

# Optimizing MCSD Portfolios

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## **Abstract**

Marginal Conditional Stochastic Dominance (MCSD) states the probabilistic conditions under which, given a specific portfolio, one risky asset is marginally preferred to another by all risk-averse investors. Furthermore, by increasing the share of dominating assets and reducing the share of dominated assets one can improve the portfolio performance for all these investors. We use this standard MCSD model sequentially to build optimal portfolios that are then compared to the optimal portfolios obtained from Chow's MCSD statistical test model. These portfolios are furthermore compared to the portfolios obtained from the recently developed Almost Marginal Conditional Stochastic Dominance (AMCSD) model. The AMCSD model restricts the class of risk-averse investors by not including extreme case utility functions and reducing the incidence of unrealistic behavior under uncertainty. For each model, an algorithm is developed to manage the various dynamic portfolios traded on the New York, Frankfurt, London, and Tel Aviv stock exchanges during the years 2000-2012. The results show how the various MCSD optimal portfolios

provide valid investment alternatives to stochastic dominance optimization. MCSD and AMCSD investment models dramatically improve the initial portfolios and accumulate higher returns while the strategy derived from Chow's statistical test performed poorly and did not yield any positive return.

## 1 Introduction

The essence of portfolio theory is to design rules that will satisfy the choices of investors to increase wealth in an environment of risk and uncertainty. In a static world the simplest model amounts to choosing a portfolio of assets that maximizes the total expected return while keeping risk at a satisfactory level. This basic model has suffered major setbacks when the variance was used to estimate portfolio risk because in order to be compatible with investors preferences, it requires that assets be normally distributed. The failure of the mean-variance paradigm became evident when it appeared that, empirically, financial assets were not normally distributed and higher moments such as skewness and kurtosis could not be easily dismissed.<sup>1</sup> Afterwards, the mean-variance model was theoretically remedied with the use of expected utility maximization.

In practice, the solution of using expected utility empirically came with the advent of Second Degree Stochastic Dominance (SSD) by Hanoch and Levy (1969), Hadar and Russell (1969), and Rothschild and Stiglitz (1970) all of whom devise rules based on the entire statistical distribution of risky assets instead of a finite number of moments. Because of this, less restrictive assumptions regarding investor's behavior were needed, with the only requirement being that the utility functions describing a rational risk-averse investor be monotonically increasing and concave. SSD compares cumulative probability distributions of

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<sup>1</sup>Furthermore, as shown by Lambert and Yitzhaki (2014), higher variance would not be so bad for low risk-averse investors.

risky assets to determinate their dominance. The main advantage of SSD lies in its ability to discriminate among existing assets. However, it lacks the power to reach portfolio optimality.

To overcome this last obstacle Shalit and Yitzhaki (1994) developed the concept of Marginal Conditional Stochastic Dominance (MCSD) in finance. Under the SSD assumptions, MCSD allows an investor holding a given portfolio to derive dominance relations between two assets. Then, a marginal increase in the share of the dominating asset at the expense of the dominated one will improve the portfolio for all risk-averse investors. A series of marginal improvements will lead to a SSD efficient portfolio regardless of whether a change in portfolio structure or a new investment opportunity is being considered. MCSD has enjoyed some success owing to the papers of Chow, Huang, and Hu (2007), Clark and Kassimatis (2012), Clark and Kassimatis (2013), Shalit and Yitzhaki (2003) and Shalit (2010) to cite a few. Most notably is that for portfolio management, Clark, Jokung, and Kassimatis (2011) used MCSD to develop a new methodology aimed at constructing SSD efficient portfolios.

Chow (2001) improved upon MCSD by introducing a statistical test to confine the number of assets to those with significant dominance relations. He basically reformulated the model to make it more suitable for statistical computation and testing. An additional issue regarding SSD and MCSD is that some extreme utility functions that satisfy the mathematical definition of risk aversion are hardly present in the real world.<sup>2</sup> To tackle this issue Denuit, Huang, Tzeng, and Wang (2014) recently developed the concept of Almost Marginal Conditional Stochastic Dominance (AMCSD). The basic idea of AMCSD is that the set of relevant utility functions is reduced by putting restrictions on the second derivative. Accordingly, Denuit et al (2014) obtained new MCSD conditions

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<sup>2</sup>For instance, a lottery with equal probabilities prizes of 999\$ or 100,000\$ does not dominate a certain prize of 1,000\$, although any rational agent will choose the lottery.

whose purpose was to increase the number of dominance relations.

