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Rational bargaining in games with coalitional externalities ☆

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Abstract

This paper provides a flexible strategic framework to analyze bargaining and values in environments with coalitional externalities. Within this framework we propose a new value that extends the Shapley value to partition function form games, the so-called Rational Belief Shapley (RBS) value. We investigate the strategic foundation of the RBS value by constructing an implementation mechanism. This mechanism extends existing models of multilateral bargaining by allowing players a higher degree of freedom to form coalitions. The same framework of bidding and renegotiation allows for natural variations of the RBS-mechanism. In this way, alternative "Shapley-like" values are obtained, and a unified platform to analyze and compare these solutions is provided.

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1. Introduction

The paper aims to provide a uniform and flexible strategic framework to analyze the interaction between agents, as well as the appropriate sharing of surplus, in economic environments with coalitional externalities, modeled as *partition function form games* by Thrall and Lucas [36]. The model of partition function form games incorporates both internal factors (the coalition actions) and external factors (the coalition structure), and thus, it provides a basis for a general analysis of cooperation and sharing issues.

Thus far, axiomatic studies have dominated the research of the area and resulted in the solution concepts proposed by Myerson [26], Bolger [8], Albizuri, Arin and Rubio [3], Pham Do and Norde [31], Ju [19], Macho-Stadler, Pérez-Castrillo and Wettstein [23], Xue [41] and Dutta, Ehlers and Kar [11]. de Clippel and Serrano [9] offer an insightful study by analyzing the structure of the games and identifying the bounds on players' payoffs as well as by comparing the properties of several solution concepts.

We believe the strategic approach to partition function form games is equally important. But it is not surprising why so little has been done on the non-cooperative foundations of the solution concepts[1] for partition function form games, given its highly combinatorial structure that significantly complicates the problem. Maskin [24] is the first one taking the non-cooperative approach, and a follow-up discussion is provided in de Clippel and Serrano [10]. Besides that, so far this issue is only partially investigated in Macho-Stadler, Pérez-Castrillo and Wettstein [22] where an implementation of their value (Macho-Stadler, Pérez-Castrillo and Wettstein [23]) is derived in environments with either purely non-negative externalities or purely non-positive externalities.

Part of the difficulty in addressing the non-cooperative foundation is the inability to make a sharp prediction of the worth of a coalition. The players outside a specific coalition can generate different types of externalities by forming different coalition structures, and moreover, the specific coalition itself can partition in several ways. In this paper, we analyze the determination of the worth of a coalition explicitly, propose the corresponding value solution concepts, and study the underlying bargaining foundations. Unlike most of the existing literature that adopts a simultaneous move perspective when determining the worth of a coalition, we explore the effect of sequential moves. A major advantage of the sequential approach is that it eliminates the coalition structures that are unlikely to be adopted by rational players. Furthermore, an intriguing aspect of our bargaining model is that the sequential procedure together with the endogenous choice of proposer in renegotiation will give rise to a value that is defined from a simultaneous move perspective. Thus, it successfully maintains the flavor of simultaneity in bargaining and its outcome.

In addition to proposing new solutions and studying their bargaining foundations, the paper delivers a general modeling framework where the bargaining procedures and game forms under consideration share a basic common structure. This helps to pin down the differences between the various sharing methods from a strategic point of view. Moreover, we also contribute to the

[1] For set value solutions to partition function form games, we refer to Huang and Sjöström [15,16].

literature by extending the multi-bidding approach of Pérez-Castrillo and Wettstein [29,30] to a more flexible and realistic model that not only generally accounts for coalitional externalities, but also allows players whose offers were rejected to bargain among themselves and form coalition structures.

The bulk of the analysis is carried out for a large class of games, where the agents have unique best response strategies. The complications presented when there are several best response strategies are tangential to our approach and can be readily resolved. We address them separately in order not to distract the readers from the essential features of the model.

We start our analysis in Section 2 by introducing the general environment of coalitional externalities, and defining three Shapley-like value concepts, the rational belief Shapley (RBS), the reverse RBS and the average RBS values. Section 3 constructs a unified strategic framework consisting of three non-cooperative mechanisms sharing a common basic bargaining structure, which implement these three values in subgame perfect equilibrium for games where the agents have unique best response strategies. In Section 4 we address the modifications necessary for the case of non-unique best responses. Section 5 concludes.

2. Coalitional externalities and the RBS value

Let $N = \{1, \dots, n\}$ be the set of players. A coalition S is a nonempty subset of N. A *partition* (or *coalition structure*) p of N is a collection of mutually disjoint coalitions whose union is N. For any coalition S, $\mathbb{P}(S)$ denotes the set of all partitions of S. A generic element of $\mathbb{P}(S)$ is denoted by p_S, where for simplicity, we use p rather than p_N. A pair (S, p) consisting of a coalition S and a partition p to which S belongs is called an *embedded coalition*. Let $\mathbb{E}(N)$ denote the set of embedded coalitions (S, p) with $S \subseteq N$.

We denote by (N, w) a *partition function form game* where $w : \mathbb{E}(N) \longrightarrow \mathbb{R}$ is called a *partition function* that assigns a real value, $w(S, p)$, to each embedded coalition (S, p). The value $w(S, p)$ represents the payoff of coalition S, given that coalition structure p forms. The set of partition function form games with player set N is denoted by PG^N.

For a partition $p \in \mathbb{P}(N)$ and $i \in N$, we denote the coalition in p to which player i belongs by $S(p, i)$. Moreover, for any $S \subseteq N$ we denote by $[S]$ the partition of S into *singletons*, $[S] = \{\{j\} | j \in S\}$. A solution on PG^N is a function f, mapping each game (N, w) in PG^N into a vector $f(N, w) = (f_i(N, w))_{i \in N} \in \mathbb{R}^N$, where $f_i(N, w)$ is the payoff to player i.

For an arbitrary partition function form game (N, w), let $v^w(N)$ denote the highest total payoff of the player set N, i.e., $v^w(N) = \max_{p \in \mathbb{P}(N)} \sum_{S \in p} w(S, p)$, which is thus the efficient outcome that players in N can achieve via internal cooperation (within coalitions) and external coordination (forming a coalition structure that generates the highest total payoff). The immediate concern is how to share $v^w(N)$ among all players in N. One natural approach is to determine what a coalition could obtain, were it to separate and act on its own, in other words what is the "disagreement point" for this coalition. This would reduce the complex environment with externalities to a standard transferable utility (TU) environment and one could employ one of the many values for TU games to resolve the equitable sharing issue.[2]

[2] A TU game is a pair (N, v), where $v : 2^N \to \mathbb{R}$ is a characteristic function satisfying $v(\emptyset) = 0$. A major solution concept for TU games is the Shapley value (Shapley [35]), ϕ, defined by $\phi_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} [v(S \cup \{i\}) - v(S)]$ for all $i \in N$.

In what follows we try to pin down a reasonable value for a coalition S using a rationale underlying the "bargaining" between S and $N\backslash S$, being inspired by the conservative (maximin) approach used by von Neumann [39] and von Neumann and Morgenstern [40] in deriving the coalitional function associated with a strategic form game. The similarity lies in that $N\backslash S$ is taken to be competing with S. The difference is that, when deriving the value of S, in von Neumann and Morgenstern [40] it is assumed that $N\backslash S$ behaves detrimentally to S without caring about what happens to $N\backslash S$ itself, whereas in the current paper we assume that $N\backslash S$ first maximizes its own payoff and then in cases where multiple such actions exist, chooses the one worst for S. Such a treatment is consistent with the standard rationality assumption used in economic analysis that an economic agent always chooses to perform the action that results in the optimal outcome for itself from among all feasible actions.

Following this perspective, a coalition S would firstly form an efficient coalition structure within itself to maximize its total wealth. If there is more than one such structure, it is to be expected to adopt the one that is most harmful to the set of the other players, $N\backslash S$. The reason is that, since $v^w(N)$ is fixed, by making $N\backslash S$ worse off, S can expect to get a larger share of the surplus when bargaining with $N\backslash S$. The same rationale can be used to derive the behavior of $N\backslash S$.

To generate a well defined coalitional value for S, the timing of the decisions of S and $N\backslash S$, whether simultaneous or sequential, matters. A simultaneous move analysis leads to a normal-form game where each coalition can choose (randomize) over the partitions it can form. Since the Nash equilibrium outcomes of this game are not necessarily unique, we run into the difficulty of having to choose one of them. A generic way of overcoming this difficulty is taking the expectation over the possible partitions generated by the players outside the coalition, as done in earlier literature (cf. Albizuri et al. [3], Macho-Stadler et al. [22]) that mostly follows an axiomatic approach. Apparently this is not convincing from a strategic point of view. In particular, such a treatment ignores important details of a game and does not seem to resolve the ambiguity in a satisfying way.

