Broadband time-reversal of optical pulses using a switchable photonic-crystal mirror

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Abstract: Recently, Chumak et al. have demonstrated experimentally the time-reversal of microwave spin pulses based on non-adiabatically tuning the wave speed in a spatially-periodic manner [Nat. Comm. 1, 141 (2010)]. Here, we solve the associated wave equations analytically, and give an explicit formula for the reversal efficiency. We discuss the implementation for short optical electromagnetic pulses and show that the new scheme may lead to their accurate time-reversal with efficiency higher than before.

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References and links
Time-reversal is one of the most spectacular and useful wave phenomena. A time-reversed pulse evolves as if time runs backward, thus eliminating any distortions or scattering that occurred at earlier times. This enables light detection, imaging and focusing through complex media [1, 2, 3, 4] with applications in diverse fields such as medical ultrasound [1, 2], communication systems and adaptive optics [5], superlensing [6], ultrafast plasmonics [7], biological and THz imaging [8], spintronics [9] and quantum information and computing [10].

For low frequency waves (e.g., in acoustics, microwave spin/electromagnetic waves etc.), time-reversal can be accomplished by electronic sampling, recording, and playing back [1, 2]. This is possible since in this frequency range, the pulse oscillates on a scale slower than electronic sampling speed. On the other hand, for high frequency waves, specifically, for optical electromagnetic waves, the pulse oscillations (and even the pulse envelope itself for sufficiently short pulses) are too short to be sampled properly by even the fastest electronic detector.

The standard solution in the optical regime is to use nonlinear processes such as Three-Wave or Four-Wave Mixing (3WM or 4WM, respectively), see e.g., [5, 11, 12, 13]. However, while such techniques have been demonstrated experimentally, they usually suffer from one or several disadvantages. In particular, they require fairly high intensities, thus, limiting on-chip integration; almost all existing schemes are narrowband whereas the schemes which are applicable to relatively short pulses are in general complex, requiring complicated setups and sometimes even cryogenic temperatures. Finally, some schemes may be difficult to apply to 2- and 3-dimensional systems.

Recent suggestions to overcome these limitations involved optical periodic structures (photonics crystals, PhCs) which are dynamically-tuned [14, 15]. However, despite being very efficient, the structures and/or modulations required for these schemes were quite challenging to realize; in addition, these schemes allow for reversal of only relatively narrow pulses.

These obstacles were recently lifted in a study by Chumak et al. [16], performed in the context of magnonic crystals, a field which has much to contribute, and gain from, the field of PhCs. In particular, it was shown in [16] that a pulse propagating in a homogeneous material can be time-reversed if the material is subject to a spatially-periodic, time-localized non-adiabatic modulation. This scheme is closely related to other non-adiabatic optical pumping
based schemes which were studied theoretically earlier in slab waveguides [17, 18], atomic vapours [19], spin wave systems [20], high-dimension PhCs [21] and especially to our recent detailed studies of zero-gap photonic crystals [22, 23].

In this article, we study the time-reversal scheme of [16] theoretically and compare it to previous non-adiabatic modulation-based schemes; we focus on time-reversal of ultrashort optical pulses, which as noted above, still has not been demonstrated experimentally. We first give a simple heuristic explanation of the dynamically-tuned PhC-based schemes [16, 22, 23] showing that the reversal originates from switching-on a bulk PhC mirror (Section 1). Then, we study the reversal process in [16] analytically, derive a simple explicit formula for the reversal efficiency in the weak-coupling limit and compare it with existing techniques (Section 2). We then briefly discuss the implementation for optical electromagnetic pulses (Section 3).

1. Principles of time-reversal using a switchable mirror

In order to understand the reversal schemes of [16, 22, 23], it is beneficial to adopt a somewhat heuristic interpretation. Recall that when a pulse is reflected by a (standard) mirror, its spatial components change their propagation direction at different times, i.e., the leading edge first and trailing edge last. Thus, the pulse effectively undergoes a U-turn, i.e., the leading edge remains the leading edge etc. Now, imagine that one could change of direction of the pulse propagation at all points in space and at the same time. Then, obviously, the leading edge will become the trailing edge, and vice versa, i.e., the pulse is (time-) reversed.

