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# Rational bidding in a procurement auction with subjective evaluations☆

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ABSTRACT

In practice, procurement auctions often involve subjective evaluations of bids, especially when consisting of quality or design parameters which are hard to quantify. We formally define a notion of subjectivity in an auction environment and analyze the implications for rational bidding behavior. Our findings explain some observed bidding behaviors that are inconsistent with standard equilibrium predictions. Finally we examine the way subjectivity facilitates the practice of favoritism on part of the auctioneer.



## 1. Introduction

The Government of India carried out the Delhi and Mumbai Airport Privatization (DMAP) auctions during the years 2003–6 with the objective of modernizing the airports. The two critical dimensions along which the bids were evaluated consisted of quality and revenue. The main feature distinguishing these from a standard auction was the difficulty to precisely quantify the quality variable.

In the DMAP auction firms submitted their bids simultaneously. A bid consisted of a technical and a monetary part. The technical part included the proposed design of the airport as well as the bidder's characteristics (size, business history, experience in airport development etc.). These firm specific features were taken as good proxies for quality (for details see Jain et al. (2007)). The monetary part was the revenue accruing to the government.

The auctioneer publicly announced a minimum quality-score prior to the submission of bids. The bids were evaluated in two rounds. In the first round, technical bids were evaluated. Each item in the technical bid was graded. The overall quality-score of the bid was given by a weighted average of these grades. If a technical bid obtained the minimum quality-score, then the bidder qualified for the subsequent round. In this round, the monetary bids of the qualified bidders were compared. The winner was the bidder whose monetary bid was the highest among those who qualified for the second round.

Setting a quality cut-off is a pervasive practice in public procurements. The setting of a minimum quality standard has an obvious implication in standard auction environments. It is a dominant strategy for the bidders to offer the minimum quality level, and hence all bidders qualify. The winner is then effectively decided by a first-price auction with respect to the monetary dimension. However, in the DMAP auction many bidders did not qualify. We believe that the difficulty of quantifying the actual quality of a bid played an important role in leading to disqualified bidders.

For example, in the DMAP auction, even though the weighting scheme was common knowledge, the exact evaluation procedure i.e. the information about "what kind of technical bid would obtain what score", was not conveyed to the bidders prior to the submission of bids. This may be due to the inability of the auctioneer to completely specify all design aspects. It might also be the case that the auctioneer is not sure of what is the "exact" design she is looking for. This means that the evaluation of technical bids was indeed subjective, a phenomenon well documented in Jain et al. (2007). More precisely, we say that the evaluation of technical bids is subjective, if there are several methods, giving rise to different rankings, to evaluate a technical bid and the auctioneer cannot commit to use one particular method.

Examples of cut-off and subjectivity are quite common: one class of examples is defense procurement of supplies and services as it often

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involves a two-stage procedure where offers are first screened on the basis of a list of minimal standards and then ranked according to price and other parameters. The minimal requirements, while described, allow discretion on the part of the procurer leading to what we see as subjective evaluation. This is the case for the U.S. Department of Defense[1] as well as for the Australian[2]; another class is government procurement in general as is the case for the European Union[3] South Africa[4] and the Philippines.[5]

The issue of subjectivity in evaluation appears in some other contexts as well. For instance, while evaluating research proposals, a funding organization may usually use any method to evaluate the proposals submitted and need not commit to any specific one. When examining offers to operate a catering service, evaluating the quality of food is again subjective in the sense we defined.

In such subjective environments the standard auction model has to be modified to take account of the uncertainty faced by the bidders. We propose a procurement auction model allowing for uncertainty arising due to the subjectivity regarding the quality evaluation process. Bidders participating in the auction bid without knowing the precise evaluation method of the technical component. They entertain beliefs regarding the evaluation methods and the strategies of the other bidders, which may be false. We define rational bidding behavior in this context (following Kalai and Lehrer (1993)). We show that in the case of correct beliefs, either all or none of the bidders qualify. Furthermore, when beliefs are not correct we show that indeed some bidders may not qualify. We also show several other properties implied by rational bidding behavior.

We also observe that subjectivity makes it impossible to impose a legal requirement on the auctioneer to commit to a particular evaluation method. This makes it possible for a dishonest auctioneer to manipulate the evaluation so as to favor one of the bidders. To analyze this phenomenon of "favoritism", we formalize a manipulation scheme, and examine the resulting equilibrium bidding on part of favored and non-favored bidders. In particular, we show that this form of favoritism may lead to inefficiency.

### *1.1. Related literature*

Che (1993) analyzed an auction with a bid-structure identical to ours with a different outcome function. While we used a lexicographic scoring rule to rank the different bids, Che (1993) used a quai-linear scoring rule for ranking. A scoring rule is a real valued function whose domain is the set of all two-tuples of technical and financial bids. The winner with the highest score wins the auction. The scoring auctions introduced by Che (1993) have been extended in different ways by Asker and Cantillon (2010), Branco (1997) and Naegelen (2002). However, none of these papers consider the implications of subjective scoring rules.

A related model is Ganuza and Pechlivanos (2000). In this model the buyer announces a design "a priori" and the firms compete on the cost parameter. They characterize optimal Bayesian incentive-compatible mechanisms. In our model the design is not specified a priori, nor is the bidding in terms of the cost parameters.

There is a large and growing literature on corruption/collusion in auctions. We only refer to some of the related papers on favoritism. In Laffont and Tirole (1991) an auctioneer, acting on behalf of a buyer, is asked to choose a firm to carry out a public project. Before the bids are submitted, the auctioneer receives a signal about the quality of the firms participating in the auction. The auctioneer can then transmit some information (not necessarily correct) to the buyer about their quality. Arozamena and Weinschelbaum (2009) consider corruption in first price auction when it is known among the other bidders that the auctioneer favors one of the bidders. The dishonest bidder is allowed to revise his bid upward or downward by the auctioneer. Burguet and Che (2004) consider a scoring auction where the relevant bids for the buyer are two dimensional, quality and price. They assume that both the bidders are dishonest — along with quality and price they bid a bribe. The auctioneer manipulates the quality bid in favor of the bidder submitting the larger bribe. In our model, the auctioneer does not manipulate the technical bid directly. Instead the evaluation procedure is manipulated to favor a preferred bidder. For further details on corruption in auctions we refer the reader to Wolfstetter and Lengwiler (2006).

The next section introduces the model. In Section 3 we define and analyze rational strategies. Favoritism is discussed in Section 4 and Section 5 concludes.

## 2. The model

The environment we consider can be described as follows: we assume that there is one auctioneer and two identical bidders (suppliers). The auctioneer wishes to procure a good, the quality of which, denoted by $q$, is variable. The auctioneer holds an auction where the bidders simultaneously submit a two-dimensional bid denoted by $(t, p) \in \Re^2_+$. The first component is the technical part of the bid and the second is the monetary part. The auctioneer evaluates the technical bid. The score that a technical bid obtains due to the evaluation is called the quality-score of that bid. The auctioneer specifies a cut-off level of the quality-score exogenously i.e. announced before the bids are submitted and this cut-off level is common knowledge. Let the cut-off quality-score be $\underline{q} > 0$. The winner of the auction is the bidder who bids the lowest monetary offer while satisfying the quality requirement. The winner carries out the project that corresponds to the winning technical bid and receives the monetary part of the bid.

