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Incentives and competitive allocations in exchange economies with incomplete markets

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Abstract

This paper concentrates on the problems created by strategic behaviour in the presence of incomplete markets. We show that the set of constrained Pareto optimal allocations as well as the set of undominated competitive equilibria are not Nash implementable. In contrast a slightly modified version of competitive equilibria namely endowment constrained competitive equilibria (ECCE) is shown to be Nash implementable by a continuous and feasible mechanism.

Keywords: Incentives; Incomplete markets; Nash implementation

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1. Introduction

In neoclassical economies with complete information and a complete set of markets there is a close relationship between market outcomes and optimality. It is best summarized by the well known welfare theorems: every competitive equilibrium is Pareto optimal and any Pareto optimal allocation can be realized as a competitive equilibrium, following an appropriate redistribution of endowments. However, this close link breaks down once the assumptions regarding the physical environment and the rules of behaviour are relaxed. The present paper focuses on

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the implications of strategic behaviour and incomplete markets, either of which can lead to non-optimal equilibria.

Incomplete markets have been extensively researched as regards existence and properties of equilibria [see Balasko and Cass (1989), Cass (1989 and 1984), Geanakoplos and Polemarchakis (1986), Geanakoplos and Mas-Collel (1989), Grossman (1977), Hart (1975), Repullo (1988), Werner (1985) for the pure exchange case, and Dreze (1974), Grossman and Hart (1979), Stiglitz (1982) and Geanakoplos et al. (1990) for economies with production; an excellent review paper is Geanakoplos (1990)]. Previous results show not only that competitive outcomes are not Pareto optimal (which is not surprising given the incomplete structure of markets), but also that they are not Pareto optimal even when restricting attention to only those allocations that can be achieved through the operation of the existing markets. As the Hart (1975) example shows, there often exists domination in the Pareto sense among different competitive equilibria (this phenomenon has been described as structural inefficiency: Stiglitz (1982)). Geanakoplos and Polemarchakis (1986) have shown that for pure exchange economies with incomplete markets the competitive outcomes are generically not constrained Pareto optimal. [1] Geanakoplos et al. (1990) have extended the result for economies with production.

These findings seem to call for outside intervention to guarantee at least that the existing markets are used efficiently. However, when judging the Pareto optimality of a given outcome, it is not sufficient to refer just to the set of some 'technologically' attainable allocations. One must consider the incentives facing the individuals comprising the economy. To go back to the purely competitive story in the Walrasian paradigm with a finite number of individuals, strategic behaviour cannot be ruled out, and the claim that the competitive equilibrium is Pareto optimal is suspect. If individuals understand the process they are taking part in, they might have an incentive to misrepresent their preferences and the resulting outcome might not be Pareto optimal. These issues were a main source of concern for the implementation literature.

The implementation literature examines the feasibility of realizing social goals. The starting point is a family of economies. A feasible alternative for a given economy is any technologically feasible allocation of commodities among the members of the economy. Social goals are modeled as social choice correspondences (SCCs), where an SCC associates with each economy a set of feasible alternatives. A central designer is assigned the task of designing a mechanism whose equilibria allocations for each economy coincide with the alternatives

[1] The 'constrained' qualification relates to the reference set with respect to which Pareto optimality is evaluated, here the set of allocations that can be achieved through changes in asset holdings and the operation of the existing (spot) markets.

prescribed for it by the SCC. An SCC will be called implementable if such a mechanism exists.

The problems and the possibility of non-implementability stem from incomplete information on the part of the designer. It is assumed the designer knows the SCC but possesses only partial, if any, information regarding the particular economy for which the mechanism will be used. Thus when designing the mechanism, strategic behaviour on the part of individuals cannot be ignored. (This sort of strategic behaviour is ruled out in the purely competitive story.) Individuals must be given the correct incentives to reveal their private information. When an SCC is not implementable, it means it cannot be attained once the problem of incentives and the possibility of strategic behaviour are not ruled out, as indeed they should not be. The optimality of an SCC cannot be discussed independently of its implementability; if it is not attainable its properties are of secondary importance.

Several results in the implementation literature provide necessary and/or sufficient conditions for the implementability of a given SCC. These conditions naturally vary with the particular equilibrium concept considered [see Maskin (1977) and Saijo (1988) for Nash implementation, Moore and Repullo (1988) for subgame perfect implementation, Palfrey and Srivastava (1991) for Nash implementation using undominated strategies, Matsushima (1988) and Abreu and Sen (1991) for virtual implementation].

