this random assignment mechanism.

Problems of this class have been analyzed in several papers. In particular, sealed-bid mechanisms have been suggested for siting noxious facilities, for incomplete information settings where each agent knows his own preferences but does not know the preferences of the other agents. Kunreuther and Kleindorfer (1986) showed that the outcomes realized by max-min strategies are efficient in those environments where each agent is indifferent as to all the outcomes, as long as he is not the host. For the case of two cities, O'Sullivan (1993) proved that symmetric Bayes-Nash equilibria of the bidding game yield an efficient outcome when the cost parameters of the two cities are independently drawn.

Ingberman (1995) added a further dimension to the problem of siting a noxious facility by relating the cost to the agents to their distance from the noxious facility. He analyzed the process of majority agreement by using an auction approach and concluded that decisions reached in this manner would not be efficient.

Rob (1989) approached the siting problem from a mechanism design point of view. The designer's role is assumed by a firm that has to decide where to build its plant. To each cost vector reported by the locations, the optimal incentive-compatible and individually rational mechanism associates a randomized decision rule and an expected compensation for each location. As is customary in such environments, the resulting mechanism could lead to inefficient outcomes.

The environment characterizing King Solomon's Dilemma is another special case of our general setup. In this environment there are several agents and a single prize that must be awarded to one of them. The designer's problem is to award the prize to the agent that values it the most without imposing any cost on that agent. Glazer and Ma (1989) assumed the agents have complete information and constructed a multi-stage mechanism whose unique subgame perfect equilibrium realizes that outcome. Perry and Reny (1999) relaxed the informational assumptions, requiring only that it is common knowledge that the agent who most values the object knows who he is. They provided a mechanism that realizes the desired outcome in iteratively weakly undominated strategies. The mechanism we have constructed would realize an efficient outcome in Nash equilibria for the complete information King Solomon's Dilemma. Moreover, it would also work in a more general framework in which each agent's welfare depended not only on whether or not he received the prize, but also on which of the other agents is the final owner. However, in most cases our mechanism would impose a monetary cost on the winner.

Jehiel et al. (1996) analyzed a more complex environment where the benefit (or cost) to an agent depends on the identity of the agent carrying out the project. In their setting, one seller has an object he wants to sell to one of  $n$  agents. First, the authors assume the seller is aware of all the benefits and show how he could construct a revenue-maximizing selling mechanism. Then in the incomplete information setting they characterize the incentive compatible and individually rational mechanisms that maximize the seller's revenue. An advantage of our simpler mechanism is that it can be applied without any knowledge of the particular environment in which it is being used. In contrast, the

outcome function of the mechanism proposed by Jehiel et al. (1996) is very sensitive to the particulars of the environment to which it is applied. However, Jehiel et al. (1996) consider both complete and incomplete information settings while we only perform our analysis when agents have complete information about each other.

For the environments considered in this section, it is in fact possible to adopt another mechanism that is different than (although similar to) the multi-bidding mechanism that achieves identical outcomes, but in which strategies have a different interpretation.<sup>7</sup> The mechanism can be informally described as follows: Each agent  $i$  announces  $(n-1)$  bids, one for each of the other agents. Furthermore, he announces whether he “really” wants to have the project. The agent with the highest net bid is chosen as the winner. We define the “net bid” of an agent as the difference between the sum of bids he makes to the others and the sum of bids the others make to him. In case of a tie, the winner is randomly chosen among those agents with the highest net bid who have announced that they “really” want the project. If there are no such agents, the winner is randomly chosen from all the agents with the highest net bid. Once the winner is determined, he pays the bids to the other agents and proceeds with the project. In this mechanism, the bids are naturally interpreted as transfer payments (positive or negative) offered for the right to carry out a particular project.

