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The provision of public goods via advertising

Existence of equilibria and welfare analysis

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This paper considers an economy where a public good is provided via advertising. The consumers' preferences are represented using the 'characteristics' approach', and the advertising has a potentially enhancing effect on the characteristics' content of a given commodity. We define competitive equilibria and show they exist. The welfare properties of the resulting allocations are analyzed, and conditions for over- and under-production of the public good are provided.

1. Introduction

The phenomenon of public goods has been extensively discussed in the economics literature. It is well known that in the presence of public goods the free market system fails to allocate resources efficiently.

For economies with public goods, one can define the notion of a Lindahl equilibrium. It can be demonstrated, under reasonable assumptions, that this equilibrium exists. Furthermore, it can be shown that any Lindahl equilibrium is Pareto optimal, and any Pareto-optimal allocation can be supported as a Lindahl equilibrium, following an appropriate redistribution of endowments.

Lindahl equilibria, however, suffer from several practical problems (the

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foremost being the need for a large number of very thin markets). They are rarely encountered in practice, despite their appealing normative properties. Several alternative methods have been suggested to decide upon the amounts of public goods provided, as well as the appropriate financing methods.

This paper is concerned with one of these alternative methods. We look at a very common public good, TV broadcasting,[1] which is financed to a large extent by advertising revenues. We address the question of whether or not firms, wishing to advertise their products by financing TV broadcasts, provide the public with an optimal quantity of the public good.

The producers hope that by advertising they will be able to raise their prices. The consumers enjoy the public good (TV broadcasts) and might end up paying higher prices for the advertised goods. Questions regarding the optimality properties of this set-up were raised in the economics literature several years ago in a series of papers [Minasian (1964), Coase (1966) and Levin (1970)]. These papers did not provide adequate answers, mainly because the theory and models of advertising and its effect on consumers were not sufficiently developed.

We model the economy following Stigler and Becker (1977). They assumed that consumers' utility functions are defined over characteristics rather than commodities. The consumers use the commodities as inputs in the 'production' of characteristics. The transformation process from commodities to characteristics depends on the information available to the consumer. The main role of advertising is to provide consumers with more information, thus making it possible for them to 'produce' more characteristics from a given amount of commodities (inputs). The advertiser, on the other hand, expects this will lead the consumer to pay a higher price for the commodity being advertised.

A concrete example of the kind of scenario we have in mind is athletic footwear. The footwear is the good, whereas the derived characteristic is the time period at which it can be worn. Advertising shows the consumers that these athletic shoes can be worn for activities other than sports, thus increasing the 'characteristic content' of a given pair of shoes. This will induce more purchases and/or higher prices.

The paper is organized as follows. In section 2 we describe the basic economy. It is shown that a competitive equilibrium exists and would generally lead to a non-optimal allocation of resources. The conditions under which over- (under-) provision of the public good would occur are carefully stated and discussed (some numerical examples are described in the appendix). In the final section, we show that our analytical and qualitative results

[1]TV broadcasts are certainly non-rivalous in consumption. The need for decoders and receivers provides for the possibility of excludability. However, we consider economies where no exclusion is exercised, and thus view TV broadcasting as a pure public good.

are robust with regard to alternative specifications of the underlying economy and offer further lines of research.

2. The basic economy

The economy consists of two identical consumers,[2] two characteristics (z_1, z_2), three private commodities (x_1, x_2, t), a public good (y), capital (k), and one firm which produces both x_1 and y. We assume there is no production of x_2.

Consumer i has initial endowments x_{i1}^*, x_{i2}^*, t^*, and k_i^* of commodities, time, and capital.

The utility function of consumer i is assumed to be of the form

$$u_i(t_i y, t^* - t_i, z_{i1}, z_{i2}), \quad i = 1, 2.$$

The $t_i y$ argument represents the amount of the public good entering the utility function. As the quality of the broadcast increases (a larger y) the marginal utility of watching TV (t_i) increases. This might lead to a larger t_i. Time not spent watching TV is used up in other leisure activities and appears as the second argument in the utility function. The two characteristics constitute the remaining arguments of the utility function.