Several papers [see Hurwicz (1979), Postlewaite and Wettstein (1989) and Schmeidler (1980)] have dealt with strategic behaviour and the problems it raises within the Walrasian paradigm. They improved on previous general results by considering more 'reasonable' mechanisms with smaller message spaces and less restrictive informational assumptions. The main emphasis was on Nash implementation and it was shown that the problems created can be overcome by designing a mechanism implementing the competitive outcomes.

The present paper addresses the issues of strategic behaviour, incentives and Nash implementability in the context of economies with incomplete markets. Our conclusions might have been different had we employed implementation concepts other than Nash. Our findings should be viewed as 'positive' results determining the feasibility of attaining certain outcomes as Nash equilibria.

As remarked before, with incomplete markets competitive outcomes do not display the nice optimality properties they display in economies with complete markets. Some of the competitive equilibria may well be Pareto comparable, and the competitive equilibria allocations would usually not be constrained Pareto optimal. This raises the question of whether or not one could design an institution, performing 'better' than the competitive markets, and implement the set of constrained Pareto optimal allocations or just the set of undominated equilibria.

We show via a counter example that neither the set of constrained Pareto optimal allocations nor the set of undominated competitive equilibria can be implemented in Nash equilibria. Next we construct a mechanism implementing the endowment constrained competitive equilibria, a slightly modified version of the competitive equilibria. [2] The mechanism constructed enjoys several economically plausible properties. The strategy spaces are finite dimensional involving the announcement of net trades and prices without any transmission of utility functions or preference parameters. Furthermore it is a continuous mechanism yielding feasible outcomes for any choice of actions by the agents.

These findings force us to reconsider the claim that competitive behaviour in an economy with incomplete markets is inefficient. They show that in economies with incomplete markets, improving upon the SCC induced by competitive behaviour runs into difficulties when taking into account the implementability of rival suggestions.

This reasoning goes one step further than the analysis in Geanakoplos and Polemarchakis (1990), whose treatment focused on the information necessary for carrying out a Pareto improving plan. They addressed the possibility of recovering it from the demand data of the individuals. However, it was implicitly assumed individuals would not behave strategically, i.e. possibly misrepresent their demands. We explicitly allow for strategic behaviour and examine its implications.

The next section introduces the economy and the necessary notation. The third section demonstrates the non-implementability of the set of constrained Pareto optimal allocations or the set of undominated competitive equilibria. In the fourth section we construct a mechanism implementing the SCC induced by competitive behaviour in economies with incomplete markets and discuss its properties. The fifth section offers conclusions and further extensions.

2. The model: definitions and notation

2.1. The economy

We consider a class of pure exchange economies denoted by E. An economy in E extends over two periods. In period one there are S possible states of nature $(\omega_1, \dots, \omega_s)$. There are K physical commodities $(k = 1, \dots, K)$ available at each date in each state of nature. At period zero there are also D real assets $(d = 1, \dots, D)$ and B financial assets $(b = 1, \dots, B)$. The payoffs of a real asset d are given by a $K \times S$ matrix $\boldsymbol{\delta}^d$, where $\boldsymbol{\delta}^d_{ks}$ denotes the amount of commodity k yielded by asset d at state s (the s column of the matrix is denoted by $\boldsymbol{\delta}^d_s$). A financial asset b is described by an S dimensional vector $\boldsymbol{\beta}^b$ where $\boldsymbol{\beta}^b_s$ denotes the monetary payoff (in 'units of account') of asset b in state s.

[2] The definition of competitive equilibria needs to be slightly altered if we want the mechanism to yield non-negative consumption allocations for any choice of actions by the agents. This observation is due to Hurwicz et al. (1982), where it is shown that the Walrasian correspondence is not implementable.

The incomplete markets aspect enters through the organization of trade. At period one there are spot markets for all commodities at each state. At period zero there is trade in commodities and period zero assets. We take the set of assets as well as the trade rules as given. This immediately raises the problem of how would a mechanism suggested by a central designer respect the constraints on exchange, implicitly imposed by the structure of the markets. We shall comment further on that when constructing the mechanism.