In this paper, adopting a sequential approach, the value of a coalition S is determined by having S move first and $N\backslash S$ move second, in the bargaining procedure previously described.[3] In this way the value of each coalition is uniquely determined by the partition(s)[4] generated.

Formally, given any partition function form game (N, w), we can define the associated *rational belief intermediate game* (N, v^w) as follows. For $\emptyset \neq S \subsetneq N$ and $p_S \in \mathbb{P}(S)$ denote $Q^{p_S} = \arg\max_{p_{N\backslash S}\in\mathbb{P}(N\backslash S)} \sum_{T\in p_{N\backslash S}} w(T, p_S \cup p_{N\backslash S})$ and $\mathbb{P}^{p_S} = \arg\min_{p_{N\backslash S}\in Q^{p_S}} \sum_{T\in p_S} w(T, p_S \cup p_{N\backslash S})$, and define $u(S, p_S) = \sum_{T\in p_S} w(T, p_S \cup p_{N\backslash S})$ for any $p_{N\backslash S} \in \mathbb{P}^{p_S}$ and $v^w(S) = \max\{u(S, p_S) | p_S \in \mathbb{P}(S)\}$. For $S = \emptyset$, we let $v^w(S) = 0$.

The *rational belief Shapley (RBS) value* $\phi^{RBS}(w)$ of (N, w) is the Shapley value of the rational belief intermediate game (N, v^w), i.e.,

$$\phi_i^{RBS}(w) \equiv \phi_i\left(v^w\right) = \sum_{S\subseteq N\backslash\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!}\left[v^w\left(S\cup\{i\}\right) - v^w(S)\right] \quad \text{for all } i \in N.$$

[3] A possible motivation for this order of moves stems from the way political bargaining unfolds. S can be a prevailing coalitional government that usually moves first. How much the coalitional government can achieve depends on the action of the opposite parties $N\backslash S$. Alternatively, one can think of the Stackelberg competition in oligopoly markets.

[4] There could exist multiple such partitions but they all generate the same outcome and hence determine the coalitional value uniquely.

These definitions follow a rationale, whereby the players comprising a coalition cooperate with each other, while acting strategically with respect to the players outside the coalition. Similar arguments underlie the notions of the α-core and β-core in Aumann [5] and Aumann and Peleg [6].

The *RBS* value incorporates a common-sense axiomatic approach. The first component consists of delineating what a coalition can expect to get (describing the intermediate game) in the presence of coalitional externalities; the second component applies the Shapley value to resolve the surplus sharing problem in an efficient and equitable way, which imparts to the *RBS* value the usual justifications (axiomatic motivation) that go along with the Shapley value. Providing the *RBS* value with a purely axiomatic basis, ignoring the strategic component underlying coalitional behavior, falls outside of the scope of this paper[5] and is a very interesting, yet challenging, topic for future research.

The rational belief Shapley value is based on a sequential move analysis where coalition S moves first and $N\backslash S$ follows. Naturally, reversing the order yields the *reverse rational belief intermediate game* (N, v^{wr}) with respect to (N, w).

For $\emptyset \neq S \subsetneqq N$ and $p_{N\backslash S} \in \mathbb{P}(N\backslash S)$ define $u(N\backslash S, p_{N\backslash S}) = \sum_{T\in p_{N\backslash S}} w(T, p_S \cup p_{N\backslash S})$ for any $p_S \in \mathbb{P}^{p_{N\backslash S}}$, and denote $\bar{Q}_{N\backslash S} = \arg\max_{p_{N\backslash S}\in\mathbb{P}(N\backslash S)} u(N\backslash S, p_{N\backslash S})$. We can then define $v^{wr}(S) = \min_{p_{N\backslash S}\in\bar{Q}_{N\backslash S}} \max_{p_S\in\mathbb{P}^{p_{N\backslash S}}} \sum_{T\in p_S} w(T, p_S \cup p_{N\backslash S})$ such that $v^{wr}(S) = 0$ when $S = \emptyset$. The *reverse rational belief Shapley* value $\phi^{rRBS}(w)$ of (N, w) is the Shapley value of the reverse rational belief intermediate game. i.e.,

$$\phi_i^{rRBS}(w) \equiv \phi_i\left(v^{wr}\right) = \sum_{S\subseteq N\backslash\{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!}\left[v^{wr}\left(S \cup \{i\}\right) - v^{wr}(S)\right]$$

for all $i \in N$.

A natural compromise between both polar cases (S moves first or last) leads to the *average rational belief Shapley* value of (N, w), denoted by $\phi^{aRBS}(w)$, which is defined by

$$\phi^{aRBS}(w) \equiv \frac{1}{2}\phi^{RBS}(w) + \frac{1}{2}\phi^{rRBS}(w).$$

This solution concept may well capture the effect of simultaneity and by its nature seems more equitable. Note also that $\phi^{aRBS}(w) = \phi(\frac{v^w+v^{wr}}{2})$.

3. The strategic bargaining games and implementation

To streamline the presentation, in this section we present the theorems and proofs for the partition function form games that satisfy the following condition. For any partition p_S of any coalition S, there is a unique partition, $\hat{p}_{N\backslash S}(p_S)$, of $N\backslash S$ that maximizes the payoff to $N\backslash S$ (i.e., $|Q^{p_S}| = 1$ for all $p_S \in \mathbb{P}(S)$) and furthermore $\arg\max_{p_S\in\mathbb{P}(S)} \sum_{T\in p_S} w(T, p_S \cup \hat{p}_{N\backslash S}(p_S))$ is a singleton.[6] Focusing on this class of games eliminates the purely technical complications created by ties without giving up any essential feature of the model. We present the modifications necessary to accommodate the case of ties in Section 4.

[5] The *RBS* value clearly satisfies efficiency, symmetry and the null-player axioms (their definitions can be found in Macho-Stadler, Pérez-Castrillo and Wettstein [23]), but these do not pin down a unique value.

[6] We are grateful to Roberto Serrano and Yair Tauman for suggesting we focus on this class of games for a more transparent presentation of our results.

3.1. *Implementing the* RBS *value*

We start by providing a strategic foundation for the *RBS* value by constructing a non-cooperative game that implements it in subgame perfect equilibria.

Such a game should provide incentives for the players to form, in equilibrium, an efficient coalition structure of N in order to achieve the *RBS* value payoff. However, it should also provide those players, in case they are rejected by the others, the opportunity and incentive to form a partition that best serves their interests, reflecting the competitive interaction between S and $N \backslash S$ underlying the *RBS* value. Therefore, in addition to the usual complexities present due to the partition function form environment, there is the delicate issue of balancing these two incentive forces that seem to work in opposite directions. Such a balance should lead $N \backslash S$ to indeed form the best coalition structure if the full cooperation of N fails to prevail, whereby S is aware that what it can achieve is subject to the other players forming a certain coalition structure. The options open to $N \backslash S$ should, however, be limited so that the incentive to cooperate at the level of the grand coalition still prevails.

The way to resolve this conflict is inspired by common behavior patterns observed in disputes, especially among political parties as well as business partners. Consider a power sharing dispute leading to the breakdown of a political party. It is usually the case that when a powerful member of a party quits the party, other members in similar situations or holding similar political opinions may follow this person and form a new party, with the first person to leave acting as its leader.[7] The leader then plays an important role in any possible renegotiation with the previous party. Thus, before quitting the party, the leader may bargain for more in return for staying. Similar scenarios are prevalent in the business world as well, where a powerful leader of a company leaves and founds a rival one, which later may merge with the original company.[8] Hence, the mechanism constructed below reflects realistic and natural bargaining processes.

We now briefly describe the strategic game in an informal way.

The players firstly participate and compete in a *bidding stage* to become a proposer. The proposer then pays to each player the bid she made. At the next stage, the proposer makes a monetary offer to every other player. The offer is accepted if all the other players agree. Upon acceptance, the proposer pays out the offers, forms the coalition structure she wishes (by accepting the offer all other players agree to abide by the proposer's instructions), and receives the total payoff generated by each embedded coalition of the partition. In case of rejection the proposer becomes temporarily *inactive*. She is momentarily left out of negotiation to "wait" while all the other players go again through the same procedure starting with a bidding stage. This process continues up to the point where a proposer's (we call this proposer player α for now, later formally i_s^*) offer is accepted by all other active players. All these players will form the coalition structure specified by player α and receive payoffs as described above. Now all rejected proposers become active again. They are given the chance to negotiate and form coalitions.

[7] There exist numerous examples. In Taiwan, James Soong (Soong Chu-yu) and his supporters quitted the KMT party and founded the People First Party, competing with the KMT. Then, both parties formed the Pan-Blue Coalition in the 2004 presidential elections. As a political leader in Japan, Ichirō Ozawa, together with his followers, was instrumental in splitting from, establishing and merging with parties. Geert Wilders, in the Netherlands, left the VVD party in 2004 and founded the PVV party in 2005 that became the third largest party in the 2010 Dutch general election.