The main feature distinguishing this environment from a standard procurement setting is the fact that the evaluation procedure cannot be precisely specified, which renders it subjective. The inability to specify it is due to the complexity of the typical evaluation procedure which tries to summarize a diverse set of hard to quantify technical attributes. That is, the bidders cannot be certain of the score of a given technical bid. We assume that the set of possible evaluation procedures consists of two functions, $q(t, \eta_1) = \eta_1 t$ and $q(t, \eta_2) = \eta_2 t$ with $0 < \eta_1 < \eta_2$. We let $\Omega = \{\eta_1, \eta_2\}$ with the auctioneer "unable"[6] to specify the actual $\eta_k$ that will be used to evaluate technical bids and assume $\Omega$ is common knowledge. We assume that the uncertainty regarding the evaluation process is resolved only after bids are submitted. Since $\eta_1 < \eta_2$ we call the evaluation procedure using $\eta_1$ strict, as it increases the expected cost of any bidder to qualify.

Each bidder incurs a cost $c(t, \theta)$ if he bids $t$ and is of type $\theta$ which is private information. The $\theta$'s are independently and identically distributed across the two bidders with support $[\underline{\alpha}, \overline{\alpha}] \subset \Re_{++}$ and a strictly increasing distribution function $F$ with a continuously differentiable density function $f$. We assume that the first and second-order partial derivatives of the cost function satisfy $c_t > 0$, $c_\theta > 0$, $c_{tt} \geq 0$, $c_{t\theta} \geq 0$ and $c_{\theta\theta} \geq 0$. The utility of bidder $i$, of type $\theta_i$, playing (bidding) $(t_i, p_i)$, if he is the winner of the auction is given by $p_i - c(t_i, \theta_i)$ and zero otherwise.

This environment generalizes the procurement setting introduced in a seminal paper by Che (1993). Without uncertainty regarding the evaluation procedure, it is a dominant strategy for each agent to submit the lowest qualifying quality. Hence, effectively, the competition is just over the monetary part and our set-up reduces to a standard first-price

[1] See http://www.acq.osd.mil/dpap/cpic/cp/docs/BBP_2-0_Comp_Guidelines_Update_(3_Dec_2014).pdf (pages 13–14).

[2] See http://www.defence.gov.au/dmo/Multimedia/DPPM-9-5247.pdf (page 4.4–4).

[3] See Lundberg and Marklund (2011), Section 1 — Institutional settings.

[4] See https://www.environment.gov.za/sites/default/files/legislations/pppfa_guideline.pdf (pages 14–16).

[5] See http://www.treasury.gov.ph/wp-content/uploads/2015/04/RFEI-Auction-and-Registry-System-for-GS-Modernization.pdf.

[6] We emphasize that the evaluation method is not part of the auctioneer's private information.

auction. The presence of uncertainty generates a significantly different environment where some agent types may not qualify and agents' strategies may depend on subjective beliefs about $\Omega$.

## 3. Rational bidding

Each bidder's strategy consists of bidding a pair $(t, p)$ as a function of $\theta$. Due to subjectivity a bidder has beliefs about the evaluation procedures as well as conjectures regarding the strategy of the other bidder. We restrict our analysis to bidders who adopt strategies that are best responses given their beliefs and conjectures. We call such best responses subjectively-rational, following Kalai and Lehrer (1993). In this section we study the bidding behavior in the auction described above. We assume that both bidders have their own subjective beliefs i.e. subjective probability distributions over the set $\Omega$. We denote the subjective probability distribution of bidder $i$ over $\Omega$ by $\mu^i$, that is $\mu^i = (\mu^i(\eta_1), \mu^i(\eta_2))$. In addition to the beliefs about the evaluation procedure, each bidder is assumed to have a conjecture[7] regarding the strategy employed by the other bidder. These notions are captured in what we define to be a joint strategy.[8]

A joint strategy of bidder $i$ is given by $s^i = ((t_i^i, p_i^i), (t_j^i, p_j^i), \mu^i)$. The first pair describes the strategy of bidder $i$, the second is the conjecture of bidder $i$ regarding bidder $j$'s strategy, denoted by $s_j^i$. The expected payoff of bidder $i$, whose cost-type is $\theta_i$, from his joint strategy $s^i$ is given by,

$$\Pi_i\left(\left(t_i, p_i, s_j^i, \mu^i\right)\middle|\theta_i\right) = (p_i - c(t_i, \theta_i))prob\left(win|t_i, p_i, s_j^i, \mu^i, \theta_i\right). \quad (1)$$

We restrict our analysis to bidders who adopt strategies that are best responses given their beliefs and conjectures, leading us to consider subjectively-rational strategies, formally defined as follows:

**Definition 1.** A strategy $(t_i^* : [\underline{\alpha}, \overline{\alpha}] \rightarrow \Re_+, p_i^* : [\underline{\alpha}, \overline{\alpha}] \rightarrow \Re_+)$ of bidder $i$ is subjectively-rational if there exists a pair $(s_j^i, \mu^i)$ for which this strategy maximizes $\Pi_i$.

We now examine the implications of the subjectively-rational property and in particular see whether it allows for different bidders to submit different technical bids. A phenomenon often observed and left unexplained by previous models. In order to do that we first examine the implication of this property regarding the beliefs of the bidders about their opponent's technical bid which turns out to be critical for the issue of submitting different technical bids. The following lemma shows that if a bidder believes that $\eta_1$ may occur, then in any subjectively rational strategy, he must believe that the other bidder always qualifies by behaving conservatively, that is, bidding $\frac{q}{\eta_1}$.

**Lemma 1.** In any subjectively rational strategy of bidder $i$ with $\mu^i(\eta_1) > 0$, it must be that $t_j^i(\theta) = \frac{q}{\eta_1}$ for all $\theta$.

**Proof.** Assume by way of contradiction that the joint strategy of bidder $i$ is,

$$s^i = \left((\cdot), \left(t_j^i(\cdot), p_j^i(\cdot)\right), \mu^i(\cdot)\right)$$

where $t_j^i$ is such that $prob\{\theta | t^{**}(\theta) = \frac{q}{\eta_2}\} = \delta$ with $\delta > 0$ and $p_j^i$ is strictly increasing. We now show that the strategy of player $i$ in $s^i$ is not subjectively rational.

By bidding $\frac{q}{\eta_1}$ the expected-payoff of bidder $i$ according to bidder $i$'s belief is:

$$\left[p^i - c\left(\frac{q}{\eta_1}, \theta\right)\right]\left[1 - F\left(\left(p_j^i\right)^{-1}\left(p^i\right)\right)\right]\left[\mu^i(\eta_1)(1-\delta) + \mu^i(\eta_2)\right] + \left[p^i - c\left(\frac{q}{\eta_1}, \theta\right)\right]\mu^i(\eta_1)\delta. \quad (2)$$

The first term is the expected pay-off of bidder $i$ in case both bidders qualify and the second is the expected pay-off in the case when only bidder $i$ qualifies. Since the second term can be made arbitrarily large by increasing $p^i$, there cannot be a subjectively-rational strategy with respect to the belief in $s^i$. Hence, in any subjectively rational strategy $t_j^i(\theta) = \frac{q}{\eta_1}$ for all $\theta$.[9] ■

Using this Lemma we show in the following proposition that when bidders' conjectures are correct, both must submit the same technical bid.

**Proposition 1.** If both bidders play subjectively rational strategies with correct conjectures, then both must be submitting the same technical bid for all realizations of types.

**Proof.** By Lemma 1 if $\mu^i(\eta_1) > 0$ then $t_j^i(\theta) = \frac{q}{\eta_1}$ for all $\theta \in [\underline{\alpha}, \overline{\alpha}]$. Since conjectures are correct bidder $j$ must indeed be submitting $\frac{q}{\eta_1}$ as his technical bid. This, in turn, implies $\mu^j(\eta_1) > 0$ which by Lemma 1 implies $t_i^j(\theta) = \frac{q}{\eta_1}$ for all $\theta \in [\underline{\alpha}, \overline{\alpha}]$. Finally, since conjectures are correct, this implies bidder $i$ must be submitting $\frac{q}{\eta_1}$ as his technical bid. Hence, if any bidder puts positive probability on $\eta_1$ and conjectures are correct both bidders submit the same technical bid for all types.