In order to perform such an extreme manipulation, one needs to abruptly reduce the transmissivity of the medium, preferably to zero, everywhere in space and for a spectral band as wide as possible. Possibly the simplest way to do that would be to open a frequency bandgap by periodically modulating the material properties. Heuristically, when the bandgap is turned on, the wave cannot propagate in any direction. Instead, the forward waves are then repeatedly converted to backward waves, then back to forward waves and vice versa. If one re-establishes the transmissivity once most of the energy of the forward wave has been converted to a backward wave, then a time-reversed pulse is released backwards. In a sense, this procedure transforms a perfectly transmitting medium into a “volume” mirror. Accordingly, in what follows we refer to these schemes as switchable mirror (SM)-based reversal schemes.

This heuristic explanation clarifies why the zero-gap-based switchable-mirror (ZGSM) is equivalent to a homogeneous-medium-based switchable-mirror (HSM). Indeed, unlike a finite-gap system, in both structures all the incident light is perfectly admitted. In addition, in both structures a gap is opened due to the modulation (see [16, 23]). It is also implied by the heuristic explanation (and later proved analytically in Section 2), that in both ZGSM and HSM, although the wave-front is reversed, it is not conjugated. Thus, the scheme can lead to perfect time-reversal only if it is complemented by a consequent step of phase-conjugation, e.g., via nearly-degenerate 4WM [22, 23]. As shown in Section 2, this may also be beneficial in order to make the scheme more efficient.

The advantage of the SM-based schemes is allowing to reverse pulses of almost unlimited wide spectrum, the only limitation being the shortness of the modulation. Also, as noted in [16, 22], the new reversal schemes, which require only a periodic modulation rather than complex optics-specific concepts, open the way for time-reversal in other wave systems for which time-reversal was not accessible before, such as atomic physics [19] and quantum computing [10].

Importantly, the HSM has several advantages over the ZGSM. First, the former is obviously simpler as essentially no fabrication is required; its performance is also practically insensitive to the modulation spatial pattern. Second, our analysis (Eq. (16) below) shows that the reversal efficiency in a HSM is about an order of magnitude higher than in a ZGSM. Third, while the frequency conversion performed in a ZGSM is purely vertical [22, 23], the frequency conver-
The Maxwell equations reduce to the 1D wave equation when a periodic grating is placed perpendicular to the direction of propagation of the pulse. In this scenario, the 1D wave equation is given by

\[ \varepsilon(x,t) = n_0^2 + \Delta \varepsilon m(t - t_0) \varepsilon_m(x). \]  

Here, \( n_0 \) is the average refractive index, the modulation is spatially periodic, i.e.,

\[ \varepsilon_m(x) = \varepsilon_m(x + d), \quad \int_0^d \varepsilon_m(x) dx = 0, \quad \max[\varepsilon_m(x)] = 1, \]  

and time-localized around \( t_0 \) with \( \max[m(t - t_0)] = m(t_0) = 1 \), i.e., the modulation essentially turns on a periodic grating perpendicular to the direction of propagation of the pulse. In this case, the Maxwell equations reduce to the 1D wave equation

\[ \partial_{xt} E(x,t) = \frac{1}{c^2} \partial_{tt} [\varepsilon(x,t) E(x,t)]. \]  

In order to solve Eq. (3), we follow the derivation given in [24], used previously to study the propagation of optical pulses in a photonic crystal with a cubic (Kerr) nonlinear response. As an aside, we note that unlike our previous studies of time-reversal in a zero-gap photonic crystal system [22, 23], which can only be studied within the deep-grating formalism [25], here, we can use the shallow-grating analysis [24]. The latter is much simpler to derive, and does not require the use of multiple scales analysis and calculation of the Floquet-Bloch modes which are required in [22, 23].

We assume the field can be written as

\[ E(x,t) = \left[ E^+(x,t)e^{ik_0x} + E^-(x,t)e^{-ik_0x} \right] e^{-i\omega_0t} + \text{c.c.}, \]  

where \( E^\pm \) represent the Slowly-Varying Envelopes (SVEs) of the forward and backward field components, respectively; \( \omega_0 = ck_0 \), \( k_0 = 2\pi n_0/\lambda_0 \) and \( n_0 \) are the carrier frequency, wavevector and refractive index of the medium, respectively, with \( \lambda_0 \) being the vacuum wavelength. Substituting the ansatz (4) into Eq. (3), neglecting the c.c. terms and the second order derivative terms, and removing the factor \( e^{-i\omega_0t} \) gives