The NE outcomes of this mechanism possess all the properties satisfied by the NE outcomes of the multi-bidding mechanism except for the fact that they are not strong NE.<sup>8</sup>

#### **IV. Conclusion**

We have addressed the problem of choosing an efficient alternative by a group of agents with conflicting interests. The underlying problem might be the siting of a major sports event, the election of a department chairman, or the location of a nuclear reactor. We assumed that the agents are fully informed and that an uninformed designer has to reach a decision that will affect all of them. We then constructed a simple and straightforward mechanism whose Nash equilibria realize any efficient outcome that satisfies intuitively appealing lower bounds on the payoffs attained in equilibrium by the participating agents. Furthermore, these equilibria were shown to be strong Nash equilibria and hence are immune to coalitional deviations.

The mechanism proposed here improves previous constructions in that it can handle a large set of environments. It involves a single stage of play with each agent being able to express his preferences by sending a multi-dimensional message.

The complete information assumption, while restrictive, can approximate realistic environments where the agents taking part in the mechanism are much better informed than the designer. For example, the designer could be a university president and the agents members of a department, one of whom should be designated as a chairman. The complete information framework allows the derivation of a powerful efficiency result and can serve as a benchmark for the study of environments with incomplete information.

The mechanism, could operate in environments with asymmetric information as well. Determining the properties of the resulting equilibria in such environments is a topic for further research.

## APPENDIX

### PROOF OF THEOREM 3:

The proof proceeds as follows: First we show that both the set of equilibrium bidding strategies and of equilibrium payoff vectors are convex. Second, we show by construction that for any agent  $i \in \mathcal{N}$  there exists an equilibrium where the payoff for any agent  $j$  other than  $i$  is  $\underline{v}^j$ . We conclude by noting that these payoff vectors are the extreme points of the set  $\mathcal{P}$  and as such any point in  $\mathcal{P}$  can be expressed as a convex combination of these vectors.

Let  $\mathbf{S}^1 = (\boldsymbol{\beta}^1, \mathbf{m}^1)$  and  $\mathbf{S}^2 = (\boldsymbol{\beta}^2, \mathbf{m}^2)$  be two strategy tuples that are NE for the bidding mechanism where  $\boldsymbol{\beta}^1 = (b_q^{1i})_{q \in \mathcal{K}, i \in \mathcal{N}}$  and  $\boldsymbol{\beta}^2 = (b_q^{2i})_{q \in \mathcal{K}, i \in \mathcal{N}}$  are the bidding strategies used and  $\mathbf{m}^1, \mathbf{m}^2$  are the  $n$ -tuples of messages sent in these equilibria. We now prove that the bidding strategies  $\boldsymbol{\beta}^\lambda = (b_q^\lambda)_{q \in \mathcal{K}, i \in \mathcal{N}}$  given by:

$$b_q^\lambda = \lambda b_q^{1i} + (1 - \lambda) b_q^{2i},$$

together with  $\mathbf{m}^1$  (or  $\mathbf{m}^2$ ) are a NE for any  $0 \leq \lambda \leq 1$ . We prove this by showing that the conditions of Lemma 5 are satisfied.

(a) The aggregate bid for any project in  $\boldsymbol{\beta}^\lambda$  is zero, since

$$B_q^\lambda = \sum_{i \in \mathcal{N}} (\lambda b_q^{1i} + (1 - \lambda) b_q^{2i}) = \lambda \sum_{i \in \mathcal{N}} b_q^{1i} + (1 - \lambda) \sum_{i \in \mathcal{N}} b_q^{2i} = 0 + 0 = 0.$$

(b) Moreover, since  $\mathbf{m}^1$  was part of a NE, it satisfies condition (b).

(c) Now we consider any efficient project, say  $q \in \mathcal{E}$ . If  $q$  is chosen, agent  $i$ 's payoff is higher than his payoff if any other project  $p \neq q$  is chosen, since:

$$v_q^i + \lambda b_q^{1i} + (1-\lambda)b_q^{2i} = \lambda(v_q^i + b_q^{1i}) + (1-\lambda)(v_q^i + b_q^{2i}) \geq$$

$$\lambda(v_p^i + b_p^{1i}) + (1-\lambda)(v_p^i + b_p^{2i}) = v_p^i + \lambda b_p^{1i} + (1-\lambda)b_p^{2i} \text{ for all } p \neq q.$$

Hence, agent  $i$ 's payoff is maximized if the efficient project  $q$  is chosen. Also note that in the two previous expressions the inequalities become equalities if project  $p$  is efficient as well.