[8] For instance, Steve Jobs left Apple and founded NeXT Computer in 1985. Eleven years later, Jobs went back to Apple with NeXT and became its leader again.

The first rejected proposer (we call this agent player β for now, later formally i_n^*) announces a take-it-or-leave-it offer to every other rejected proposer. If player β's offer is rejected the set of rejected proposers forms the singletons partition, whose stand-alone payoffs are subject to the coalition structure formed by the active players before, and the game ends. If the take-it-or-leave-it offer is accepted unanimously, then they form a coalition structure specified by player β, and the game enters the *renegotiation stage*.[9]

Player β makes an offer to player α. If it is accepted, player β chooses a partition of N and collects the surplus generated by that partition. Otherwise, if it is rejected, the game enters its final stage where player α chooses a partition for the set of all active players. Observing that final choice, player β chooses a partition for the set of all rejected proposers and both α and β collect the surpluses generated by the partitions they have chosen.

Note that in all cases the final payoffs to all players are calculated by taking into account all the bids and offers made throughout the game.

We now formally describe the strategic game to implement the *RBS* value.

Mechanism A If there is only one player $\{i\}$ in the game, she simply receives $w(\{i\}, \{\{i\}\})$. When there are two or more players, the mechanism is defined recursively. The mechanism proceeds with a sequence of up to n "identical" rounds possibly augmented by a renegotiation stage. Each of these rounds consists of three stages: a bidding stage, a proposal stage, and an acceptance stage. These rounds terminate whenever the acceptance stage ends with agreement, and in the case of no agreement when round n is reached.

The rounds determine for any set of (active) players S (with $s = |S| > 1$) a proposer in S. If the offer made by the proposer to the players in S is rejected, the proposer joins the set of previously rejected proposers and the remaining players in S form the new set of active players and another round starts.

If the offer is accepted and $S = N$ the game ends. In the case where the offer is accepted (in the case where $s = 1$ the "offer" is vacuously accepted) and $S \neq N$ the first rejected proposer makes an offer to all the other rejected proposers. If this offer is rejected all the rejected proposers $N \backslash S$ are partitioned into singletons and the game ends. If this offer is accepted the game enters a renegotiation stage with details to be provided later.

Each round played by S proceeds as follows (the first round starts with $S = N$).

Stage 1: Each player $i \in S$ makes $s - 1$ bids $b_j^i \in \mathbb{R}$ with $j \neq i$, to compete for being the proposer. For each $i \in S$, define the *net bid* to player i by $B^i = \sum_{j \neq i} b_j^i - \sum_{j \neq i} b_i^j$. Let $i_s^* \in \operatorname{argmax}_{i \in S}(B^i)$ where in case of a non-unique maximizer any of these maximal bidders is chosen to be the proposer with equal probability. Once i_s^* has been chosen, player i_s^* pays every player $j \in S \backslash \{i_s^*\}$ the amount $b_j^{i_s^*}$.

Stage 2: Player i_s^* proposes a payment $x_j^{i_s^*} \in \mathbb{R}$ to every player $j \in S \backslash \{i_s^*\}$.

Stage 3: The players other than i_s^*, sequentially,[10] either accept or reject the proposal. If at least one player rejects the proposal, then it is rejected. In case of a rejection player i_s^* joins the set of rejected proposers and the players in $S \backslash \{i_s^*\}$ proceed to the next round and go through the same

[9] For other applications of a multi-stage negotiation process of offers and counter-offers see Serrano and Vohra [33,34] and Einy and Wettstein [12] that implement various bargaining sets.

[10] Any sequential ordering of the players will suffice, e.g., ordering by the indices of the players.

three stages again. When an overall acceptance occurs we distinguish between the case where $s = n$ and the case $s < n$.

Case 1: $s = n$.
Player i_n^* chooses a partition $p_N^{i_n^*}$ and the game ends. The final payoff to each $j \in N\backslash\{i_n^*\}$ is given by $b_j^{i_n^*} + x_j^{i_n^*}$. The final payoff to i_n^* is then $\sum_{T \in p_N^{i_n^*}} w(T, p_N^{i_n^*}) - \sum_{j \neq i_n^*} b_j^{i_n^*} - \sum_{j \neq i_n^*} x_j^{i_n^*}$.

Case 2: $s < n$.
Player i_s^* chooses a partition $p_S^{i_s^*}$ and pays $x_j^{i_s^*}$ to every player $j \in S\backslash\{i_s^*\}$. Then, the first rejected proposer i_n^* proposes a vector of payments $y_k^{i_n^*} \in \mathbb{R}$ to every player $k \in (N\backslash S)\backslash\{i_n^*\}$. These players, sequentially, either accept or reject the proposal. If at least one player in $(N\backslash S)\backslash\{i_n^*\}$ rejects it, then it is rejected. In this case the game ends. The partition of N generated is $p_S^{i_s^*} \cup [N\backslash S]$. The payoff to each $j \in S\backslash\{i_s^*\}$ is given by $(\sum_{t=s}^n b_j^{i_t^*}) + x_j^{i_s^*}$, the payoff to i_s^* is given by $(\sum_{t=s+1}^n b_{i_s^*}^{i_t^*}) + \sum_{T \in p_S^{i_s^*}} w(T, p_S^{i_s^*} \cup [N\backslash S]) - \sum_{j \in S\backslash\{i_s^*\}} b_j^{i_s^*} - \sum_{j \in S\backslash\{i_s^*\}} x_j^{i_s^*}$, the payoff to any i_k^*, $k \in \{s+1, \ldots, n-1\}$ is $\sum_{t=k+1}^n b_{i_k^*}^{i_t^*} + w(i_k^*, p_S^{i_s^*} \cup [N\backslash S]) - \sum_{j \in N\backslash\{i_k^*, \ldots, i_n^*\}} b_j^{i_k^*}$, and the payoff to i_n^* is $w(i_n^*, p_S^{i_s^*} \cup [N\backslash S]) - \sum_{j \in N\backslash\{i_n^*\}} b_j^{i_n^*}$. If the proposal is accepted by all players in $(N\backslash S)\backslash\{i_n^*\}$, then i_n^* chooses a partition $p_{N\backslash S}^{i_n^*}$ of $N\backslash S$ and the game enters the renegotiation stage as follows.

Stage 4: Player i_n^*, as the "representative" of $N\backslash S$, makes an offer $z_{i_s^*}^{i_n^*}$ to i_s^* who is the "representative" of S. If i_s^* accepts, then i_n^* chooses a partition $\bar{p}_N^{i_n^*}$ as the final partition of N and the game ends. The payoff to each $j \in S\backslash\{i_s^*\}$ remains $(\sum_{t=s}^n b_j^{i_t^*}) + x_j^{i_s^*}$. The payoff to i_s^* is $(\sum_{t=s+1}^n b_{i_s^*}^{i_t^*}) + z_{i_s^*}^{i_n^*} - \sum_{j \in S\backslash\{i_s^*\}} b_j^{i_s^*} - \sum_{j \in S\backslash\{i_s^*\}} x_j^{i_s^*}$. The payoff to any i_k^*, where $k \in \{s+1, \ldots, n-1\}$, is still $\sum_{t=k+1}^n b_{i_k^*}^{i_t^*} + y_{i_k^*}^{i_n^*} - \sum_{j \in N\backslash\{i_k^*, \ldots, i_n^*\}} b_j^{i_k^*}$. The payoff to i_n^* is $\sum_{T \in \bar{p}_N^{i_n^*}} w(T, \bar{p}_N^{i_n^*}) - z_{i_s^*}^{i_n^*} - \sum_{k=s+1}^{n-1} y_{i_k^*}^{i_n^*} - \sum_{j \in N\backslash\{i_n^*\}} b_j^{i_n^*}$. If i_s^* rejects, then the game enters stage 5.