\[ 2ik_0 \partial_x E^+ + 2i(\omega_0 + \omega_0^3/2) \partial_x E^+ = 2i \left( k_0 \partial_x E^- - \omega_0 n_0^2/c^2 \partial_x E^- \right) e^{-2ik_0x} \]

\[ + \Delta \varepsilon \frac{\varepsilon_m(x)}{c^2} \left[ 2\partial_t m \left( \partial_t E^+ - i\omega_0 E^+ \right) - m \left( 2i\omega_0 \partial_t E^+ + \omega_0^3 E^+ \right) \right] \]

\[ + \Delta \varepsilon \frac{\varepsilon_m(x)}{c^2} \left[ 2\partial_t m \left( \partial_t E^- - i\omega_0 E^- \right) - m \left( 2i\omega_0 \partial_t E^- + \omega_0^3 E^- \right) \right] e^{-2ik_0x}. \]
The spatial modulation $\varepsilon_m(x)$ couples the forward and backward field components. When $k_0$ is close to the first bandgap, i.e., when $k_0 = k^{(s)} + \delta k$ (with $k^{(s)} \equiv \pi/d$ or equivalently, $\lambda^{(s)}_0 = 2n_0 d$), the $j = \pm 1$ components of the grating are close to the phase mismatch between the forward and backward field components, so that the coupling becomes most efficient. Following [24], we now expand the spatial part of the modulation as a Fourier series as follows

$$\varepsilon_m(x) = \sum_{j=1}^{m} \varepsilon_m^{(j)} e^{\frac{2\pi i j x}{d}} + c.c., \quad \varepsilon_m^{(j)} = \frac{1}{d} \int_0^d \varepsilon_m(x) e^{\frac{2\pi i j x}{d}} dx. \quad (6)$$

For a weak grating, $\Delta \varepsilon \ll 1$, it is justified to take only the $j = \pm 1$ components of the grating [24]; this is equivalent to setting

$$\varepsilon_m(x) = 2\varepsilon_m^{(1)} \cos\left(\frac{2\pi x}{d}\right). \quad (7)$$

Substituting Eq. (7) in Eq. (5), neglecting all the fast-oscillating terms and separating into two sets of equations gives

$$2ik_0 \partial_t E^+(x,t) + 2 \left(ik_0 \frac{n_0}{c} - (\partial_t m - i\omega_0 m) \frac{\Delta \varepsilon \varepsilon_m^{(1)}}{c^2} e^{-2i\delta ks}\right) \partial_x E^+ = \Delta \varepsilon \frac{\varepsilon_m^{(1)}}{c^2} e^{-2i\delta ks} \omega_0 (\omega_0 m + 2i\partial_t m) E^-, \quad (8)$$

$$-2ik_0 \partial_x E^-(x,t) + 2 \left(ik_0 \frac{n_0}{c} - (\partial_t m - i\omega_0 m) \frac{\Delta \varepsilon \varepsilon_m^{(1)*}}{c^2} e^{2i\delta ks}\right) \partial_t E^- = \Delta \varepsilon \frac{\varepsilon_m^{(1)*}}{c^2} e^{2i\delta ks} \omega_0 (\omega_0 m + 2i\partial_t m) E^+. \quad (9)$$

For modulations longer than the pulse period, the dominant term on the Right-Hand-Side is the first. Furthermore, for a weak modulation, one can neglect the correction to the pulse velocity, i.e., the second term inside the parentheses on the Left-Hand-Side. These assumptions lead to the following coupled equations

$$\partial_t E^+(x,t) + \frac{n_0}{c} \partial_x E^+ = i\kappa e^{-2i\delta ks} m(t) E^-, \quad (10)$$

$$\partial_x E^-(x,t) - \frac{n_0}{c} \partial_t E^- = -i\kappa^* e^{2i\delta ks} m(t) E^+, \quad (11)$$

where

$$\kappa \equiv \Delta \varepsilon \frac{\varepsilon_m^{(1)}}{2cn_0} \omega_0, \quad (12)$$

is a complex coupling coefficient. The final form of the coupled equations (10)-(11) is the same as in [24] except for the nature of the modulation. Indeed, whereas in [24] the modulation is induced by the traveling pulses and thus is spatio-temporally-localized, in the current context, the modulation occurs everywhere at the same time, but for a brief moment.