In this paper, we apply the three different MCSD approaches to test active portfolio management. Our purpose is to show in practice how a series of small portfolio improvements leads to portfolio optimization for all risk-averse investors. Although MCSD was applied successfully to active portfolio management we are providing here a new comparison of three approaches related to MCSD. Our data consists of historical returns for the 2000-2012 period from four financial markets to test our optimization algorithms written in MATLAB.

In the next section we provide the theoretical framework of the three MCSD models. In section 3, we outline the optimization methodology. In section 4, we present the data and in Section 5 we show the main results. Section 6 concludes the paper as well as the implications of our findings.

## 2 MCSD Theory

We begin by providing the theoretical background for the three models we use to construct optimal portfolios, namely the original Shalit and Yitzhaki (1994) MCSD, the model derived from Chow's (2001) statistical test for MCSD, and Denuit et. al.'s (2014) recent model of AMCSD. The three models are rooted in the concept of SSD that expresses the probabilistic conditions under which all risk-averse investors prefer one risky asset to another. SSD was developed independently by Hadar and Russell (1969), Hanoch and Levy (1969) and, Rothschild and Stiglitz (1970) who derived the necessary and sufficient conditions by using the asset's cumulative probability distributions functions (CDF) as we now present. Let us consider a risk-averse investor who maximizes the expected utility of asset returns  $EU(r)$  where  $U$  is non-decreasing and concave. Given two risky assets with random returns  $r_k$  and  $r_j$  and CDF  $F_k$  and  $G_j$  the SSD necessary and sufficient conditions are stated as follows:

**Theorem 1.** :  $E_F [U (r_k)] \geq E_G [U (r_j)]$  if and only if  $\int_{-\infty}^x [G_j (r_j) - F_k (r_k)] \geq 0$  for all  $x$  and all concave  $U$ .

It should be noted that these conditions are not so practical when applied to optimizing portfolios as they require infinite comparisons of CDFs, their intersections, and their areas under these CDFs. A more accessible and intuitive alternative for using CDFs was provided by Shorrocks (1983) who developed the absolute (generalized) Lorenz curves to rank distributions and derive appropriate necessary and sufficient conditions for SSD.<sup>3</sup> To see this equivalence, let us define the absolute Lorenz curve as the function relating conditional mean return to the cumulative probability of getting that return, namely:

$$L(\xi) = \int_{-\infty}^x r dF(r) \text{ for all } x, \quad (1)$$

where  $\xi$  is the cumulative probability  $\xi = \int_{-\infty}^x dF(r)$ . Gattswith (1977) has simplified the notation of the Lorenz curve as  $L(\xi) = \int_0^\xi F^{-1}(\omega) d\omega$  and thus only one equation is needed to formulate the Lorenz curve. As shown by Thistle (1989), the SSD conditions using the Lorenz become:

**Theorem 2.** :  $E_F [U (r_k)] \geq E_G [U (r_j)]$  if and only if  $L_k (\xi) - L_j (\xi) \geq 0$  for all  $\xi \in (0, 1]$ ,

where  $L_k (\xi)$  and  $L_j (\xi)$  are the absolute Lorenz curves of asset  $k$  and asset  $j$ , respectively. In other words, the absolute Lorenz of the dominating asset must not lie below the absolute Lorenz curve of the dominated one. The main reason to prefer the SSD concept over some other alternatives is that there is no need for restrictive assumptions regarding the utility function and the distribution of the risky assets. Nonetheless, there are major shortcomings, the

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<sup>3</sup>In welfare economics, relative Lorenz curves are used to measure income inequality and wealth distribution. In financial economics, absolute Lorenz curves are used to rank distributions and measure the risk and return of assets and portfolios. See Shalit (2014).

main one being SSD's ineptitude to obtain optimal portfolios. Indeed, once an optimum portfolio is attained it can always be improved by altering the allocation increasing the dominating asset and shortening the dominated asset and raising the portfolio expected return.