Stage 5: The two parties, S and $N\backslash S$, stay separate but are allowed to change their partitions. Player i_s^* moves first and chooses a partition of S, $\tilde{p}_S^{i_s^*}$, possibly the partition $p_S^{i_s^*}$ chosen before. After observing the move of i_s^*, i_n^* chooses a partition $\tilde{p}_{N\backslash S}^{i_n^*}$, possibly the partition $p_{N\backslash S}^{i_n^*}$ chosen before, and the game ends. Hence, the final partition of N is $(\tilde{p}_S^{i_s^*} \cup \tilde{p}_{N\backslash S}^{i_n^*})$. The payoff to each $j \in S\backslash\{i_s^*\}$ is again $(\sum_{t=s}^n b_j^{i_t^*}) + x_j^{i_s^*}$. The payoff to i_s^* is $(\sum_{t=s+1}^n b_{i_s^*}^{i_t^*}) + \sum_{T \in \tilde{p}_S^{i_s^*}} w(T, \tilde{p}_S^{i_s^*} \cup \tilde{p}_{N\backslash S}^{i_n^*}) - \sum_{j \in S\backslash\{i_s^*\}} b_j^{i_s^*} - \sum_{j \in S\backslash\{i_s^*\}} x_j^{i_s^*}$. The payoff to any i_k^*, where $k = s+1, \ldots, n-1$, is $\sum_{t=k+1}^n b_{i_k^*}^{i_t^*} + y_{i_k^*}^{i_n^*} - \sum_{j \in N\backslash\{i_k^*, \ldots, i_n^*\}} b_j^{i_k^*}$. The payoff to i_n^* is $\sum_{T \in \tilde{p}_{N\backslash S}^{i_n^*}} w(T, \tilde{p}_S^{i_s^*} \cup \tilde{p}_{N\backslash S}^{i_n^*}) - \sum_{k=s+1}^{n-1} y_{i_k^*}^{i_n^*} - \sum_{j \in N\backslash\{i_n^*\}} b_j^{i_n^*}$.

To analyze the subgame perfect equilibrium (SPE) outcome of the above game, we first study the subgame following an agreement being reached within the set of active players S.

Lemma 3.1. *In any SPE of the game following an acceptance of the offer made by i_s^*, $s < |N|$, the payoff to every player i_k^*, with $k \in \{s+1, \ldots, n-1\}$, is $w(i_k^*, p_S^{i_s^*} \cup [N\backslash S]) + \sum_{t=k+1}^n b_{i_k^*}^{i_t^*} -$*

$\sum_{j\in N\setminus\{i_k^*,\dots,i_n^*\}} b_j^{i_k^*}$, *where* $p_S^{i_s^*}$ *is the partition formed by* S *following the acceptance of the offer made by* i_s^*. *The payoff to* i_s^* *is* $v^w(S) + (\sum_{t=s+1}^{n} b_{i_s^*}^{i_t^*}) - \sum_{j\in S\setminus\{i_s^*\}} b_j^{i_s^*} - \sum_{j\in S\setminus\{i_s^*\}} x_j^{i_s^*}$, *and the payoff to* i_n^* *is* $v^w(N) - v^w(S) - \sum_{k=s+1}^{n-1} w(i_k^*, p_S^{i_s^*} \cup [N\setminus S]) - \sum_{j\in N\setminus\{i_n^*\}} b_j^{i_n^*}$.

The proof is provided in Appendix A.

In the next theorem, we use the following strict zero-monotonicity condition. A partition function form game w is called *strictly zero-monotonic* if for all $i \in N$, all $\emptyset \neq S \subseteq N\setminus\{i\}$, and all $p_{N\setminus(S\cup\{i\})} \in \mathbb{P}(N\setminus(S\cup\{i\}))$, all $p_{N\setminus S} \in \mathbb{P}(N\setminus S)$, all $p_{N\setminus\{i\}} \in \mathbb{P}(N\setminus\{i\})$,

$$\max_{p_{S\cup\{i\}}\in\mathbb{P}(S\cup\{i\})} \sum_{T\in p_{S\cup\{i\}}} w(T, p_{S\cup\{i\}} \cup p_{N\setminus(S\cup\{i\})}) > \left(\max_{p_S\in\mathbb{P}(S)} \sum_{T\in p_S} w(T, p_S \cup p_{N\setminus S})\right) + w\big(\{i\}, \{i\} \cup p_{N\setminus\{i\}}\big).$$

Basically, this implies that the maximal surplus that $S \cup \{i\}$ can obtain exceeds the sum of the maximal surplus that S can obtain and the stand-alone value of i. This type of condition is common in the literature[11] of implementing cooperative solution concepts: in environments that satisfy it players have an incentive to join coalitions.

We will show that for any strictly zero-monotonic game (N, w), the SPE outcomes of the game generated by Mechanism A coincide with the *RBS* value payoff vector, $\phi^{RBS}(N, w)$.

Theorem 3.2. *Mechanism A implements the rational belief Shapley value for a strictly zero-monotonic partition function form game in SPE.*

The proof is provided in Appendix A. To better understand the proof, here we provide an informal sketch of the reasoning underlying the behavior of the rejected proposers, which is a key feature of the mechanism. One readily sees that only the first rejected proposer, i_n^*, can potentially get some extra payoff beyond her stand-alone payoff given the coalition structure formed by S, because she is the proposer for all the rejected players at the take-or-leave stage. By the strict zero-monotonicity, we can deduce that any subsequent proposer will make an offer that is accepted, as otherwise he will become a rejected proposer, and will only receive a stand-alone payoff, as shown in Lemma 3.1. Hence, reasoning backwards, the first proposer i_n^*, if her offer is rejected by $N\setminus\{i_n^*\}$, will in fact end up with her stand-alone payoff, too. Consequently, despite that the first rejected player is seemingly endowed with a large advantage, it is actually impossible for her to exploit it. Then, in equilibrium, i_n^* will also make an offer that is accepted by all players in $N\setminus\{i_n^*\}$.

It should be noted that while the power to make "a take-it-or-leave-it" offer does not enable i_n^* to actually derive any direct benefits, it does enhance all subsequent proposers' positions when making an offer. It helps any such proposer to make an offer not exceeding the coalitional value of the remaining active players, as defined by the intermediate game, because such a coalitional value is the most these players can obtain were they to reject the offer, given all rejected proposers will form a coalition structure maximizing their payoffs.

This mechanism differs from the ones in Macho-Stadler et al. [22] used to implement average values for partition function form games. Rather than use a lottery, the current mechanism allows

[11] See, among others, Pérez-Castrillo and Wettstein [29], Macho-Stadler, Pérez-Castrillo and Wettstein [22], Vidal-Puga and Bergantiños [38], Vidal-Puga [37], and Hart and Mas-Colell [14].

the players to decide themselves upon the coalition structure. Another difference is the renegotiation stage inherent in our mechanism that allows for ample possibilities of implementing various solutions. Furthermore, in our setup, there is no need to employ different mechanisms depending on the nature of externalities.

The *RBS* value can be implemented with a simplified version of Mechanism A by taking out the renegotiation stage. We adopt Mechanism A for two reasons. Firstly, it shows that the *RBS* value is robust to certain forms of renegotiation. It provides a second chance to reach the equilibrium outcome: if the first proposer goes off equilibrium path at the offering stage, then she can use the renegotiation stage to reach the *RBS* value. This robustness with respect to "mistakes" has been emphasized, in general, when discussing desirable properties of mechanisms (see Postlewaite and Wettstein [32]). Secondly, it provides a convenient basis for constructing similar mechanisms to implement the other values, as shown in the following subsection, and thus we can compare these values at the same platform.

3.2. *Implementing the* reverse *and the* average RBS *values*

A straightforward, yet mechanical, way of implementing the *reverse RBS* value is to modify Mechanism A by having the rejected proposers move first and the accepted proposer act as a follower in choosing partitions. We are interested in the possibility of implementing the *reverse RBS* value without changing the order of moves in the basic mechanism. Consider a natural adjustment at the renegotiation stage such that it is player i_s^*, instead of i_n^*, who makes the renegotiation offer. Surprisingly, this suffices to implement the *reverse RBS* value.

Mechanism B The mechanism is the same as Mechanism A except for stage 4.

Stage 4: Player i_s^* makes a renegotiation offer $z_{i_n^*}^{i_s^*}$ to i_n^*. If i_n^* accepts, then i_s^* chooses a partition $\bar{p}_N^{i_s^*}$ as the final partition of N and the game ends. The payoff to each $j \in S\backslash\{i_s^*\}$ is $(\sum_{t=s}^n b_j^{i_t^*}) + x_j^{i_s^*}$. The payoff to i_s^* is $(\sum_{t=s+1}^n b_{i_s^*}^{i_t^*}) - \sum_{j\in S\backslash\{i_s^*\}} b_j^{i_s^*} - \sum_{j\in S\backslash\{i_s^*\}} x_j^{i_s^*} + \sum_{T\in\bar{p}_N^{i_s^*}} w(T, \bar{p}_N^{i_s^*}) - z_{i_n^*}^{i_s^*}$. The payoff to any i_k^*, where $k = s+1, \ldots, n-1$, equals $\sum_{t=k+1}^n b_{i_k^*}^{i_t^*} + y_{i_k^*}^{i_n^*} - \sum_{j\in N\backslash\{i_k^*,\ldots,i_n^*\}} b_j^{i_k^*}$. The payoff to i_n^* is $z_{i_n^*}^{i_s^*} - \sum_{k=s+1}^{n-1} y_{ik^*}^{i_n^*} - \sum_{j\in N\backslash\{i_n^*\}} b_j^{i_n^*}$. If the renegotiation offer is rejected by i_n^*, then the game enters stage 5 which is the same as in Mechanism A.

