An Optimal Patent Policy in a Dynamic Model of Innovation

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Abstract: We assess patent policies where R&D aims at higher quality and the firms engage in strategic behavior. The innovation realized in one period affects the cost of realizing innovations in subsequent periods. The two firms compete in prices, and only one can innovate in each period. We compare the optimal innovation pattern to the second best innovation pattern, involving strategic behavior. Both are superior to the innovation pattern realized in an unregulated market. The market performance can be improved through the introduction of patent protection, and we characterize an optimal protection policy in the form of novelty requirements.

1. Introduction

Innovations are prevalent in all areas of modern economic societies. However, they are not free and result from costly R&D undertaken by individuals and firms who anticipate a stream of profits in respect to the appropriation of rewards awaiting innovators there exist well-known problems. It is commonly recognized that the social rate of return on innovations exceeds the private rate of return on innovations. This calls for various corrective measures. The patent system is one of the most common remedies used to alleviate the discrepancy between the two rates of return.

The patent system has pronounced effects on how the economy is functioning and has been paid a great deal of attention to in the literature on economics. Particular emphasis has been placed on the design of an optimal patent policy – a policy that would try to balance off the inefficiencies resulting from the monopoly position assumed by the patent holder and the gains to society from the patented innovations.

Apart from the obvious static distortions created by the presence of monopoly, there exist intertemporal distortions. Innovations do not occur in vacuum and patents granted today may hinder the development of future innovations. As Arrow (1962) puts it, today's innovations are the inputs for the innovations of tomorrow.

Many papers have analyzed the merits and deficiencies of the patent system and the following short survey covers only some of them. Nordhaus (1969) determined the optimal time horizon over which patent protection should be granted, with the view of balancing the monopoly distortions created by the presence of a patent and the rewards needed to generate the innovation in the first place.

However, patent protection entails more than just a time dimension. No matter how long a patent may be, it would not be very effective if a slightly modified version of the innovation could be freely marketed or used. Klemperer (1990) and Gilbert and Shapiro (1990) focused precisely on that issue adding the notion of the dimension of patent breadth, interpreted as the variety of the products protected. On the breadth dimension there are clearly costs incurred by society from barring the free production of close variants of the patent. These authors examined the optimal mix of breadth (how encompassing the patent protection is) and length in the process of when providing the innovator with a given reward, ignoring the discussion of the total reward's size.

Apart from the breadth issue there is the height issue. If the smallest improvement on an existing patent can be freely used and marketed, the patent awarded will lose much of its effectiveness. These social costs and gains have to be taken into consideration and van Dijk (1992) examines this third feature of protection.

Patents, even when granted, can be violated and an integral part of the patent system is

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the enforcement mechanism. The way patents are enforced plays an important role in determining the overall effect of the patents' system. Waterson (1990) introduced the concept of trial costs and damages awarded, and examined the connection between the patent system and the variety of products offered to consumers.

The methods of analysis in the above papers can be broadly categorized into general equilibrium type arguments (Nordhaus, Klemeperer, Gilbert and Shapiro) versus game theoretic models (van Dijk, Waterson). The first do not take explicit account of the strategic considerations of the innovating firms whereas the other papers model the (small number of) firms in a very specific game form. The game theoretic models, which also play a significant role in the literature on timing of innovations and patent licensing, emphasize the role of strategic considerations. R&D efforts of one firm are closely related to its beliefs regarding the actions of an opponent firm.

As mentioned before, patents do not occur in vacuum. The implications of patent protection in the context of a dynamic sequence of innovations are extremely important. Patents granted today may hinder the development of future patents and this obvious social cost must be taken into account. Chou & Shy (1991), Judd (1985) and Grossman & Helpman (1991 (chapters 3 & 4)) present dynamic general equilibrium models with the sequence of innovations. Chou and Shy (1991) and Judd (1985) provide explicit discussions of patent length and its welfare implications. These models are clearly general equilibrium models abstracting from the strategic considerations that would motivate the firm's actions.

The models closest to ours are those by Green and Scotchmer (1995) and Chang (1995). They recognize the dynamic structure of innovation and study the relationship between the patent breadth and the incentives to innovate at each stage of the development process. They analyze a two stage model where the second stage innovation can occur only if the first one has taken place. The innovations are undertaken by two distinct firms. The scope of patent protection affects the split of profits among the different innovators and their relative bargaining positions. Game theoretic models are used to analyze the issues of licensing and cooperative agreements and characterize the optimal protection level.