There are N (we assume $N \geq 3$) consumers ($i = 1, \ldots, N$). A consumer is completely specified by a utility function $u^i : R_+^{K(S+1)} \to R$ and an initial endowment $w^i \in R_{++}^{K(S+1)}$ (w_0^i and w_s^i in R_{++}^K will denote the initial endowments in period zero and state s respectively.) $W = \sum_{i=1}^N w^i$ will denote the aggregate endowment. The u^is are assumed to be strictly quasi-concave, strictly increasing and continuously differentiable. $z^i \in R^{K(S+1)}$ and $a^i \in R^{B+D}$ will denote a net trade and a portfolio for consumer i (z_0^i and z_s^i will denote the net trades in period zero and in state s respectively. The first B components of a^i denote the holdings of financial assets and the last D, the holdings of real assets).

An endowment constrained competitive equilibrium (ECCE) is a pair of price vectors $q^* \in R^{B+D}$ (asset prices) and $p^* \in R_+^{K(S+1)}$ (commodity prices $p^* = (p_0^*, p_1^*, \ldots, p_S^*)$ where $p_i^* \in R_+^K$), a collection of net trades $(z^{*i})_{i=1}^N$ and a collection of portfolios $(a^{*i})_{i=1}^N$ that satisfy the following:

(i) For every $i = 1, \ldots, N$ (z^{*i}, a^{*i}) maximizes $u^i(w^i + z^i)$ over the set:

$$B^i(p^*, q^*) = \left\{ (z^i, a^i) \mid p_0^* z_0^i + q^* a^i \leq 0,\ w^i + z^i \leq W, \right.$$

$$\left. p_s^* z_s^i \leq p_s^* \sum_{d=1}^{D} a_d^i \delta_s^d + \sum_{b=1}^{B} a_b^i \beta_s^b,\ s = 1, \ldots, S \right\}$$

(ii) $\sum_{i=1}^N a^{*i} = 0$; $\sum_{i=1}^N z^{*i} = 0$.

The only difference between this definition and the usual definition of a competitive equilibrium is the constraint $w^i + z^i \leq W$. Both definitions coincide for interior allocations.

The net trade z^* would be called an ECCE allocation.

2.2. The mechanism

A mechanism consists of an n-tuple of strategy sets, one for each individual and an outcome function mapping n-tuples of strategies into feasible allocations. We view the mechanism as a game form with the actual payoffs determined by the utility functions of the participating individuals. Nash equilibria are defined in the customary way, and we are interested in the set of allocations generated by the

Nash equilibria. Detailed discussion and notation can be found in the implementation papers cited above. In order not to burden the reader with unnecessary notation we skip this more general formulation and describe our particular mechanism in Section 4.

3. Constrained Pareto optimal allocations and undominated competitive equilibria allocations are not implementable

In order to show that the SCC induced by the constrained Pareto optimal allocations is not implementable, we construct an economy (*EE*). In *EE* a monotonic change in preferences [3] at a constrained Pareto optimal allocation leads to an economy *EE'* where this allocation is constrained sub-optimal, thus eliminating it from the SCC. However, as is well known (Maskin, 1977), a necessary condition for the implementability of an SCC is that such a monotonic change of preferences should leave the alternative (at which it took place) in the SCC. [4] Thus the SCC induced by constrained Pareto optimal allocations is indeed not implementable. *EE* will have a unique competitive equilibrium where individuals end up consuming their initial endowments. It can be easily verified that this allocation is constrained Pareto optimal. *EE'* will possess two competitive equilibria, the one with no trade as well as a second equilibrium dominating it. This second equilibrium serves to show that the initial allocation is no longer constrained Pareto optimal (it can be achieved via the existing spot markets and appropriate asset holdings in period zero).

It is also clear that this example serves to demonstrate the non-implementability of the SCC induced by undominated competitive equilibria. This SCC will contain the original allocation by virtue of it being the unique competitive equilibrium. However, this allocation will not be part of the SCC in the transformed economy, due to the existence of the second equilibrium which dominates the original equilibrium.

Description of the economy *EE*: the economy extends over two periods. There are two possible states in period one, and two physical commodities. In period zero there are two real assets

$$\boldsymbol{\delta}^1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{\delta}^2 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}.$$

[3] A change of preferences at a given allocation is called monotonic if, at the new profile of preferences, the upper contour set of each individual at that allocation is contained in the upper contour set prior to the change of preferences.

[4] A necessary condition for Nash implementability is Maskin monotonicity. An SCC satisfies the Maskin monotonicity condition if whenever an allocation is prescribed by the SCC for a given profile of preferences, the SCC goes on prescribing it for any profile of preferences obtained from the given profile by a monotonic change of preferences at x.