If $\mu^i(\eta_2) = \mu^j(\eta_2) = 1$ then both bidders must be submitting the $\frac{q}{\eta_2}$ for all cost-types. ■

In the light of this result, any Bayes-Nash equilibrium notion would fail to generate different technical bids in equilibrium, since conjectures must be correct in a Bayes-Nash equilibrium. Note that in our model there are pure strategy Bayes-Nash equilibria, however, all of them have bidders submit identical technical bids. To explain the submission of different technical bids (as observed in DMAP), we must assume that a bidder's conjecture about an opponent's strategy may not be correct. Resorting to diametrically opposed beliefs trivially leads to different technical bids but as the following proposition shows even when both bidders attach positive probability to $\eta_1$, subjectively rational strategies allow for submitting different technical bids for all possible types. Hence it is not necessary to invoke any type of irrational behavior in order to explain the observation of different technical bids.

**Proposition 2.** If bidders' conjectures are not correct and $\mu^i(\eta_1) > 0, i = 1, 2$, there exist two joint strategies in which both bidders' strategies are subjectively rational and the technical bids offered are different.

**Proof.** The technical bids submitted can be either $\frac{q}{\eta_1}$ or $\frac{q}{\eta_2}$ by subjective rationality. We now show that there exist $p^*$, $p^{**}$ and $\mu^2(\eta_2) = \delta$ with $0 < \delta < 1$ such that in the following pair of joint strategies $s^1 = ((\frac{q}{\eta_1}, p^*(\cdot)), (\frac{q}{\eta_1}, p^*(\cdot)), \mu^1(\eta_1) = 1)$, $s^2 = ((\frac{q}{\eta_2}, p^{**}(\cdot)), (\frac{q}{\eta_1}, p^*(\cdot)), \mu^2(\eta_2))$, bidders own strategies are subjectively rational.

Consider first $s^1$. In this joint strategy bidder 1 believes that both bidders will qualify and face a standard auction in the monetary dimension. Hence, $p^*$ given by

$$p^*(\theta) = c\left(\frac{q}{\eta_1}, \theta\right) + \frac{1}{[1-F(\theta)]}\int_\theta^{\overline{\alpha}} c_\theta\left(\frac{q}{\eta_1}, r\right)[1-F(r)]dr$$

[7] Conjecturing a given strategy is equivalent to putting on it probability 1. Also note that in our analysis a bidder's beliefs and conjectures do not vary with the bidder's type.

[8] The notion of joint strategy was introduced by Kalai and Lehrer (1993). There a joint strategy lists strategies of a player and the strategies conjectured to be played by his opponents. We have augmented their notion of a joint strategy by including a player's belief about which game is being played, which in our model is the belief about which evaluation procedure may be used.

[9] Note that the proposition holds even when $p_j^i$ is not strictly monotone, since the proof hinges on the unboundedness of the second term in Eq. (2).

is subjectively rational, see Che (1993). Since bidder 1's expected payoff is non-negative, submitting $\frac{q}{\eta_1}$ is subjectively rational as well.

Now consider $s^2$.

Let bidder 2's monetary bid be denoted by $p^2$. Given the belief of bidder 2 in $s^2$ the expected-payoff of bidder 2 with type $\theta$ for each of the two possible technical bids is given as follows:

By bidding $\frac{q}{\eta_1}$ the expected-payoff of bidder 2, according to 2's belief is:

$$\left[p^2 - c\left(\frac{q}{\eta_1}, \theta\right)\right]\left[1 - F(p^{*-1}(p^2))\right]. \tag{3}$$

By bidding $\frac{q}{\eta_2}$ the expected-payoff of bidder 2 according to 2's belief is:

$$\left[p^2 - c\left(\frac{q}{\eta_2}, \theta\right)\right]\left[1 - F(p^{*-1}(p^2))\right]\mu^2(\eta_2). \tag{4}$$

If the maximum expected-payoff in Eq. (3) is higher than that of Eq. (4) then bidder 2 bids $\frac{q}{\eta_1}$ and argmax of Eq. (3) as the financial bid. Otherwise, bidder 2 will bid $\frac{q}{\eta_2}$ and argmax of Eq. (4). We will show that bidder 2's optimal bid is $\frac{q}{\eta_2}$ for all $\theta$ for some $\mu^2(\eta_2) < 1$.

Step 1 Note that $p^*$ is a continuous and strictly increasing function and that we can restrict attention to $p^2 \in [p^*(\underline{\alpha}), p^*(\overline{\alpha})]$ [10]. Hence, both maximization problems have a solution since a continuous function always attains a maximum over any closed interval.

Step 2 Let $p^2(\theta)$ maximize Eq. (3). Bidder 2 can be made strictly better off for some $\mu^2(\eta_2)$ less than 1, by bidding $\frac{q}{\eta_2}$ as the technical bid and using $p^2(\theta)$ for the monetary bid.

To do that we have to show the following:

$$\left[p^2(\theta) - c\left(\frac{q}{\eta_1}, \theta\right)\right]\left[1 - F(p^{*-1}(p^2(\theta)))\right] < \left[p^2(\theta) - c\left(\frac{q}{\eta_2}, \theta\right)\right]\left[1 - F(p^{*-1}(p^2(\theta)))\right]\mu^2(\eta_2).$$

Since $[1 - F(p^* - 1(p^2(\theta)))] \geq 0$ it is enough to show that:

$$\left[p^2(\theta) - c\left(\frac{q}{\eta_1}, \theta\right)\right] < \left[p^2(\theta) - c\left(\frac{q}{\eta_2}, \theta\right)\right]\mu^2(\eta_2). \tag{5}$$

Note that by definition $p^2(\theta) - c(\frac{q}{\eta_1}, \theta) \geq 0$ for all $\theta$.

This implies $p^2(\theta) - c(\frac{q}{\eta_2}, \theta) > 0$ for all $\theta$, since $\frac{q}{\eta_2} < \frac{q}{\eta_1}$. Collecting terms in Eq. (5),

$$\frac{p^2(\theta) - c\left(\frac{q}{\eta_1}, \theta\right)}{p^2(\theta) - c\left(\frac{q}{\eta_2}, \theta\right)} < \mu^2(\eta_2).$$

We note that for each $\theta$ the ratio $\frac{b - c(\frac{q}{\eta_1}, \theta)}{b - c(\frac{q}{\eta_2}, \theta)}$ is a well defined function of $b$ for $b \geq c(\frac{q}{\eta_1}, \theta)$. In fact this function is differentiable in $[c\left(\frac{q}{\eta_1}, \theta\right), p^*(\overline{\alpha})]$. The derivative is given by $\frac{c\left(\frac{q}{\eta_1}, \theta\right) - c\left(\frac{q}{\eta_2}, \theta\right)}{[b - c(\frac{q}{\eta_2}, \theta)]^2}$ which is positive since $c\left(\frac{q}{\eta_1}, \theta\right) > c\left(\frac{q}{\eta_2}, \theta\right)$. Hence for al $\theta$ the function is maximized at $p^*(\overline{\alpha})$. Hence, for all $\theta$

$$\frac{p^*(\overline{\alpha}) - c\left(\frac{q}{\eta_1}, \theta\right)}{p^*(\overline{\alpha}) - c\left(\frac{q}{\eta_2}, \theta\right)} \geq \frac{p^2(\theta) - c\left(\frac{q}{\eta_1}, \theta\right)}{p^2(\theta) - c\left(\frac{q}{\eta_2}, \theta\right)}.$$

Observe that $\frac{p^*(\overline{\alpha}) - c\left(\frac{q}{\eta_1}, \theta\right)}{p^*(\overline{\alpha}) - c\left(\frac{q}{\eta_2}, \theta\right)} < 1$ for all $\theta$. Also this function is continuous in $[\underline{\alpha}, \overline{\alpha}]$ and hence attains a maximum $M < 1$. Then we have $1 > M \geq \frac{p^2(\theta) - c\left(\frac{q}{\eta_1}, \theta\right)}{p^2(\theta) - c\left(\frac{q}{\eta_2}, \theta\right)}$ for all $\theta$. Now set $\mu^2(\eta_2) = \delta$ such that $M < \delta < 1$.

