Egalitarian Equivalence under Asymmetric Information[*]

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Abstract

We propose a definition of egalitarian equivalence that extends Pazner and Schmeidler's (1978) concept to environments with incomplete information. If every feasible allocation rule can be implemented by an incentive compatible mechanism (as, for instance, in the case of non-exclusive information), then interim egalitarian equivalence and interim incentive efficiency remain compatible, as they were under complete information. When incentive constraints are more restrictive, on the other hand, the two criteria may become incompatible.

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1 Introduction

Fairness and efficiency are two criteria often adhered to by policy makers, arbitrators settling disputes, managers deciding on compensation packages, and feature constantly in economic and social debates. These properties were first studied in a complete information setting. In economic environments, prominent solution concepts include the notions of fair outcomes, which are both efficient and envy-free (Foley, 1967, and Varian, 1974), and egalitarian equivalent allocations (Pazner and Schmeidler, 1978).[1]

In many applications, agents already hold private information at the time of selecting an outcome. The role of information is indeed one of the central topics in economics since the end of the sixties. However, most effort has been devoted to understanding what is achievable in the presence of informational constraints[2] and trying to find feasible mechanisms that maximize the revenue of an agent.[3] Few papers discuss and apply criteria to select a socially appealing incentive compatible mechanism (some related literature is discussed towards the end of this introduction).

In this paper, we extend the egalitarian principle captured by the concept of egalitarian equivalence to pure exchange economies with asymmetric information. An allocation is egalitarian equivalent in an economy with complete information (see Pazner and Schmeidler, 1978) with respect to a reference bundle if all agents are indifferent between the proposed allocation and a common bundle that is proportional to the reference bundle. That is, measuring the agents' surplus in terms of the reference bundle, all obtain the same surplus. In a similar spirit, we say that a mechanism is interim egalitarian equivalent if all the agents are indifferent, in expected terms given their private information, between the proposed mechanism and receiving a fixed proportion of the reference bundle, in each possible profile of types.[4]

Under complete information, egalitarian equivalent allocations that are also (ex-post) Pareto efficient always exist. Under asymmetric information, efficiency requires to take into account both the gains from insurance and the agents' incentives to possibly misrepresent their information. This idea is captured by the notion of interim incentive efficiency defined by Holmström and Myerson (1983). Our main result (Proposition 1) states that mechanisms that are both interim egalitarian equivalent and interim incentive efficient also exist in economies where the social planner or the society as a whole can

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[1]Other attempts to capture fairness and efficiency in the theory of social choice include the maximization of social welfare orderings, such as the egalitarian minimum, the utilitarian sum, or the Nash product. A major difference compared to the notions of envy-freeness and egalitarian equivalence, is that utilities must have some cardinal content in order to escape impossibility results of the kind first proved by Arrow. Efficiency and fairness are also of great concern in the theory of cooperative games. A prominent example of such concern is given by the Shapley value for characteristic functions with transferable utilities and its various extensions to more general environments (see McLean, 2002, for a survey).

[2]The revelation principles (Gibbard, 1973, Green and Laffont, 1977, and Myerson, 1979) have been a powerful tool in this task.

[3]See, for instance, the development of auctions, contract theory, or the principal-agent literature.

[4]Pérez-Castrillo and Wettstein (2006) use the ideas behind the concept of egalitarian equivalence to propose an ordinal Shapley value for economic environments.

implement any feasible allocation rule[5] via an incentive compatible mechanism (as, for instance, in the case of Postlewaite and Schmeidler's (1986) non-exclusive information - see Corollary 1). When incentive constraints are more restrictive, on the other hand, the two criteria may become incompatible. This is somehow reminiscent of the incompatibility between concepts of equity and efficiency under complete information in economic environments that are more general than classical pure exchange economies (see Pazner and Schmeidler, 1974, and Maniquet, 1999).

We now briefly discuss some related literature. Mirrlees (1971) is a first classic paper where a social choice criterion is applied under asymmetric information to select a desirable incentive compatible mechanism. The social objective he follows is to maximize the sum of the agents' utilities (or a common transformation of those utilities), in the utilitarian tradition. His methodology has been followed since then in the literature on optimal taxation. In most papers, there is a large population, which implies that all possible types (representing, for instance, the agents' productivity or their cost of effort) are present in the population. Another classic paper where a utilitarian principle is applied to select an incentive compatible mechanism is Myerson and Satterthwaite's (1983) bilateral trade problem. Here, only two agents are interacting and only one pair of types (interpreted as reservation prices) is actually realized. The utilitarian criterion is applied ex-ante, i.e. behind the veil of ignorance and using the relative likelihood of each possible pair of types. There is a more recent literature that is developing at the intersection of computer science and economics that looks for strategy-proof mechanisms that maximize a worst-case scenario index, in order to guarantee, for instance, a minimal percentage of the maximal total surplus in every possible realization of the types (see e.g. Guo and Conitzer, 2009, Moulin, 2009, and references therein). Surprisingly, there are almost no papers that propose axiomatic discussions of social choice criteria under incomplete information.[6] The only published paper that we are aware of is Nehring (2004) who proves that ex-ante utilitarianism is the only interim social welfare ordering that is both consistent with interim Pareto comparisons and that extends the ex-post utilitarian criterion.[7] de Clippel (2010a) shows that his approach cannot be used to extend any other classical social welfare ordering from the ex-post to the interim stage. de Clippel (2010b) follows a different methodology – trying to characterize a social welfare function that satisfies extensions of Kalai's (1977, Theorem 1) axioms – to obtain a notion of egalitarianism under incomplete information. The solution discussed in the present paper can be seen as an adaptation of this criterion to economic environments with the objective of avoiding

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[5]A feasible allocation rule determines a feasible way of sharing the total endowment of the economy for each type profile that comes with a strictly positive probability.

[6]Incomplete information means that the agents already hold some private information when the social decision has to be made. There is a large literature on social choice under uncertainty, discussing for instance problems of insurance, but it focuses only on the case of symmetric uncertainty.

[7]The axiomatic results in the theories of bargaining and social choice share some common features under complete information, the Nash product being also a natural social welfare ordering, for instance. While it is still unclear whether these similitudes survive the presence of asymmetric information, it is worth mentioning that there are some partial axiomatic results that extend Nash's (1950) bargaining theory (see Harsanyi and Selten, 1972; Myerson, 1984; Weidner, 1992).

interpersonal comparisons of utilities.

