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There has been a great deal of research in recent years investigating the question of whether or not there exist institutions (game forms) for which the set of equilibria will coincide with the set of Walrasian equilibria. In this paper we show the existence of a game form that is feasible, both for equilibrium and disequilibrium strategies, continuous, and for which the set of Nash equilibria coincides with the set of (constrained) Walrasian equilibria for all pure exchange economies. The game form allows agents to behave strategically both with respect to their preferences and their initial endowments.

1. INTRODUCTION

The goal of the literature on implementation is to define the limits imposed on a society due to incentives. In the abstract, a society can rank various social choice functions, or correspondences, associating economic outcomes with possible economic environments. This ranking can be done whether or not the social choice rules being compared can be achieved. In evaluating the performance of a particular institutional structure, however, we would like to know which of the social choice correspondences are achievable. That the performance of an existing institutional structure is less attractive than some alternative is not particularly interesting if the alternative performance involves agents behaving in ways that are not in their self interest unless we have some means to force them to do so. If, for example, the alternative involves the agents revealing information truthfully, we should expect failure unless we have provided the agents with the incentives to truthfully reveal. In considering alternative social choice correspondences, we would like to restrict our attention to those that can be implemented, that is, to those that are plausibly consistent with the self-interested behaviour of agents in some institutional setting.

The Walrasian correspondence has played a central role in implementation theory, primarily because it plays a central role in economic theory in general. From a normative perspective, Walrasian allocations have desirable properties; under particular assumptions, they will be individually rational and Pareto efficient. From a positive point of view, Walrasian outcomes are often taken as a benchmark—outcomes that we might expect to arise in a world devoid of frictions or transactions costs with many agents. While the desirable normative properties, individual rationality and Pareto efficiency, hold even in the case that there are small numbers of agents, it is not normally believed that even in the absence of transactions costs, markets would yield Walrasian outcomes.

Since the desirable properties of the Walrasian correspondence apply regardless of the number of agents in an economy, we would like to know whether the presumed unattainability of the Walrasian outcomes is due to the particular form of markets that were considered or whether small numbers of agents prevent the possibility of Walrasian equilibria regardless of institutional structure. Formally, we want to know whether there exists a game form such that for any pure exchange economy, the set of Nash equilibrium allocations are the same as the set of Walrasian allocations.

We want the game form to have certain properties that we consider desirable. First, we would like the game form to yield non-negative quantities of the commodities for every n-tuple of strategies the agents choose.[1] We will call a game form that satisfies this property *feasible*. Hurwicz, Maskin and Postlewaite (1982) have shown that if we want a game form to be feasible and to have among its equilibria the Walrasian allocations, then a slightly larger set than the Walrasian allocations must also be among the Nash equilibria. This slightly larger set is the constrained Walrasian allocations. A precise definition of a constrained Walrasian allocation is given below, but intuitively it is the same as a Walrasian allocation if each trader's demand for a good is constrained to be no greater than the total endowment of the good. Like Walrasian allocations, these allocations are individually rational and Pareto efficient.

A second property we want in our game form is that it be continuous: small changes in an agent's strategy choice should not lead to large changes in the resulting allocation.[2] Continuity of the outcome function of a game form is highly desirable if we are to seriously consider the game form. This paper, as well as many concerned with implementation, implicitly assumes that the economic environments are characterized by complete information. This is a reasonable research strategy, since we know that the presence of asymmetric information by itself may make it impossible to achieve Walrasian equilibria.[3] We are interested in the degree to which small numbers of agents present problems as well. We take complete information to be an abstraction, however. We would like a game that performs well (that is, generates Walrasian equilibria for each economy) and is robust to small misspecifications of the characteristics of the agents in the economy. If the outcome function is continuous, then a strategy that is a best response to a known strategy vector of the others when there is no uncertainty is nearly a best response to a probability distribution over strategies that differ only slightly from the known strategy vector. In a sense, a Nash equilibrium in the "no uncertainty" case would be an epsilon equilibrium for a perturbed environment in which there was a small amount of uncertainty.