Hence, bidder 2's strategy in $s^2 = \left(\left(\frac{q}{\eta_1}, p^*(\cdot)\right)\left(\frac{q}{\eta_2}, p^{**}(\cdot)\right), (1-\delta, \delta)\right)$ is subjectively rational and hence the proof follows. ■

**Remark.** To generate submission of different technical bids, conjectures of at least one of the players must be false (see Proposition 1). Hence, the condition that bidders' conjectures are not correct cannot be relaxed. Also note that in the proof the joint strategies constructed require that only one bidder's conjecture is wrong.

As we noted in Proposition 2, the result that technical bids are different is to be expected when beliefs of the two bidders regarding the evaluation procedure are very different. We want to show, this can happen also when beliefs are somewhat similar that is both bidders attach a strictly positive probability to $\eta_1$. Note that $\delta$ depends on all the parameters of the model and need not be arbitrarily close to 1.

In Lemma 1 we have seen that subjective rationality of bidder $i$ together with the assumption $\mu^i(\eta_1) > 0$ necessarily lead to $t^i_j(\theta) = \frac{q}{\eta_1}$ for all $\theta$. Now we wish to provide an explanation why $i$ may believe the monetary bid strategy of $j$ to be $p^*$. However, relative to Lemma 1 we need more assumptions. We assume that: If according to $i$'s belief and $i$'s conjecture about $j$'s belief, $j$ is playing a game that has a symmetric equilibrium, then $i$ conjectures that $j$ plays a symmetric equilibrium strategy. We refer to this assumption as (*SYM*) and obtain the following proposition.

**Proposition 3.** Let $i$ play a subjectively rational strategy and $\mu^i(\eta_1) > 0$. Let (*SYM*) hold.

Furthermore, assume the following:

(SR) $i$ conjectures it is common knowledge that subjective rationality is satisfied.

(UA) $i$ conjectures it is common knowledge that $\mu^i(\eta_1) > 0$ and $\mu^j(\eta_1) > 0$.

Then, $p^i_j(\theta) = p^*(\theta)$ for all $\theta$.

**Proof.** Since $\mu^i(\eta_1) > 0$ by Lemma 1 $t^i_j(\theta) = \frac{q}{\eta_1}$ for all $\theta$. Since $i$ conjectures $\mu^j(\eta_1) > 0$, $i$ conjectures that $j$ conjectures that $i$ decides technical bids according to $t(\theta) = \frac{q}{\eta_1}$ for all $\theta$. Now it follows from Lemma 1, (*SR*) and (*UA*) that: By UA and SR bidder $i$ conjectures it is common knowledge that both players conjecture that the other player submits a technical bid equal to $\frac{q}{\eta_1}$.

Hence, according to $i$'s beliefs and conjectures, $j$ is playing an $\eta_1$ game[11]. It is a well known result that $p^*$ is the unique symmetric Bayes-Nash equilibrium in an $\eta_1$ game. Hence, by (*SYM*) the proof follows.[12] ■

[10] Note that $p^*(\overline{\alpha}) = c(\frac{q}{\eta_1}, \overline{\alpha})$ by L'Hopital's rule.

[11] An $\eta_k$ game is the monetary part of the auction environment where both bidders submitted $\frac{q}{\eta_k}$ for all $\theta$ as a technical bid.

[12] We note that if $\mu^i(\eta_1) = 0$, subjective rationality of bidder $i$ does not impose any restriction on bidder $i$'s belief about the technical bid of bidder $j$. Hence if $\mu^i(\eta_1) = 0$ then it is not necessary that bidder $i$ believes that $j$ acts according to the $\eta_1$ game.

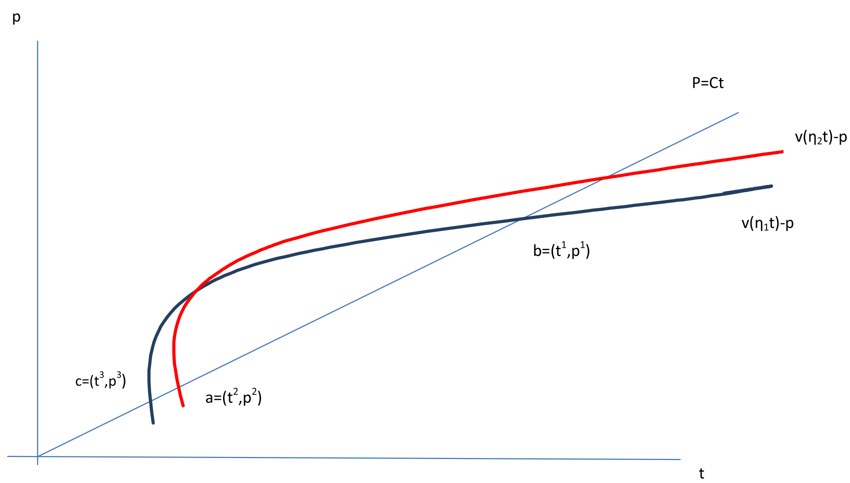


**Fig. 1.** Commitment and time inconsistency.

**Remark.** We have not provided an equilibrium notion for our environment but note that any reasonable equilibrium notion should imply that strategies are subjectively rational. One such notion, that of self-confirming equilibria was introduced in Dekel et al.(2004) for Bayesian games[13]. This equilibrium, while motivated by learning, could be applied to our one-period game setting where there is no scope for learning. Following the terminology in Dekel et al. (2004) a type profile is given by $(\eta, \theta_1, \theta_2)$ where $\eta \in \Omega$, $\theta_i \in [\underline{\alpha}, \overline{\alpha}]$. The conjecture of $i$ about $j$ is given by $s_j^i$, and the belief of $i$ about the type profile is $\mu^i(\eta) f(\theta_i) f(\theta_j)$. The signaling function $y_i$ is taken to be constant, since our game is a one-period game with no scope for learning. A self-confirming equilibrium is now defined as in Dekel et al. (2004), note that their conditions $(i)$ and $(iii)$ are vacuously satisfied in our environment and condition $(ii)$ coincides with our definition of subjective rationality. Hence, any pair of subjectively rational strategies is a self-confirming equilibrium, and they are all equally plausible. A natural question is then which of them are reasonable. Finding an answer is equivalent to determining which beliefs and conjectures, appearing in a self-confirming equilibrium, are more plausible. Proposition 3 provides a set of sufficient conditions on conjectures to guarantee there is a unique set of conjectures common to all self-confirming equilibria.

One reason the auctioneer may not commit might be due to a form of time inconsistency, which we now proceed to discuss. Using one evaluation procedure leads in equilibrium to an outcome that is less desirable (with this evaluation) than the equilibrium outcome associated with an alternate evaluation procedure.