The transition from commodities to characteristics is given by

$$z_{i1} = f_i(t_i) x_{i1} \quad \text{and} \quad z_{i2} = x_{i2}, \quad i = 1, 2.$$

Thus, we identify the second commodity with the second characteristic. As regards the first commodity, the transformation into the first characteristic is non-trivial. It will depend on the information available to the consumer. The information is derived from TV advertising. We assume the transformation process takes on the above-specified multiplicative form, so that a larger t_i serves to increase the characteristics' content of x_{i1}. f_i is assumed to be differentiable and strictly increasing.

Hence, the utility function in the commodities space would be

$$u_i(t_i y, t^* - t_i, f_i(t_i) x_{i1}, x_{i2}), \quad i = 1, 2.$$

It is assumed to be differentiable, strictly increasing in its four arguments, and strictly quasi-concave in t_i, x_{i1}, and x_{i2}.

Even though we have assumed consumers are identical and hence $u_1 = u_2$ and $f_1 = f_2$, we maintain the consumer's index on these functions to distinguish between the consumers when writing down the Pareto-optimality conditions.

[2]They will have identical preferences as well as identical initial endowments and share holdings.

The firm produces x_1 and y via the production functions $g(k_{x1})$ and $h(k_y)$, respectively.

An allocation for the economy would be a non-negative tuple of numbers: $x_{11}, x_{12}, x_{21}, x_{22}, t_1, t_2, y, k_{x1}, k_y$.

Feasibility is defined in the obvious way and an allocation is Pareto optimal if there exists no other feasible allocation where no individual is worse off and at least one individual is strictly better off.

Any interior Pareto-optimal allocation would satisfy the following condition (assuming the necessary differentiability requirements):

$$\frac{u_{11}t_1}{u_{13}f_1(t_1)} + \frac{u_{21}t_2}{u_{23}f_2(t_2)} = \frac{g'}{h'}, \tag{1}$$

where u_{ij} denotes the partial derivative of u_i with respect to its jth argument.

It is the usual optimality condition in economies with public goods: 'The sum of the individual Marginal Rates of Substitution (MRS_i) equals the Rate of Technical Substitution (RTS).'

Note that for a symmetric Pareto-optimal allocation this condition reduces to

$$\frac{2u_{11}t_1}{u_{13}f_1(t_1)} = \frac{g'}{h'}.$$

We prove that competitive equilibria for the economy described above exist and examine the properties satisfied by the resulting allocations. To this effect, we study the problems that are solved by the consumers and the producer.

2.1. The consumers' problem

The consumers treat the commodity prices as given, ignoring the effect that viewing time may have on the prices charged. On the other hand, they take into account the informational content of the broadcast when choosing the viewing time. We formulate the consumers' problems as follows

$$\max_{t_i, x_{i1}, x_{i2}} \; u_i(t_i y, t^* - t_i, f_i(t_i)x_{i1}, x_{i2})$$

$$\text{s.t. } p_1 x_{i1} + p_2 x_{i2} = I_i, \quad i = 1, 2,$$

$$t_i, x_{i1}, x_{i2} \geqq 0,$$

$$t_i \leqq t^*.$$

(I_i denotes the income of consumer i.)

2.2. The x_1 and y producer's problem

The producer behaves competitively, by which we mean the prices of the relevant characteristic and the production factor are treated as given.

In order to formulate the producer's problem one needs to specify the producer's beliefs concerning the relationship between the public good level, y, and the viewing time, t. We assume the beliefs are given by $t_p(y) = \alpha y$. Since consumers are identical, it is not necessary to specify consumers' specific beliefs. We require that in equilibrium the producer's beliefs coincide with the times chosen by the consumers.[3]

The producer will therefore solve:

$$\max_{k_{x1}, k_y} q_1 f_1(\alpha h(k_y)) g(k_{x1}) - w(k_{x1} + k_y).^4$$

2.3. The competitive equilibrium

A competitive equilibrium is a feasible allocation $\beta = (x_{11}, x_{12}, x_{21}, x_{22}, t_1, t_2, y, k_{x1}, k_y)$, prices p, q_1, w, and α, all non-negative, that satisfy

(i) $t_1 = t_2$,[5]

(ii) $\alpha y = t_1$,

(iii) $p_1 = q_1 f_1(t_1)$,

(iv) Given the prices p_1, p_2, q_1, w, and α, the allocation β solves the problems of the consumers and the producer described above.