If the outcome function is not continuous, this will not necessarily be the case. In fact, it is not at all clear how general games with discontinuous outcome functions can be extended to a case in which an agent is uncertain about the strategies chosen by the other agents, no matter how small the uncertainty. Models in which some characteristics were not common knowledge would have to be dealt with by other mechanisms.

It can be argued that if there is uncertainty, it should be modeled directly. There has been recent work extending results on implementation to models with uncertainty (see, e.g. Palfrey and Srivastava (1986*b*), Postlewaite and Schmeidler (1986) and Wettstein (1986)). The introduction of uncertainty makes the problem significantly more difficult

1. Schmeidler (1980), and subsequently Hurwicz (1979), constructed game forms that implemented the Walrasian correspondence but did not satisfy this condition.

2. The game forms of Schmeidler (1980) and Hurwicz (1979) were continuous. Hurwicz, Maskin and Postlewaite (1982) presented game forms that were feasible but not continuous.

3. To see this, it is enough to construct examples with asymmetric information in which the self-selection constraints are not satisfied. Such examples are not difficult to construct. See, for example, Blume and Easley (1983) or Palfrey and Srivastava (1986*a*).

and more obscure. We ask for continuity of the outcome function as a (partial) substitute for the direct modelling of uncertainty.

The last property we ask of our game form is that it be immune to strategic behaviour with respect to initial endowments as well as to preferences. The first property we listed above that we would like our game form to satisfy was feasibility: every n-tuple of strategies should yield a feasible allocation. Since the set of allocations that is feasible depends upon the (aggregate) initial endowment in the economy, the game form must depend in some way upon the agents' endowments. If the game form depends upon the initial endowments, agents may be able to misrepresent their endowments, or otherwise behave strategically with respect to them. If the endowments are publicly available for all to see, such manipulation may not be possible. Howevever, even if the endowments are physical goods (as opposed to such endowments as abilities), the endowments may not be publicly observable and there may be the potential for strategic behaviour. We ask that the game form be immune to strategic behaviour in the following sense. We include as part of an agent's strategy the revelation of his endowment. We assume (implicitly) that the endowments are physical commodities and overstatement of endowment is physically impossible. An agent can, however, withhold goods from the market (understate his endowment) and consume the amount withheld, in addition to whatever is the outcome of the game form directly.[4]

We will show that there are game forms that satisfy the above desiderata. In the next section, we present the formal model and definitions followed by a game form that is feasible and continuous and which takes individuals' endowments as unknown. We show that the game form implements the constrained Walrasian equilibria. The last section of the paper contains a discussion of the game form and related literature.

2. THE MODEL AND THEOREMS

The economy:

n = the number of individuals ($n \geqq 3$)

l = the number of goods

w^i = the initial endowment of individual $i (w^i \in R^l_{++})$

$w = \sum_{i=1}^n w^i$

$\gtrsim_i$ = the preference relation of individual i, defined on $R^l_+ - \{w^i\}$ induced by a real, strictly quasi-concave, strictly monotone increasing, and continuously differentiable function defined on R^l_+ and an initial endowment $w^i \in R^l_{++}$ ($>_i$ will denote the strong preference relation)

z^i = the net trade of individual $i (z^i \in R^l)$

$(p, z) \in R^l_+ \times R^{ln}$ is a *constrained Walrasian equilibrium* if:

(i) $p \cdot z^i = 0, i = 1, \ldots, n$
(ii) $y >_i z^i$ and $w^i + y \leqq w$ imply $p \cdot y > 0, i = 1, \ldots, n$
(iii) $\sum_{i=1}^n z^i = 0$.

z will be called a *constrained Walrasian allocation.*

4. Hurwicz, Maskin, and Postlewaite (1982) treat the topic of manipulation of endowments in detail. The primary differences between that work and the present is that they dealt with endowment manipulation for arbitrary social choice correspondences, while we deal only with the constrained Walrasian correspondence. On the other hand, the solutions to the endowment problem in that work are discontinuous, while here we ask for continuity.

Vector inequalities are defined as follows: $x \leqq y$ if and only if $x_i \leqq y_i$ for all $i = 1, \ldots, l$; $x < y$ if and only if $x \leqq y, x \neq y$; $x \ll y$ if and only if $x_i < y_i$ for all $i = 1, \ldots, l$.