There are two steps to show Mechanism B implements the *reverse RBS* value in SPE. Lemma 3.3 shows that in any SPE of the subgame following an acceptance by S with $s = |S| < n$, the payoff to the set of rejected proposers $N\backslash S$ equals $v^{wr}(N\backslash S)$.[12] The proof is omitted as it is similar to that of Lemma 3.1. Theorem 4.2 shows that any SPE outcome of the mechanism coincides with the *reverse RBS* value, relying on the self-duality of the Shapley value.

Lemma 3.3. *In any SPE of the game following an acceptance by S with $s < |N|$, the payoff to every player i_k^*, with $k \in \{s+1, \ldots, n-1\}$, is $w(i_k^*, p_S^{i_s^*} \cup [N\backslash S]) + \sum_{t=k+1}^n b_{i_k^*}^{i_t^*} - \sum_{j\in N\backslash\{i_k^*,\ldots,i_n^*\}} b_j^{i_k^*}$. The payoff to i_s^* equals $v^w(N) - v^{wr}(N\backslash S) + (\sum_{t=s+1}^n b_{i_s^*}^{i_t^*}) - \sum_{j\in S\backslash\{i_s^*\}} b_j^{i_s^*} -$*

[12] We adopt the assumption that when facing the same payoffs, players prefer to be in a larger coalition, as appearing in Moldovanu and Winter [25] and Vidal-Puga and Bergantiños [38]. It implies that if i_n^* receives the same payoff by standing alone or being the representative of $N\backslash S$, then i_n^* prefers the latter.

$\sum_{j\in S\setminus\{i_s^*\}} x_j^{i_s^*}$, *and the payoff to* i_n^* *equals* $v^{wr}(N\setminus S) - \sum_{k=s+1}^{n-1} w(i_k^*, p_S^{i_s^*} \cup [N\setminus S]) - \sum_{j\in N\setminus\{i_n^*\}} b_j^{i_n^*}$.

Theorem 3.4. *Mechanism B implements the* reverse RBS *value for a strictly zero-monotonic partition function form game in SPE.*

The proof is provided in Appendix A.

To implement the *average RBS* value, one can modify Mechanism A with an exogenous "flip a coin" stage such that both parties have the same probability to be the first mover or follower. To construct an implementation mechanism with an endogenous adjustment, building on Ju and Wettstein [20], we could have the accepted proposer i_s^* and the first rejected proposer i_n^* bid for the right to be the first mover (or follower). Unfortunately, this does not lead to the desired outcome in the current context where externalities prevail.

Instead, in the same spirit of Mechanism B, we investigate what happens if both players bid for the right to make the renegotiation offer. Quite nicely, such a mechanism yields an average of the *RBS* value and the *reverse RBS* value as the unique SPE outcome.

Mechanism C The rules of stages 1, 2, and 3 are the same as in Mechanism A or Mechanism B. The difference arises from stage 4 onwards, due to an extra bidding stage in renegotiation.

Stage 4 (bidding for renegotiation): i_n^* and i_s^* simultaneously submit bids $\tilde{b}_{i_s^*}^{i_n^*}$ and $\tilde{b}_{i_n^*}^{i_s^*}$, respectively. The player with the larger (net) bid pays the bid to the other player and becomes the proposer in renegotiation. In case of identical bids, the proposer is randomly chosen.

Stage 5 (renegotiation offer): Dependent on whether the proposer is i_n^* or i_s^*, the game proceeds as in Mechanism A (when i_n^* is the proposer) or Mechanism B (when i_s^* is the proposer). If i_n^* is the proposer, then i_n^* makes the renegotiation offer $z_{i_s^*}^{i_n^*}$ to i_s^*. If the offer is accepted, then i_n^* can unite S and $N\setminus S$ together and organize all players into a partition $\bar{p}_N^{i_n^*}$. The payoffs to all players can be readily derived by taking into account the bids paid or received at stage 4. If the offer is rejected, then the games enters stage 6. A similar process applies if i_s^* is the proposer from stage 4.

Stage 6: This stage is identical to stage 5 in Mechanism A or Mechanism B.

Theorem 3.5. *Mechanism C implements the* average RBS *value for a strictly zero-monotonic partition function form game in SPE.*

The proof can be readily constructed along the similar lines in those of Theorem 3.2 and Theorem 3.4, together with the definition of the *average RBS* value.

4. Implementation in the case of multiple best responses

In games where for any partition p_S of any coalition S there exist multiple partitions of $N\setminus S$ that maximize the payoff to $N\setminus S$, the previous mechanisms still weakly implement the corresponding values, however there exist other SPE outcomes. In this section we present a mechanism for implementing the *RBS* value when there are multiple best responses. Moreover, we will indicate how this can be done for the other two values.

First of all, we still use the same renegotiation stage as in Mechanism A. It is not difficult to see that i_n^* has an incentive to adopt a coalition structure, in case of a tie, that is worst to S. Next, to prevent non-credible threats, i_n^* is allowed to move again after the offer was rejected to ensure i_n^* will indeed choose an action among those that are equally favorable to $N\backslash S$. However, there still exists the problem of multiple SPE outcomes. If i_s^* rejects the renegotiation offer, i_n^* is again indifferent as to the coalition structure she picks and i_s^* might believe that i_n^* would pick a coalition structure that is actually better for i_s^*. This problem is addressed by introducing a small cost for switching between partitions.

Prior to the formal description of the mechanism, we introduce a new variable related to the gain of switching from one partition to another, which is denoted by $\hat{\epsilon}$ and defined as follows. Consider the following set of numbers $|\sum_{T\in p_S} w(T, p_S \cup p_{N\backslash S}) - \sum_{T\in p'_S} w(T, p'_S \cup p_{N\backslash S})|$, for any $\emptyset \neq S \subseteq N$, any $p_{N\backslash S} \in \mathbb{P}(N\backslash S)$, and any $p_S, p'_S \in \mathbb{P}(S)$. If this set contains positive numbers, let $\hat{\epsilon}$ be the smallest of them.

Mechanism D The mechanism is identical to Mechanism A throughout stages 1–4. At stage 5, which is reached if the renegotiation offer at stage 4 is rejected by i_s^*, the players in S remain with the partition $p_S^{i_s^*}$, whereas player i_n^* if she wishes can, by incurring a small cost $\epsilon > 0$,[13] choose a partition $\tilde{p}_{N\backslash S}^{i_n^*}$ different from $p_{N\backslash S}^{i_n^*}$ for $N\backslash S$ and the game ends.

If i_n^* chooses to keep the partition $p_{N\backslash S}^{i_n^*}$ as before, the final partition of N is $p_S^{i_s^*} \cup p_{N\backslash S}^{i_n^*}$. The payoff to each $j \in S\backslash\{i_s^*\}$ is given by $(\sum_{t=s}^n b_j^{i_t^*}) + x_j^{i_s^*}$. The payoff to i_s^* is given by $(\sum_{t=s+1}^n b_{i_s^*}^{i_t^*}) + \sum_{T\in p_S^{i_s^*}} w(T, p_S^{i_s^*} \cup p_{N\backslash S}^{i_n^*}) - \sum_{j\in S\backslash\{i_s^*\}} b_j^{i_s^*} - \sum_{j\in S\backslash\{i_s^*\}} x_j^{i_s^*}$. The payoff to any $i_k^* \in (N\backslash S)\backslash\{i_n^*\}$ is $\sum_{t=k+1}^n b_{i_k^*}^{i_t^*} + y_{i_k^*}^{i_n^*} - \sum_{j\in N\backslash\{i_k^*,\dots,i_n^*\}} b_j^{i_k^*}$. The payoff to i_n^* is given by $\sum_{T\in p_{N\backslash S}^{i_n^*}} w(T, p_S^{i_s^*} \cup p_{N\backslash S}^{i_n^*}) - \sum_{k\in(N\backslash S)\backslash\{i_n^*\}} y_k^{i_n^*} - \sum_{j\in N\backslash\{i_n^*\}} b_j^{i_n^*}$.

If i_n^* chooses to change the partition into $\tilde{p}_{N\backslash S}^{i_n^*}$, the final partition of N is $p_S^{i_s^*} \cup \tilde{p}_{N\backslash S}^{i_n^*}$. The payoffs are as before with $(p_S^{i_s^*} \cup \tilde{p}_{N\backslash S}^{i_n^*})$ replacing $(p_S^{i_s^*} \cup p_{N\backslash S}^{i_n^*})$ for all players, except for player i_n^* who receives $\sum_{T\in p_{N\backslash S}^{i_n^*}} w(T, p_S^{i_s^*} \cup \tilde{p}_{N\backslash S}^{i_n^*}) - \sum_{k\in(N\backslash S)\backslash\{i_n^*\}} y_k^{i_n^*} - \sum_{j\in N\backslash\{i_n^*\}} b_j^{i_n^*} - \epsilon$.