In the next section, we present the framework and the classical definitions while, in Section 3, we introduce the notion of interim egalitarian equivalence. In Section 4 we prove our existence and uniqueness result and, in Section 5, we present an economic example that further illustrates our concept and shows that interim egalitarian equivalence and interim incentive efficiency may be incompatible in the presence of exclusive information. Finally, in Section 6, we highlight additional properties of the solution, suggest a weaker concept for those economies where interim egalitarian equivalent and interim incentive efficient mechanisms do not exist, and further discuss our approach.

2 The Framework and Standard Definitions

An *economy* is a 6-tuple

$$(N, L, (T_i)_{i \in N}, \pi, e, (u_i)_{i \in N}),$$

where N is the set of agents, L is the set of goods, T_i is agent i's set of possible types, $\pi \in \Delta(T)$ ($T = \times_{i \in N} T_i$) is the common prior describing the relative probability of the types, $e \in \mathbb{R}_+^L \setminus \{0\}$ is the aggregate endowment of the economy in each possible state t, and $u_i : \mathbb{R}^L \times T \to \mathbb{R}$ is a concave, continuous and strongly increasing utility function that represents the preferences of agent i (lotteries are evaluated according to the expected utility criterion). For notational convenience, we also denote by N, L and T the number of elements in the corresponding sets. We assume without loss of generality that each type of each agent comes with a strictly positive probability, i.e. for all $t_i \in T_i$ and all $i \in N$ there exists t_{-i} such that $\pi(t_i, t_{-i}) > 0$.

Since types are private information, it may be profitable for the agents to communicate before agreeing on an allocation. Formalizing this idea, a *mechanism* is a function $\mu : \times_{i \in N} M_i \to \mathbb{R}^{L \times N}$, where M_i is any finite set of "messages." Agents are assumed to play a Bayes-Nash equilibrium in the game induced by the mechanism. The revelation principle (Myerson, 1979) allows us, without loss of generality, to restrict attention to direct mechanisms (i.e. $M_i = T_i$, for each $i \in N$) for which truth-telling forms a Bayes-Nash equilibrium, that is mechanisms that are incentive compatible. To formally define this property, note that if all the other agents report their types truthfully, then agent i's expected utility when reporting t_i' in the direct mechanism μ, while being of type t_i, is

$$U_i(\mu, t_i'|t_i) = \sum_{t_{-i} \in T_{-i}} \pi(t_{-i}|t_i) u_i(\mu_i(t_i', t_{-i}), t),$$

where $\pi(t_{-i}|t_i)$ denotes the conditional probability of t_{-i} given t_i. For simplicity, we will write $U_i(\mu|t_i)$ instead of $U_i(\mu, t_i|t_i)$. The mechanism μ is *incentive compatible* if

$$U_i(\mu|t_i) \geq U_i(\mu, t_i'|t_i)$$

for each t_i, t_i' in T_i and each $i \in N$. A mechanism μ is *incentive feasible* if it is incentive compatible and *feasible*, that is $\sum_{i \in N} \mu_i(t) \leq e$, for all $t \in T$.

Efficiency is a prerequisite for any cooperative solution. Its content was first formalized under incomplete information by Holmström and Myerson (1983). An incentive compatible mechanism μ' *interim Pareto dominates* an incentive compatible mechanism μ if $U_i(\mu'|t_i) \geq U_i(\mu|t_i)$ for all $t_i \in T_i$ and all $i \in N$, with at least one of the inequalities being strict. A mechanism is *interim incentive efficient* if it is incentive feasible, and it is not interim Pareto dominated by any other incentive feasible mechanism.

3   Interim Egalitarian Equivalence

Efficiency is a necessary condition for a cooperative solution to be appealing, but it is not sufficient, as it remains silent regarding the distribution of the gains derived from cooperation. Pazner and Schmeidler (1978) made an interesting proposal to select a subset of Pareto efficient allocations under complete information, i.e. when the type sets are singletons. In order to obtain a solution that depends only on the ordinal information encoded in the agents' preferences, they proposed to measure cooperative gains in the space of goods following the direction given by a reference bundle $d \in \mathbb{R}_+^L \setminus \{0\}$. For each allocation $a \in \mathbb{R}_+^{L \times N}$ and each agent i, let λ_i^a be the real number defined by the following equation:[8]

$$u_i(a_i) = u_i(\lambda_i^a d).$$

The allocation a is *egalitarian equivalent* (along d) if $\lambda_i^a = \lambda_j^a$ for all $i, j \in N$. Pazner and Schmeidler proposed to restrict attention to those allocations that are Pareto efficient and egalitarian equivalent, and prove existence and uniqueness under mild assumptions.[9]

The purpose of our paper is to extend Pazner and Schmeidler's solution to environments with incomplete information (for any finite set T_i, $i = 1, \ldots, n$), and study its properties. One may be tempted to simply look for the mechanism that associates to each t a Pareto efficient egalitarian equivalent allocation in that ex-post economy. This way to proceed is wrong for at least two reasons. First, that mechanism need not be incentive compatible, and thereby impossible to implement in practice. Second, it does not exploit the possibility of mutually beneficial insurance. In other words, it would be incompatible with interim incentive efficiency in most economies. Agents know only their own type when choosing the mechanism. The solution concept should thus be based on their preferences at that point in time (*interim*, and not *ex-post*). Let $d \in \mathbb{R}_+^L \setminus \{0\}$ be the reference vector.[10] For each incentive compatible mechanism μ and each type t_i of each agent i, let $\lambda_i^\mu(t_i)$ be the real number defined by the following equation:

$$U_i(\mu|t_i) = U_i(\lambda_i^\mu(t_i)d|t_i).$$

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[8]We omit the vector t of types in the equation, since it is assumed to be common knowledge in this paragraph.

[9]Sprumont and Zhou (1999) characterize the set of Pareto efficient and egalitarian equivalent allocations for large economies by means of three properties: Efficiency, a Lower Bound on the level of utility obtained, and a Solidarity axiom that reflects behavior with respect to changes in population.

[10]The definitions and the results obtained in this paper extend to the case where d varies with t, at the cost of heavier notations.