Proposition 1. Given the assumptions:

For $i = 1, 2$

(i) $u_i(t_i y, t^* - t_i, f_i(t_i) x_{i1}, x_{i2})$ *is continuous, increasing, and strictly quasi-concave in* t_i, x_{i1} *and* x_{i2};

(ii) f_i *is continuous and increasing;*

(iii) x_{i1}^*, x_{i2}^* *and* k_i^* *are strictly positive;*

and

(iv) $q_1 f_1(\alpha h(k_y)) g(k_{x1}) - w(k_{x1} + k_y)$ *is continuous and concave in* k_{x1} *and* k_y *for any non-negative* q_1, α *and* w;

[3] Hence we allow the beliefs to differ from the time chosen for out-of-equilibrium actions. This is similar to the treatment of beliefs in general equilibrium models of monopolistic competition [Negishi (1960–1961), Arrow and Hahn (1971)].

[4] Since $f_1 = f_2$ and consumers are identical, we may use either one in specifying the producer's problem.

[5] This is because we want the consumers to face the same price for x_1. Given our assumptions regarding preferences and the fact the consumers are identical, it is obvious that t_1 would indeed equal t_2. We will address the issue of different prices and non-identical consumers in the final section.

(v) *h is increasing, continuous, concave and* $h(0) > 0$;
(vi) *g is increasing, continuous and concave with* $g(0) = 0$;
there exists a competitive equilibrium.

Proof. We adopt the abstract game approach [Shafer and Sonnenschein (1975) and Debreu (1982)].

We first 'bound' the economy. The set of all pairs (x_{i1}, x_{i2}) individual i can get in a feasible allocation is bounded; the same holds for the set of all feasible (k_{x1}, k_y) choices. Let D be a compact and convex set in R^2_+ which contains these sets in its interior.

There are four players: the two consumers, the producer, and a market player. The game can be described as follows. The market player chooses three prices (q_1, p_2, w) and α, the producer chooses k_{x1} and k_y, and the consumers choose the x_{ij}'s and t_i's. The payoff functions are the utility functions of the consumers and the profit function for the producer (this profit function will incorporate the beliefs regarding the relationship between t and y). The market player's payoff is the usual value of excess demand and an additional term inducing the 'correct' choice for α.

Formally the game is comprised of:

(a) *Strategy sets.* The strategy sets for the consumers are $A_i = [0, t^*] x(R^2_+ \cap D)$, $i = 1, 2$. The strategy set for the x_1 and y producer is $A_3 = R^2_+ \cap D$. The strategy set for the market player is

$$A_4 = S^3 x[0, t^*/h(0)] \ (S^3 \text{ the unit simplex in } R^3).$$

Let $A = \pi^4_{i=1} A_i$, and let a denote a generic element of A.

(b) *Feasibility correspondences:*[6]

$$\phi_1(a) = \{x \in A_1 \mid q_1 f_1(\alpha h(k_y))(x_{11} - x^*_{11}) + p_2(x_{12} - x^*_{12}) \leqq 0.5\pi_1 + wk^*_1\},$$

$$\phi_2(a) = \{x \in A_2 \mid q_1 f_2(\alpha h(k_y))(x_{21} - x^*_{21}) + p_2(x_{22} - x^*_{22}) \leqq 0.5\pi_1 + wk^*_2\},$$

where

$$\pi_1 = q_1 f_1(\alpha h(k_y)) g(k_{x1}) - w(k_{x1} + k_y\},$$

$$\phi_i(a) = A_i, \quad \text{for } i = 3, 4.$$

(c) *Payoff functions:*

$$\mu_1(a) = u_1(t_1 y, t^* - t_1, f_1(t_1) x_{11}, x_{12}),$$

$$\mu_2(a) = u_2(t_2 y, t^* - t_2, f_2(t_2) x_{21}, x_{22}),$$

[6]In an abstract game the strategies available to player i may depend on the strategies chosen by other players. The ith feasibility correspondence will map A into A_i, indicating the subset of A_i from which player i may choose.