To correct upon SSD lacunae, Shalit and Yitzhaki (1994) developed the concept of MCSD, which as mentioned previously, provides dominating and dominated assets conditional upon holding a portfolio. For all risk-averse expected utility maximizers, MCSD provides the probabilistic rules for one asset to marginally dominate another one and improve upon the initial portfolio. Consider a risk-averse investor holding a portfolio  $p$  of  $n$  risky assets defined by the shares  $\alpha \equiv \{\alpha_i\}$  such that  $\sum_{i=1}^n \alpha_i = 1$ . The assets yield risky returns  $r \equiv \{r_i\}$  and the portfolio return is obtained by  $p = \sum_{i=1}^n \alpha_i r_i$ . Expected utility maximizers who want to improve their current positions, but without the trouble of reorganizing their entire portfolios, can marginally alter some of their holdings. Usually MCSD provides the conditions for dominance in the case of two assets. To marginally increase the share of dominating asset  $j$ , one can marginally decrease the share of dominated asset  $k$ , such as:  $d\alpha_k = -d\alpha_j$ . Accordingly, portfolio return changes as  $dp = d\alpha_k (r_k - r_j)$  and the change in expected utility becomes:  $dE[U(p)] = E[U'(p) d\alpha_k (r_k - r_j)]$ , leading to:

$$\frac{dE[U(p)]}{d\alpha_k} = \int_{-\infty}^{\infty} U'(t) [\mu_k(t) - \mu_j(t)] f_{\alpha}(t) dt \quad (2)$$

where  $\mu_i(t)$  is the conditional expected return on asset  $i$  given the portfolio return  $t$ , i.e.,  $\mu_i(t) = E(r_i | p = t)$  and  $f_{\alpha}(t)$  is the *pdf* of portfolio returns  $t$ . (See Shalit and Yitzhaki (1994)). Asset  $k$  dominates asset  $j$  if, and only if Equation (2) is non-negative and is increasing the share of asset  $k$  on account of asset  $j$  increasing expected utility. Since  $U'(t)$  is positive for all returns Equation (2) can be expressed in terms of Absolute Concentration Curves (ACCs) which

are defined as the cumulative expected returns on an asset conditional on the return on the portfolio  $\alpha$ . Namely,

$$ACC_i^\alpha(\xi) = \int_{-\infty}^p \mu_i(t) f_\alpha(t) dt \quad (3)$$

where  $p$  is the return implicitly defined by the *CDF* of the portfolio and  $\xi$  the cumulative probability:

$$\xi = \int_{-\infty}^p f_\alpha(t) dt. \quad (4)$$

**Theorem 3.** (*MCS**D*) *Given portfolio  $\alpha$  asset k dominates marginally asset j for all risk-averse investors if and only if*

$$ACC_k^\alpha(\xi) \geq ACC_j^\alpha(\xi) \quad (5)$$

Proof: (See Shalit and Yitzhaki (1994))

MSCD conditions are provided for two assets given a portfolio. As shown by Shalit and Yitzhaki (2003) in the case of  $m$  assets, the conditions are derived by trying to maximize the Lorenz of Equation (1) which is written as:

$$L(\xi) = \int_{-\infty}^p t f_\alpha(t) dt = \sum_i^n \alpha_i \int_{-\infty}^p r_i f_\alpha(t) dt = \sum_i^n \alpha_i ACC_i^\alpha(\xi) \quad (6)$$

For a given portfolio  $\{\alpha_0\}$ , an alternative portfolio  $\{\alpha_1\} = \{\alpha_0 + d\alpha\}$  is preferred by all risk-averse investors if portfolio  $\{\alpha_1\}$  leads to a higher Lorenz i.e.,:

$$\sum_i^n \frac{\partial L(\xi)}{\partial \alpha_i} d\alpha_i = \sum_i^n ACC_i^\alpha(\xi) d\alpha_i \geq 0 \quad \text{for all } \xi \quad (7)$$

subject to  $\sum_i^n d\alpha_i = 0$ . One of the advantages of MCS*D* is that a series of marginal improvements will eventually lead to an optimal SSD portfolio. The main shortcomings of the method is that the inclusion of abnormal concave

utility functions are compulsory. Another less important one is the lack of a statistical test, which, however was later devised by Chow and which is described as follows.