This means that agent i of type t_i is indifferent between participating in the mechanism μ and receiving the fixed proportion $\lambda_i^\mu(t_i)$ of the bundle d in each possible type profile (for the other agents). We propose a criterion according to which an incentive feasible mechanism μ is "equitable" if, at any possible interim event, all the agents obtain the same (interim) gains (as measured along the vector d).

Definition 1 *An incentive compatible mechanism μ is* interim egalitarian equivalent *if, for all $t \in T$ with $\pi(t) > 0$, we have: $\lambda_i^\mu(t_i) = \lambda_j^\mu(t_j)$ for all $i, j \in N$.*

The next section is devoted to the study of interim incentive equitable mechanisms, defined as follows:

Definition 2 *A mechanism is* interim incentive equitable *(IIE) if it is both interim egalitarian equivalent and interim incentive efficient.*

4 Sufficient Condition for the Existence and Uniqueness of IIE Mechanisms

We start by establishing the existence and essential uniqueness of IIE mechanisms in environments where the incentive constraints are not really restricting what the social planner or the society as a whole can implement. We need two new definitions and a lemma before stating and proving this result formally.

Definition 3 *A non-empty subset $B \subset T$ is* common knowledge *if $\pi(t_i, \hat{t}_{-i}) = 0$, for all $i \in N$, and all $(t_i, \hat{t}_{-i}) \in T \setminus B$ for which there exists $t_{-i} \in T_{-i}$ such that $t \in B$.*

We denote by $\hat{T}$ the support of π and let $B(t)$, for any $t \in \hat{T}$, be the minimal common knowledge event that contains t. Since the intersection of any two common knowledge events containing t is a common knowledge event containing t and $\hat{T}$ is a common knowledge event, such an event $B(t)$ exists and is given by the intersection of all common knowledge events containing t.

Lemma 1 *Let $t^* \in \hat{T}$, then, for any $t \in B(t^*)$, there exists a finite sequence $(t^s)_{s=1}^S$ in $B(t^*)$ such that $t^1 = t^*, t^S = t$, and for all $s \in \{1, \ldots, S-1\}$, $t_j^{s+1} = t_j^s$ for some $j \in N$.*

Proof. Let B be the set of $t \in \hat{T}$ for which there exists a finite sequence $(t^s)_{s=1}^S$ in $\hat{T}$ such that $t^1 = t^*, t^S = t$, and for all $s \in \{1, \ldots, S-1\}$, $t_j^{s+1} = t_j^s$ for some $j \in N$. It is straightforward to check that B is a common knowledge event, and hence $B(t^*) \subseteq B$. Moreover, all the elements in the previous sequence $(t^s)_{s=1}^S$ are also in $B(t^*)$. ∎

Definition 4 *An* allocation rule *is a function $a : \hat{T} \to \mathbb{R}_+^{L \times N}$. It is* feasible *if $\sum_{i \in N} a_i(t) \leq e$, for all $t \in \hat{T}$. For each $i \in N$ and each $t_i \in T_i$, $U_i(a|t_i)$ will denote agent i's expected utility when of type t_i, should the allocation rule a be implemented truthfully:*

$$U_i(a|t_i) = \sum_{t_{-i} \in T_{-i} \text{ s.t. } t \in \hat{T}} \pi(t_{-i}|t_i) u_i(a_i(t), t).$$

Proposition 1 *Let $(N, L, (T_i)_{i \in N}, \pi, e, (u_i)_{i \in N})$ be an economy such that, for any feasible allocation rule a, there exists an incentive feasible mechanism μ such that*

$$U_i(\mu|t_i) = U_i(a|t_i),$$

for all $t_i \in T_i$ and all $i \in N$. Then an IIE mechanism exists. In addition, agents are indifferent between any two IIE mechanisms. In particular, if $U_i(\cdot|t_i)$ is strictly concave, for each $i \in N$ and each $t_i \in T_i$, then there exists a unique IIE mechanism.[11]

Proof. Denote by $\mathcal{B}$ the set of minimal common knowledge events for π. For each $B \in \mathcal{B}$, define:

$$\lambda_B = \max \{\lambda | \exists \text{ feasible allocation rule } a \text{ s.t. } U_i(a|t_i) = U_i(\lambda d|t_i) \text{ for all } i \in N, \ t \in B\}.$$

Such a λ_B exists because the set on the right-hand side is nonempty (for instance, $a = 0$ is a feasible allocation rule), bounded from above (because it is impossible to sustain unbounded levels of utility), and closed (because utility functions are continuous). Let a_B be a feasible allocation rule such that $U_i(a_B|t_i) = U_i(\lambda_B d|t_i)$ for all $i \in N$ and all $t \in B$, [12] and let a^* be the allocation rule defined as follows:

$$a^*(t) = a_{B(t)}(t),$$

for each $t \in \hat{T}$, where $B(t)$ is the minimal common knowledge event that contains t. Finally, following the assumption of this proposition, let μ be an incentive feasible mechanism that gives the same interim utility to all the agents of all types as the allocation rule a^*. We conclude the first part of this proof by showing that μ is an IIE mechanism. By construction, $\lambda_i^\mu(t_i) = \lambda_j^\mu(t_j)$ for all $i, j \in N$ and all $t \in \hat{T}$, and hence we need only check that μ is interim incentive efficient. Suppose on the contrary that there exists an incentive feasible mechanism ν that interim Pareto dominates μ, i.e. $U_i(\nu|t_i) \geq U_i(\mu|t_i)$, for all $t_i \in T_i$ and all $i \in N$, with at least one of the inequalities being strict. Suppose for instance that $U_j(\nu|\bar{t}_j) > U_j(\mu|\bar{t}_j)$. Let also $\bar{t}_{-j} \in T_{-j}$ be such that $\bar{t} \in \hat{T}$ and $u_j(\nu_j(\bar{t}), \bar{t}) > u_j(\mu_j(\bar{t}), \bar{t})$. The mechanism ν restricted to $\hat{T}$ can be thought of as an allocation rule. Let us modify it to construct a new allocation rule a' as follows. At least one of the components of $\nu_j(\bar{t})$, let's say l, is strictly positive, because $u_j(\nu_j(\bar{t}), \bar{t}) > u_j(0, \bar{t})$. Let then $a'_j(\bar{t})$ be the bundle obtained by decreasing $\nu_j^l(\bar{t})$ by a small amount ε, while keeping the other components constant. For each $i \in N \setminus \{j\}$, let $a'_i(\bar{t})$ be the bundle obtained by increasing $\nu_i^l(t)$ by $\varepsilon/n - 1$, while keeping the other components constant. Finally, for all $t \in \hat{T} \setminus \{\bar{t}\}$ and all $i \in N$, let $a'_i(t) = \nu_i(t)$. If $\varepsilon > 0$ is small enough, then $U_i(a'_i|\bar{t}_i) > U_i(\mu|\bar{t}_i)$, for all $i \in N$, and $U_i(a'_i|t_i) \geq U_i(\mu|t_i)$, for all $t_i \in T_i$ and all $i \in N$. We can now use any type of any agent who is strictly better off under the new allocation rule to improve yet other types of other agents by further modifying the previous allocation