There are no financial assets. The initial endowments of the individuals are given by: $w^1 = (w_0^1, w_1^1, w_2^1)$ and $w_2 = (w_0^2, w_1^2, w_2^2)$ with

$$w_0^1 = w_0^2 = (0, 0); \quad w_1^1 = (2, 2); w_2^1 = (4, 4)$$
$$w_1^2 = (4, 4); w_2^2 = (2, 2).$$

Their preferences are given by: [5]

$$u^1 = 0.5\left(\left(x_{11}^1\right)^{0.5} + \left(x_{12}^1\right)^{0.5}\right) + \left(x_{21}^1\right)^{0.5} + \left(x_{22}^1\right)^{0.5}$$
$$u^2 = 0.5\left(\left(x_{11}^2\right)^{0.5} + \left(x_{12}^2\right)^{0.5}\right) + \left(x_{21}^2\right)^{0.5} + \left(x_{22}^2\right)^{0.5}$$

We can normalize prices at each state with $q_1 = p_{11} = p_{21} = 1$ (p_{ij} denotes the price of commodity j in state i of period one), and look for q_2, p_{12} and p_{22} that would clear the markets.

If $p_{12} \neq p_{22}$ then the two distinct budget constraints can be collapsed into one constraint. However the resulting equilibrium prices imply $p_{12} = p_{22}$ in contradiction to our original assumption. Hence there cannot exist an equilibrium with $p_{12} \neq p_{22}$.

If $p_{12} = p_{22}$ then the second asset's price must be p_{12} as well. In this case trading in assets does not change the consumption possibilities in period one. The equilibrium prices in period one are then $p_{12} = p_{22} = 1$, and the individuals stay with their initial endowments. It can be easily verified that this allocation is constrained Pareto optimal. (See the Appendix for the detailed calculations.) Being the only equilibrium it also belongs to the SCC induced by undominated competitive equilibria.

Now consider a new economy EE' where the second individual's preferences remain the same, whereas individual one's preferences undergo a monotonic transformation at the point (2, 2, 4, 4). [6] The new utility function for one is given by:

$$\frac{2^{0.5}}{4}\ln\left(x_{11}^1\right) + 0.5\left(x_{12}^1\right)^{0.5} + \left(x_{21}^1\right)^{0.5} + \ln\left(x_{22}^1\right)$$

[5] This economy clearly violates the assumption of strictly positive endowments. Furthermore no actual consumption will take place in period zero and individuals' preferences are defined only on period one's consumption. This facilitates the calculation of equilibria. A suitably modified version of the mechanism constructed in Section 4, with no period zero consumption component, would still implement the ECCE for this economy.

The fact that there are only two individuals in the economy (whereas one needs three individuals for the implementation results that will follow) is intended to simplify the presentation. We can get the same conclusions for any n-replica of this economy.

[6] The upper contour set of the individual at that point shrinks. (2, 2, 4, 4) is preferred to anything it was preferred to, prior to the change.

With this utility function the old equilibrium, with effectively no trade in assets and having the individuals stay with their initial endowments, still exists and yields utility levels of:

$$u^1 = 4.3385 \text{ and } u^2 = 4.8284.$$

However, following this change there is another competitive equilibrium with prices given by:

$$q_1 = p_{11} = p_{21} = 1;\ q_2 = 1.0541;\ p_{12} = 1.0965;\ p_{22} = 1.0347$$

and allocations given by:

$d_1^1 = -58.7503;\ d_2^1 = 55.7360$

$x_{11}^1 = 2.8517;\ x_{12}^1 = 3.3817;\ x_{21}^1 = 3.3818;\ x_{22}^1 = 3.5545$

$b_1^2 = 58.7507;\ b_2^2 = -55.7363$

$x_{11}^2 = 3.1483;\ x_{12}^2 = 2.6183;\ x_{21}^2 = 2.6182;\ x_{22}^2 = 2.4455.$

The utility levels at this competitive equilibrium are

$u^1 = 4.3971$ and $u^2 = 4.8781$

So the allocation (2, 2, 4, 4) and (4, 4, 2, 2) is now dominated by the second competitive equilibrium. This allocation is hence constrained sub-optimal and drops out of the SCC induced by constrained Pareto optimal allocations. The same holds for the SCC induced by undominated competitive equilibria. Thus both SCCs are indeed not implementable.