The mechanism:

The strategy space for individual i is

$$M^i = ((R_{++}^{ln} \cap A^i) \times R^l \times R_{++}^l \times R_{++}^2)$$

where

$$A^i = \{x \in R_{++}^{ln} \mid x^i \leqq w^i\}.$$

A generic element of $M = M_1 \times \ldots \times M_n$ will be denoted by $((v_1^i, \ldots, v_n^i), z^i, p^i, (r^i, q^i))$ where:

$v^i = (v_1^i, \ldots, v_n^i)$ denotes the initial endowment profile announced by individual i (v_j^i corresponds to i's announcement of the initial endowment of individual j). We assume the mechanism does not know the initial endowment; the restriction imposed by A^i is that individuals cannot overstate their own endowments.

z^i = net trade announced by individual i

p^i = price vector announced by individual i

(r^i, q^i) = two positive numbers determining "fines" associated with discrepancies in individual announcements and weights assigned to the agents' demands.

Before formally defining the mechanism which will continuously implement the constrained Walrasian equilibrium correspondence, we will describe it in words.

First, the mechanism takes the announced v_i^is and treats them as the true initial endowments. From these endowments a set of bundles $(\alpha^1, \ldots, \alpha^n)$ for the agents is constructed as follows. A weighted average is taken of the prices the agents announce. The weight for an individual is proportional to the variance of the prices announced by other individuals. Thus, if all agents other than a given agent announce the same price, that given agent cannot affect the price since the weight attached to his announcement will be 0. Also, an agent can reduce other agents' weights by announcing the mean of the prices announced by other agents.

The bundle that agent i is then assigned, α^i, is then determined by projecting his announced net trade onto the price hyperplane determined by the weighted average price above. The allocation that results from this process may not be feasible. In this case, the bundles α^i are shrunk (that is, the agents are rationed) the minimal amount so as to make them feasible. The rationing is continuous and further, any single agent can reduce the amount he will be rationed through the r^i component of his strategy choice.

The bundles described above are modified in one more way so as to give the agents an incentive to correctly announce their initial endowments. Each agent's initial endowment is announced by every other agent. To the bundles described above, penalties and rewards are subtracted and added. The rewards increase as an agent announces an endowment for himself that is higher than that announced for him by others. The penalties are levied to cover the rewards. These adjustments are continuous as a function of the strategy choices. Their effect is to induce each agent to announce for himself as high an endowment as possible (consistent with the constraint that he cannot overstate his endowment), and to announce for other agents the endowments they announce for themselves.

This process yields the allocation that results from agents' strategy choices. The only equilibria of the game have agents truthfully announcing their endowments and trading to a constrained Walrasian equilibrium given these endowments. We turn now to the formal definition of the game.

Let $v=\sum_{i=1}^n v_i^i$ be the true "aggregate" endowment vector. Based on the p^i's, an average price vector, $\bar{p}$, is constructed as follows. Define:

$$a_i=\sum_{t,t'\neq i}\|p^t-p^{t'}\|^2, \qquad a=\sum_{i=1}^n a_i$$

$$b_i=\begin{cases}\dfrac{a_i}{a} & a>0\\ \dfrac{1}{n}, & a=0\end{cases}$$

and finally,

$$\bar{p}=\sum_{i=1}^n b_i p^i.$$ [5]

Now a set B^i is defined for each individual:

$$B^i(\bar{p}, v_i^i, v)=\{z\in R^l \mid \bar{p}\cdot z=0,\, z+v_i^i\geqq 0,\, z+v_i^i\leqq v\}.$$ [6]

Define y^i to be the closest point to z^i in B^i (B^i is a closed convex nonempty set), and define the set A as follows:

$$A=\{r\in R_{++}\mid r\cdot r^i\leqq 1,\, i=1,\ldots,n \text{ and } r\cdot\textstyle\sum_{i=1}^n r^i(y^i+v_i^i)\leqq v\}.$$

Let r^* be defined as $r^*=\max_{r\in A} r$.

The set of n final bundles (one for each individual) constructed in this stage is:

$$\alpha^i=r^*r^i(y^i+v_i^i), \qquad i=1,\ldots,n.$$

We turn now to the adjustments that provide incentives for truthful revelation of endowments.