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[11] More precisely, any two IIE mechanisms coincide on $\hat{T}$, since the definition of a mechanism over type profiles that come with a zero probability is irrelevant.

[12] We note that a_B is a feasible allocation rule and as such is defined over $\hat{T}$. However, the only relevant information about a_B is the values it assumes on the set B.

rule in the same manner. By Lemma 1, repeating the argument finitely many times, one can derive a feasible allocation rule that gives higher interim utilities to all agents than a^* over $B(\bar{t})$, thereby contradicting the maximality of $\lambda_{B(\bar{t})}$. Hence μ is in fact interim incentive efficient, and hence IIE.

Let us now focus on the essential uniqueness of IIE mechanisms. Let μ and ν be two IIE mechanisms. Using the notations from the first part of the proof, it must necessarily be the case that $\lambda_B^\mu = \lambda_B^\nu$, for all minimal common knowledge events B. Indeed, suppose on the contrary that there exists a minimal common knowledge event B such that $\lambda_B^\mu > \lambda_B^\nu$. Then the mechanism ν' that is equal to ν on $T \setminus B$ and to μ on B is incentive feasible and Pareto dominates ν, thereby contradicting the fact that ν is interim incentive efficient. Hence, indeed, $\lambda_B^\mu = \lambda_B^\nu$, for all minimal common knowledge events B, and agents are indifferent between any two IIE mechanisms. Suppose now, in addition, that $U_i(\cdot|t_i)$ is strictly convex, for each $i \in N$ and each $t_i \in T_i$. For every $t \in \hat{T}$, let $a(t) = \frac{\mu(t)+\nu(t)}{2}$. The allocation rule a is feasible and, given the strict concavity of the utility functions, it interim Pareto dominates both μ and ν. Indeed, $U_i(a|t_i) \geq U_i(\mu|t_i)$ since

$$U_i(a|t_i) = U_i(\frac{\mu+\nu}{2}|t_i) \geq \frac{1}{2}[U_i(\mu|t_i) + U_i(\nu|t_i)] = U_i(\mu|t_i),$$

for each $t_i \in T_i$ and all $i \in N$. Moreover, if μ and ν are distinct, then at least one of the inequalities is strict. The assumption of the proposition would then imply that one can construct an incentive feasible mechanism that interim Pareto dominates μ, thereby contradicting its interim incentive efficiency. We must thus conclude that $\mu(t) = \nu(t)$ on $\hat{T}$. ∎

Remark 1 *It is clear from the proof that the λ's associated with all agents of all types must coincide over minimal common knowledge events B, that is, $\lambda_i^\mu(t_i) = \lambda_j^\mu(t_j)$ for all $t \in B$ and $i, j \in N$, if μ is an IIE mechanism and B is a minimal common knowledge event. However, the gains that agents obtain can vary across more general events. An obvious illustration of this phenomenon is given by the special case of complete information, where $\hat{T}$ is "diagonal" or, equivalently, each profile of types in $\hat{T}$ is uniquely determined by any of its components. Corollary 1 below will show that Proposition 1 applies in this case, although it should be clear already that the IIE mechanisms will coincide on $\hat{T}$ with Pazner and Schmeidler (1978) egalitarian equivalent allocations in the corresponding ex-post economies. Hence, indeed, for any given $t \in \hat{T}$, we will have $\lambda_i^\mu(t_i) = \lambda_j^\mu(t_j)$ over all agents i and j at an IIE mechanism μ, but this common factor λ will often vary with $t \in \hat{T}$.*

Non-exclusive information, as first defined by Postlewaite and Schmeidler (1986), provides a natural class of information structures for which the assumption of Proposition 1 is automatically satisfied.

Definition 5 *The agents in an economy have* non-exclusive information *(NEI) if, for any agent i and any $t_{-i} \in T_{-i}$, there exists a unique $t_i^* \in T_i$ such that $\pi(t_i^*, t_{-i}) > 0$.*

NEI means that the pooled information of any $n-1$ agents uniquely determines the profile of types.

Corollary 1 *If the agents in the economy have NEI, then an IIE mechanism exists. In addition, agents are indifferent between any two IIE mechanisms. In particular, if $U_i(\cdot|t_i)$ is strictly concave for each $i \in N$ and each $t_i \in T_i$, then there exists a unique IIE mechanism.*[13]

Proof. The corollary directly follows from Proposition 1 after showing that its assumption is satisfied. Let thus a be any feasible allocation rule. Let then μ be the mechanism defined as follows: $\mu(t) = a(t)$ for all $t \in \hat{T}$ and $\mu(t) = \min_{\hat{t} \in \hat{T}} \mu(\hat{t})$ for all $t \notin \hat{T}$. If all agents report truthfully, then no agent can gain by deviating from reporting his true type since, by NEI, any deviation will yield a type profile that does not belong to $\hat{T}$, and result in receiving a smaller or equal bundle. Hence, the mechanism μ is incentive feasible. By construction, we also have $U_i(\mu|t_i) = U_i(a|t_i)$, for all $t_i \in T_i$ and all $i \in N$, as desired. ∎

There are other instances that have been discussed in the literature where incentive constraints can be circumvented. It should be clear that Proposition 1 would apply in those cases as well, even though we will not phrase these results formally here, because they require slightly different frameworks. For instance, there are environments where the true state of the world is commonly known by the designer and the agents at the time of implementing the agreements.[14] In such cases, incentive constraints are irrelevant, and yet information is relevant at the time of selecting an agreement. As an illustration, the payment of an insurance contract depends on the observable losses incurred, or the payment of a financial asset (e.g. equities or options) is contingent on the realization of some observable events. There are more general situations where Proposition 1 or a variant may apply. For instance, Riordan and Sappington (1988) showed how a public ex post signal that is correlated with agents' types may render the initial information asymmetry inconsequential.