Our model deals with a dynamic structure where one innovation facilitates the other and examines the optimal structure of protection, taking into account the static as well as the dynamic distortions introduced by the presence of patents. The bulk of the paper will deal with the case of one dimension-either height or quality (modeled as a reduction in the marginal cost of production). It is a game theoretic model that enables us to capture the strategic considerations behind the undertaking of R&D expenditures. Protection can be described as the level of novelty required for granting a patent to the upgraded product. When a patent is granted, innovators are entitled to use their innovation exclusively within a spesified time period.

The main features differentiating our analysis from those of the previous models are concerned with technology of innovation and preferences. We also introduce three time periods to allow for a broader set of interactions among firms.

In G&S (1995) and Chang (1995), innovation technology was discrete, the choice being whether to innovate by a given fixed amount at some given cost or not. Furthermore, the second innovation was entirely dependent on the first one. On the other hand our innovation choice is continuous and the firms choose profit maximizing levels of innovation, taking into consideration the associated costs and future consequences of their actions. There is no "first" and "second" innovation, even though the earlier innovations in the sequence reduce the cost of subsequent innovations.

In the above works the main question was whether or not to innovate. The design of the patent was intended to achieve a profit distribution among the firms that would induce the first firm to innovate. In our model the firms have to decide not only whether or not to innovate but how much to innovate. Our design of protection, in addition to simply determining whether or not to innovate, intends to induce optimal amounts of innovation. Besides the benefits associated with innovation vary continuously and are derived from the aggregate demand curves stated as is standard in the innovation literature.

We consider a three period model where firms take turns innovating. This allows the first firm to return to the market and appropriate some of the rents made possible by its own innovation

as well as by those generated by the competitor.

G&S (1995) and Chang (1995) analyzed the optimality of various agreements signed by the firms. The benefits associated with a more appropriate division of profits were weighed against the costs of reduced competition in the product markets. We assume such agreements are not allowed and briefly comment on the results of full cooperation by firms.

In the second section we introduce the model and compare the innovation patterns realized by market equilibria (ME) with non-binding protection levels to socially optimal innovation patterns. In the third section we examine various scenarios of protection and cooperation. We characterize the emerging innovation patterns and compare them to the ME. The fourth section discusses the length of protection parameter. The fifth section consists of several numerical examples demonstrating the points rose in the previous discussions. The sixth section presents conclusions and suggests further directions of research.

2. The Model

We consider a three period model. The demand curve facing the industry in each period is $P = f(Q)$. There are two firms (1 and 2) in the industry, each having identical marginal costs of production given by c. They compete via prices and at the start of period 1 the price is c (see Figure 1). In period 1, firm 1 may choose to innovate. The innovation is viewed as an upgrading the product so that the same amount could provide more of the characteristic (see Grossman and Helpman (1991, chapter 4)). This can be alternately formulated as a lower marginal cost of production. The desired reduction of cost (x_1) is associated with an R&D cost of $g(x_1)$[1]. g is assumed to be convex. We assume the innovation chosen is not drastic[2]. Therefore quantity and price will not change following the innovation, and the profits of firm 1 will increase (ignoring of course the R&D cost). If firm 1 decides to innovate in the amount of x_1, where $x_1 \geq D_1$ (an innovation must exceed a certain minimum in order for it to be patentable[3]), it is granted a patent. The patent protection is in the form of D_2 and specifies the minimal degree of novelty required from the upgraded product[4]. x_2, the cost reduction undertaken by firm 2, must meet or exceed it for it to be patentable. If firm 2 does not pursue any R&D activity in period 2, its marginal cost of production will be c or $c - x_1 + B$ depending on whether or not we have backward protection denoted by B. Backward protection can be thought of as a constraint on producing and marketing lower quality products by competitors. If large enough, the backward protection would prohibit competitors to use any production process, whose unit cost is below c. We usually assume that the backward protection is large enough to insure the relevant unit cost of the competitor to be c. Therefore firm 2 realizes zero profits if it chooses not to innovate.

Not all ratios can be used for the following analysis. In order to determine, in which categories certain stakeholder groups are best off, we use three basic ratios for three stakeholder groups. After that the ratios having an influence on the formation of basic ratios will be exploited, using formulas given by Mereste (1987). These ratios reflect the main reasons for explored differences between different enterprise groups.

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[1] A more general formulation would specify g as time dependent and two distinct arguments corresponding to innovations from current and last period, i.e. $g_i(x_i, x_{i-1})$.