$$\mu_3(a) = q_1 f_1(\alpha h(k_y))g(k_{x1}) - w(k_{x1} + k_y),$$

$$\mu_4(a) = q_1 f_1(t_1)(x_{11} - x_{11}^* + x_{21} - x_{21}^* - g(k_{x1})) + w(k_{x1} + k_y - k_1^* - k_2^*) + p_2(x_{12} - x_{12}^* + x_{22} - x_{22}^*) + 2\alpha - \frac{\alpha^2 h(k_y)}{t_1}.$$

The strategy sets are compact and convex. The feasibility correspondences are continuous and convex-valued (the budget correspondence is well-behaved since profits are non-negative and initial endowments are strictly positive). The payoff functions are all continuous and quasi-concave in the relevant choice variables. Hence, the abstract game has an equilibrium. By the market player's payoff function we see that in equilibrium $\alpha = t_1/h(k_y)$. Since the consumers are identical and face identical convex choice sets with strictly quasi-concave payoff functions, $t_1 = t_2$ in equilibrium, demonstrating that in equilibrium αy equals the times chosen by the consumers. Given the specification of the payoff functions it is obvious that the equilibrium actions for the game solve the consumers' and producer's problems in the bounded economy. Adding up the budget constraints [recalling that in equilibrium $\alpha h(k_y) = t_1 = t_2$], we derive that in the equilibrium for the game:

$$q_1 f(t_1)(x_{11} - x_{11}^* + x_{21} - x_{21}^* - g(k_{x1})) + p_2(x_{12} - x_{12}^* + x_{22} - x_{22}^*)) + w(k_{x1} + k_y - k_1^* - k_2^*) = 0.$$

Hence, the value of excess demand is non-positive for any choice of q_1, p_2 and w. This implies that the excess demand in each market is non-positive, which when combined with the fact that the value of excess demand in equilibrium is zero and p_1, p_2 and w are all positive (recall that preferences as well as production functions are assumed to be strictly monotone) shows that all the markets clear.

It can be shown in the usual manner that the consumers' and producer's actions would solve their respective problems even in the non-bounded economy and thus we have demonstrated the existence of a competitive equilibrium. Q.E.D.

We now show that, in general, the market allocation would lead to a non-optimal level of the public good. The nature of the deviation (i.e. whether there is too little y or too much y) will depend upon the preferences of the individuals and the effectiveness of advertising. The larger the marginal utility of the public good relative to the marginal utility of the produced private commodity, the more likely we are to encounter underprovision of the public good and vice versa. The more effective the advertising is (a larger f'_1) the more likely we are to encounter overprovision of the public good and vice versa. The precise conditions for the economy with identical individuals are given in Proposition 2. Note that since the individuals are treated symmetrically in the market allocations, we use individual one's utility function to describe the conditions.

From the producer's problem we see that the producer's optimal (interior) choices must satisfy

$$f'_1 \alpha h' g(k_{x1}) = f_1(\alpha h(k_y))g'. \tag{2}$$

The marginal productivity of k is equated over its alternative uses.

Proposition 2. If the market allocation satisfies $u_{11}y^* \neq u_{13}f'_1x^*_{11}$, then it is not Pareto optimal. Furthermore, if $u_{11}y^* > (<) u_{13}f'_1x^*_{11}$, then there is too little (much) y produced [i.e. individuals could be better off by allocating more (less) of K to the production of y].

Proof. Eq. (2), which is satisfied by the market allocation, can be written as

$$\frac{g'}{h'} = \frac{\alpha f'_1 g(k^*_{x1})}{f_1(\alpha h(k^*_y))}.$$

Note that $g(k^*_{x1}) = 2x^*_{11}$, $h(k^*_y) = y^*$ and in equilibrium $\alpha y^* = t^*_1$; thus,

$$\frac{g'}{h'} = \frac{2f'_1 x^*_{11} t^*_1}{y^* f_1(t^*_1)}.$$

Any Pareto-optimal allocation must satisfy (1). Since the competitive equilibrium allocation treats the two identical individuals symmetrically, (1) reduces to

$$\frac{g'}{h'} = \frac{2u_{11}t^*_1}{u_{13}f_1(t^*_1)}.$$

Thus, if $f'_1x^*_{11}/y^* \neq u_{11}/u_{13}$ or, in other words, $u_{11}y^* \neq u_{13}f'_1x^*_{11}$, the market allocation would not be Pareto optimal.