The existence result in Proposition 1 might not seem surprising at first sight given Pazner and Schmeidler's (1978) existence result under complete information, and given that incentive constraints are assumed not to be restricting what the social planner or the society as a whole can implement. Even so, asymmetric information plays a significant role in the definition of our notion of equity, and this is what makes the existence result interesting and more challenging to prove than in the special case of complete information. Also, the result comes in contrast to previous results on fair allocations. Fairness is another classical ordinal notion of equity for exchange economies that combines efficiency with envy-freeness (see Foley, 1967, and Varian, 1974). Fair allocations are known to exist in well-behaved exchange economies (e.g. the competitive equilibrium with equal income leads to a fair allocation). de Clippel (2008) proposed a natural extension of these definitions to problems that involve asymmetric information, and showed that interim

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[13]See footnote 11.

[14]That assumption was used for instance by Wilson (1978) and more recently by de Clippel (2007) (see also references therein) in their study of the core under incomplete information, and by de Clippel and Minelli (2004) in their study of bargaining under incomplete information.

envy-freeness may be incompatible with interim incentive efficiency, even if incentive constraints can be overlooked.[15]

Existence of interim equitable mechanisms is no longer guaranteed when incentive constraints are truly restricting the set of allocation rules that can be implemented. Indeed, moving from fairness to egalitarian equivalence while consistent with efficiency in NEI environments may clash with interim efficiency once incentive constraints are relevant - see the example in the next section. Problems of existence are rather usual for equity concepts when considering very general environments (see Pazner and Schmeidler (1974) in the case of fairness with production, see de Clippel (2008) in the case of fairness under incomplete information, or Maniquet (1999) in the case of the 'no domination' criterion with non-convex preferences).

5   An Economic Example

This section demonstrates that when some agent has exclusive information, IIE mechanisms may exist for some configurations of the parameters while not for others. The reason for non existence is the conflict between equity, efficiency and the provision of incentives, captured by our notion of interim incentive equitability.

We consider the following environment[16]. There are two commodities – money and another consumption good. The aggregate amounts available to share are M and Q, respectively. There are two agents, 1, 2. Agent 1 has no private information, and hence has only one possible type, $T_1 = \{*\}$, while agent 2 has two possible types, $T_2 = \{L, H\}$, with $\pi(*, L) = p$, and $\pi(*, H) = 1 - p$. For instance, there are two possible states of nature (representing, say, the intrinsic quality for agent 2 of the consumption good to be shared), low and high, and while agent 2 knows which state prevails, agent 1 is uncertain about it. The preferences of the agents are given by:

Types	$e(.)$	$u_1(q_1, m_1)$	$u_2((q_2, m_2), .)$
$(*, L)$	(Q, M)	$m_1 + 2\sqrt{q_1}$	$m_2 + v_L q_2$
$(*, H)$	(Q, M)	$m_1 + 2\sqrt{q_1}$	$m_2 + v_H q_2$

A mechanism is a function that associates vectors (q_{it}, m_{it}) $(i = 1, 2)$, with each report $t \in \{L, H\}$ from agent 2.

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[15]The example from de Clippel (2008) does not immediately fit the model of the present paper, because it was written under the assumption that the true state of the world is commonly known at the time of implementing the agreements. Also, the aggregate amount of money to be shared was changing with the state of the world. It is easy, though, to construct a similar example with NEI that would fit our current framework. In that example, an IIE mechanism exists (as predicted by Proposition 1), while interim envy-freeness and interim incentive efficiency are incompatible. Details are available upon request from the authors.

[16]We thank an anonymous referee for simplifying the example we used in a previous version of the paper. The referee's suggestion to restrict attention to the direction $d = (0, 1)$ streamlined the derivation of the results regarding existence and non-existence of IIE mechanisms.

Proposition 2 *Suppose that $Q > \frac{1}{v_L^2}$. Then an IIE mechanism exists along the reference vector $d = (0,1)$ if and only if $v_L \leq (1-p)v_H$ or $Q \leq \frac{p^2}{(v_L - (1-p)v_H)^2}$.*

Proof. We show in the Appendix that the set of interim incentive efficient mechanisms is the union of the three following regions[17]:

Region 1

$$q_{1H} = \frac{1}{v_H^2}, \; q_{1L} = \frac{1}{v_L^2}, \text{ and}$$

any allocation of money satisfying $m_{2L} - m_{2H} \in \left[\frac{1}{v_L} - \frac{v_L}{v_H^2}, \frac{v_H}{v_L^2} - \frac{1}{v_H} \right]$.

Region 2

$$q_{1H} = \frac{1}{v_H^2}, \quad \begin{array}{l} \text{any } q_{1L} \geq \frac{1}{v_L^2} \text{ if } v_L \leq (1-p)v_H \\ \text{any } q_{1L} \in \left[\frac{1}{v_L^2}, \frac{p^2}{(v_L - (1-p)v_H)^2} \right] \text{ if } v_L > (1-p)v_H \end{array}, \text{ and}$$

any allocation of money satisfying $m_{2L} - m_{2H} = v_H \left(q_{1L} - \frac{1}{v_H^2} \right)$.

Region 3

$$\text{any } q_{1H} \in \left[\frac{(1-p)^2}{(v_H - pv_L)^2}, \frac{1}{v_H^2} \right], \; q_{1L} = \frac{1}{v_L^2}, \text{ and}$$

any allocation of money satisfying $m_{2L} - m_{2H} = v_L \left(\frac{1}{v_L^2} - q_{1H} \right)$.