We can now state the following lemma and theorem, but omit their proofs because of the similarity to those of Lemma 3.1 and Theorem 3.2.

Lemma 4.1. *In any SPE of the game following an acceptance by S with $s = |S| < n$, the payoff from that point on (so here we provisionally skip the bids and offers which are irrelevant for this subgame) to every player i_k^*, with $k \in \{s+1, \dots, n-1\}$, equals $w(i_k^*, p_S^{i_s^*} \cup [N\backslash S])$, the payoff to i_s^* equals $\sum_{T\in p_S^{i_s^*}} w(T, p_S^{i_s^*} \cup p_{N\backslash S}^{i_n^*})$, and the payoff to i_n^* equals $v^w(N) - \sum_{T\in p_S^{i_s^*}} w(T, p_S^{i_s^*} \cup p_{N\backslash S}^{i_n^*}) - \sum_{k=s+1}^{n-1} w(i_k^*, p_S^{i_s^*} \cup [N\backslash S])$, where $p_{N\backslash S}^{i_n^*} \in \mathbb{P}^{p_S^{i_s^*}}$.*

Theorem 4.2. *Mechanism D implements the* RBS *value for a strictly zero-monotonic partition function form game in SPE.*

[13] If $\hat{\epsilon}$ exists, $\epsilon < \hat{\epsilon}$, and otherwise ϵ can be any positive number.

The reason why Mechanism D leads the set of rejected players $N \backslash S$ to choose a partition that, while best for them, minimizes the payoff to the others, lies in the subtle interaction between the strategic possibilities within the renegotiation stage and the option for i_n^* to change partition one more time, at some cost, if no agreement is reached in renegotiation. Due to the renegotiation stage, rejected players will adopt a partition detrimental to their opponents. The option of changing partition at cost ϵ lends credibility to the threat since the rejected players would not switch to another partition even if renegotiation is unsuccessful.

To obtain the *reverse RBS* value and *average RBS* value in this case, one can use mechanisms B and C by assuming lexicographic preferences[14] similar to those in Abreu and Rubinstein [1], Pérez-Castrillo and Wettstein [28], and Hyndman and Ray [17].

5. Conclusion

We provided a general framework to analyze strategic bargaining and values in environments with coalitional externalities. We started by using a basic rationale for determining the value of a coalition and obtained three distinct values of a coalition in such a setting. These coalitional values generated, in turn, three Shapley-like values. To examine their strategic foundation we constructed mechanisms implementing them. The common feature is a multi-bidding and offering mechanism to which we added coalition formation and renegotiation stages. Two major innovations are the ability to form coalitions for the set of active players as well as the set of rejected players and the option for renegotiation.

There are several immediate extensions to the analysis in the paper. One could further modify the renegotiation stages to generate a large family of values and then study their plausibility in light of existing social norms and institutions. For example, if we disallow players to change partitions after rejecting the offer in renegotiation, this will lead to "pessimistic" values in equilibrium. Likewise, one may study a mechanism to implement a "Shapley" value defined for altruistic players, like the benevolent assumption of benignness used in Hyndman and Ray [17]. One possible way to carry out the implementation is by introducing "bribes". Other extensions may include changing the rule according to which the "leader" of the rejected proposers is determined. For instance, if the last rejected proposer is made the leader, it can be shown that the counterpart of the equal surplus value in TU games for partition function form games is the outcome implemented. It would also be of interest to see what happens if the leader is randomly selected, or, alternatively, determined via yet another multi-bidding procedure. Furthermore, while keeping the basic analytical framework, one could extend the model by following a process of sequential formation of coalitions for rejected players.

One can explore other behavior patterns in bargaining. For instance, it is interesting to study the case where players in $N \backslash S$ behave in more sophisticated ways. That is, any action taken by S may trigger a chain of reactions by the players in $N \backslash S$ who may be supposedly farsighted players. The mechanism presented in the paper may also be used to analyze the global games (Gilboa and Lehrer [13]) from a strategic perspective.

Finally, the values constructed and the mechanisms implementing them can be used to analyze concrete settings, in environmental economics (see Ambec and Ehlers [4] that address a river sharing problem), international economics (see Aghion, Antràs and Helpman [2] that analyzes among other things the effects of coalitional externalities on the pattern of trade agreements

[14] A player cares first and foremost about his own payoff but when indifferent, prefers a choice that minimizes the opponent's payoff.

negotiated and formed among countries), industrial organization (see Bloch [7] for a survey of coalition formation in industrial organization, and Jelnov and Tauman [18] for determining the private value of a patent).

Appendix A

Proof of Lemma 3.1. We split the proof into the following parts by solving the game backwards.

Part (a). In any SPE at stage 5, player i_n^* adopts the unique partition $\hat{p}_{N\setminus S}(\tilde{p}_S^{i_s^*})$, where $\{\hat{p}_{N\setminus S}(\tilde{p}_S^{i_s^*})\} = Q^{\tilde{p}_S^{i_s^*}} = \mathbb{P}^{\tilde{p}_S^{i_s^*}}$, for any partition $\tilde{p}_S^{i_s^*}$ chosen by i_s^* at stage 5.

Part (b). In any SPE at stage 5, player i_s^* adopts the unique partition p_S^*, where $\{p_S^*\} = \arg\max_{p_S \in \mathbb{P}(S)} u(S, p_S)$, taking into account the response of i_n^* (derived in Part (a)) at stage 5. Hence, the payoffs from stage 5 are given by $v^w(S)$ to i_s^* and $v^{wr}(N\setminus S)$ to i_n^*.

Part (c). In any SPE at stage 4, i_s^* accepts any offer $z_{i_s^*}^{i_n^*} > v^w(S)$, and rejects any $z_{i_s^*}^{i_n^*} < v^w(S)$. The reason is that i_s^* can, at stage 5 by Part (b), secure the payoff $v^w(S)$. In case $v^w(N) > v^w(S) + v^{wr}(N\setminus S)$, in any SPE i_n^* makes an acceptable offer. Therefore, the SPE strategy for i_n^* is to offer $v^w(S)$ to i_s^* and for i_s^* is to accept any offer $z_{i_s^*}^{i_n^*} \geq v^w(S)$. Hence the SPE payoffs at this stage are $v^w(S)$ to i_s^* and $v^w(N) - v^w(S)$ to i_n^*, since following the acceptance by i_s^*, i_n^* will form a partition generating the surplus $v^w(N)$. In case $v^w(N) = v^w(S) + v^{wr}(N\setminus S)$, there exists additional SPE strategies where i_n^* offers less than $v^w(S)$ and is thus rejected. However, the final payoffs for this stage to i_s^* and i_n^* would still be $v^w(S)$ and $v^w(N) - v^w(S)$, respectively. Note that $v^w(N) < v^w(S) + v^{wr}(N\setminus S)$ is not possible by the definition of $v^w(N)$.

Part (d). In any SPE at stage 3, given the partition $p_S^{i_s^*}$ chosen by i_s^*, any i_k^*, with $k \in \{s+1, \ldots, n-1\}$, accepts any offer $y_{i_k^*}^{i_n^*} > w(i_k^*, p_S^{i_s^*} \cup [N\setminus S])$. This is because a rejection would only yield $w(i_k^*, p_S \cup [N\setminus S])$. Clearly, if $y_{i_k^*}^{i_n^*} < w(i_k^*, p_S \cup [N\setminus S])$ then it is rejected.

Part (e). In any SPE at stage 3, i_n^* adopts the following strategy. Firstly, if i_s^* chooses a partition p_S such that $v^w(N) - \sum_{k=s+1}^{n-1} w(\{i_k^*\}, p_S \cup [N\setminus S]) - v^w(S) < w(\{i_n^*\}, p_S \cup [N\setminus S])$, then i_n^* makes an unacceptable offer to some player(s) i_k^*, with $k \in \{s+1, \ldots, n-1\}$, since i_n^* would be better off with the stand alone payoff. Secondly, if p_S is such that $v^w(N) - \sum_{k=s+1}^{n-1} w(\{i_k^*\}, p_S \cup [N\setminus S]) - v^w(S) > w(\{i_n^*\}, p_S \cup [N\setminus S])$, then in any SPE i_n^* offers $w(\{i_k^*\}, p_S \cup [N\setminus S])$ to all players i_k^*, with $k \in \{s+1, \ldots, n-1\}$ which will be accepted. Obviously, by making such an offer, the payoff to i_n^* from this stage onwards (using the previous parts of the proof) would be $v^w(N) - \sum_{k=s+1}^{n-1} w(\{i_k^*\}, p_S \cup [N\setminus S]) - v^w(S)$ which is greater than $w(\{i_n^*\}, p_S \cup [N\setminus S])$, the payoff when making an unacceptable offer. Thirdly, if p_S is such that $v^w(N) - \sum_{k=s+1}^{n-1} w(\{i_k^*\}, p_S \cup [N\setminus S]) - v^w(S) = w(\{i_n^*\}, p_S \cup [N\setminus S])$, then i_n^* is indifferent between the two choices and thus may adopt any in an SPE.