[2] See Romano (1991) for the definition of drastic and nondrastic discovery.

[3] If the innovation is not patentable, it can be immediately imitated since there is no one time period protection and hence it yields zero profits.

[4] $D_i = 0$ corresponds to the no-protection case, whereas $D_i = \infty$ provides the maximal protection and amounts to giving the protection to the idea, all upgraded products originating from the initial innovation are protected.
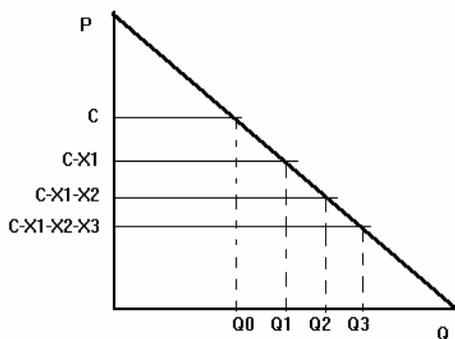
Fig. 1. The Demand and the Innovation

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Firm 1 will be able to innovate again in period 3, the last period of the game. It can further reduce the marginal cost from its period 2 level by x_3 ($x_3 \geq D_3$), with the reduction cost given by $g(x_3)$. In this case it can sell at the marginal cost of the second firm and make positive profits; firm 2 will make zero profits.

If firm 1 chooses not to innovate then similarly to stage 2, it faces zero profits.

We present the above discussion in the game tree depicted in Figure 2. The game is solved backwards in the usual manner and subgame perfect equilibrium can be easily calculated.
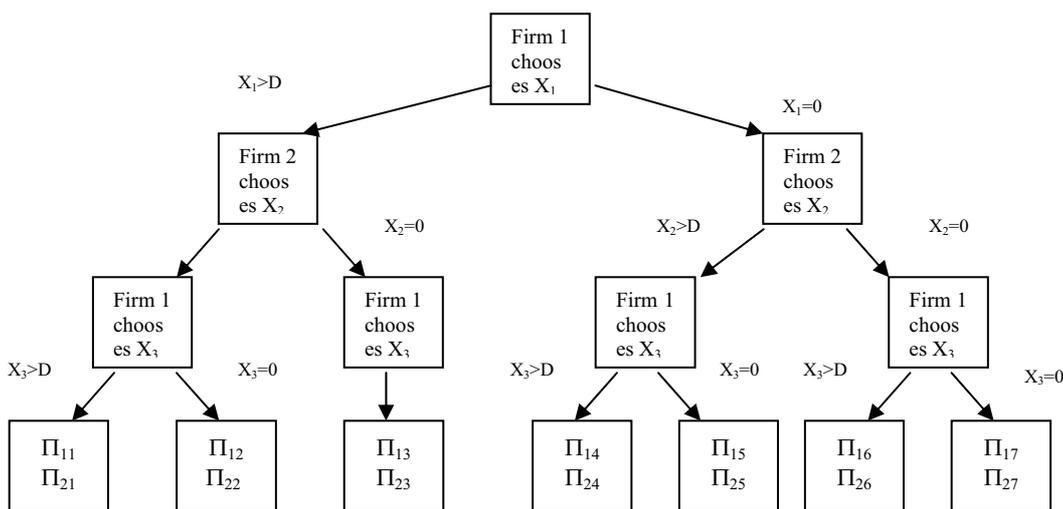

Fig. 2. The Game Tree

Where:

$\Pi_{11}=Q_0X_1+Q_2X_3-g(X_1)-g(X_3)$

$\Pi_{21}=Q_1X_2-g(X_2)$

$\Pi_{12}=Q_0X1-g(X_1)$

$\Pi_{22}=2Q_1X_2-g(X2)$

$\Pi_{13}=2Q_0X1+Q_0(X_1+X_3)-g(X_3)-g(X_3)$

$\Pi_{23}=0$

$\Pi_{14}=Q_1X3-g(X_3)$

$\Pi_{24}=Q_0X_2-g(X_2)$

$\Pi_{15}=0$

$\Pi_{25}=2Q_0X_2-g(X_2)$

$\Pi_{16}=Q_0X_3-g(X_3)$

$\Pi_{26}=0$

$\Pi_{17}=0$

$\Pi_{27}=0$

Casual observations of the market for home entertainment systems, like Nintendo and Sega, reveal behavior patterns similar to those outlined above. The firms "take turns" in introducing new lines of games (games that can be thought of as providing more entertainment at a lower marginal cost). In the computer software market several competing companies take turns in improving the product offered, introducing updated version of spreadsheets and word processors.