4. The ECCE allocations are implementable: construction of the mechanism

The strategy space of each individual i would be $S^i = R^{B+D} \times R_+^{K(S+1)} \times R^{B+D} \times R^{K(S+1)} \times R_{++}$ with generic element $(q^i, p^i, a^i, z^i, m^i)$. Individual i's choices can be interpreted as suggested prices for assets (q^i) and commodities (p^i), requested trades in assets and commodities $(a^i$ and $z^i)$ and a parameter m^i used in the process of averaging the possibly conflicting demands of all the individuals.

The mechanism will operate as follows: the prices announced by the individuals are averaged in such a way that if all individuals except for a single one announce the same price, this will be the average price constructed and hence this single individual's announcement will have no effect on the price constructed.

The next step determines the trades in assets. The trades announced by the individuals are modified so as to satisfy the two requirements of asset markets clearing and individual feasibility. (It must be the case for all individuals that in period zero the value of the assigned portfolio cannot exceed the value of the initial endowment. Furthermore, for any of the S possible states in period one the aggregate wealth (combining the initial endowment and the portfolio's payoff) of an individual is non-negative and bounded from above by the value of the aggregate endowment W. The value is based on the average price previously determined.) The modification is done in such a way that each individual can get portfolios arbitrarily close to any portfolio within the individually feasible set (respecting the wealth constraints outlined above).

The next step entails the construction of budget sets for each individual. These budget sets are based on the prices and asset trades derived previously, and incorporate the aggregate endowment constraint ($w^i + z^i \leq W$) as well. The net trades in commodities announced by the individuals are projected onto the corresponding budget sets. These resulting net trades, however, might not be feasible in the aggregate. In that case, the commodity bundles assigned to the individuals are scaled down just enough to guarantee aggregate feasibility, assuming physical commodities can be freely disposed of.

We show that in the economies we consider, the allocations generated by Nash equilibria for this mechanism coincide with the set of ECCE allocations. Thus competitive outcomes in pure exchange economies with incomplete markets can indeed be implemented.

We note that the allocations generated by the mechanism both in and out of equilibrium always coincide with allocations that can be achieved through the operation of the existing markets. The trades assigned satisfy the same set of budget constraints across all the individuals. So if there are any technical reasons or constraints on exchange that were the cause for the incomplete market structure to start with, they are respected by the designer as well.

We now formally describe the mechanism and prove the equivalence between Nash and competitive equilibrium outcomes.

Step 1. We construct a pair of weighted average price vectors as follows. Define

$$\boldsymbol{\alpha}^i = \sum_{t,t' \neq i} |p^t - p^{t'}|^2; \quad \boldsymbol{\alpha} = \sum_{i=1}^{N} \boldsymbol{\alpha}^i$$

$$\boldsymbol{\beta}^i = \begin{cases} \dfrac{\boldsymbol{\alpha}^i}{\boldsymbol{\alpha}} & \boldsymbol{\alpha} > 0 \\ \dfrac{1}{n} & \boldsymbol{\alpha} = 0 \end{cases}$$

and

$$\bar{p} = \sum_{i=1}^{N} \boldsymbol{\beta}^i p^i.$$

The weighted average of the asset price vectors is similarly constructed and is denoted by $\bar{q}$.

It is evident from the construction that if all individuals except for one announce the same price vector, that would be the average price vector constructed. The result is independent of that single individual's message since, when taking the average, a zero weight is attached to it.

Step 2. The announced trades in assets are adjusted so as to clear the asset markets and guarantee individual feasibility. In order to do that, the following minimization problem is solved, thus generating the desired asset trades.

$$\operatorname{Min} \sum_{i=1}^{N} | \eta^i a^i - a^i |^2 \cdot m^i$$

$$\eta^i$$

S.T.

$$\sum_{i=1}^{N} \eta^i a^i = 0$$

$$\bar{q}(\eta^i a^i) \le \bar{p}_0 w_0^i, \; i = 1, \dots, N$$

$$0 \le \bar{p}_s \sum_{d=1}^{D} (\eta^i a_d^i) \delta_s^d + \sum_{b=1}^{B} (\eta^i a_b^i) \beta_s^b + \bar{p}_s w_s^i \le \bar{p}_s W_s,$$

$$i = 1, \dots, N; \; s = 1, \dots, S.$$

This minimization problem will always have a solution $(\eta^{*i})_{i=1}^N$, and the asset trade assigned to individual i would be $\bar{a}^i = \eta^{*i} a^i$. The announced a^is will coincide with the assigned ones if they happen to satisfy all the feasibility constraints and sum up to zero. Furthermore, note that if individual i announces an a^i that satisfies the various feasibility constraints, then by announcing an m^i large enough, the solution to this minimization problem would entail an η^i arbitrarily close to one and hence generate an asset trade (for i) arbitrarily close to the one announced by individual i.

