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Forming efficient networks

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Abstract

We suggest a simple sequential mechanism whose subgame perfect equilibria give rise to efficient networks. Moreover, the payoffs received by the agents coincide with their Shapley value in an appropriately defined cooperative game.

Keywords: Networks; Implementation; Shapley value

JEL classification: C71; C72

1. Introduction

Recently, there has been a surge of interest in economic environments described via graphs, where activity takes the form of creating links among agents. A major concern then is the attainment of efficient networks (i.e., networks that maximize the output produced).

We suggest a simple sequential mechanism that adapts to the network environment proposal made in Pérez-Castrillo and Wettstein (2001).[1] The subgame perfect equilibria (SPE) of the mechanism generate efficient networks. Moreover, the payoffs received by the agents coincide with their Shapley values in an appropriately defined cooperative game.

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[1] See also Mutuswami et al. (2004).

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Jackson and Wolinsky (1996) were the first to analyze the problem of generating efficient networks. They focused on the tension between efficiency and stability. Dutta and Mutuswami (1997) and Mutuswami and Winter (2002) resolve this tension through the use of mechanisms, assuming that the planner has information regarding the value function. Navarro and Perea (2002) suggest a bargaining procedure among pairs of agents whose SPE outcomes coincide with the value appearing in Jackson and Wolinsky (1996).

An approach similar to ours can be found in Currarini and Morelli (2000), who propose two sequential-move mechanisms whose SPE outcomes generate efficient networks. The payoff configuration in their mechanisms is endogenously generated but is highly asymmetric. Also, Matshbayashi and Yamakawa (2004) propose a mechanism to share the cost of constructing the network and show that some equilibria involve both efficient networks and equitable allocation rules.

The advantages of our proposal are the following: the payoff division is equitable; the efficiency and equity properties of the mechanism hold for a large class of environments; and the mechanism is simple and does not require out-of-equilibrium free disposal.

2. Forming networks

Let $N=\{1,\ldots,n\}$ be the set of agents. For any $S\subseteq N$, let g^S denote the set of all subsets of S of size 2. A *graph* or network, denoted generically by g, is some subset of g^N. If $g\subset g^S$ where $S\subsetneq N$, we say that g is a graph restricted to S. A graph, therefore, is a structure of *bilateral relations* among agents. Agents i and j have a bilateral relation iff $\{i,j\}\in g$. We refer to the subset $\{i,j\}$ of g as the *link* between i and j and denote it by (ij). We let G_S denote the set of all graphs involving links just between members of S: $g\in G_S$ and $(ij)\in g$ imply that $\{i,j\}\subset S$.

Given a graph g, agents i and j are *connecteding* if there exists a sequence of agents $i=i_0,i_1,\ldots,i_K=j$ such that $(i_k i_{k+1})\in g$ for all $k=0,\ldots,K-1$. Let $N(g)\equiv\{i|$ there exists j such that $(ij)\in g\}$ denote the set of agents who have at least one bilateral relation. The graph $h\subset g$ is a *connected component* of g if all agents in $N(h)$ are connected to each other in h, and for all $i\in N(h)$, $j\in N\backslash N(h)$, $(ij)\notin g$. The set of all connected components of g is denoted by $C(g)$.

A *value function* is a mapping v: $G_N\rightarrow R$. The value of g represents the total surplus produced by agents when they form a set of bilateral relationships represented by g. We restrict attention to value functions satisfying *component additivity* [i.e., $v(g)=\sum_{h\in C(g)}v(h)$]. Component additivity rules out externalities between different components. We let V denote the set of component additive value functions. Given $v\in V$, a graph g is *strongly efficient* if $v(g)\geq v(g')$ for all $g'\in G_N$. An *allocation rule* is a mapping Y: $V\times G\rightarrow R^n$ satisfying $\sum_{i\in N}Y_i(v,g)=v(g)$. An allocation rule Y is *component-balanced* if $\sum_{i\in N(h)}Y_i(v,g)=v(h)$ for every $h\in C(g)$.

An example of an allocation rule is proposed by Jackson and Wolinsky (1996), which associates to each graph the Shapley value of a transferable utility game associated with the graph. Formally, fix v. For any graph g and $S\subseteq N$, let $g|S\equiv\{(ij)|(ij)\in g$ and $\{i,j\}\subseteq S\}$ denote the restriction of g to S. The Jackson–Wolinsky allocation rule for any graph g, which we denote by ϕ^g, is:

$$\varphi_i^g(N,v)=\sum_{S\subseteq N\setminus\{i\}}\frac{|S|!(n-|S|-1)!}{n!}[v(g|S\cup\{i\}))-v(g|S)].$$

Jackson and Wolinsky (1996) note that this allocation rule may arise naturally if the allocations result from bargaining between agents. However, this bargaining is not modeled explicitly.

The previous value equates the worth of a coalition S to the surplus generated by looking at the restriction of g to S. However, agents in S can form many other graphs besides $g|S$ and, ideally, one would like to take this into consideration. A natural possibility is to associate to each coalition S the *maximum surplus that can be derived by the members of S acting on their own*. One can now consider the transferable utility game (N,W) defined by $W(S)=\max\{v(g)|g\in G_S\}$ for all $S\subset N$ and the corresponding Shapley value. The game (N,W) is superadditive; we denote by ϕ its Shapley value:

$$\phi_i(N,W)=\sum_{S\subseteq N\setminus\{i\}}\frac{|S|!(n-|S|-1)!}{n!}[W(S\cup\{i\})-W(S)].$$

This Shapley value is the same as the value considered in Jackson (2003) for the player-based flexible allocation network case.