The Pareto Optimum

A Pareto optimal pattern of innovation is one that maximizes the sums of consumer and producer surpluses over the three periods. It ignores all constraints imposed by the market structure, and is the solution of the following maximization problem[1] (the PO problem):

$$\underset{x_1,x_2,x_3}{Max}\ 3\int_{c-x_1}^{c}f^{-1}(t)dt+2\int_{c-x_1-x_2}^{c-x_1}f^{-1}(t)dt+\int_{c-x_1-x_2-x_3}^{c-x_1-x_2}f^{-1}(t)dt-g(x_1)-g(x_2)-g(x_3)\quad(1)$$

The solution of this problem is the first best allocation and is not attainable given the structure of the operating markets.

The Second Best (SB)

Realizing the innovator acts strategically, and sells the product at a price equal to the marginal cost of the closest competitor, we introduce the concept of a Second Best allocation. A second best allocation maximizes the sum of consumer and producer surpluses when a product is priced at the marginal cost of the competitor. Thus, it solves the following maximization problem (the SB problem):

$$\underset{x_1,x_2,x_3}{Max}3x_1f^{-1}(c)+2\int_{c-x_1}^{c}(f^{-1}(t)-f^{-1}(c))dt+2x_2f^{-1}(c-x_1)+\int_{c-x_1-x_2}^{c-x_1}(f^{-1}(t)-f^{-1}(c-x_1))dt+$$

$$x_3f^{-1}(c-x_1-x_2)-g(x_1)-g(x_2)-g(x_3)\quad(2)$$

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[1]We assume that there is no discount factor associated with the social welfare gains realized in the three periods.

The first order conditions satisfied for the PO problem are (we assume now and in all subsequent problems that the second order conditions are satisfied):

$$f^{-1}(c-x_1) + f^{-1}(c-x_1-x_2) + f^{-1}(c-x_1-x_2-x_3) = g'(x_1)$$

$$f^{-1}(c-x_1-x_2) + f^{-1}(c-x_1-x_2-x_3) = g'(x_2) \qquad (3)$$

$$f^{-1}(c-x_1-x_2-x_3) = g'(x_3)$$

The first order conditions for the SB problem are:

$$f^{-1}(c) + f^{-1}(c-x_1) + f^{-1}(c-x_1-x_2) - x_2(f^{-1}(c-x_1))' - x_3(f^{-1}(c-x_1-x_2))' = g'(x_1)$$

$$f^{-1}(c-x_1) + f^{-1}(c-x_1-x_2) - x_3(f^{-1}(c-x_1-x_2))' = g'(x_2) \qquad (4)$$

$$f^{-1}(c-x_1-x_2) = g'(x_3)$$

From these conditions it is clear that welfare maximizing innovation patterns decline over time.

The firms acting in the market are taken to be profit maximizers and we analyze the innovation patterns induced by the subgame perfect equilibrium in the game described above. The equilibria will naturally depend on the protection parameters. We will refer to the situation of no protection (or more generally non binding protection levels) as the market equilibrium (ME) (in this scenario any innovation is awarded a one time period protection, there is no threshold the innovation must meet). The protection equilibrium (PE) would be the innovation pattern realized in the presence of binding protection levels. There are gains in cooperation and so we would next examine the innovation patterns realized when firms cooperate.

In all the cases the resulting innovation patterns are compared to the solutions of the PO and SB problems.

The Market Equilibrium (ME)

The first order conditions generated by the market equilibrium can be determined as follows:

The determination of x_2:

$$Max \ f^{-1}(c-x_1) \cdot x_2 - g(x_2) \Rightarrow g'(x_2) = f^{-1}(c-x_1) \qquad (5)$$

hence:

$$\frac{dx_2}{dx_1} = \frac{-(f^{-1}(c-x_1))'}{g''(x_2)} \qquad (6)$$

The determination of x_1 and x_3:

$$Max \ f^{-1}(c) \cdot x_1 + f^{-1}(c-x_1-x_2(x_1)) \cdot x_3 - g(x_1) - g(x_3) \Rightarrow$$

$$g'(x_3) = f^{-1}(c-x_1-x_2(x_1)) \qquad (7)$$

$$g'(x_1) = f^{-1}(c) - (f^{-1}(c-x_1-x_2(x_1)))' (1 + \frac{dx_2}{dx_1}) \cdot x_3$$