Part (f). In any subgame perfect equilibrium, at stage 3 i_s^* always chooses a partition p_S such that $v^w(N) - \sum_{k=s+1}^{n-1} w(\{i_k^*\}, p_S \cup [N\setminus S]) - v^w(S) \geq w(\{i_n^*\}, p_S \cup [N\setminus S])$.

Now suppose that i_s^* at stage 3 chooses p'_S such that $v^w(N) - \sum_{k=s+1}^{n-1} w(\{i_k^*\}, p'_S \cup [N\setminus S]) - v^w(S) < w(\{i_n^*\}, p'_S \cup [N\setminus S])$. Then, firstly, it follows that $\sum_{T \in p'_S} w(T, p'_S \cup [N\setminus S]) < v^w(S)$. For, supposing that $\sum_{T \in p'_S} w(T, p'_S \cup [N\setminus S]) \geq v^w(S)$, we would find that

$$\sum_{T\in p'_S} w\big(T, p'_S \cup [N\backslash S]\big) + \sum_{k=s+1}^{n-1} w\big(\{i_k^*\}, p'_S \cup [N\backslash S]\big) + w\big(\{i_n^*\}, p'_S \cup [N\backslash S]\big)$$
$$\geq v^w(S) + \sum_{k=s+1}^{n-1} w\big(\{i_k^*\}, p'_S \cup [N\backslash S]\big) + w\big(\{i_n^*\}, p'_S \cup [N\backslash S]\big)$$
$$> v^w(N).$$

This is a contradiction because by definition $v^w(N) \geq \sum_{T\in p_N} w(T, p_N)$ for all $p_N \in \mathbb{P}(N)$. Secondly, by Part (e) it follows that at stage 3 i_n^* would make an offer to be rejected by players in $(N\backslash S)\backslash\{i_n^*\}$, which results in the payoff $\sum_{T\in p'_S} w(T, p'_S \cup [N\backslash S])$ to i_s^*. However, choosing p_S such that $v(N) - \sum_{k=s+1}^{n-1} w(\{i_k^*\}, p_S \cup [N\backslash S]) - v^w(S) \geq w(\{i_n^*\}, p_S \cup [N\backslash S])$ will lead to payoff $v^w(S)$ to i_s^* on the basis of parts (a) to (e), which is higher than $\sum_{T\in p'_S} w(T, p'_S \cup [N\backslash S])$.

Hence, taking into account the relevant bids paid or received, parts (a)–(f) readily imply the equilibrium payoffs for i_s^*, i_k^* with $k \in \{s+1, \ldots, n-1\}$, and i_n^* as stated in the lemma. □

Proof of Theorem 3.2. Let (N, w) be a strictly zero-monotonic partition function form game. The proof consists of two parts, first we show that the *RBS* value payoff is indeed an equilibrium outcome, and then we show that every SPE yields the payoff vector $\phi^{RBS}(N, w)$.

To show that the *RBS* value payoff $\phi^{RBS}(N, w)$ is indeed an SPE outcome, we explicitly construct a profile of strategies constituting an SPE, yielding the *RBS* value, $\phi^{RBS}(N, w)$, as an outcome. We describe the strategies followed by the players for any part of the game they might participate in. Let each player $i \in N$ take the following strategy. We distinguish between the cases where i belongs to the set of active players, and the case where i is a rejected proposer. Let S be the (current) set of active players.

Case 1: player $i \in S$, $|S| > 1$.

At stage 1, player $i \in S$ bids $b_j^i = \phi_j(S, v_S^w) - \phi_j(S\backslash\{i\}, v_{S\backslash\{i\}}^w)$ for every $j \in S\backslash\{i\}$, where the game $(S\backslash\{i\}, v_{S\backslash\{i\}}^w)$ is defined by $v_{S\backslash\{i\}}^w(T) = v^w(T)$ for all $T \subseteq S\backslash\{i\}$.

At stage 2, if i is chosen as the proposer, i offers $x_j^i = \phi_j(S\backslash\{i\}, v_{S\backslash\{i\}}^w)$ to every $j \in S\backslash\{i\}$.

At stage 3, if $i = i_s^*$ and his offer is accepted by all players in $S\backslash\{i\}$, then i chooses the partition p_S^*; if $j \in S\backslash\{i\}$ is the proposer, then i accepts any offer that is greater than or equal to $\phi_i(S\backslash\{j\}, v_{S\backslash\{j\}}^w)$ and rejects any offer strictly smaller than $\phi_i(S\backslash\{j\}, v_{S\backslash\{j\}}^w)$.[15]

At stage 4, player i (in case $i = i_s^*$) accepts any offer greater than or equal to $v^w(S)$ and rejects any offer strictly less than $v^w(S)$.

At stage 5, player i (in case $i = i_s^*$) chooses the partition p_S^*.

Case 2: player $i \in N\backslash S$. i will remain inactive until the offer of the proposer in S was accepted by the other players in S. Once S has reached an agreement, i proceeds as follows.

At stage 3, if $i = i_n^*$, i offers $y_j^i = w(\{j\}, p_S^{i_s^*} \cup [N\backslash S])$ to every $j \in (N\backslash S)\backslash\{i\}$, given $p_S^{i_s^*}$ adopted by S. Moreover, if the offer is accepted by every $j \in (N\backslash S)\backslash\{i\}$, then i forms the coalition structure $\hat{p}_{N\backslash S}(p_S^{i_s^*})$ within $N\backslash S$, where $\hat{p}_{N\backslash S}(p_S^{i_s^*}) \in Q^{p_S^{i_s^*}}$. If $i \in \{i_{s+1}^*, \ldots, i_{n-1}^*\}$, then i

[15] Note that in the case where $|S| = 1$ player i is perceived as a proposer whose offer (to himself), $w(\{i\}, \{\{i\}\} \cup \hat{p}_{N\backslash\{i\}}(\{\{i\}\})) = \phi_i(\{i\}, v_{\{i\}}^w)$, is accepted.

accepts any offer that is no less than $w(\{i\}, p_S^{i_s^*} \cup [N\backslash S])$ and rejects it otherwise. Note that if $N\backslash S = \{i\}$, as the only rejected player, i accepts $w(\{i\}, p_{N\backslash\{i\}}^{i_{n-1}^*} \cup \{\{i\}\})$.

At stage 4, if $i = i_n^*$, i makes the offer $v^w(S)$ to i_s^*.

At stage 5, if $i = i_n^*$, i chooses the partition $\hat{p}_{N\backslash S}(\tilde{p}_S^{i_s^*})$ where $\tilde{p}_S^{i_s^*}$ is the partition chosen at this stage by i_s^*.

Obviously this profile of strategies yields acceptance when $S = N$, and all players obtain their *RBS* value payoffs. To check that the above strategies constitute an SPE, first note that according to Lemma 3.1 the strategies adopted at stages 4 and 5 and the strategies adopted by the rejected proposers at stage 3 are on a subgame perfect equilibrium path. To show that the strategies of the active players at stages 1, 2 and 3 are on a subgame perfect equilibrium path we proceed by induction. The induction assumption is that these are equilibrium strategies, whenever the game reaches a stage with s active players comprising the set S. This is "trivially" true for $s = 1$.