In order to solve equations (10)-(11), we follow the procedure outlined in [22, 23]. Specifically, we first transform each equation into a frame moving with each of the pulses, namely, we define $x^{(f,b)} = x \mp vt$. We then assume that the coupling is weak and neglect the coupling term on the RHS of Eq. (10). In this case, the solution of Eq. (10) is simply

$$E^+(x^{(f)},t) = E^+(x - vt, 0) \equiv E_0^+(t), \quad (13)$$
where \( E_0^+(t) \) is the incident pulse profile. Substituting in Eq. (11) then gives

\[
E^-(x^b(t), t) = i v \kappa^* e^{2i\delta \kappa^*} \int_{-\infty}^{t} e^{-2i\delta \kappa^*} e^{2i\delta \kappa^*} m(t' - t_0) E^+(x^b(t') - 2v t') dt'.
\]  

Eq. (14) shows that the backward wave is given by the convolution of the forward wave with the modulation. Thus, as noted in [22, 23], this reversal scheme yields a wave-front reversal rather than a complete time-reversal (which requires also the conjugation of the envelope).

The convolution integral can be solved exactly for a unit amplitude Gaussian pulse and a Gaussian modulation

\[
E_0^+(x / \sqrt{2T_p}) = e^{-\frac{x^2}{2T_p^2}}, \quad m(t - t_0) = e^{-(t - t_0)^2 / 4T_{mod}^2}.
\]  

In this case, following [23], it can be shown that for \( t_0 = 0 \) and \( \delta k = 0 \), the wave-front of the reversed component is given by

\[
|E^-(x + vt)| = \sqrt{\frac{\Delta n}{n_0}} \frac{T_{mod}T_p}{\sqrt{T_p^2 + 4T_{mod}^2}} E_0^+ \left( \frac{x + vt}{\sqrt{T_p^2 + 4T_{mod}^2}} \right).
\]  

where \( \Delta n \cong \frac{|\Delta n|}{2 \sqrt{2\pi}} \) is the depth of the refractive index modulations. Note that at the time at which the modulation is maximal (i.e., at \( t = t_0 = 0 \)), the width of the gap opened by the modulation is given approximately by \( \frac{2\pi\Delta n}{n_0} \) [26]. Thus, the reversal efficiency is proportional to the width of the gap opened by the modulation, in agreement with our interpretation of the scheme (see Section 1).

By comparing the reversal efficiency (16) to that obtained in a 1D zero-gap PhC [23, Ch. VI], it is seen that the reversal in the HSM is about one to two orders of magnitude more efficient (in terms of intensity) than in a ZGSM. Furthermore, in comparison to the early suggestion of time-reversal via non-adiabatic 4WM [17, 18], the reversal efficiency in our case is almost an order of magnitude more efficient. Thus, by splitting the time-reversal into two consecutive steps (envelope reversal via a SM and phase conjugation), one gains flexibility, simplicity (e.g., by allowing to work with a single pump pulse) and efficiency. In fact, with relatively small intensity, the phase conjugation step can be used also for amplifying the reversed signal, thus allowing for 100% or more time-reversal of ultrashort optical pulses. Finally, the SM-based techniques can be implemented with a wider variety of modulation techniques, including linear modulators [14, 15, 16].

### 3. Implementation

As discussed in detail in [23], the pulse duration dictates the required modulation technique. For pulses longer than a few picoseconds, the non-adiabatic modulation can be done electronically, e.g., with linear modulators. Thus, a time-reversal mirror can be built from a standard material such as LiNbO\(_3\) which is spatially-modulated in a periodic manner. A 100% reversal efficiency can be easily obtained using index modulations on the scale of \( 10^{-3} \) [22, 23].

For shorter pulses, the required modulations can be performed via a nonlinear process such as Cross-Phase Modulation or carrier injection in a pump-probe configuration. One way to ensure that the pump induces an index modulation in the \( x \) direction only is to confine the probe (signal) into a thin waveguide and send a much wider and much shorter intensity-modulated pump at right angles to the waveguide (see e.g., [17, 18]). The intensity modulation can be performed by sending a single pump pulse through a waveguide array or by interference of two pump beams. An alternative is to use a structure with a periodically-varying Kerr coefficient [27, 28], preferably, with a changing sign (see [29] for a review).

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