Consider the following environment: the bidder's cost is given by $c(t, \theta) = t\theta$ where $\theta$ is uniformly distributed on Eqs. (11) and (2). The buyer's utility is given by $v(q) - p$. The expected payment in equilibrium of the buyer, if the auctioneer uses $\eta_1$ is: $2\int_1^2 \left( \int_\theta^2 \frac{q}{\eta_1} r dr \right) d\theta$ which can be written as $C\frac{q}{\eta_1}$ with $C > 0$. Similarly if the auctioneer uses $\eta_2$ it is $C\frac{q}{\eta_2}$.

We now show there exist $\eta_1, \eta_2, \underline{q}$ such that:

(i) The buyer using $\eta_2$ ex-ante prefers the equilibrium outcome generated by $\eta_1$, denoted by $(t^1, p^1)$ to the equilibrium outcome generated by $\eta_2$, denoted by $(t^2, p^2)$.

(ii) The buyer using $\eta_1$ ex-ante prefers the equilibrium outcome generated by $\eta_2$, denoted by $(t^2, p^2)$ to the equilibrium outcome generated by $\eta_1$, denoted by $(t^1, p^1)$ (Fig. 1).

Choose $\eta_1, \underline{q} = t^1\eta_1$ and draw the indifference curve of the buyer through $b$, given by $v(\eta_1 t) - p = const$. Assuming $v(\cdot)$ satisfies standard regularity conditions, there will exist another intersection point of this indifference curve with the line $p = Ct$ to the left of $b$, denote it by $c$. Consider now the indifference curve through $c$ given by $v(\eta_2^* t) - p = const$. where $\eta_2^* t_3 = \underline{q}$. Note that $\eta_2^* > \eta_1$, and hence the indifference curve we drew through $c$ intersects the line $p = Ct$ to the right of $b$. Thus, $v(\eta_2^* t^3) - p^3 < v(\eta_2^* t^1) - p^1$. By continuity of $v(\eta t) - p$ in $\eta$, $t$ and $p$, there exists an $\eta_2 < \eta_2^*$, close enough to $\eta_2^*$ such that $t^2 = \frac{q}{\eta_2}$ and $p^2 = Ct^2$ with $v(\eta_2 t^2) - p^2 < v(\eta_2 t^1) - p^1$ and $v(\eta_1 t^1) - p^1 < v(\eta_1 t^2) - p^2$ establishing (i) and (ii).

The auctioneer in our model, as in Che (1993), did not impose an upper bound on the monetary bids. This enabled us to characterize an equilibrium and indeed have firms submitting different technical bids in equilibrium. Note that again as in Che (1993) the bids used in subjectively rational strategies were in fact bounded (Lemma 1 and Proposition 2). While imposing a large enough bound in Che (1993) would have no effect on the equilibrium behavior, the imposition of an a priori bound (no matter how large) in our model would have some implications, which we examine in the next subsection.

### 3.1. Bounded monetary bids

Recall that in Lemma 1 we found that when bidder $i$ believes the strict evaluation may occur, subjective rationality imposes certain restrictions on conjectures. The fact monetary bids were not bounded, played a crucial role in the proof. Putting an upper bound, $\overline{v}$, on permissible bids makes, as the following example shows, it difficult to uniquely pin down the conjectures.

**Example.** Suppose $c(t, \theta) = t\theta$, $\theta \in [1, 2]$, $F(\theta) = \theta - 1$. Then the monetary bid functions for the Nash equilibria in pure strategies (denoted by $p^*$ and $p^{**}$) are as follows. For the $\eta_1$ game, $p^*(\theta) = \frac{1}{2-\theta}\int_\theta^2 \frac{q}{\eta_1} r dr$. For the $\eta_2$ game, $p^{**}(\theta) = \frac{1}{2-\theta}\int_\theta^2 \frac{q}{\eta_1} r dr$.

Suppose also that $\overline{v}$ is such that $[p^{**}(\overline{\alpha}) - c\left(\frac{q}{\eta_2}, 1\right)] < [\overline{v} - c(\frac{q}{\eta_1}, 2]$. Hence, there exists a $0 < \delta < 1$, such that, $[p^{**}(\overline{\alpha}) - c\left(\frac{q}{\eta_2}, 1\right)] < [\overline{v} - c\left(\frac{q}{\eta_1}, 2\right]\delta$. Let $\mu^i(\eta_1) = \delta$. Then there are best responses to both $s_j^i = \left(\frac{q}{\eta_1}, p^*(\cdot)\right), \mu^i(\eta_1) = 1$ and $(s_j^i)' = \left(\frac{q}{\eta_2}, p^{**}(\cdot)\right), (\mu^i)'(\eta_1) = \delta$.

[13] Note that the self-confirming equilibrium is merely an example and our analysis throughout deals just with subjectively rational strategies.

However, even when monetary bids are bounded, subjective rationality has important implications. In what follows we assume that $\overline{v} \geq c\left(\frac{q}{\eta_1}, \overline{\alpha}\right)$, thus all types would find it profitable to participate in the auction.

Subjectively rational bids (monetary and technical) clearly satisfy: $p \in \left[c\left(, \frac{q}{\eta_2}, \underline{\alpha}\right), \overline{v}\right]$ and $t \in \left\{\frac{q}{\eta_1}, \frac{q}{\eta_2}\right\}$. We now explicitly describe the payoff function, whose derivatives will feature in the proof of Proposition 4. We let $R$ denote the range of $p_j^i$ and $R^c$ denote the set of bids that exceed the maximum bid according to $p_j^i$. We also let $prob\left\{\theta | t_j^i(\theta) = \frac{q}{\eta_2}\right\} = \delta$ and $K = prob\{\theta | p^i \leq p_j^i(\theta)\}$.

Then the pay-off function of bidder $i$ is given by,

$$\Pi^i\left(\left(p^i, t^i\right), s_j^i, \mu_i\right) = \begin{cases} 0 & if\left(t^i, p^i\right) \in \left\{\frac{q}{\eta_1}\right\} \times R^c \\ \left[p^i - c\left(t^i, \theta_i\right)\right]\mu^i(\eta_1)\delta & if\left(t^i, p^i\right) \in \left\{\frac{q}{\eta_1}\right\} \times R \\ \left[p^i - c\left(t^i, \theta_i\right)\right]\left[K + (1-K)\mu^i(\eta_1)\delta\right] & if\left(t^i, p^i\right) \in \left\{\frac{q}{\eta_2}\right\} \times R^c \\ \left[p^i - c\left(t^i, \theta_i\right)\right]K\mu^i(\eta_2)\delta & if\left(t^i, p^i\right) \in \left\{\frac{q}{\eta_1}\right\} \times R \end{cases}.$$

We also assume that the $t_j^i$ are weakly decreasing and that for all $i$, $p_j^i(\cdot)$ is strictly increasing for $j = 1, 2$. Call this assumption (MON).

We also assume that for each $t^i \in \{\frac{q}{\eta_1}, \frac{q}{\eta_2}\}$ there is a unique local maximum pay-off for agent $i$. Call this assumption (UNIQUE).

The following proposition shows these assumptions imply that in subjectively rational strategies technical bids are constant and do not depend on $\theta$.

**Proposition 4.** Suppose (MON) and (UNIQUE) hold. Then $t_i^i(\cdot) = \frac{q}{\eta_1}$ or $t_i^i(\cdot) = \frac{q}{\eta_2}$, for any subjectively rational strategy.

**Proof.** Suppose bidder $i$ bids $\frac{q}{\eta_1}$ and $\frac{q}{\eta_2}$ for two non-empty intervals.

Consider the technical bid $\frac{q}{\eta_2}$.