## 2.1 Chow's Statistical Test

The main issue not addressed by standard MCSD is whether dominance relations can be established from a sample of asset returns. Chow (2001) answered this question by developing a procedure that tests whether ACCs intersect statistically. The test is described as follows: Define anew the ACCs and the Lorenz to be more suitable for statistical computations. From Equation (4) the inverse  $p = F_\alpha^{-1}(\xi)$  defines portfolio return  $p$  for the probability  $\xi$ . Furthermore let  $I(t)$  be an index function mapping 1 for  $t \leq p$  and 0 otherwise. Hence, the ACC for asset  $i$  can be written:

$$ACC_i^\alpha(r_i|t \leq F^{-1}(\xi)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_i I(t) f_\alpha(t, r_i) dr_i dt. \quad (8)$$

Equation (8) allows us to construct a test that checks whether two unique ACCs intersect. Consider a sample of  $n$  assets and  $T$  time periods. We choose a set of  $S$  target returns from the portfolio distribution as indicated by  $\{p_s | s \in [1, 2, \dots, S]\}$ . Every target return is a test point that checks the intersection of ACCs. From Equation (8) we can compute the ACC of asset  $i$ , at a target point  $p_s$  by averaging all the observations of vector  $r_i I(p_s)$  which is calculated as:

$$\widehat{ACC}_i(r_i|t \leq p_s) = \frac{1}{T} \sum_{j=1}^T r_{i,j} I_j(p_s) = \overline{r_i I(p_s)}. \quad (9)$$

Using Equation (9) we then estimate the difference between two ACCs at the target return  $p_s$  as:



$$\widehat{\Phi}^{k-j}(p_s) = \widehat{ACC}_k(r_k|t \leq p_s) - \widehat{ACC}_j(r_j|t \leq p_s) \quad (10)$$

This allows us to obtain a vector of length  $S$  for all the target returns. The next step is to estimate the standard deviations using Rao's theorem and computing:

$$\widehat{\sigma} \left[ \widehat{\Phi}^{k-j}(p_s) \right] = \frac{1}{T} \sqrt{\widehat{var} [r_k I(p_s)] + \widehat{var} [r_j I(p_s)] - 2\widehat{cov} [r_k I(p_s), r_j I(p_s)]} \quad (11)$$

Finally, we obtain the  $Z$  statistic of the estimator of (10) by dividing with the standard deviation in Equation (11) :

$$Z^{k-j}(\tau_t) = \frac{\widehat{\Phi}^{k-j}(p_s)}{\widehat{\sigma} \left[ \widehat{\Phi}^{k-j}(p_s) \right]} \quad (12)$$

When parts of ACCs lie close to each other, conventional statistical inference methods cannot distinguish weak dominance relations from intersections. In this case, Chow suggests to obtain the critical values for every test point by using the Studentized Maximum Modulus distribution, with the normal approximation for large samples. The critical value given  $S$  test points and significance level  $\alpha$  can be computed by the following approximation:

$$SMM(\alpha, S, \nu \rightarrow \infty) \rightarrow Z \left( 0.5 \left( 1 - (1 - \alpha)^{\frac{1}{S}} \right) \right) \quad (13)$$

Using the values obtained by (12) and (13) it is possible to test the dominance relations for the desired significance level using the following rule: Asset  $k$  dominates asset  $j$  if  $Z^{k-j}(\tau_t) \geq SMM(\alpha, S, \nu \rightarrow \infty)$  for all  $t$  and asset  $j$  dominates asset  $k$  if  $Z^{k-j}(\tau_t) \leq -SMM(\alpha, S, \nu \rightarrow \infty)$  for all  $t$ . For other cases there are no significant dominance relations either because ACCs intersect or lie close to each other.

This statistical test can help us make real-life decisions when investment portfolios need to be revised. Though, generally, such a test can limit the number of pairs of assets with dominance relations only to those with strong and significant dominance and thus be very helpful for portfolio management. In practice, however, some questions remain unanswered regarding the desired significance of the test and its power. Indeed, one doesn't know what significance levels will produce the best operative results, as lower significance levels will be less discriminative and higher levels will show poor dominance relations to choose from.