Step 3. Based on the constructed prices and asset trades the following n budget sets are constructed:

$$B^i\left(\bar{p}, \bar{q}, (\bar{a}^i)_{i=1}^N\right) = \left\{ z \in R^{K(S+1)} \;\middle|\; \begin{array}{l} z + w^i \le W \quad z + w^i \ge 0 \\ \bar{p}_0 z_0 + \overline{q a}^i \le 0 \\ \bar{p}_s z_s \le \bar{p}_s \sum_{d=1}^{D} \bar{a}_d^i \delta_s^d + \sum_{b=1}^{B} \bar{a}_b^i \beta_s^b \end{array} \right\}.$$

Let y^i be the closest point in B^i (a non-empty convex set) to z^i. These y^is may not be feasible in the aggregate; to take care of that problem the following set J is defined:

$$J = \left\{ m \in R_{++} \mid m m^i \le 1, \; i = 1, \dots, N; \; m \sum_{i=1}^{n} m^i (y^i + w^i) \le W \right\}$$

and we let $m^* = \operatorname{Max}_{m \in J} m$.

The n final bundles assigned to the individuals by the mechanism are: $\gamma^i = m^* m^i (y^i + w^i)$, $i = 1, \dots, N$.

Theorem. *For any economy in E, the set of Nash allocations for the mechanism constructed above coincides with the set of ECCE allocations.*

Proof. First we show that any ECCE allocation can be realized as a Nash equilibrium. Let the allocation $(x^{*1},\dots,x^{*n})$ with associated asset trades $(a^{*1},\dots,a^{*n})$ and prices p^* (for commodities) and q^* (for assets) be an ECCE allocation. The following n-tuple of strategies forms a Nash equilibrium, yielding that allocation. $p^i = p^*$; $q^i = q^*$; $a^i = a^{*i}$; $z^i = x^{*i} - w^i$; $m^i = 1$, $i = 1,\dots,n$ for this n-tuple of strategies $\bar{p} = p^*$; $\bar{q} = q^*$; $m^* = 1$; $\gamma^i = x^{*i}$.

When assigning the asset trades we get $\boldsymbol{\eta}^i = 1$. (The a^{*i}s are feasible and sum up to zero by virtue of being part of an ECCE allocation). The z^is are contained in the constructed budget sets and thus $y^i = z^i$. Since these net trades are part of an ECCE allocation they are feasible in the aggregate and $m^* m^i = 1$.

It remains to show this is a Nash equilibrium. No individual i can change the $\bar{p}$ and $\bar{q}$ by altering p^i and q^i. By changing the announced asset trades and/or the announced net trades the individual might indeed generate a different final bundle γ^i. This bundle however would still satisfy $\gamma^i + w^i \le W$ and together with the constructed asset trade would satisfy all the budget constraints, evaluated at prices $\bar{p}$ and $\bar{q}$. The bundle allocated by the mechanism, prior to the deviation, formed part of an ECCE allocation and hence must be weakly preferred to the bundle generated by the planned deviation. So no individual i can get a strictly preferred outcome by deviating and the suggested n-tuple of strategies indeed forms a Nash equilibrium. □

The proof that any Nash equilibrium is an ECCE is accomplished via the following series of lemmata.

Let $\bar{p}^*$; $\bar{q}^*$; $\bar{a}^{*i}$; m^{*i}; m^{**}; γ^{*i} be the values generated at a Nash equilibrium point.