If $i$, of type $\theta$, bids $\frac{q}{\eta_2}$ then whenever $i$ qualifies, so does the other bidder and hence, $i$'s monetary bid must lie in the range of $p_j^i$ whenever $i$ bids $\frac{q}{\eta_2}$.

Thus, $p_i^i(\theta) = p_j^i(z)$ for some $z \in (\underline{\alpha}, \overline{\alpha})$. Now consider the derivative of the pay-off function of bidder $i$ with respect to $p$ at the price $p_j^i(z)$ when the technical bid is given by $\frac{q}{\eta_1}$ and $i$ bids a price in the range of $p_j^i$. It is given below.

$\frac{d\pi_i(p^i, \theta; \frac{q}{\eta_1})}{dp^i} | p^i = p_j^i(z) = -[p_j^i(z) - c(\frac{q}{\eta_1}, \theta)]\ f(z)\frac{dz}{dp_j^i} + [1 - F(z)] + [p_j^i(z) - c(\frac{q}{\eta_1}, \theta)] f(z)(\frac{dz}{dp_j^i(z)}) + F(z)\mu^i(\eta_1)\delta$. Since, $-[p_j^i(z) - c(\frac{q}{\eta_2}, \theta)] f(z)\frac{dz}{dp_j^i(z)} + [1 - F(z)] = 0$, $\frac{d\pi_i(p^i, \theta; \frac{q}{\eta_1})}{dp^i}|_{p^i = p_j^i(z)} > 0$, if $[p_j^i(z) - c(\frac{q}{\eta_1}, \theta)] \geq 0$. By the continuity of the cost function, there exits an $\varepsilon$ such that for all $\theta' \in (\theta - \varepsilon, \theta)$, $\frac{d\pi_i(p^i, \theta'; \frac{q}{\eta_1})}{dp^i}|_{p^i = p_j^i(z)} > 0$. If $t_i^i(\theta') = \frac{q}{\eta_1}$ then by (UNIQUE) $p_i^i(\theta') > p_i^i(\theta)$. This violates (MON). If $[p_j^i(z) - c(\frac{q}{\eta_1}, \theta)] < 0$, again we have violation of (MON). Hence, $t_i^i$ must be left continuous.

Now let $t_j^i(\theta) = \frac{q}{\eta_1}$. Let $p_i^i(\theta) = p_j^i(z')$, for some $z' \in (\underline{\alpha}, \overline{\alpha})$. Then $\frac{d\pi_i(p^i, \theta; \frac{q}{\eta_1})}{dp^i}|_{p^i = p_j^i(z')} = 0$. By continuity of the cost function there exists an $\varepsilon > 0$ such that for all $\theta' \in (\theta, \theta + \varepsilon)$, $\frac{d\pi_i(p^i, \theta'; \frac{q}{\eta_1})}{dp^i}|_{p^i = p_j^i(z')} < 0$. Hence $t_i^i$ must be right continuous.

Hence, if $i$ bids a price in the range of $p_j^i$ then his technical bid function is continuous, meaning the bidder bids either $\frac{q}{\eta_1}$ or $\frac{q}{\eta_2}$.

Hence, it follows that if bidder $i$ bids both technical bids then it must be that $i$'s monetary bid is $\overline{v}$ for the types for which his technical bid is $\frac{q}{\eta_1}$. Then by (MON) $i$ bids $\overline{v}$ for $\theta$ for which he bids $\frac{q}{\eta_2}$. Hence, by (MON) bidder $i$ must bid $\overline{v}$ for all the $\theta$ s for which he bids $\frac{q}{\eta_2}$. Since $p_j^i$ is strictly monotonic this means that the expected pay-off of bidder $i$ by bidding $\frac{q}{\eta_2}$ and $\overline{v}$ is zero. Which violates subjective rationality because by bidding below $\overline{v}$ and in the range of $p_j^i$, bidder $i$ can earn a positive expected pay-off. Hence our claim follows. ■

A natural question is then whether there exists a Bayes-Nash equilibrium with different technical bids for this bounded environment. The following proposition shows this is impossible if the monetary bids are continuous and strictly increasing.[14]

**Proposition 5.** There does not exist a Bayes-Nash equilibrium with continuous, strictly increasing monetary bid functions, and different constant technical bids.

**Proof.** Assume by way of contradiction there exists such an equilibrium where bidder 1's technical bid is $\frac{q}{\eta_1}$ and bidder 2's is $\frac{q}{\eta_2}$ and their monetary bids are $p^1$ and $p^2$.

We first show it must be the case that $p^2(\overline{\alpha}) = p^1(\overline{\alpha}) = \overline{v}$. Let $p^2(\overline{\alpha}) < \overline{v}$. Consider first the situation, $p^1(\overline{\alpha}) < p^2(\overline{\alpha})$. This is not possible in a Bayes-Nash equilibrium, since in that case bidder 2 with $\theta$ close enough to $\overline{\alpha}$ could increase his payoff by lowering his monetary bid to lie within the range of $p^1$. Now assume that $p^1(\overline{\alpha}) \geq p^2(\overline{\alpha})$. Suppose, $p^1(\overline{\alpha}) = p^2(\overline{\alpha})$. Then, the expected payoff of bidder 1 for the type $\overline{\alpha}$ is $[p^1(\overline{\alpha}) - c(\frac{q}{\eta_1}, \overline{\alpha})]\mu^1(\eta_1)$. Since $p^1(\overline{\alpha}) < \overline{v}$, $[p^1(\overline{\alpha}) - c(\frac{q}{\eta_1}, \overline{\alpha})]\mu^1(\eta_1) < [\overline{v} - c(\frac{q}{\eta_1}, \overline{\alpha})]\mu^1(\eta_1)$. Hence, bidder 1 will deviate to $\overline{v}$ for the type $\overline{\alpha}$. If $p^1(\overline{\alpha}) > p^2(\overline{\alpha})$ then by continuity this inequality holds for $\theta$ close enough to $\overline{\alpha}$ which implies that bidder 1's monetary bid should be constant in this range contradicting monotonicity. Hence, $p^2(\overline{\alpha}) = p^1(\overline{\alpha}) = \overline{v}$ Then bidder 2's expected pay-off for the type $\overline{\alpha}$, is 0. This cannot happen in equilibrium, since, $\overline{v} - c(\frac{q}{\eta_2}, \overline{\alpha}) > 0$, implies bidder 2 can increase his payoff by bidding a price $p$ in the interior of the range of $p^1$ with $c(\frac{q}{\eta_2}, \overline{\alpha}) < p < \overline{v}$. ■

Proposition 5 is the counterpart of Proposition 1 and shows that when conjectures are correct the imposition of a bound on monetary bids also does not generate different technical bids. Furthermore, since it becomes harder to predict the subjectively rational outcomes it is not clear whether an auctioneer would like to impose a bound.

Returning to our original unbounded environment, note that so far we have assumed the auctioneer and the bidders are honest. In particular, the auctioneer does not take advantage of the subjectivity aspect prevalent in the evaluation procedure. Unfortunately, it has been often observed (see Jain et al. (2007) for the case of the DMAP auction) that this need not be the case. We proceed to analyze a particular element of corruption in the following section.

## 4. Rational bidding in the presence of favoritism

The difficulty, due to subjectivity, of enforcing a particular evaluation method, makes it possible for the auctioneer to bias the ranking procedure in favor of one of the bidders. Rational behavior in such environments will naturally be quite different than that observed in the previous section. In what follows we analyze rational behavior and outcomes in the presence of favoritism. The difference in outcomes is due, in part, to the fact, bidders have now more information regarding the auction game.