Furthermore, note that if $u_{11}y^* > u_{13}f'_1x^*_{11}$, then $f'_1x^*_{11}/y^* < u_{11}/u_{13}$, and the market allocation satisfies $g'/h' < 2u_{11}t^*_1/u_{13}f_1(t^*_1)$ so that the sum of the *MRS*s exceeds the *MRT*, and individuals can be better off by producing more y. Increasing production of y is beneficial because consumers at the margin value this more (through increasing the utility from the consumption of the public good) than the loss of utility incurred by having to reduce the output of the other good. However, increasing the production of y would reduce the firm's profits, since it has to bear all the cost of advertising (i.e.

the provision cost of the public good), and the resulting increase in revenues would not justify it. Thus the market mechanism would end up providing a sub-optimal amount of the public good.

A similar argument demonstrates there is too much y if the inequality is reversed. Q.E.D.

3. Summary and conclusions

We have studied an economy where a public good (TV broadcasting) is provided as a by-product of advertising by producers of private commodities. The consumers' preferences were defined over characteristics and the advertising activity influenced the transformation from commodities to characteristics.

We modelled the economy so as to capture the basic ingredients of preferences, production, and advertising. A competitive equilibrium taking into account the producer's beliefs regarding consumers' behaviour was defined and shown to exist. It was demonstrated that, in general, the resulting market allocation would be non-optimal.

The analytical as well as the qualitative conclusions we have reached in our basic economy extend to more complex environments. If we allow for more commodities, characteristics, and producers, the notation would be more cumbersome, but the analysis and the results would basically remain intact.

Allowing for non-identical consumers is more involved since it raises the problem that different consumers may be willing to pay different prices. By assuming that the producer can distinguish between the different consumers and charge each one accordingly, we can formulate a slightly modified producer's problem. As in section 2, competitive equilibria would exist and the nature of the deviation from Pareto optimality will be determined by individuals' preferences and advertising effectiveness.

Owing to the great amount of literature on advertising [see Nelson (1974)] and the various market structures (not necessarily competitive) several questions remain open which indicate further directions for research.

First there is a need for analysis of other advertising models to study the existence and welfare properties of appropriately defined market allocations. Of particular interest would be the examination of advertising as a signal [Kihlstrom and Riordan (1984), Milgrom and Roberts (1986) and Horstmann and MacDonald (1989)], where advertising affects consumer beliefs on quality of commodities. It would require a careful specification of the environment and the information structure. Preliminary findings suggest that the distortions would be similar to the ones appearing in this paper.

Second, it has been recognized that various other (non-market) methods of providing for public goods may also lead to non-optimal allocations. It would be of interest to check how the market allocations derived in our paper fare with respect to these alternative methods.

Finally, in our model the production of the public good was carried out by the producers of the advertised goods. One might consider a set of firms that produce the 'public good' in order to sell it to the producers (for advertising purposes). In this case it would be interesting to examine the effect that the structure of the public good industry (number of firms, competitive or non-competitive) would have on the optimality properties of the resulting market allocations.

Appendix

The purpose of this appendix is to construct a numerical example demonstrating the workings of the economy described in section 2. We will calculate Pareto-optimal allocations as well as market allocations. It will be shown that as we make the public good less and less desirable we move from underproduction to overproduction.

In Case A, where the public good is relatively desirable (a 0.25 exponent on y), the competitive equilibrium would lead to underproduction of the public good, whereas in Case B (a 0.05 exponent on y) the public good would be overproduced.