## 2.2 Almost Marginal Conditional Stochastic Dominance

Some of the issues incurred with MCSD deal the inclusion of extreme utility functions in the set of risk-averse investors. Most of these extreme functions do not appear in the real world and hence should not be considered when optimizing portfolios of risky assets. Denuit, Huang, Tzeng, and Wang, (2014) developed "Almost Marginal Conditional Stochastic Dominance" (AMCSD) that applies the dominance relations to a smaller set of risk-averse investors by excluding those extreme utility functions. Thus, AMCSD reduces the set of risk-averse agents by restricting the second derivative of the utility function. This can be seen as follows: Let  $U^1$  be the entire set of increasing and concave utility functions that are defined on portfolio return  $p$ . A new set,  $U^2$  is defined using a single parameter  $\epsilon \in (0, 0.5)$  that limits set of utility functions:

$$U^2(\epsilon) \equiv \{U(p) \in U^1 | \forall p : -U''(p) \leq \inf(-U''(p))(\epsilon^{-1} - 1)\} \quad (14)$$

Where  $\inf(\cdot)$  is the infimum function. It can be easily shown that  $U^2(0) = U^1$  and  $U^2(0.5) = \emptyset$ . In that case,  $U^2$  can be considered as a set without extreme behavior functions, with the critical limit being set by a sole parameter  $\epsilon$ .

By using Equation (4) and taking into consideration the fact that portfolio return  $p$  can be obtained for the probability  $\xi$  by applying  $p = F_\alpha^{-1}(\xi)$  we can redefine the ACCs to be a function of  $p$  instead of  $\xi$ , and can define the set of portfolio returns in which there is a “transgression” of the standard MCSD criteria as :  $\Omega \equiv \{p | ACC_k^\alpha(p) - ACC_j^\alpha(p) < 0\}$

**Theorem 4.** : (AMCSD) *For any utility function in  $U^2$ , the expression in (2) is non-negative if, and only if:*

$$\left\{ \begin{array}{l} \int_{-\infty}^{\infty} - [ACC_k^\alpha(p) - ACC_j^\alpha(p)] |p \in \Omega \leq \epsilon \int_{-\infty}^{\infty} |ACC_k^\alpha(p) - ACC_j^\alpha(p)| dp \\ \mu_k \geq \mu_j \end{array} \right. \quad (15)$$

Proof: See Denuit et al (2014)

We have shown that it is possible to derive dominance relations on a confined set of risk-averse utility functions. It is likely that by using the criteria described in (15) we can better model the behavior of real-world investors, and thus produce better performance. Still a valid question remains regarding what the proper value for  $\epsilon$  is in limiting the extreme behavior utility functions. Levy, Leshno, and Leibovitch (2010) used a series of laboratory experiments to derive the value of 0.3. In the following we will try to shed some light on this issue.

### 3 Managing the MCSD Portfolios

We now present the rules directing the investment algorithm for the three portfolios. It is assumed that market participants behave rationally in the sense that, learning from historic returns, investors derive the dominance relations and then, buy the dominating stocks and sell the dominated ones. For each MCSD

approach we start with two portfolios, one where the assets are weighted according to the market index of that specific stock exchange and the other where all assets are weighted equally, i.e., the so called 1/N portfolio.

### 3.1 Moving Window, Sample Size, and Computation Procedure

Following Chow’s suggestion that a sample of 600 is the minimal size needed to achieve satisfying test power, the sample size upon which the dominance relations are established is set to 600 observations,. Going beyond 600 observations would probably introduce irrelevant and noisier data into the analysis. To be more consistent with the activity of a real-life portfolio manager, we assume that the portfolio changes every 30 observations, which accounts for approximately a month and a half of daily trades. For every data set, the first day a portfolio changes is with the 601-th observation, since dominance relations are established with the 1- 600 observations. The second trading day is with the 631-th observation since the sample used to compute dominance includes the 31-630 observations, and so on. After composing the new portfolio, it is used for the next 30 days until a new portfolio is formed, and the process is renewed all over again.

For every update point, i.e., observations 601,631, etc., a sample of 600 previous daily returns of  $K$  stocks and the initial portfolio consisting either in the market index, either the 1/N portfolio is used. With this data we construct the marginal change of  $\alpha^\Delta$  of  $K$  elements and compute the new portfolio returns for the next 30 days as follows: If  $R_p$  are the returns on the initial portfolio, then the returns on the new portfolio are  $R_p^* = R_p + \sum_{i=1}^K \alpha_i^\Delta r_i$ . This method was chosen because we ignore the exact weights of stocks in the initial market index portfolio and know only their returns. We restrict the marginal change to be

in the  $\pm 100\%$  interval in order to avoid infinite loops and stay close to real life portfolios. We now explain how the  $\alpha^\Delta$  vector is constructed.