Since there is no subjectivity about evaluating the monetary bids, we consider favoritism only in the technical dimension. The evaluation

[14] Our model may admit other equilibria with less well-behaved monetary bidding functions. Nevertheless, our findings demonstrate that results pertaining to the characterization of monotone equilibria, on which the literature on games with incomplete information focuses, may be harder to obtain in the presence of uncertainty.

of the technical part of the bid is altered so that bidder 2 always qualifies and if possible bidder 1 is disqualified. That is, if bidder 1 submits $t_1 = \frac{q}{\eta_2}$ then the auctioneer uses $\eta_1$, bidder 2 submits $\frac{q}{\eta_1}$ and 1 does not qualify. If bidder 1 submits $t_1 = \frac{q}{\eta_1}$ then the auctioneer uses $\eta_2$, bidder 2 submits $\frac{q}{\eta_2}$ and both qualify. Note that this favoritism does not involve transmission of wrong information as in Laffont and Tirole (1991) or direct manipulation of bids as in Burguet and Che (2004). The subjectivity we introduced makes it possible to favor a seller without exposing the auctioneer to legal sanctions[15]. This collusion is common knowledge[16] among the bidders and hence joint strategies will now consist just of technical and monetary bids, there is no need to specify beliefs regarding the auctioneer.

Note that every bidder now has more information relative to the basic model discussed in the earlier section in which there is no collusion. Hence, we would expect a rational player to take into account this information while deciding on their bidding strategies. A typical joint strategy that incorporates this information can be written as,

$$s^i = \left(\left(t_i^i, p_i^i\right), \left(t_j^i, p_j^i\right)\right).$$

Because of the collusion in any rational strategy it must be that bidder 1 submits $t_1^1 = \frac{q}{\eta_1}$ and bidder 2 submits $t_2^2 = \frac{q}{\eta_2}$ for all $\theta$.

We now proceed to describe the monetary part, $p_1^*$ and $p_2^*$, of subjectively rational strategies. We will also show that these strategies may lead to the choice of an inefficient bidder.

**Proposition 6.** Consider the following joint strategies, $s^1 = \left(\left(\frac{q}{\eta_1}, p_1^*\right)\left(\frac{q}{\eta_2}, p_2^*\right)\right)$ and $s^2 = \left(\left(\frac{q}{\eta_1}, p_1^*\right), \left(\frac{q}{\eta_2}, p_2^*\right)\right)$.

Where the functions $p_1^*$ and $p_2^*$ solve the following system of differential equations:

$$p_j^{*-1'}(p_i) = \frac{1-F\left(p_j^{*-1}(p_i)\right)}{f\left(p_j^{*-1}(p_i)\right)\left(p_i - c\left(\frac{q}{\eta_i}, p_j^{*-1}(p_i)\right)\right)}, i = 1, 2. \quad (6)$$

Then $\left(\frac{q}{\eta_1}, p_1^*\right), \left(\frac{q}{\eta_2}, p_2^*\right)$ are subjectively rational strategies.

**Proof.** By subjective rationality $p_1^*(\underline{\alpha}) = p_2^*(\underline{\alpha})$ and $p_1^*(\overline{\alpha}) = p_2^*(\overline{\alpha})$. If bidder $i$ bids below $p_j^*(\underline{\alpha})$ then he wins for sure. But by increasing a small amount he increases pay-off without reducing his probability of winning. Similarly, bidder $i$ will not bid above $p_j^*(\overline{\alpha})$. If he bids above $p_j^*(\overline{\alpha})$ then he does not win for sure.

In view of this argument it follows that the auction reduces to a lowest price auction where bidder 1's cost lies in the set $V_1 = \left\{c\left(\frac{q}{\eta_1}, \theta\right) | \theta \in \underline{\alpha}, \overline{\alpha}]\right\}$ and bidder 2's cost lies in the set $V_2 = \left\{c\left(\frac{q}{\eta_2}, \theta\right) | \theta \in [\underline{\alpha}, \overline{\alpha}]\right\}$. Since cost functions are continuous, $V_1$ and $V_2$ are intervals[17]. A typical element in $V_i$ is denoted by $v_i$. The transformed distribution functions over the intervals $V_1$ and $V_2$ are $F_1$ and $F_2$. The payoff function of bidder $i$ is $p_i - v_i$ if he wins and zero otherwise. By Lizzeri and Persico (2000), strictly increasing and differentiable subjectively rational strategies exist. The strategies $p_1^{**}$ and $p_2^{**}$ are solutions to the following system of differential equations,[18]

$$p_j^{**-1'}(p_i) = \frac{1-F_j\left(p_j^{**-1}(p_i)\right)}{f_j\left(p_j^{**-1}(p_i)\right)\left(p_i - p_j^{**-1}(p_i)\right)}, i = 1, 2.$$

Writing these equations as functions of cost-type gives us the system of differentiable Eq. (6). ■

The subjectively rational price bid functions in Proposition 6 are not the same. The next proposition shows that the honest bidder bids a higher price for every realization of cost-type.

**Proposition 7.** The subjectively rational monetary bidding functions satisfy $p_1^*(\theta) > p_2^*(\theta)$ for all $\theta \in (\underline{\alpha}, \overline{\alpha})$.

**Proof.** First we show that if $p_1^*$ and $p_2^*$ intersect in the interior of $[\underline{\alpha}, \overline{\alpha}]$ then they will intersect at most once. We know that $p_j^{*-1'}(p_i) = \frac{1-F(p_j^{*-1}(p_i))}{f(p_j^{*-1}(p_i))(p_i - c\left(\frac{q}{\eta_i}, p_j^{*-1}(p_i)\right)}$. Suppose $p_1^* - 1(p) = p_2^* - 1(p) = \theta$ then since $t_1^* > t_2^*$, we obtain:

$$p_2^{*-1'}(p) = \frac{1-F(\theta)}{f(\theta)(p - c(t_1^*, \theta))} > p_1^{*-1'}(p) = \frac{1-F(\theta)}{f(\theta)(p - c(t_2^*, \theta))}.$$

In other words, $\frac{dp_2^*(\theta)}{d\theta} < \frac{dp_1^*(\theta)}{d\theta}$. This means that if the bidding functions intersect, then the bidding function of bidder 1 cuts the bidding function of bidder 2 from below. Hence the intersection takes place at most once in $(\underline{\alpha}, \overline{\alpha})$.

Now we prove our claim. Assume for the sake of contradiction that there exists $\theta$ such that $p_1(\theta) \le p_2(\theta)$. This means that for all $x \in (\underline{\alpha}, \theta)$ we must have $p_1(x) < p_2(x)$. Let $p_1(\underline{\alpha}) = p_2(\underline{\alpha}) = \underline{p}$.

This implies that for all $p$ close enough to $\underline{p}$,

$$p_1^{*-1}(p) > p_2^{*-1}(p). \quad [*]$$

Since $F$ is a strictly increasing distribution function, $1 - F(p_1^* - 1(p)) < 1 - F(p_2^* - 1(p))$ for all $p$ close enough to $\underline{p}$. Note that $F(p_1^* - 1(p))$ and $F(p_2^* - 1(p))$ are different functions because they are different compositions. For the sake of convenience let $H(p) = [1 - F(p_1^* - 1(p))] - [1 - F(p_2^* - 1(p))]$. Clearly, $H(\underline{p}) = 0$. Note that for all $p$ close enough to $\underline{p}$, $H(p) < 0$. We prove the following claim, which helps us to reach a contradiction.

**Claim 1.** For every $\epsilon > 0$ there exists $p(\epsilon) \in (\underline{p}, \underline{p} + \epsilon)$ such that $\frac{dHp(\epsilon)}{dp} < 0$.