Interim egalitarian equivalence along the reference bundle $d = (0,1)$ implies that $\lambda_1 = \lambda_2(L)$ and $\lambda_1 = \lambda_2(H)$. Thus, $\lambda_1 = \lambda_2(L) = \lambda_2(H)$, which implies

$$m_{2L} + v_L q_{2L} = m_{2H} + v_H q_{2H}.$$

The incentive compatibility constraints for Agent 2 are

$$m_{2L} + v_L q_{2L} \geq m_{2H} + v_L q_{2H} \text{ and } m_{2H} + v_H q_{2H} \geq m_{2L} + v_H q_{2L}$$

Hence we obtain $m_{2L} + v_L q_{2L} \geq m_{2L} + v_H q_{2L}$. This implies, since $v_L < v_H$, that $q_{2L} = 0$.

There can not exist IIE mechanisms in Regions 1 or 3 because $q_{1L} = \frac{1}{v_L^2} > 0$ in those regions. In Region 2, the candidate allocations involve $q_{1L} = Q$, $q_{2L} = 0$, $q_{1H} = \frac{1}{v_H^2}$ and $q_{2H} = Q - \frac{1}{v_H^2}$. They are IIE allocations if and only if there is a λ such that

$$pm_{1L} + 2p\sqrt{Q} + (1-p)m_{1H} + 2(1-p)\frac{1}{v_H} = \lambda M \tag{1}$$

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[17] Note that the resource constraints $q_{1t} + q_{2t} = Q$ and $m_{1t} + m_{2t} = M$ hold in all the three regions. Destroying part of the endowment can never be part of an interim incentive efficient mechanism, since giving it to agent 1 rather than disposing of it would constitute an incentive compatible Pareto improvement.

$$m_{2H} + v_H \left(Q - \frac{1}{v_H^2} \right) = \lambda M \tag{2}$$

$$m_{2L} = \lambda M. \tag{3}$$

We notice that any allocation that satisfies (1), (2) and (3) also satisfies $m_{2L} - m_{2H} = v_H \left(Q - \frac{1}{v_H^2} \right) = v_H \left(q_{1L} - \frac{1}{v_H^2} \right)$ as required to the allocations in Region 2. Equations (2) and (3) provide m_{2H} and m_{2L} once we know λ. We use these equations and $m_{1L} = M - m_{2L}$ and $m_{1H} = M - m_{2H}$ to obtain the following equivalent equation to (1):

$$2p\sqrt{Q} + (1-p)v_H \left(Q - \frac{1}{v_H^2} \right) + 2(1-p)\frac{1}{v_H} = (2\lambda - 1) M. \tag{4}$$

Equation (4) always has a solution, with $\lambda \geq \frac{1}{2}$. Therefore, an IIE allocation exists if and only if the condition on q_{1L} in Region 2 holds, which amounts to $v_L \leq (1-p)v_H$ or $Q < \frac{p^2}{(v_L - (1-p)v_H)^2}$, as claimed. ∎

6   Concluding Discussion

We start by discussing some additional properties of the interim equitable solution that associates with each economy the set of mechanisms that are interim incentive equitable. First, the proposal is invariant to affine transformations of the interim utilities, i.e. changing the utility function u_i of any agent $i \in N$ in t by multiplying it with a strictly positive coefficient that may vary with t_i and/or adding a real number that may vary with t_i, does not affect the solution. Second, the interim equitable solution satisfies Myerson's (1984) probability invariance axiom, since it depends on the probabilities only through the computation of interim utilities. One could even have considered a more general framework with the agents' ordinal interim preferences as exogenous variables (allowing, for instance, for non-expected utility), instead of deriving those from the expected utility criterion applied to ex-post utilities. Indeed, both interim incentive efficiency and interim egalitarian equivalence depend only on those interim preferences, and the interim equitable solution is then ordinally invariant in this more general framework. Third, the interim equitable solution is anonymous, meaning that renaming the agents, or even their types, will not change their payoffs. Fourth, the solution is also monotonic, meaning that increasing the total endowment e cannot make any agent of any type worse off (assuming that d does not vary with e). Fifth, we can also offer a weak comparison of the level of interim satisfaction achieved at mechanisms in the interim equitable solution and the level of satisfaction achieved for egalitarian equivalent allocations in the ex-post economies. Let $\lambda^*(t)$ be the level reached at any Pareto efficient and egalitarian equivalent allocation, in the ex-post economy obtained should t be realized. Let $\lambda^*(E)$ be the level reached by any mechanism in the solution on the minimal common knowledge event E. If π satisfies the NEI condition, then $\lambda^*(E) \geq \min_{t \in \hat{T} \cap E} \lambda^*(t)$ for each minimal common knowledge event E. Notice that the inequality is most often strict, because of the possibility of mutually beneficial insurance. On the other hand, the inequality does not extend to economies

that do not satisfy NEI, because the incentive constraints can be so severe that it is impossible to guarantee even the minimum of the ex-post levels. Sixth, it is easy to check that the interim equitable solution satisfies a solidarity property with respect to both changes of population and in preferences. Suppose first that we move from an economy to an alternative economy with fewer agents and all other exogenous variables remaining unchanged. Then, it must be that all the agents in the smaller economy either suffer, or that they all benefit from the change, on each event that is common knowledge among them.[18] Next, changing the preference of any type of an agent (in terms of taste and/or risk aversion) will have an impact in the same direction on all the other agents independently of their information, and on all other types of the agent whose preference changed. In some cases, one can say more as to whether the impact is positive or negative. Consider for instance an economy with NEI and private values, meaning that the agents' utility functions do not vary with the other agents' types.[19] Then it is easy to check that an increased risk aversion for one type of one agent will have a *negative* impact on the others - resources are reallocated so as to compensate that type of that agent for his increased risk aversion. Changing the risk aversion of some types may change the set of incentive feasible mechanisms in non-trivial ways, and so the sign of the comparative static cannot be systematically determined in the absence of NEI. Even so, one can say that they either all benefit, or all suffer.

While we have listed a few appealing properties of the interim equitable solution, finding an axiomatic characterization remains an open question. Our work suggests that the natural place to start would be to derive a characterization on the class of economies with NEI, where the interim equitable solution is always non-empty. Even in that case, the problem is likely to be non-trivial given the lack of results under complete information in the first place. One direction that is definitely worth investigating is determining whether Sprumont and Zhou's (1999) result extends to our framework, in the hope of characterizing the interim equivalent solution when $d = e$ in economies with a continuum of agents. Indeed, they showed under complete information that this is the only solution satisfying a solidarity axiom, similar to the one described at the end of the previous paragraph, and a property of equal split lower bound, requiring that no agent is worse off than with his fair share of the total endowment. Notice that the interim equivalent solution with $d = e$ does satisfy the equal split lower bound, since the constant mechanism that splits the total endowment equally independently of the agent's reports is incentive feasible.