The restricted graph $g|S$ is not necessarily the graph that maximizes the surplus for S even if g itself is strongly efficient. The two approaches outlined above are thus bound to give different results.

The *bidding mechanism* can be considered as a model of network formation in which a bargaining process is modeled explicitly. By playing this mechanism, connected components are formed sequentially. In stage 1, the agents bid to choose the proposer. In stage 2, the proposer offers a vector of payments to all other agents and chooses a connected component he wants to form. The offer is accepted if all the other agents agree. In case of acceptance, the connected component is formed and the agents outside it proceed to play the same game again among themselves. In the case of rejection, all the agents other than the proposer play the same game again. Formally, the bidding mechanism operates as follows:

If there is only one agent i (say), he can only form the empty graph, $g=\emptyset$ and, therefore, he obtains $v(\emptyset)=0$.[2]

Given the rules for at most $n-1$ agents, the mechanism for $N=\{1,\ldots,n\}$ works as follows:

$t=1$: Each agent $i\in N$ makes bids $b_j^i\in\Re$, one for every $j\in N$, with $\sum_{j\in N}b_j^i=0$. Agents bid simultaneously.
For each $i\in N$, define the *aggregate bid* to agent i by $B_i=\sum_{j\in N}b_i^j$. Let $\alpha=\text{argmax}_i(B_i)$, where an arbitrary tie-breaking rule is used in the case of a nonunique maximizer. Once the winner α has been chosen, every agent $i\in N$ pays b_α^i and receives B_α/n.
$t=2$: Agent α chooses a subset of agents S_α (such that $\alpha\in S_\alpha$), a graph $g_\alpha^*\in G_{S\alpha}$ (such that g_α^* is connected on S_α) and offers $x_i^\alpha\in\Re$ to every $i\in N\setminus\{\alpha\}$.
$t=3$: The agents in $N\setminus\{\alpha\}$, sequentially, either accept or reject the offer. If an agent rejects it, then the offer is rejected. Otherwise, the offer is accepted.

If the offer is accepted, then the final payoff to agent $i\in S_\alpha\setminus\{\alpha\}$ is $x_i^\alpha-b_\alpha^i+B_\alpha/n$; agent α receives $v(g_\alpha^*)-\sum_{i\neq\alpha}x_i^\alpha-b_\alpha^\alpha+B_\alpha/n$ and agents in $N\setminus S_\alpha$ receive $x_i^\alpha-b_\alpha^i+B_\alpha/n$ plus what they obtain in the game

[2] Component additivity implies that the value of an isolated player (and therefore, the empty graph) is zero.

played by $N \backslash S_\alpha$. If the offer is rejected, the final payoff to α is $-b_\alpha^\alpha + B_\alpha/n$ and the final payoff to any $i \neq \alpha$ is the sum of $-b_\alpha^i + B_\alpha/n$ and the payoff obtained in the game played by $N \backslash \{\alpha\}$.

Theorem 1. *At any SPE of the bidding mechanism, a strongly efficient graph is always formed. Moreover, the payoffs to the agents are uniquely given by the Shapley value* ϕ.

Proof. The proof proceeds via induction on the number of agents. The theorem holds for the case of $n=1$. We assume it holds for all $m \leq n-1$ and show that it also holds for $m=n$.

Consider a game (N,v) with n agents. We claim (under the induction hypothesis) that the following strategies are the unique SPE of the mechanism and furthermore generate the Shapley value ϕ:

At $t=1$, each agent i, $i \in N$, announces $b_j^i = \phi_i(N \backslash \{j\}, W) - \phi_i(N, W)$ for every $j \neq i$ and $b_i^i = W(N) - W(N \backslash \{i\}) - \phi_i(N, W)$.

At $t=2$, agent i, if he is the proposer, chooses a subset of agents S_i and a graph (a connected component) g_i^* such that $v(g_i^*) + W(N \backslash S_i) = W(N)$. Moreover, he offers $x_j^i = \phi_j(N \backslash \{i\}, W)$ to every $j \in S_i \backslash \{i\}$, and $x_j^i = \phi_j(N \backslash \{i\}, W) - \phi_j(N \backslash S_i, W)$ to every $j \notin S_i$.

At $t=3$, agent i, if agent $j \neq i$ is the proposer and $i \in S_j$, accepts any offer greater than or equal to $\phi_i(N \backslash \{j\}, W)$ and rejects it otherwise. If agent $j \neq i$ is the proposer and $i \notin S_j$, agent i accepts any offer greater than or equal to $\phi_i(N \backslash \{j\}, W) - \phi_i(N \backslash S_j, W)$ and rejects it otherwise.

By the induction argument, all *SPE* must have all agents different from the proposer behave according to the strategy described in $t=3$. The proposer at $t=2$ will not make any offer larger than those prescribed by the strategies at $t=2$. Moreover, given the payoff structure, he would be best off if he forms a component that is part of an efficient graph. It can also be shown that in any *SPE*, the aggregate bid to each agent should be zero and, as a result, an agent's payoff is the same regardless of who is chosen as the proposer. The proposed bids at $t=1$ are the only feasible bids that satisfy all of these requirements.[3] □

Finally, Mutuswami et al. (2004) proved that the equilibrium bidding strategies in the mechanism satisfy very strong properties. In particular, (a) they are unique; (b) they are robust to deviations by coalitions of agents; and (c) they are maxmin strategies at the bidding stage.

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[3] For a more detailed proof, see Pérez-Castrillo and Wettstein (2001) and Mutuswami et al. (2004).

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