### 3.2 Pairing the Stocks

We choose a pair of stocks whose dominance relation is defined according to the model at hand. Using the sample data, two matrices are constructed: a Boolean dominance matrix and a dominance strength matrix. The Boolean dominance matrix indicates whether the  $i$ -th stock dominates the  $j$ -th stock with “1” in the  $i$ -th column and  $j$ -th row and “0” elsewhere. The dominance strength matrix quantifies the strength of the dominance relations. For example, in MCSD and AMCSD this value is the maximum difference between the ACCs of the two stocks. In the Chow test this value is the maximum of the Z-statistic for the two stocks. Once the matrices are computed, all pairs are sorted with respect to their dominance strength index. A pair is chosen that has the largest strength index value under two conditions: the pair was not used in the previous 10 iterations in order to avoid internal loops, and none of the stocks in the pair reached the  $\pm 100\%$  limit. After a pair is chosen, the marginal change is obtained when the share of dominating stock is increased at the expense of the dominated stock. Thereafter, the Boolean dominance relation matrix and dominance strength matrix is computed again and the whole process is reiterated. Unless it is terminated due to lack of pairs with definite dominance relations, the process is restricted to a maximum of 1000 iterations.

### 3.3 Marginal Change

After a pair with a defined dominance is selected, a subroutine is conducted to compute the best marginal change of the weight of the dominating stock on account of the weight of the dominated stock. First, the weight of the

dominating stock is increased by 50% and the weight of the dominated stock is subsequently decreased by 50%. Then, the dominance relations are computed for those two stocks again. If the dominance relation persists, then the change is preserved; otherwise it is discarded. Next, the whole process is reiterated with a change of 25%. At every step  $n$ , the change is  $1/2^n$ . This method allows us to get close enough to the point where dominance relations disappear within 20 fixed steps. For every step the  $\pm 100\%$  limit is checked and the process stops if this weight exceeds this interval.

## 4 The Data

The research was performed in four different stock exchanges: For the Tel-Aviv stock exchange, we use daily prices for the period 03.01.2000 - 28.06.2012. The stocks were chosen from the constituents of the Tel-Aviv-100 index, and the index itself was taken as one of the initial portfolios. The constituents weights were computed by the market shares in the index. The New York stock exchange was represented with the S&P-500 index, and its constituents were the subsequent stocks listed for the period 03.01.2000 - 31.12.2011. The London stock exchange had a smaller sample for the period 03.03.2003 - 30.12.2011 and included the FTSE-100 index, and its constituents. The Frankfurt stock exchange had a similar sample for the period 02.01.2003 - 30.12.2011. The difference between the Frankfurt exchange and the other exchanges is the incongruence of the initial portfolio index DAX-30 and the list of the assets based on the DAX-100. The reason for this disparity is the lack of accessible data regarding the DAX-100 returns.<sup>4</sup> All the returns were corrected for dividends and splits. Assets with more than half of the data missing were eliminated and therefore in some mar-

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<sup>4</sup>The data for the Tel-Aviv Stock Exchange data were obtained from its website and for the other three datasets were obtained from Yahoo! finance.

kets the final number of assets was less than 100. For the missing data, a linear interpolation was made using the prices before and after the gap. After these corrections, the daily return for day  $t$  was computed as  $r_t = P_t/P_{t-1} - 1$ .

## 5 Optimization Results

To check which MCSD managed portfolio approach produces statistically significant excessive returns, we used two tests: the simple mean difference test and the sign test. We begin to show the first test. Let  $r_p^M$  and  $r_p^I$  be the returns of the actively managed and initial portfolio, respectively. We set the null hypothesis as  $H_0 : E(r_p^M - r_p^I) \leq 0$  and use the statistic  $\bar{r}_p^M - \bar{r}_p^I$  where  $\bar{r}_p^M$  and  $\bar{r}_p^I$  are the sample means. Hence, we use the standard Z-test with standard deviation  $\hat{\sigma}(r_p^M - r_p^I)/\sqrt{N}$  as follows:

$$Z_{st} = \frac{\bar{r}_p^M - \bar{r}_p^I}{\hat{\sigma}(r_p^M - r_p^I)/\sqrt{N}} . \quad (16)$$

The sign test counts the instances that returns of the managed portfolio exceed those of the initial portfolio. When there is no significant increase in returns, the probability of sampling an occurrence with greater returns must be not different from 0.5. Thus, under the null  $H_0 : Prob(r_p^M - r_p^I > 0) \leq 0.5$  we compute the subsequent Z-statistic as follows:

$$Z_{st} = \frac{N^+ - 0.5N}{0.5\sqrt{N}} \quad (17)$$

where  $N^+$  is the number of instances that the managed portfolio has outperformed the initial one. Since the procedures use the first 600 observations to establish dominance, MCSD-managed portfolios started from the 601st observation.