As we have shown interim equitable solutions may not exist when incentive constraints truly restrict the set of allocation rules that can be implemented. A common way to proceed when there is tension between efficiency and equality, is to maximize the minimum of the indices one aims at equalizing, and to refine that criterion by applying a lexico-

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[18]Under complete information, the egalitarian equivalent rule satisfies the stronger property that all the agents gain when considering fewer agents. This stronger property does not always hold under incomplete information, because deleting an agent changes the set of common knowledge events and incentive constraints may become more restrictive.

[19]Restricting attention to the case of private values makes it simpler to understand what it means to increase an agent's risk aversion for some type.

graphic argument. Formally, let α be the function that associates with each vector of real numbers the vector obtained by ordering its components decreasingly. Then, an interim incentive efficient mechanism μ is said to be weakly interim equitable if it maximizes $\alpha(\lambda^\mu)$ according to the lexicographic ordering over the set of interim incentive efficient mechanisms. The set of weakly interim equitable mechanisms is always non-empty, and coincides with the interim equitable solution on economies where it exists.

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Appendix: Characterization of Interim Incentive Efficiency for the Economy in Section 5

We characterize the set of interim incentive efficient mechanisms through a series of claims. First, notice that the two incentive constraints are:

$$m_{2H} + v_H q_{2H} \geq m_{2L} + v_H q_{2L}, \tag{5}$$

$$m_{2L} + v_L q_{2L} \geq m_{2H} + v_L q_{2H}. \tag{6}$$

Claim 1 *Interim incentive efficiency implies $q_{1H} \leq \frac{1}{v_H^2}$. In addition, $q_{1H} = \frac{1}{v_H^2}$ if (6) is not binding.*

Proof. Indeed, from any mechanism, change q_{2H} (resp. q_{1H}) by a small $+\delta$ (resp. $-\delta$) and simultaneously change m_{2H} (resp. m_{1H}) by an amount $-\delta v_H$ (resp. $+\delta v_H$). The utility obtained by both types of agent 2 and constraint (5) do not change. Constraint (6) is relaxed if $\delta > 0$. Finally, agent 1's utility level increases with the change when $\delta > 0$ and $q_{1H} > \frac{1}{v_H^2}$ or when $\delta < 0$ and $q_{1H} < \frac{1}{v_H^2}$. Therefore, $q_{1H} > \frac{1}{v_H^2}$ cannot be part of an IIE mechanism. Also, $q_{1H} < \frac{1}{v_H^2}$ cannot be part of an IIE mechanism if (6) is not binding. ■

Claim 2 *Interim incentive efficiency implies $q_{1L} \geq \frac{1}{v_L^2}$. In addition, $q_{1L} = \frac{1}{v_L^2}$ if (5) is not binding.*

Proof. Similarly as before, from any allocation, change q_{2L} (resp. q_{1L}) by a small $+\delta$ (resp. $-\delta$) and simultaneously change m_{2L} (resp. m_{1L}) by an amount $-\delta v_L$ (resp. $+\delta v_L$). Agent 2's utility and constraint (6) do not change. Constraint (5) is relaxed if $\delta < 0$. Agent 1's utility increases when $\delta < 0$ and $q_{1L} < \frac{1}{v_L^2}$ or when $\delta > 0$ and $q_{1L} > \frac{1}{v_L^2}$. ■

Claim 3 *Both incentive constraints (5) and (6) can not bind simultaneously at an interim incentive efficient mechanism.*

Proof. From Claims 1 and 2, $q_{1L} \geq \frac{1}{v_L^2} > \frac{1}{v_H^2} \geq q_{1H}$. If (5) and (6) were both to hold with equality, then $q_{1L} = q_{1H}$, and one would reach a contradiction.

We now analyze the three possible regions where interim incentive efficient mechanisms can lie: no binding incentive constraints, or only one binding constraint. To the equations identifying the interim incentive efficient mechanisms below, we always have to add the obvious requirements $q_{1t} + q_{2t} = Q$ and $m_{1t} + m_{2t} = M$ for $t = H, L$. ■

Claim 4 *In Region 1, where no incentive constraint is binding, the IIE allocations are characterized by:*

$$q_{1H} = \frac{1}{v_H^2}, \ q_{1L} = \frac{1}{v_L^2}, \ \text{and}$$

any allocation of money satisfying $m_{2L} - m_{2H} \in \left[\dfrac{1}{v_L} - \dfrac{v_L}{v_H^2}, \dfrac{v_H}{v_L^2} - \dfrac{1}{v_H} \right].$

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Proof. The allocations must be ex-post Pareto efficient if incentive constraints are not relevant, while the condition on $m_{2L} - m_{2H}$ rewrites the constraints (5) and (6) for those values of q_{1H} and q_{1L}. ■

Claim 5 *In Region 2, where constraint (5) is binding, the IIE allocations are characterized by:*
(a) If $v_L \leq (1-p)v_H$

$$q_{1H} = \frac{1}{v_H^2}, \ any \ q_{1L} \geq \frac{1}{v_L^2},$$

any allocation of money that involves $m_{2L} - m_{2H} = v_H \left(q_{1L} - \frac{1}{v_H^2} \right)$.

(b) If $v_L > (1-p)v_H$, same conditions as in (a) except that $q_{1L} \in \left[\frac{1}{v_L^2}, \frac{p^2}{(v_L - (1-p)v_H)^2} \right]$.