Lemma 1. *Given the prices* $\bar{p}^*$; $\bar{q}^*$ *individual i can get arbitrarily close to any net trading plan* (a, z) *in* $R^{B+D}xR^{K(S+1)}$

$$\bar{p}_0^* z_0 + \bar{q}^* a \le 0$$

that satisfies
$$\bar{p}_s^* z_s \le \bar{p}_s^* \sum_{d=1}^{D} a_d \boldsymbol{\delta}_s^d + \sum_{b=1}^{B} a_b \boldsymbol{\beta}_s^b$$

$$z + w^i \ge 0;\ z + w^i \le W;\ z + w^i \ne 0.$$

Proof. The a component satisfies the feasibility constraints in the minimization problem determining the $\boldsymbol{\eta}^i$s. Announcing a large enough m^i would generate an asset trade, for individual i, arbitrarily close to a. Hence the individual could generate any of a sequence of asset trades $(a_t)_{t=1}^{\infty}$ converging to a. It is easy to verify that there exists a sequence of $(z_t)_{t=1}^{\infty}$ with z_t in the budget set corresponding to a_t such that z_t converges to z. (Note that we assumed $z + w^i$ has strictly positive elements.)Announcing a large enough m^i makes m^* very small. As a result all the terms, except for the ith term, in the sum $m\Sigma_{i=1}^N m^i(y^i + w^i)$ are driven to zero. Hence the individual can get arbitrarily close to any of those z_ts. So in effect the individual can get arbitrarily close to z. □

Lemma 2. $m^{**}m^{*i} = 1$ *for* $i = 1, \dots, N$.

Proof. Assume by way of contradiction that there exists an individual i for which $m^{**}m^{*i} < 1$. Hence the final bundle individual i received in the competitive equilibrium is strictly better than the Nash equilibrium outcome. Since the competitive equilibrium trading plan satisfies the requirements of Lemma 1, we have reached a contradiction to the Nash property. (Preferences are continuous and the individual can get arbitrarily close to that better outcome.) □

Lemma 3. $\Sigma_{i=1}^N(y^i + w^i) = \Sigma_{i=1}^N w^i$

Proof. As before, we conclude that at a Nash equilibrium each individual must satisfy the budget constraints (of step 3) with equality. If there exists a component j such that: $\Sigma_{i=1}^N(y^i + w^i)j < \Sigma_{i=1}^N w^i$ (note that we cannot have an opposite inequality by the definition of J), we must have a coordinate k for which the situation is reversed. (Summing up over the budget constraints it must be the case that $\bar{p}\Sigma_{i=1}^N(y^i + w^i) = \bar{p}\Sigma_{i=1}^N w^i$.) This contradicts the definition of J.

So far we have shown that the net trades assigned by the mechanism satisfy the budget constraints at prices $\bar{p}^*$ and $\bar{q}^*$. It remains to be shown that individuals are maximizing their utility.

Suppose that there exists a trading plan (a, z) that satisfies the budget constraints for individual i as well as $z + w^i \leq W$ and is preferred to γ^{*i}. Hence it satisfies the requirements of Lemma 1 and individual i can attain a net trading plan arbitrarily close to z which, by continuity of preferences, would be preferred to γ^{*i}, contradiction to it being part of a Nash equilibrium. Hence the Nash allocation forms an ECCE allocation. □

5. Conclusions

We have started by showing that certain SCCs, which are considered 'better' than the SCC induced by competitive behaviour in economies with incomplete markets, are not Nash implementable. This implies that an intervention, be it through the construction of any mechanism whatsoever, will not be able to generate only constrained Pareto optimal allocations or select just the undominated equilibria.

Then we proceeded to show that the competitive outcomes when viewed as an SCC can be Nash implemented. Thus, even though not possessing any obvious optimality properties, they can be realized despite strategic behaviour on part of the individuals. This strategic behaviour might in turn undermine attempts to improve on the competitive outcomes, in the same way it prevented the Nash implementation of the above mentioned SCCs.

It remains the topic of future research to examine the implications strategic behaviour (in these economies) might have when considering solution concepts other than Nash. These solution concepts do lead to a much larger class of implementable SCCs but involve highly complex mechanisms. It remains to be seen to what degree they can be simplified when dealing with these economic environments.

The extension to economies with production is not as straightforward as one might expect. The firm's objectives in such an environment are not well defined and as a result several behaviour criteria have been suggested. It would be important to discuss their implementability and once this has been established consider the possibility of implementing something 'better'.