To examine the welfare properties of the ME, we consider a hypothetical planner who

chooses the optimal x_1 and x_3 levels, taking as given the $x_2(x_1)$ function yielded by the market scenario. The first order conditions of such a hypothetical planner (when maximizing the same objective function as the SB) are given by:

$$f^{-1}(c) + f^{-1}(c - x_1) + f^{-1}(c - x_1 - x_2) + (f^{-1}(c - x_1))' \cdot \frac{dx_2}{dx_1} - f^{-1}(c - x_1))' \cdot x_2 +$$

$$f^{-1}(c - x_1 - x_2) \cdot \frac{dx_2}{dx_1} - (f^{-1}(c - x_1 - x_2))' \cdot (1 + \frac{dx_2}{dx_1}) \cdot x_3 =$$

$$g'(x_1) + g'(x_3) \cdot \frac{dx_2}{dx_1} \tag{8}$$

and:

$$f^{-1}(c - x_1 - x_2(x_1)) = g'(x_3) \tag{9}$$

Evaluating the first condition under the ME scenario, we see that the LHS exceeds the RHS (the first term on the RHS cancels with the first and last terms of the LHS, the second term on the RHS cancels the sixth term on the LHS, and all the remaining terms on the LHS are positive). Hence, the ME allocation can be improved by choosing a larger x_1. This will lead to increased levels of x_2 and x_3 as well. This proves the following proposition:

Proposition 1: The profit maximizing innovation levels generated by the ME are suboptimal and can be Pareto improved by adopting larger innovation levels.

Since it can be easily established that the PO innovation levels exceed the SB innovation levels we get:

Corollary 1: The innovation levels induced by the PO problem exceed those of the ME.

A natural way to improve the market performance is to use binding protection levels. Awarding a patent only innovations exceeding a specified threshold. These protection measures are analyzed in the next section.

3. Protection and Cooperation

Protection Equilibrium (PE)

The protection parameters can be thought of as playing a role similar to that seen in minimum patentability standards (see La Manna (1992) for the analysis of such standards in a one period model). Their effect on actual innovation levels would depend on whether or not they are binding, as well as the amount of profits realized when adhering to them.

In the case where the specified protection levels (D_1, D_2, D_3) are binding and admit non negative profits along the innovation path, the firms would innovate up to the D amounts. If this leads to negative profits, the firms, whose profits become negative when innovating up to the D levels, would avoid innovating, and end up with zero profits.

The optimal sequence of protection levels is the solution of a well defined maximization problem. However this problem is dependent on the values assumed by the cost and demand parameters, and there is little economic insight to be gained by characterizing the general solution. Henceforth we provide results on the desirability of some degree of protection, on the one hand, and the non desirability of extreme levels of protection that would leave only one innovator in the industry, on the other.

Claim 1: If the two firms realize strictly positive profits in the market equilibrium, there

are protection levels that lead to a welfare improving innovation pattern.

Proof: Set protection levels D_1 and D_2 slightly higher than the innovation levels x_1 and x_2 realized by both firms in the ME in periods 1 and 2.

Note that in period 3 there is no need to set a protection level, since at this period the innovating firm maximizes the SB objective function when maximizing its profits.

However it is possible that increased protection levels that generate zero profits at each period would yield too much R&D. This is seen in:

Claim 2: Let the demand function facing the industry be linear. Let x_1, x_2 and x_3 denote the SB innovation levels, D_1 and D_2 be the innovation levels with zero period profits and

$$e = \frac{g'(x)x}{g(x)}$$ be the elasticity of scale of g.

Then

(i) $e < (>) 1 + \dfrac{Q_2}{Q_0} + \dfrac{Q_3}{Q_0}$ implies $x_1 > (<) D_1$ (10)

(ii) $e < (>) 1 + \dfrac{Q_3}{Q_1}$ implies $x_2 > (<) D_2$ (11)

Proof: Zero profits in period 1 imply that $D_1 Q_0 = g(D_1)$, hence $g'(D_1) = e Q_0$. For x_1 we have that $g'(x_1) = Q_0 + Q_1 + Q_3$, so $e < (>) 1 + \dfrac{Q_2}{Q_0} + \dfrac{Q_3}{Q_0}$ implies that $g'(x_1) > (<) g'(D_1)$ and therefore $x_1 > (<) D_1$. (ii) is similarly proved.

Of course if the case is that $e > 1 + \dfrac{Q_2}{Q_0} + \dfrac{Q_3}{Q_0}$ then the D_i levels stemming from zero profits will exceed the SB levels in every period.