Let x_+ denote the vector derived from x by replacing each nonpositive coordinate of x by zero.

Let

$$\delta_i=\left|\left(v_i^i-\frac{\sum_{j\neq i} v_i^j}{n-1}\right)_+\right|$$

$$\delta=1+\sum_{i=1}^n \delta_i$$

$$\delta_{-i}=\sum_{j\neq i}\delta_j$$

$$\alpha_{-i}=\sum_{j\neq i}\alpha^j$$

and

$$q=\sum_{i=1}^n q^i$$

$$v_{-i}=\frac{\sum_{j\neq i} v^j}{n-1}.$$

The final bundle $\tilde{y}^i$ allocated to individual i is:

$$\tilde{y}^i=\left(\left(1-\frac{\delta_{-i}}{\delta}\right)\alpha^i+\frac{\delta_i}{\delta}\alpha_{-i}\right)\left(1+|v^i-v_{-i}|\frac{q_i}{q}\right)^{-1}.$$

5. Note that $\bar{p}$ is a continuous function of $(p^i)_{i=1}^n$ even though the b_i's are not continuous.

6. We assume free disposability. It is quite possible that this assumption could be dropped with the cost being a more complicated mechanism than described in this paper.

Theorem 1. *For the mechanism described above, the set of constrained Walrasian allocations is contained in the set of Nash allocations.*

Proof. Let $(x^1, \ldots, x^n, p)$ be a constrained Walrasian equilibrium where x^i denotes the net trade of individual i.

We shall show that this allocation can be achieved as a Nash allocation. The following n-tuple of strategies will yield this allocation:

$$r^i = q^i = 1; \qquad p^i = p; \qquad z^i = x^i; \qquad (v_1^i, \ldots, v_n^i) = (w^1, \ldots, w^n).$$

We will show that these strategies constitute a Nash equilibrium. Note that $\bar{p} = p$, $r^* = 1$, $\alpha^i = \tilde{y}^i = x^i$. In this case z^i is actually contained in B^i; this implies that each individual gets the final bundle $x^i + w^i$, or a net trade x^i. Individual i cannot change $\bar{p}$ by changing his announced price (changing p^i yields $a > 0$ and $a_i = 0$; thus, the new p^i cannot change the average).

Announcing a different net trade $\tilde{x}^i$ and/or a different endowment profile may, at most, yield him a final bundle b^i that satisfies

$$b^i + w^i \leqq w$$

$$p \cdot b^i = 0.$$

If $b^i >_i x^i$, by definition of a constrained Walrasian equilibrium, we must have

$$p \cdot b^i > p \cdot x^i.$$

However, $p \cdot x^i = 0$, thus, we reach a contradiction.

As regards changes in the endowment profile announced by individual i we should note the following: the v_i^i component, since he is now announcing his true endowment, can only be adjusted downwards. This in turn would lead to a shrinking of the B^i set, or in other words a smaller set of net trades to choose from. Changes in the other components may only effect the transition from α^i to $\tilde{y}^i$, creating a scaling down of the α^i's, whereas prior to the change $\alpha^i = \tilde{y}^i$.

Notice that changing r^i cannot help individual i since the best he could hope for is $r^* r^i = 1$ and this is already satisfied.

We have shown that the ith individual cannot improve his position by changing his own strategy while the strategies of the others remain fixed. This is true for all i, hence, we are at Nash equilibrium and $(x^1, \ldots, x^n)$ is a Nash allocation. ‖

Theorem 2. *The set of Nash allocations is contained in the set of constrained Walrasian allocations.*

Proof. We will prove this theorem using a series of lemmas.

Lemma 1. *Given the price p determined by the mechanism and a net trade z that satisfies:*

(i) $p \cdot z = 0$,
(ii) $z + v_i^i \leqq v$,
(iii) $0 \leqq z + v_i^i$,

individual i can secure himself a net trade arbitrarily close to this z.