Proof. To show that, we recall that the utility levels achieved by any interim incentive efficient allocation in Region 2 must satisfy, in addition to the resource constraints, $q_{1t} + q_{2t} = Q$ and $m_{1t} + m_{2t} = M$ for $t = H, L$, the following 5 equations:

$$u_1 = pm_{1L} + 2p\sqrt{q_{1L}} + (1-p)m_{1H} + 2(1-p)\sqrt{q_{1H}} \tag{7}$$

$$u_{2H} = m_{2H} + v_H q_{2H} \tag{8}$$

$$u_{2L} = m_{2L} + v_L q_{2L} \tag{9}$$

$$m_{2H} + v_H q_{2H} = m_{2L} + v_H q_{2L} \tag{10}$$

$$m_{2L} + v_L q_{2L} > m_{2H} + v_L q_{2H} \tag{11}$$

Using the resource constraints we obtain:

$$u_1 = p(M - m_{2L}) + 2p\sqrt{Q - q_{2L}} + (1-p)(M - m_{2H}) + 2(1-p)\sqrt{Q - q_{2H}}$$

and finally:

$$u_1 = p(M - m_{2L}) + 2p\sqrt{Q - q_{2L}} + (1-p)(M - m_{2H}) + 2(1-p)\sqrt{Q - q_{2H}}.$$

Thus, we get the following 5 equations:

$$u_1 = p(M - m_{2L}) + 2p\sqrt{Q - q_{2L}} + (1-p)(M - m_{2H}) + 2(1-p)\sqrt{Q - q_{2H}}$$

$$u_{2H} = m_{2H} + v_H q_{2H}$$

$$u_{2L} = m_{2L} + v_L q_{2L}$$

$$m_{2H} + v_H q_{2H} = m_{2L} + v_H q_{2L}$$

$$m_{2L} + v_L q_{2L} > m_{2H} + v_L q_{2H}.$$

Interim incentive efficiency implies that $q_{1H} = (1/v_H)^2$ and $(1/v_L)^2 \leq q_{1L}$. Also note that one cannot obtain a Pareto improvement by just changing the m allocation, neither

can an improvement be realized by changing q_{1H} or by increasing q_{1L}. So in order to characterize interim incentive efficient allocations we need to determine the conditions under which a decrease in q_{1L}, which amounts to an increase in q_{2L}, cannot lead to a Pareto improvement either. Increasing q_{2L} by $\delta > 0$ and decreasing m_{2L} by δv_L leaves both types of agent 2 with the same utility level and does not violate the incentive constraint for agent 2 of type L. It does, however cause the incentive constraint for agent 2 of type H to be violated, since the RHS increases by $-\delta v_L + \delta v_H$. To offset that increase, the utility of agent 2 of type H must be increased by at least the same amount, and since we want to maximize the gain of agent 1, it should be increased by $-\delta v_L + \delta v_H$. The efficient way to realize that raise is to increase m_{2H} by $-\delta v_L + \delta v_H$ (since efficiency in this region requires $q_{1H} = (1/v_H)^2$). All these changes alter u_1 by $du_1 = \delta p v_L - \frac{\delta p}{\sqrt{Q - q_{2L}}} - (1-p)\delta(v_H - v_L)$. If the initial allocation was efficient, the utility of agent 1 must decrease. Thus it must be that:

$$\delta p v_L - \frac{\delta p}{\sqrt{Q - q_{2L}}} - (1-p)\delta(v_H - v_L) \le 0$$

or

$$\frac{p}{\sqrt{Q - q_{2L}}} \ge -(v_H - v_L) + p v_H = v_L - (1-p)v_H$$

which holds for any q_{2L} (thus any q_{1L}) if $v_L \le (1-p)v_H$.

If $v_L > (1-p)v_H$, this change will not lead to an improvement if $\frac{p^2}{q_{1L}} \ge (v_L + (p-1)v_H)^2$ or $q_{1L} \le \frac{P^2}{(v_L+(p-1)v_H)^2}$ (note that the LHS is indeed greater than $\frac{1}{v_L^2}$).

To conclude note that the requirement $m_{2L} - m_{2H} = v_H\left(q_{1L} - \frac{1}{v_H^2}\right)$ rewrites (using the resource constraint) the incentive constraint for agent 2 of type H. ∎

Claim 6 *In Region 3, where constraint (6) is binding, the IIE allocations are characterized by:*

$$any\ q_{1H} \in \left[\frac{(1-p)^2}{(v_H - p v_L)^2}, \frac{1}{v_H^2}\right],\ q_{1L} = \frac{1}{v_L^2},\ and$$

$$any\ allocation\ of\ money\ that\ involves\ m_{2L} - m_{2H} = v_L\left(\frac{1}{v_L^2} - q_{1H}\right).$$

Proof. Proceeding as in the previous proof we get that the utility levels achieved by any interim incentive efficient allocation in Region 3 must satisfy equations (7), (8), (9), and

$$m_{2H} + v_H q_{2H} > m_{2L} + v_H q_{2L}$$

$$m_{2L} + v_L q_{2L} = m_{2H} + v_L q_{2H}.$$

Interim incentive efficiency implies that $q_{1H} \le (1/v_H)^2$ and $q_{1L} = (1/v_L)^2$. Similar to before we need to show that a decrease in q_{2H} cannot lead to a Pareto improvement . To that effect we decrease q_{2H} by $\delta > 0$, increasing m_{2H} by δv_H. Similar to before this leads

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to a violation of the incentive constraint for agent 2 of type L. The RHS of the constraint increases by $-\delta v_L + \delta v_H$. The "best" way to restore the constraint is to increase m_{2L} by $\delta(v_H - v_L)$. These changes alter u_1 by $du_1 = -p\delta(v_H - v_L) - (1-p)\delta v_H + \frac{\delta(1-p)}{\sqrt{Q-q_{2H}}}$. If the initial allocation was efficient, the utility of agent 1 must decrease. It must be the case then that

$$-p\delta(v_H - v_L) - (1-p)\delta v_H + \frac{\delta(1-p)}{\sqrt{Q-q_{2H}}} \leq 0$$

or

$$\frac{(1-p)}{\sqrt{Q-q_{2H}}} \leq p(v_H - v_L) + (1-p)v_H = v_H - pv_L$$

which holds since we are in the region where $q_{1H} \geq \frac{(1-p)^2}{(v_H - pv_L)^2}$ (note that the RHS is indeed smaller than $\frac{1}{v_H^2}$).

To conclude note that the requirement $m_{2L} - m_{2H} = v_L \left(\frac{1}{v_L^2} - q_{1H} \right)$ rewrites (using the resource constraint) the incentive constraint for agent 2 of type L. $\blacksquare$