Cooperation Equilibrium (CE)

The basic issue is how cooperation affects welfare and whether cooperation would increase the amount of innovative activity. Joint action of both firms would try to maximize[1]:

$$f^{-1}(c)x_1 + f^{-1}(c - x_1)x_2 + f^{-1}(c - x_1 - x_2)x_3$$

$$-g(x_1) - g(x_2) - g(x_3)$$

(This assumes that the firms other than the innovating firm get the cost reduction within a one period delay. This will be referred to as the CE (12) scenario.)

The first order conditions are:

$$f^{-1}(c) - (f^{-1}(c - x_1))'x_2 - (f^{-1}(c - x_1 - x_2))'x_3 = g'(x_1)$$

$$f^{-1}(c - x_1) - (f^{-1}(c - x_1 - x_2))'x_3 = g'(x_2) \qquad (13)$$

$$f^{-1}(c - x_1 - x_2) = g'(x_3)$$

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[1]Note that in terms of protection, CE(1) may result from a large D2.

The relationship between ME innovation levels and those realized under cooperation is given by:

Proposition 2: The innovation levels realized under CE(1) exceed those of ME.

Proof: Taking the derivative of the objective function (1), and treating the $x_2(x_1)$ function generated by the ME as given, we get:

$$f^{-1}(c)-(f^{-1}(c-x_1))'x_2+f^{-1}(c-x_1)\frac{dx_2}{dx_1}-(f^{-1}(c-x_1-x_2))'(1+\frac{dx_2}{dx_1})x_3$$
$$=g'(x_1) \tag{14}$$

However, if we plug in the values generated by the ME allocation, we see that the LHS exceeds the RHS. Hence cooperation would lead to increased innovation levels compared to the ME.

In the above specification we have implicitly assumed that the firms can use the cost innovation within a one period delay. Alternately, we could assume that the other firms continue producing at marginal cost c (they have no access to the cost innovation). This will be referred to as the CE(2) scenario. Under CE (15) they maximize:

$$3f^{-1}(c)x_1+2f^{-1}(c)x_2+f^{-1}(c)x_3$$
$$-g(x_1)-g(x_2)-g(x_3)$$

This yields the following first order conditions:

$$3f^{-1}(c)=g'(x_1)$$
$$2f^{-1}(c)=g'(x_2)$$
$$f^{-1}(c)=g'(x_3)$$

Proposition 3: If the demand function is linear, and the innovations realized under CE(2) are not drastic, then the CE(2) innovation levels in periods 1 and 2 exceed the CE(1) innovation levels.

Proof: Assume that the demand function is of the form P=A-BQ. If c is the marginal cost prior to any innovations, the assumption of non drastic innovations implies that $x_1+x_2+x_3 \le A-c$. Examining the first order condition for x_1 under CE(1) and CE(2) we see that since $(x_2+x_3)\cdot\frac{1}{B} \le (A-c)\cdot\frac{1}{B}=f^{-1}(c)$, the x_1 level under CE(2) exceeds the x_1 level under CE(1). Similarly the x_2 level under CE(2) exceeds the CE(1) level.

When considering the level of innovation in the third period, the x_3 of CE(1) clearly exceeds that of CE(2).

The difference between these two cooperation scenarios lies in the amount of backward protection, and we conclude that increasing backward protection induces larger investments in R&D during the earlier periods.

The innovation levels under CE(2) can also be compared to the innovation levels under PE, with zero profits at each period.

Claim 3: If the elasticity of scale of g is greater than 3 (i.e. $\frac{g'(x_1)x_1}{g(x_1)} > 3$) the innovation levels at CE(2) are below those realized under PE with zero period profits.

Proof: Denote the innovation levels of CE(2) by x_1, x_2 and x_3 and let D_1, D_2 and D_3 be those of the PE. To show that $D_1 > x_1$ note that from the F.O.C. for PE one gets: $f^{-1}(c)D_1 = g(D_1)$

and hence $g'(D_1) > 3f^1(c)$ (since elasticity of scale exceeds 3). This implies $x_1 < D_1$ (since $g'(x_1)=3f^1(c)$). For D_2 we have $f^1(c-D_1)D_2=g(D_2)$ and so $g'(D_2) > 3f^1(c-D_1) > 2f^1(c)$ and once more it implies $D_2 > x_2$, and similarly $D_3 > x_3$.