Proof. Declaring a large enough r^i, individual i brings about a very small r^* (since $r^* r^i \leqq 1$), thus, almost nullifying the effect of the other individuals in the sum

$$r^* \sum_{i=1}^n r^i(y^i + v_i^i).$$

So, if he announced the net trade z, he can get as close to z as he wishes. (His announcing z will yield $y^i = z$ since z is in B^i). ||

Lemma 2. *At a Nash equilibrium point we must have* $r^* r^i = 1$, $i = 1, \ldots, n$.[7]

Proof. Assume by way of contradiction that there exists an individual i for which $r^* r^i < 1$. Hence $\alpha^i = r^* r^i(y^i + v_i^i) \leqq y^i + v_i^i$. Thus individual i would strictly prefer a strategy n-tuple that leads to $\alpha^i = y^i + v_i^i$. (An increase in α^i translates into an increase in $\tilde{y}^i$). By Lemma 1 individual i could get arbitrarily close to the net trade y^i, and since his preferences are continuous, could make himself strictly better off. This contradicts the fact we began with a Nash Equilibrium. ||

Lemma 3. *At a Nash equilibrium point*

$$\sum_{i=1}^n (y^i + v_i^i) = \sum_{i=1}^n v_i^i.$$

Proof. Suppose by contradiction, that this inequality does not hold. Then, there must be a component j such that

$$\sum_{i=1}^n (y^i + v_i^i)_j < \sum_{i=1}^n (v_i^i)_j.$$

(We cannot have an inequality in the other direction by the definition of A). However, $p(\sum_{i=1}^n (y^i + v_i^i)) = p \sum_{i=1}^n v_i^i$, where p is the average price prevailing at the Nash equilibrium.

Thus, there must be a component k that satisfies

$$\sum_{i=1}^n (y^i + v_i^i)_k > \sum_{i=1}^n (v_i^i)_k.$$

But, this contradicts the definition of A; hence, we must have the equality

$$\sum_{i=1}^n (y^i + v_i^i) = \sum_{i=1}^n v_i^i. \quad ||$$

Lemma 4. *At a Nash equilibrium point we must have*

(i) $v^i = v^j$ *for all* i *and* j,
(ii) $v_i^i = w^i$ *for all* $i = 1, \ldots, n$.

Proof. To show (i) note that if $v^i \neq v_{-i}$ for some i then no choice of q_i could give rise to an equilibrium (q_i must be strictly positive but the ith individual would like to make it as small as possible). Hence in equilibrium $v^i = v_{-i}$ for all $i = 1, \ldots, n$ which implies all the individuals must be announcing the same profile. We know by assumption that $v_i^i \gg 0$ and $v_i^i \leqq w^i$ for all $i = 1, \ldots, n$. To show (ii) assume by way of contradiction that $v_i^i \neq w^i$ for some i. In that case individual i could adjust his announced v_i^i upwards; this would enlarge his B^i set and with a suitable change in r^i, guarantee him at least the same level of utility as that prior to the change. However, this change will make δ^i strictly positive, whereas δ_{-i} is still zero. By announcing a q_i small enough he would be able to "nullify" the effect of $|v^i - v_{-i}|$ becoming positive. As a result he would become better off, due in a sense to the commodities he "collects" from the others via the $(\delta_i/\delta)\alpha_{-i}$ term. (Note that α_{-i} can never be the zero vector).

7. We are grateful to G. Tian for an observation that simplified the proof of this lemma.

Now we are ready to prove the theorem. Suppose $(x^1, \ldots, x^n)$ is a net trade allocation corresponding to a Nash equilibrium with price p. By Lemma 4 we know $v_i^i = w^i$. Hence the mechanism operates from the correct initial endowments and $(x^1, \ldots, x^n)$ is a "true" net trade allocation. Furthermore, stage 2 does not alter the allocations constructed in stage 1.

By Lemma 2 and the definition of B_i, $p \cdot x^i = 0$, $i = 1, \ldots, n$. By Lemma 3 we get $\sum_{i=1}^n x^i = 0$. All that remains to be shown is that each individual is maximizing his utility.

Suppose there exists a point $y \succ_i x^i$ that satisfies $y + w^i \leqq w$ and $p \cdot y \leqq 0$. Because of monotonicity of preferences, it will be enough to confine ourselves to the case $p \cdot y = 0$.