Therefore  $\beta^\lambda$  and  $\mathbf{m}^1$  constitute a NE. From the above calculations we also see that the payoff vector associated with the  $\beta^\lambda$  equilibrium is the convex combination with weights  $\lambda$  and  $(1-\lambda)$  of the payoffs corresponding to the  $\mathbf{S}^1$  and  $\mathbf{S}^2$  equilibria. Thus, we proved that the sets of equilibrium bidding strategies and of equilibrium payoffs are convex.

We now proceed to construct the “extremal” equilibrium points. Take any agent  $j \in \mathcal{N}$ . We claim that the following bidding strategies constitute the best equilibrium for agent  $j$ :

$$b_q^i = v_q^i - \underline{v}^i \text{ for } i \neq j, q \in \mathcal{K},$$

$$b_q^j = \sum_{i \neq j} \underline{v}^i - \sum_{i \neq j} v_q^i \text{ for } q \in \mathcal{K}.$$

Also, the players' strategy include a vector of messages such that only efficient projects are chosen (for example,  $m^i = q$  for all  $i \in \mathcal{N}$ , where  $q \in \mathcal{E}$ ).

It is easy to check that the previous bid vector of any agent belongs to  $\mathbb{H}^k$ . To show this is an equilibrium we prove that all the conditions of Lemma 5 are satisfied. Condition (a) holds since with these bids the aggregate bid for any project is indeed zero.



Condition (c) is satisfied by construction. It remains to show that condition (b) of Lemma 5 holds; that is, given these strategies, each agent's payoff is maximized if any efficient project is carried out. The payoff for any agent  $i \neq j$  is  $\underline{v}^i$  and hence does not depend on the project chosen and is maximized for any choice of an efficient project. Agent  $j$  receives the difference between the total surplus and the previous payoffs. Therefore, agent  $j$  prefers that an efficient project be carried out. Hence, all the conditions of Lemma 5 are satisfied and the previous strategies are part of a NE that yields a payoff vector that agent  $j$  prefers the most.

The payoff vectors corresponding to the equilibria we have constructed are the extremal points of the set  $\mathcal{P}$  and as such any point  $\mathbf{p}$  in  $\mathcal{P}$  can be expressed as a convex combination of these vectors. Applying the same convex combination to the bidding strategies yields a NE of the game with the associated payoff vector coinciding with  $\mathbf{p}$ .

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## FOOTNOTES

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<sup>1</sup> An analysis of the voting methods and Vickrey-Clarke-Groves mechanisms, can be found, for example, in Jean-Jacques Laffont (1988).

<sup>2</sup> Our mechanism is much simpler and more intuitive than general mechanisms constructed in the Nash implementation literature, which are designed to handle a large class of environments, as in Eric Maskin (1999).

<sup>3</sup> Jackson and Moulin (1992) also describe a simple modification of the mechanism so that the unique subgame perfect equilibrium coincides with the desired outcome.

<sup>4</sup> Since the bids of every agent sum up to zero, the sum of the aggregate bids is also zero.

<sup>5</sup> We chose the equal sharing rule to allocate the possible surplus  $B_\alpha \geq 0$  resulting from the bids made to the winning project. Any other sharing rule would generate the same results.

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<sup>6</sup> Game forms for which the set of NE coincides with the set of Strong NE have also been constructed for implementing competitive outcomes (Bezalel Peleg (1996)) and taxation methods (Nir Dagan et al. (1999)).

<sup>7</sup> A similar bidding procedure was used by David Pérez-Castrillo and David Wettstein (2001) as part of a non-cooperative implementation of the Shapley value.

<sup>8</sup> For more details on this mechanism, the reader is referred to Pérez-Castrillo and Wettstein (2000).