## 5.1 MCS D

At first, in every stock exchange, we use the market index as the initial portfolio and apply the standard MCS D portfolio management to check whether the method can improve portfolio performance. In financial theory the market index is often considered an efficient portfolio, hence, this test amounts to checking market efficiency. The results for the two tests are given in Table 1:

**Table 1: Excess Returns of MCS D Managed Portfolios**

	Tel-Aviv	New York	London	Frankfurt
$\bar{r}_p^{MCS D} - \bar{r}_p^I$	0.000367	-0.000037	0.000081	0.000417
p-value of the mean diff. test	0.076045	0.552780	0.402263	0.125767
p-value of the sign test	0.079495	0.508111	0.167100	0.042489
Observations	2470	2419	1631	1699

The results show that in three out of the four stock markets the portfolio management algorithm using the MCS D criterion produced greater average daily returns than the initial portfolio. In two of the markets the improvement was statistically significant. In the Tel-Aviv and the Frankfurt exchanges the algorithm produced an additional 0.0367% and 0.0417% to the average daily return, which amounts to approximately to 9.175% and 10.425% in annual returns. In the US market, the managed portfolio produced poorer results than the initial one, although those results were not statistically significant. We have established that the management algorithm efficiency depends upon which market it is applied and it generally improved the initial portfolio.

## 5.2 MCS D with the Integrated Chow Test

Although the purpose of the Chow test is to improve the performance of the standard MCS D, questions remain as to the validity of the parameters used. For



one thing, what is the optimal value of the significance level  $\alpha$ ? As we would like to find only viable dominance relations, a small  $\alpha$  should be chosen. However, a small  $\alpha$  would lead to poor test power and viable dominance relations could be ignored due to statistical “noise”. Another question regards the number of points that should be used in the simultaneous test. In his original work, Chow used simulated data and came to the conclusion that 10 points received higher power than 20 points. Finally, the location of these points seems completely arbitrary. To tackle these issues we used values of  $\alpha$  of 0.05, 0.1, 0.2 and 0.3 and checked anew the notion that 10 points were better than 20 points. Finally, two approaches were used to locate the test points: either to place them equally on the return distribution or else to place them on the equal quantiles.

The combinations of these parameters produced different results and we reached the conclusion that using 10 test points instead of 20 yields better test power. Furthermore, placing the test points at equally spaced quantiles results in a better test power. Finally, for the significance level the best results were obtained using the greater values of  $\alpha$ . In Table 2, we show the results using 10 test points placed at equally spaced quantiles, and setting  $\alpha = 0.3$ .

**Table 2: Chow Test-Managed Portfolios**

	Tel-Aviv	New York	London	Frankfurt
$\bar{r}_p^{Chow} - \bar{r}_p^I$	0.000267	-0.000082	0.000105	0.000195
p-value of the mean diff. test	0.058611	0.679168	0.299266	0.205474
p-value of the sign test	0.000069	0.014841	0.021305	0.000697
Observations	2470	2419	1631	1699

The results show the same pattern as with the MCSD-managed portfolios, but the statistically significant changes are much smaller. As compared with the MCSD managed portfolios, Chow’s test algorithm produced only two thirds

of the excess returns in Tel-Aviv and half of the excess returns in Frankfurt. In order to determine the best portfolio management, we calculated the statistics for the excess returns using Chow’s test in lieu of the regular MCSD and compared them as shown in Table 3.

**Table 3: Chow Test vs Standard MCSD-Managed Portfolios**

	Tel-Aviv	New York	London	Frankfurt
$\bar{r}_p^{Chow} - \bar{r}_p^{MCSD}$	-0.000100	-0.000045	0.000025	-0.000221
p-value of the mean diff. test	0.672852	0.585092	0.464774	0.817302
p-value of the sign test	0.358609	0.572597	0.284505	0.623767
Observations	2470	2419	1631	1699

It seems that in three of the four markets, the portfolios managed according to Chow’s test resulted in worse performances than those portfolios managed by the standard MCSD even when we used the best combination of test parameters. The results confirm what was evident in Table 2 i.e., that in most cases MCSD with an integrated Chow test reduces the average daily excess results although not always statistically significantly.