Case A. The two consumers' utility functions are given by

$$u_i=(t_i y)^{0.25}+(20-t_i)^{0.5}+(t_i^{0.25}x_{i1})^{0.5}+x_{i2}^{0.5},\quad i=1,2.$$

So we have:

$$f_i(t)=t_i^{0.25}.$$

The consumers' initial endowments are $x_{i1}^*=0$, $x_{i2}^*=4$, $k_i^*=10$, $t^*=20$ and each consumer owns half the shares of the firm. The production functions are given by

$$g(k_{x1})=k_{x1}^{0.5};\qquad h(k_y)=1+k_y.$$

In the symmetric Pareto-optimal allocation the utility level for both individuals is 10.2196 and

$$k_{x1}=6.5700;\qquad k_y=13.4300,$$

$$t_i=7.0160;\qquad x_{i1}=1.2816;\qquad x_{i2}=4,\quad i=1,2.$$

Calculating the competitive equilibrium. We set the price of x_2 at 1. In order to determine the other equilibrium values we solve the following system of equations (since the consumers are identical, we look for a symmetric equilibrium):

$$2(1+k_y)^{0.25}t_1^{-0.75}+t_1^{-0.875}x_{11}^{0.5}-4(20-t_1)^{-0.5}=0,$$

$$t_1^{0.25}x_{12}=x_{11}p_1^2,$$

$$p_1x_{11}+x_{12}=10w+4+0.5(p_1k_{x1}^{0.5}-w(k_{x1}+k_y)),$$

$$(p_1/(1+k_y))k_{x1}^{0.5}=4w,$$

$$p_1k_{x1}^{-0.5}=2w,$$

$$2x_{12}=8,$$

$$k_{x1}+k_y=20.$$

The solution of these equations yields:

$$k_{x1}=14.0000; \quad k_y=6.0000; \quad t_1=6.3614,$$

$$x_{11}=1.8708; \quad x_{12}=4.0000,$$

$$p_1=1.8427 \quad \text{and} \quad w=0.2462.$$

Using these values we can recover α and q_1 by using

$$\alpha=t_1/(1+k_y) \quad \text{and} \quad q_1=p_1/((\alpha(1+k_y))^{0.25})$$

which yield $\alpha=0.9088$ and $q_1=1.1603$. Finally, the producer's profit in equilibrium is given by

$$\text{profit}=p_1k_{x1}^{0.5}-w(k_{x1}+k_y)=1.9699.$$

The competitive equilibrium is not Pareto optimal. Looking for a Pareto-optimal allocation where individual two's utility is 10.0000 we find that

$$k_{x1}=6.5816; \quad k_y=13.4184,$$

$$t_1=7.0909; \quad t_2=6.9345; \quad x_{11}=1.4432,$$

$$x_{12}=4.4893; \quad x_{21}=1.1223; \quad x_{22}=3.5107,$$

and

$$u_1=10.4262.$$

Therefore, the first individual can be made better off by having more y produced.

Case B. Let the two consumers' utility functions be given by

$$u_i=(t_i y)^{0.05}+(20-t_i)^{0.5}+(t_i^{0.25}x_{i1})^{0.5}+x_{i2}^{0.5}, \quad i=1,2.$$

Except for this change in preferences, the economy would be identical to the economy in Case A.

The symmetric Pareto-optimal allocation would yield the utility level 8.9264 for both individuals:

$$k_{x1}=18.5071; \qquad k_y=1.4929$$

and

$$t_i=2.1648; \qquad x_{i1}=2.1500; \qquad x_{i2}=4.0000, \quad i=1,2.$$

The competitive equilibrium would yield

$$k_{x1}=14.0000; \qquad k_y=6.0000,$$

$$t_i=2.0698; \qquad x_{i1}=1.8708; \qquad x_{i2}=4.0000, \quad i=1,2,$$

$$p_1=1.6014; \qquad w=0.2140; \qquad q_1=1.3351; \qquad \alpha=0.2957,$$

and

$$\text{profit}=1.7120.$$

At this allocation individual two's utility would be given by 8.8754. Calculating a Pareto-optimal allocation where individual two's utility is 8.8754 we get

$$k_{x1}=17.6014; \qquad k_y=2.3986,$$

$$t_1=2.1565; \qquad t_2=2.1265; \qquad x_{11}=2.1332,$$

$$x_{12}=4.0607; \qquad x_{21}=2.0622; \qquad x_{22}=3.9393$$

and

$$u_1=8.9327.$$

In this case the first individual can be made better off by producing less y.

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