Assume now that the prescribed strategies constitute equilibrium strategies for any stage with s active players. To show they are equilibrium strategies for an arbitrary set of $s+1$ active players note the following: The strategies of active players at stage 3 are on an SPE path by the induction assumption; the strategy of the proposer at stage 2 is on an SPE path for a proposer that wishes to make an acceptable offer (the payments offered are the minimal ones that would generate an acceptance). The reason that any proposer, other than i_n^*, would like to make an acceptable offer is due to the strict zero-monotonicity. Such a proposer will receive, by the induction assumption, in case the offer is rejected, $w(\{i_{s+1}^*\}, p_S^{i_s^*} \cup [N\backslash S])$. On the other hand, if his offer were accepted he would have received the larger payoff $v^w(S \cup \{i_{s+1}^*\}) - v^w(S)$.[16] Moreover, by backwards induction, the first proposer, i_n^*, can foresee that in case of rejection, she will reach the renegotiation stage as a singleton and then would get $v^w(N) - v^w(N\backslash\{i_n^*\})$. However, this is the same payoff she would receive were she to make the offer at stage 2 that leads to acceptance. Hence making an acceptable offer by the first proposer is part of an SPE. It remains to verify that stage 1 strategies are on an SPE path. By the balanced contributions property (Myerson [27]) we know that

$$\phi_i\big(N, v^w\big) - \phi_i\big(N\backslash\{j\}, v^w_{N\backslash\{j\}}\big) = \phi_j\big(N, v^w\big) - \phi_j\big(N\backslash\{i\}, v^w_{N\backslash\{i\}}\big)$$

for all $i, j \in N$, and consequently, $B^i = 0$ for all $i \in N$. Thus, the proposer is randomly determined, furthermore the payoff of each player is his *RBS* value. Any deviation from the bids made by player i from $(b_j^i)_{j\neq i}$ to $(b_j'^i)_{j\neq i}$, with $b_j'^i \neq b_j^i$ for at least some $j \in N\backslash\{i\}$, will lead to one of the following two cases. Case 1: Some player other than i becomes the proposer, in which case player i's payoff does not change. Case 2: Player i remains the proposer in which case it must be that $\sum_{j\neq i} b_j'^i \geq \sum_{j\neq i} b_j^i$. Hence, Player i's payoff could not increase in this case either. Thus, Player i has no incentive to deviate from this bidding strategy at stage 1.

Hence we have shown that the induction assumption is satisfied for any s, and the case $s = n$ shows this strategy profile is an SPE for the game induced by Mechanism A.

To show that every SPE yields the payoff vector $\phi^{RBS}(N, w)$, we proceed by induction as well. The induction assumption is that whenever the game reaches a round with s active players comprising the set S, then in any SPE of the game, S partitions according to p_S^* and the payoff to any player j in S from that stage onwards is $\phi_j^{RBS}(S, v^w)$. The induction assumption holds for $s = 1$, because when $s = 1$ the "offer" is vacuously accepted and by Lemma 3.1 the $n - 1$

[16] One can readily see that the strict zero-monotonicity implies that for all $i \in N$, all $\emptyset \neq S \subseteq N\backslash\{i\}$ and all $p_S \in \mathbb{P}(S)$, $v^w(S \cup \{i\}) > v^w(S) + w(\{i\}, p_S \cup [N\backslash S])$.

rejected proposers reach the partition $\hat{p}_{N\setminus\{i_1^*\}}(\{i_1^*\})$ and the payoff to $\{i_1^*\}$ is indeed $\phi_{i_1^*}^{RBS}(i_1^*, v^w)$. We now assume it holds for any set of $s-1$ active players. To show it holds for any set of s active players we proceed by a series of claims.[17]

Claim 1. *In any SPE a player $j \in S\setminus\{i_s^*\}$ does not accept any offer below $\phi_j^{RBS}(S\setminus\{i_s^*\}, v^w)$ and accepts any offer above $\phi_j^{RBS}(S\setminus\{i_s^*\}, v^w)$. Furthermore, the proposer i_s^* does not make offers larger than these values.*

It follows from the induction assumption. Moreover, i_s^* will not offer strictly more than these values since a larger payoff could be achieved by offering slightly less.

Claim 2. *In any SPE i_s^* offers $x_j^{i_s^*} = \phi_j^{RBS}(S\setminus\{i_s^*\}, v^w)$ to every player $j \in S\setminus\{i_s^*\}$, and each player $j \in S\setminus\{i_s^*\}$ accepts the offer. Furthermore, the final partition of S is given by p_S^*.*

By the induction assumption and the same reasoning as in the proof of Lemma 3.1, i_s^* has no incentive to make an offer to be rejected. Then, by Claim 1, the only SPE equilibrium strategies leading to acceptance are the ones described above. Furthermore, note that, by the SPE behavior in stages 4 and 5, the partition formed by S is p_S^*.

Claim 3. *In any subgame perfect equilibrium the net bids are all zero, i.e. $B^i = 0$ for all $i \in S$. Any player $i \in S$ is indifferent with respect to the identity of the proposer.*

Let $\Psi = \{i \in S \mid B^i = \max_{j\in S}(B^j)\}$, if $\Psi = S$ all net bids are zero since $\sum_{i\in S} B^i = 0$. To show that $\Psi = S$ we note $\Psi \neq S$ cannot happen in an SPE. If $\Psi \neq S$, any player $i \in \Psi$ can obtain a larger payoff by changing his bids. Choose a player $j \notin S\setminus\Psi$. Let player $i \in \Psi$ change his bids by announcing $b'^i_k = b^i_k + \epsilon$ for all $k \in \Psi\setminus\{i\}$, and $b'^i_j = b^i_j - |\Psi|\epsilon$ for j, and $b'^i_l = b^i_l$ for all $l \in S\setminus\{\Psi \cup \{j\}\}$. The new net bids lead to the same set Ψ with a lower aggregate payment for player i if chosen as the proposer, which contradicts the fact the original bids formed part of an SPE. Then, since all net bids are the same, if there is a player i that could gain from player j being the proposer, he could slightly adjust his bids and increase his payoff, which is impossible if the bids were formed part of an SPE.

Claim 4. *In any SPE, the payoff of player $i \in S$ from that round (where S is the set of active players) onwards is $\phi_i^{RBS}(S, v^w)$.*

By Claim 2 if player i is the proposer his payoff is $v^w(S) - v^w(S\setminus\{i\}) - \sum_{j\in S\setminus\{i\}} b^i_j$. If player $j \in S\setminus\{i\}$ is the proposer, player i's payoff is $\phi_i^{RBS}(S\setminus\{j\}, v^w) + b_i^j$. By Claim 3 all these payoffs are the same, hence, player i's payoff is

$$\frac{1}{s}\Bigg(\sum_{j\in S\setminus\{i\}} \phi_i^{RBS}\big(S\setminus\{j\}, v^w\big) + b_i^j + v^W(S) - v^w\big(S\setminus\{i\}\big) - \sum_{j\in S\setminus\{i\}} b^i_j\Bigg)$$
$$= \frac{1}{s}\Bigg(\sum_{j\in S\setminus\{i\}} \phi_i^{RBS}\big(S\setminus\{j\}, v^w\big) + v^w(S) - v^w\big(S\setminus\{i\}\big)\Bigg),$$

[17] As noted before, there are two types of equilibria, both leading to identical payoff distributions. In the proof we concentrate on the equilibrium where the first proposer makes an acceptable offer.

since the net bid of any player i is zero (Claim 3). Readily one can verify that the last expression equals $\phi_i^{RBS}(S, v^w)$ (see Myerson [27]). □

Proof of Theorem 3.4. By Lemma 3.3 the payoffs to S and $N\backslash S$ in any SPE of the subgame starting from stage 3 of Mechanism B are $v^w(N) - v^{wr}(N\backslash S)$ and $v^{wr}(N\backslash S)$, respectively. Therefore, the players in S when bidding for the proposer of S and when making their offers, once chosen as the proposer, to the other players, actually compete for obtaining the surplus $v^w(N) - v^{wr}(N\backslash S)$. The proof thus proceeds as in Theorem 3.2, with the underlying "characteristic function", describing the payoff to coalition S, given by $v^w(N) - v^{wr}(N\backslash S)$ rather than $v^w(S)$.

Before proceeding we recall the definition of a dual game and the self-duality of the Shapley value for TU games (cf. Kalai and Samet [21]). Given a TU game $v \in TU^N$, its dual game (N, v^d) is defined by $v^d(S) = v(N) - v(N\backslash S)$ for all $S \subseteq N$. A solution concept f satisfies self-duality if $f(v) = f(v^d)$ for every $v \in TU^N$. It is well known that the Shapley value possesses self-duality. Applying this to our setup we let the "original" game be given by $v(S) = v^{wr}(S)$, the dual game is then $v^d(S) = v^w(N) - v^{wr}(N\backslash S)$ for all $S \subseteq N$. Furthermore, $\phi(N, v^d) = \phi(N, v^{wr})$ which is $\phi^{rRBS}(N, w)$.

Let (N, w) be a strictly zero-monotonic partition function form game. To show that the *reverse RBS* value payoff $\phi^{rRBS}(N, w)$ is indeed an SPE outcome, one proceeds as in the proof of Theorem 3.2 and can readily verify that $\phi(N, v^d)$ is an SPE outcome, which by self-duality is $\phi^{rRBS}(N, w)$. To show that every SPE yields the payoff vector $\phi^{rRBS}(N, w)$, i.e., $\phi(N, v^d)$, we proceed by induction following arguments similar to those appearing in Theorem 3.2. □

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