Now, by Lemma 1, individual i can secure for himself a net trade arbitrarily close to y. Since individuals are announcing their true endowments and since preferences are continuous, he can secure a net trade preferred to x^i, a contradiction to $(x^1, \ldots, x^n)$ being a Nash equilibrium allocation.

We have shown that $y \succ_i x^i$ and $y + w^i \leqq w$ imply $p \cdot y > 0$ for all $i = 1, \ldots, n$. Therefore, $(x^1, \ldots, x^n, p)$ is a constrained Walrasian allocation. ‖

Combining the two theorems we conclude that the set of Nash allocations coincides with the set of constrained Walrasian allocations.

3. DISCUSSION

1. In the definition of the game, agents were allowed to under-report their own initial endowments. The amount that an agent under-reported was consumed by that agent. We can think of this as withholding goods from the market. An alternative case is that in which an agent cannot withhold goods from the market to consume, but rather can only destroy a portion of his initial endowment. Clearly if agents' preferences are monotonic and no agent has an incentive to withhold goods, no agent has an incentive to destroy goods either. Thus, our game can be modified so that goods not reported disappear rather than get consumed by the under-reporting agent and the outcome would be the same: the set of Nash equilibrium allocations coincides with the set of constrained Walrasian equilibrium allocations. In fact, we could simplify the endowment part of the game if the withheld goods are destroyed rather than consumed. In that case it would suffice to use just part of our mechanism. Furthermore, each agent could be required to announce only his own endowment. In this case no destruction would occur in equilibrium. Any individual could get a net trace arbitrarily close to any net trade he achieved via destruction while announcing his true endowment. Since he will be able to consume in addition the amount withheld and destroyed, he will be strictly better off.[8]

2. Benassy (1986) suggested that discontinuities in the strategic outcome function may be an essential element for competitiveness. In light of our result, there must be differences between the two models. Benassy made two assumptions about strategic market games that are not satisfied by our game form. The first is that he assumed that the markets for goods are separate and that the quantity of a good that an agent receives depends only on the messages (strategies) sent in that market. Our outcome function doesn't have this property; altering your demand or supply for some good may affect the final quantities of other goods. Second, Benassy assumes that each agent has a strategy that guarantees that the outcome is individually rational regardless of the strategies sent by other agents. In our game form, there is no such strategy. Any Nash equilibrium will

8. Tian (1989) constructed a mechanism for public goods economies similar to this.

be individually rational, but there is no strategy that an agent can choose that uniformly guarantees individual rationality.

We are not arguing that our game form is somehow the "better"; we are only pointing out that the discontinuities in market games that Benassy demonstrated result from quite specific assumptions he made about what constituted a market game.

3. The techniques used in this paper can probably be applied to economies with public goods and production. The particular way that rationing is done to maintain continuity would, of course, have to be modified.

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REFERENCES

BENASSY, J.-P. (1986), "On Competitive Market Mechanisms", *Econometrica*, **54**, 95–108.

BLUME, L. and EASLEY, D. (1983), "On the Game-Theoretic Foundation of Market Equilibrium with Asymmetric Information" (mimeo, Cornell University).

HURWICZ, L. (1979), "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points", *Review of Economic Studies*, **46**, 217–225.

HURWICZ, L., MASKIN E. and POSTLEWAITE, A. (1982), "Feasible Implementation of Social Choice Correspondences by Nash Equilibria" (unpublished mimeo).

PALFREY, T. and SRIVASTAVA, S. (1986a), "Private Information in Large Economies", *Journal of Economic Theory*, **39**, 34–58.

PALFREY, T. and SRIVASTAVA, S. (1986b), "Implementation with Incomplete Information in Exchange Economies" (unpublished mimeo).

POSTLEWAITE, A. and SCHMEIDLER, D. (1986), "Implementation in Differential Information Economies", *Journal of Economic Theory*, **39**, 14–33.

SCHMEIDLER, D. (1980), "Walrasian Analysis via Strategic Outcome Functions", *Econometrica*, **48**, 1585–1593.

TIAN, G. (1989), "Implementation of the Lindahl Correspondence by a Single-Valued, Feasible, and Continuous Mechanism", *Review of Economic Studies*, **56**, 613–621.

WETTSTEIN, D. (1986), "Implementation Theory in Economies with Incomplete Information" (unpublished mimeo).