Welfare comparisons are more difficult since the CE(2) scenario leads to a much more pronounced loss of consumer surplus.

4. Length of Protection

Thus far we have assumed that protection is awarded for the entire period. The period in our model can be interpreted as the time interval necessary for the other firm to come up with its innovation. We assume its length to be exogenously determined.

In this section we discuss the relaxation of the "entire period protection scenario". Reducing the protection length will lead to lower levels of R&D. The welfare implications of such a move are harder to gauge, and depend on the specific parameters of the problem.

Like in the original Nordhaus (1969) analysis there are two conflicting forces. On the one hand, shorter protection levels generate larger amounts of consumer surplus, but on the other they lead to reduced profits for the innovators, which in turn lead to lower R&D expenditures.

Our treatment of the time dimension differs from that of Gilbert and Shapiro (1990) and Klemperer (1990) due to the added feature of novelty requirements. Prolonging the protection time, which leads to larger profits for the innovators may pave the way for larger novelty requirements. This may somewhat compensate for the loss in consumer surplus. A full treatment of these issues is outside the scope of this paper and is a topic for future work.

5. Numerical Examples

We present several numerical calculations demonstrating the impact of various patent policies on R&D expenditures and social welfare. We assume demand is given by: Q=300-P; the initial marginal cost of production (c) is 250 and the cost of innovation is given by: $g(x) = 3 \cdot x^2$.

The results are summarized in Table 1 and in Figure 3. The main findings can be stated as follows: A "permissive" patent policy with regard to quality (low novelty requirements) leads to low levels of R&D and innovation. The innovation levels (10.3, 10.0, 11.7) , (see, line no. 2, Table 1), are very low in comparison to the second best social optimum (line no. 1, Table 1). The other scenarios in this table indicate that:

(i) There is a wide range of low protection levels (0 < D < 10) which are not binding and have no impact on the level of R&D and innovation.

(ii) Raising the level of patent protection (above the minimal level of D=10) will, up to a point, increase R&D expenditures as well as social welfare. In our example it holds for values up to 25.

(iii) There is a discontinuity when the level of patent protection rises to above 25. At this point there is a drop in both R&D expenditures as well as welfare levels. The firms switch from the innovation path in Figure 2 to a path located to the right of it.

(iv) The outcome of cooperation is given in line 9. This situation is preferred to the ME. There is however, a level of patent protection such that ME leads to higher levels of innovation and welfare.

(v) In scenario 4 we study the largest protection level, generating positive investments in R&D. The same holds for scenario 10. The difference is that in scenario 4 the length of protection is the same in each period, while in scenario 10, it is possible to set different lengths of protection for each period. In both cases the overall profits of each firm equal zero. Though, in the first case, the first period's profits are negative, while the third period's profits are positive. The firm invests in R&D even though the direct profits are negative. This investment is essential for future R&D and innovations, which will that leads to higher profits.

(vi) The innovation levels maximizing welfare are decreasing over time (scenario 1). An opposite trend exists in the case of ME (line no. 2) and zero profits (line 10). Binding protection

leads to equal levels of innovation in each period. Note that cooperation does lead to decreasing innovation levels over time, but these innovations are much smaller than those that are socially desired.

A graphical representation of these results can be found in Figure 3.

Table 1

R&D policy for various scenarios
Q=300-P; c=250; g(x)=3x^2

No. of Sce-nario	Scenario	X_1	X_2	X_3	Welfare	Profits Firm 1	Profits Firm 2
1	Second Best	72.3	54.8	29.5	8,900	-12,067	-2,308
2	D=1	10.3	10.0	11.7	2,700	608	303
3	D=15	15	15	15	3,713	600	300
4	D=25	25	25	25	5,313	0	0
5	D=26	26	0	8	2,964	2,080	0
6	D=40	40	0	8	3,328	1408	0
7	D=50	50	0	8	3,108	208	0
8	D=55	0	0	0	0	0	0
9	Joint Ac-tion	25	17	8	2,916	2,916	-
10	Zero Prof-its	16.7	22.2	29.6	3,674	0	0

*Note, in scenario 9 (Joint Action), the D is low. In scenario 10 the period profits are zero.
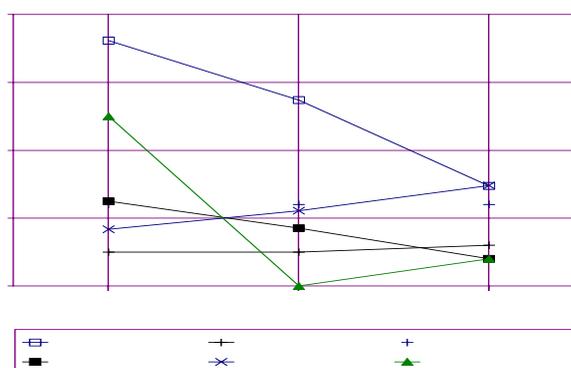