### 5.3 AMCSD

In the AMCSD procedure, only one parameter needs to be valued, i.e., the utility set restriction factor  $\epsilon$ . Increasing this parameter confines the utility set that we are taking into account and increases the number of dominance relations pairs. In some recent work by Levy, Leshno, and Leibovitch (2010), a value of 0.3 was obtained. In the present paper we use the values of 0.1, 0.2, 0.3 and 0.4 to see how portfolio performance changes. It seems that the optimal value of the  $\epsilon$  parameter is 0.4. In most cases, the mean excess returns are increasing monotonically and become more significant when  $\epsilon$  rises. In Table 4 we present

the AMCSD results of excess returns for the initial portfolio and in Table 5 we exhibit the results for the AMCSD portfolio vs the MCSD-managed portfolio.

**Table 4: AMCSD-Managed Portfolios**

For $\epsilon=0.4$	Tel-Aviv	New York	London	Frankfurt
$\bar{r}_p^{AMCSD} - \bar{r}_p^I$	0.000619	0.000100	0.000252	0.000794
p-value of the mean diff. test	0.044458	0.401202	0.279602	0.118175
p-value of the sign test	0.045498	0.086559	0.022446	0.000741
Observations	2470	2419	1631	1699

**Table 5: AMCSD- vs MCSD- Portfolios**

	Tel-Aviv	New York	London	Frankfurt
$\bar{r}_p^{AMCSD} - \bar{r}_p^{MCSD}$	0.000252	0.000137	0.000172	0.000377
p-value of the mean diff. test	0.153745	0.243020	0.199042	0.182856
p-value of the sign test	0.018193	0.001225	0.006194	0.076160
Observations	2470	2419	1631	1699

The results clearly show that AMCSD can produce better portfolio returns in all markets and that those changes are statistically significant (at least, according to the sign test). The managed portfolios in Tel-Aviv, New York, London, and Frankfurt exchanges produce average excess daily returns of 0.0619%, 0.01%, 0.0252%, and 0.0794% , which amounts to annual mean returns of 15.475%, 2.5%, 6.3% and 19.85%. Moreover, when AMCSD portfolio management is used instead of MCSD we observe significant improvements in all four markets.

## 5.4 Changing the Initial Portfolio

As a test for robustness, we used the various MCSD procedures on the 1/N portfolio as the initial portfolio. Every asset in the portfolio is equally weighted from all the assets in the specific exchange. The results are not reported here and are available from the authors upon request. These results are somewhat astonishing since in all the markets and for all the procedures, the improvements were significantly lower than when the index portfolio was used as the initial portfolio. These results raise the following questions: Does this indicate that the 1/N portfolio is more efficient than the market index portfolio? This is quite possible since MCSD has far less room for improvement. Although these results to some extent contradict the general conclusions of De Miguel et. al. (2009) that 1/N naive portfolios are inefficient, we reiterate that the improvements provide portfolios that are marginally stochastic dominant. We have shown that improving performance strongly depends upon the initial portfolio and more specifically whether it is located near a local (or even global) optimum if such optimum exists.

## 5.5 The Market Effect

If we examine the mean excess return in the various exchanges, a pattern seems to prevail in all methods. The best results are obtained in the Tel-Aviv and Frankfurt exchanges followed by London and then New York. One possible explanation for this pattern is that the level of market competition and efficiency can affect the results of the managing algorithm. Namely, more efficient markets like New York and London exhibit fast price adjustments, and automatic trading systems perform poorly.

## 6 Conclusion

In this paper we used three different, yet interconnected, methods to manage portfolios. As shown, the standard MCSD procedure provides quite good results at least in the less efficient markets. Using Chow’s test in MCSD resulted in poorer portfolio improvement probably due to the test power which ignores assets with growth potential. It may be possible to improve the test by deriving its optimal parameters by using 10 test points instead of 20 and locating them based on quantiles. Finally, we found evidence that using AMCSD improves the performance of the portfolio, beyond the standard MCSD. The reason is that if forced to satisfy “bizarre” concave utility functions, we ignore assets with improving returns.

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