**Proof.** Fix an $\epsilon > 0$. By the Mean Value Theorem, there exists $p(\epsilon) \in (\underline{p}, \underline{p} + \epsilon)$ such that $\epsilon \frac{dHp(\epsilon)}{dp} = H(\underline{p} + \epsilon) - H(\underline{p}) = H(\underline{p} + \epsilon) < 0$. Since $\epsilon > 0$, $\frac{dHp(\epsilon)}{dp} < 0$. ■

From the above claim it follows that there exists a $p$ close enough to $\underline{p}$ such that $p_1^* - 1(p) > p_2^* - 1(p)$ and $\frac{dH(p)}{dp} < 0$. This means that, −

[15] In the DMAP auction one of the bidders sued the evaluation committee for favoritism but was unable to establish it. In fact, the court ruled that the final decision made by the committee should prevail.

[16] If bidder 1 is not certain whether the auctioneer colludes with bidder 2, then the setup is equivalent to the basic model since bidder 1 does not know which evaluation procedure will be used.

[17] We assume the two intervals have a non-empty intersection to rule out trivial solutions.

[18] Note that an implicit assumption for this existence result is that for every $t$ the inverse of $c(t, \theta)$ is twice continuously differentiable. Together with the assumption of continuous differentiability of $f$ then $F_i$ has density $f_i$ and $f_i$ is continuously differentiable, for $i = 1, 2$. This differentiability assumption is required in order to use the results in Lizzeri and Persico (2000) in our model. An example of a cost function with these properties is $c(t, \theta) = t\theta$.

$f(p_1^{*-1}(p))p_1^{*-1'}(p) + f(p_2^{*-1}(p))p_2^{*-1'}(p) < 0$. Hence,

$$f(p_1^{*-1}(p))p_1^{*-1'}(p) > f(p_2^{*-1}(p))p_2^{*-1'}(p) > 0. \quad [**]$$

Observe that the first order condition for bidder $i$ can be written as,

$$f\left(p_j^{*-1}(p_i)\right)p_j^{*-1'}(p_i)\left(p_i - c\left(t_i^*, p_i^{*-1}(p_i)\right)\right) = \left[1 - F\left(p_j^{*-1}(p_i)\right)\right],$$

$$\Rightarrow f\left(p_j^{*-1}(p_i)\right)p_j^{*-1'}(p_i)p_i - \left[1 - F\left(p_j^{*-1}(p_i)\right)\right] = f\left(p_j^{*-1}(p_i)\right)p_j^{*-1'}(p_i)c\left(t_i^*, p_i^{*-1}(p_i)\right).$$

Hence,

$$c\left(t_i^*, p_i^{*-1}(p_i)\right) = p_i - \frac{\left[1 - F\left(p_j^{*-1}(p_i)\right)\right]}{f\left(p_j^{*-1}(p_i)\right)p_j^{*-1'}(p_i)}. \quad [***]$$

Since $[1 - F(p_2^* - 1(p))] > [1 - F(p_1^* - 1(p))]$ for all $p$ close enough to $\underline{p}$, by [**] we can choose $p$ such that $\frac{[1-F(p_2^{*-1}(p))]}{f(p_2^{*-1}(p))p_2^{*-1'}(p)} > \frac{[1-F(p_1^{*-1}(p))]}{f(p_1^{*-1}(p))p_1^{*-1'}(p)} > 0$. This observation together with [***] gives,

$$\begin{aligned} c\left(t_1^*, p_1^{*-1}(p)\right) &= p - \frac{\left[1 - F(p_2^{*-1}(p))\right]}{f(p_2^{*-1}(p))p_2^{*-1'}(p)} < c\left(t_2^*, p_2^{*-1}(p)\right) \\ &= p - \frac{\left[1 - F(p_1^{*-1}(p))\right]}{f(p_1^{*-1}(p))p_1^{*-1'}(p)}. \end{aligned}$$

Therefore $c(t_1^*, p_1^* - 1(p)) < c(t_2^*, p_2^* - 1(p))$. Since $t_1^* > t_2^*$ we must have $p_1^* - 1(p) < p_2^* - 1(p)$. This is a contradiction to [*]. ■

From Proposition 7 it follows that with positive probability bidder 1 may not win the auction even if bidder 1 is relatively more efficient than the dishonest one. Hence, favoritism may lead to inefficiency.

## 5. Concluding remarks

We analyzed bidding behavior in a subjective environment, describing a class of auctions where the bidders do not precisely know how their bids will be evaluated, by introducing subjectivity. We assumed bidders behave rationally given their beliefs and derived properties of beliefs, strategies and outcomes that are satisfied in any reasonable equilibrium. These properties explained observable bidding behavior that contradicts predictions for standard auctions.

In particular, we have shown that while technical bids must be identical in a standard auction, they may well differ in the subjective environments we consider. This might happen also when both bidders believe a strict evaluation is possible. Furthermore, by imposing more assumptions regarding the opponent's bidding behavior we can determine the conjectures about the monetary part of the opponent's strategy as well.

Finally we introduced a modified environment allowing for favoritism on part of the auctioneer. This led to better informed bidders and we provided a characterization of rational bidding strategies taking into account the new information structure.

Subjectivity might prevail in any auction environment entailing a “less than fully specified” evaluation of some of the bid components. Such environments include, and are not limited to, scoring auctions (Che (1993), Asker and Cantillon (2010)) and innovation races (Pérez-Castrillo and Wettstein (forthcoming)). The analysis of the implications of subjectivity and how they diverge from standard equilibrium predictions in such environments remain the topic of further research.

## References


Arozamena, L., Weinschelbaum, F., 2009. The effect of corruption on bidding behavior in first price auctions. Eur. Econ. Rev. 53, 645–657.

Asker, J., Cantillon, E., 2010. Procurement when price and quality matter. RAND J. Econ. 41, 1–34.

Branco, F., 1997. The design of multidimensional auctions. RAND J. Econ. 28, 63–81.

Burguet, R., Che, Y.K., 2004. Competitive procurement with corruption. RAND J. Econ. 35, 50–68.

Che, Y.K., 1993. Design competition through multidimensional auctions. RAND J. Econ. 24, 668–680.

Dekel, E., Fudenberg, D., Levine, D.K., 2004. Learning to play Bayesian games. Games Econ. Behav. 46, 282–303.

Ganuza, J.J., Pechlivanos, L., 2000. Heterogeneity-promoting optimal procurement. Econ. Lett. 67, 105–112.

Jain, R., Raghuram, G., Gangwar, R., 2007. Airport privatization in India: lessons from the bidding process in Delhi and Mumbai. Working Paper. No. 2007-05-01. Indian Institute of Management, Ahmedabad, India.

Kalai, E., Lehrer, E., 1993. Subjective equilibrium in repeated games. Econometrica 61, 1231–1240.

Laffont, J.J., Tirole, J., 1991. Auction design and favoritism. Int. J. Ind. Organ. 9, 9–42.

Lizzeri, A., Persico, N., 2000. Uniqueness and existence of equilibrium in auctions with a reserve price. Games Econ. Behav. 30, 83–114.

Lundberg, S., Marklund, P.O., 2011. The pivotal nature of award methods in green public procurement. Environ. Econ. 2, 64–73.

Naegelen, F., 2002. Implementing optimal procurement auctions with exogenous quality. Rev. Econ. Des. 7, 135–153.

Pérez-Castrillo, D., Wettstein, D., Discrimination in a new model of contests with two-sided asymmetric information. Int. Econ. Rev. (forthcoming).

Wolfstetter, E., Lengwiler, Y., 2006. Corruption in procurement auctions. In: Dimitri, N., Piga, G., Spagnolo, G. (Eds.), Handbook of Procurement Theory and Practice for Managers. Cambridge University Press, Cambridge, pp. 412–429.