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Appendix: Competitive equilibrium in the economy *EE*

The budget set facing individual 1 in *EE* is given by:

$$b_1^1 + q_2 \cdot b_2^1 = 0$$

$$x_{11}^1 + p_{12} \cdot x_{12}^1 = \left(2 + b_1^1\right) + p_{12} \cdot \left(2 + b_2^1\right)$$

$$x_{21}^1 + p_{22} \cdot x_{22}^1 = \left(4 + b_1^1\right) + p_{22} \cdot \left(4 + b_2^1\right)$$

or (eliminating asset one holdings)

$$x_{11}^1 + p_{12} \cdot x_{12}^1 = \left(2 - q_2 \cdot b_2^1\right) + p_{12} \cdot \left(2 + b_2^1\right)$$

$$x_{21}^1 + p_{22} \cdot x_{22}^1 = \left(4 - q_2 \cdot b_2^1\right) + p_{22} \cdot \left(4 + b_2^1\right)$$

If p_{12} does not equal p_{22} then q_2 does not equal p_{22} either and we can eliminate b_2^1 and get a single budget constraint facing individual 1 which is given by:

$$x_{11}^1 + \pi_2 \cdot x_{12}^1 + \pi_3 \cdot x_{21}^1 + \pi_4 \cdot x_{22}^1 = 2 + \pi_2 \cdot 2 + \pi_3 \cdot 4 + \pi_4 \cdot 4$$

where

$$\pi_2 = p_{12}; \; \pi_3 = \frac{p_{12} - q_2}{q_2 - p_{22}}; \; \pi_4 = \pi_3 \cdot p_{22}.$$

Individual 2 will face a similar budget constraint with different endowments. However, as can be easily verified, the market clearing prices in such a case would be given by: $\pi_2 = 1$; $\pi_3 = 2$; $\pi_4 = 2$, which implies that $p_{12} = p_{22}$ in contradiction to our original assumption. So we must have $p_{12} = p_{22}$ and then it follows that the equilibrium allocation is given by the initial endowments.

This allocation (given by the initial endowments) is also constrained Pareto optimal in *EE*. To see that, we consider allocations generated by arbitrary asset holdings in period zero. (Note that the asset holdings are not entirely arbitrary, since they cannot lead to violation of the feasibility constraints in period 1.)

Denoting the holdings of asset j by individual i by b_j^i we get the following pairs of budget constraints.
For individual 1:

$$x_{11}^1 + p_{12} \cdot x_{12}^1 = 2 + b_1^1 + p_{12} \cdot \left(2 + b_2^1\right)$$

$$x_{21}^1 + p_{22} \cdot x_{22}^1 = 4 + b_1^1 + p_{22} \cdot \left(4 + b_2^1\right),$$

and for individual 2:

$$x_{11}^2 + p_{12} \cdot x_{12}^2 = 4 + b_1^2 + p_{12} \cdot \left(4 + b_2^2\right)$$

$$x_{21}^2 + p_{22} \cdot x_{22}^2 = 2 + b_1^2 + p_{22} \cdot \left(2 + b_2^2\right).$$

Adding up the budget constraints for state 1, recalling that the commodity markets must be cleared, yields:

$$b_1^1 + b_1^2 + p_{12} \cdot \left(b_2^1 + b_2^2\right) = 0,$$

and similarly (from state 2 budget constraints)

$$b_1^1 + b_1^2 + p_{22} \cdot \left(b_2^1 + b_2^2\right) = 0.$$

The spot markets at state 1 would imply the following demands on part of the individuals:

$$x_{11}^1 = \frac{2 + b_1^1 + p_{12} \cdot \left(2 + b_2^1\right)}{1 + \dfrac{1}{p_{12}}}$$

$$x_{11}^2 = \frac{4 + b_1^2 + p_{12} \cdot \left(4 + b_2^2\right)}{1 + \dfrac{1}{p_{12}}}$$

Market clearing would require:

$$\frac{6+b_1^1+b_1^2+p_{12}\cdot 6+p_{12}\cdot\left(b_2^1+b_2^2\right)}{1+\dfrac{1}{p_{12}}}=6$$

which implies $p_{12}=1$. Similarly $p_{22}=1$.

Hence, following asset reallocations, the individuals obtain the following consumption bundles:

$$x_{11}^1=x_{12}^1=2+\frac{b_1^1+b_2^1}{2};\ x_{21}^1=x_{22}^2=4+\frac{b_1^1+b_2^1}{2}$$

and

$$x_{11}^2=x_{12}^2=4+\frac{b_1^2+b_2^2}{2};\ x_{21}^2=x_{22}^2=2+\frac{b_1^2+b_2^2}{2}.$$

Hence the initial endowments do form a constrained Pareto optimal allocation. One individual's gain through asset reallocations is the other individual's loss, and it is impossible to make both of them better off. (Since $p_{12}=1$ we have $b_1^1+b_2^1=-(b_1^2+b_2^2)$.)

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