Fig. 3. Innovation over Time

In summary, a patent policy with low protection levels is inefficient. Infinite protection, i.e., patent protection on an idea, leads to the CE(2) scenario and results in higher innovation levels. Setting protection levels such that the firm's profits are close to zero, increases the level of R&D. However, this suggestion entails some risk, since setting the protection level too high would cause a sharp drop in R&D expenditures.

Table 2

R&D policy for various scenarios
$$g(x) = 3x^5$$

No. of Scenario	Scenario	X_1	X_2	X_3	Welfare	Profits Firm 1	Profits Firm 2
1	Second Best	1.80	1.63	1.37	410.95	92.0	49.9
2	D=1	1.36	1.36	1.37	376.3	111.8	55.9
3	D=2.04	2.04	2.04	2.04	316.9	0	0
4	D=2.1	2.10	0	1.37	253.8	39.4	0
5	Joint Action	1.78	1.61	1.35	395.99	395.99	--
6	Zero Profits	2.02	2.04	2.06	314.6	0	0

*Note, in scenario 5 (Joint action), the D is low. In scenario 6 the period profits are zero.

Table 2 demonstrates the effect of a high elasticity cost function. As expected, a policy of zero period profits leads to excessive amounts of R&D.

Table 3

The Effects of the Protection Length

The Vari-able	Second Best	Market Equilibrium		Protected Eq. Zero profits		Protected Eq. Same Di	
		T=0.5	T=1	T=0.5	T=1	T=0.5	T=1
x_1	72.3	4.6	10.3	8.3	16.7	10	25
x_2	54.8	4.5	10	9.7	22.2	10	25
x_3	29.5	4.9	11.7	11.3	29.5	10	25
Welfare	8,900	1,322	2700	2,461	3,674	2725	5,313

Results:
Longer length of protection increases R&D.
It is not true that shorter period raises welfare. (Considering the impact on R&D)
The effect of longer protection length is significant especially when it is accompanied by a patent policy of increasing D (protection).

6. Conclusions

We provided a framework for analyzing patent protection policies. The framework was dynamic in nature and incorporated the strategic aspects of the firm's behavior. The complexity of the problem made it difficult to derive general policy recommendations, and most of the results are sensitive to underlying cost and demand data.

We show that imposing some novelty requirements (not too large) improves the performance of the market equilibrium. The optimal level of novelty requirements is sensitive to the cost structure, as shown in claim 2. This shows that optimal setting of novelty requirements involves more than just, technical evaluations of significant changes, worthwhile of a patent. (A practice, which is often followed in real life patent authorities.)

Casual observations indicate there is no fixed set of rules by which to judge patent cases. Decisions are very discretionary in nature. This is consistent with the findings of this paper which are also very "context sensitive". Nevertheless the paper helps to isolate factors like cost structure

and demand parameters, which may increase or decrease protection parameters.

Cooperation between firms increases the R&D levels as compared to when there is no co-operation. Depending on the cost structure, cooperation may lead to lower R&D expenditures than of the PE as shown in claim 3.

It is worth pointing out that our protection scenario serves to increase R&D by the imposition of a lower bound on the improvement size and not through increasing profits for the innovator.

When comparing our results to those of G&S (1995) and Chang (1995) we see that it may not be necessary for one firm to "subsidize" another in order to obtain an optimal sequence of innovations. It is also the case that zero profits are not always desirable and under certain circumstances lead to excessive investment in innovations.

This work leaves open several further directions of research. The joint determination of protection length and novelty requirements has to be explored in great depth. We have also restricted our attention to quality improvements, but the closely related issue of "variety" improvements should also be examined in this dynamic setting.

Various arrangements which give rise to bargaining on the division of profits generated by a given innovation are analyzed in G&S (1995). Its implications in our dynamic setting deserve attention.

Empirical findings would shed light on our assumption regarding the exogenous length of the protection period. It is often the case that patents become obsolete prior to expiration of protection, due to new innovations. A careful study of this topic would, of course, entail the specification of a cost function g which depends on both innovation size and time required to achieve it.

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