Mean-Gini Analysis of Stochastic Externalities: The Case of Groundwater Contamination[1]

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Abstract. The mean-Gini approach is used to analyze stochastic externalities generated by agricultural production. The model addresses the problem of groundwater pollution caused by excessive fertilizer application. Inherent in the mean-Gini approach to expected utility maximization is a two-fold value: the simplicity of the two-parameter mean-variance model and satisfaction of necessary and sufficient conditions for stochastic dominance. Price and quantity policy recommendations to control externalities are formulated based upon the relative assessment of uncertainty by the regulatory authority and the farmers. Using the Gini as a measure of risk allows for the quantification of control policy measures under differentiated risk aversion and multiple sources of pollution. The model shows that when producers underestimate uncertainty, quota policies restricting fertilizer are more efficient than tax policies in reducing groundwater contamination.

Key words. Stochastic externalities, water pollution policies, stochastic dominance.

1. Introduction

This paper presents the mean-Gini approach for analyzing production externalities generated in the presence of uncertainty. Specifically, I use this approach to address the problem of groundwater pollution caused by crop fertilizers.

Fertilizers are used to increase crop production. During the growing cycle any fertilizer not taken up by the plant is absorbed by the environment. There are two types of losses associated with this natural process. The first relates to whether the crop efficiently extracts the necessary nutrients from the soil solution. The second type of loss is a consequence of the stochastic environment of agricultural production, because fertilizer must be applied prior to knowing the growing conditions. Solutions to these problems of fertilizer externalities may be approached by controlling the method and degree of fertilizer applications, which leads to internalization of external effects. If governments were to regulate fertilizer by imposing price or quantity control policies, producers would be forced to use fertilizer optimally by choosing effective compounds and/or improving the timing of application.

The paper establishes the conditions under which a quota policy on inputs application can be more effective than a price policy in reducing the level of damage caused by stochastic externalities. Weitzman (1974) addresses this question and shows how, under uncertainty, the preference for quantity control over price regulation depends upon the difference in the marginal benefit

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and cost functions. The same type of analysis was undertaken by Koenig (1985), who advocates *ad valorem* taxes to correct externalities in the presence of uncertainty. The specific problem of water pollution under uncertainty has been addressed by Shortle and Dunn (1986), who conclude that a tax-cum-subsidy policy on fertilizer use reduces externalities more efficiently than a quota policy on application or a direct policy on estimated run-offs.

The approach taken here is to extend the stochastic externalities problem by using the mean-Gini model instead of expected utility maximization. Under a range of conditions based upon probability distributions, the two approaches are mutually compatible. Indeed, Gini mean difference is a measure of risk that, in conjunction with the mean, is used to establish necessary and sufficient conditions for stochastic dominance.[2] In a sense, the Gini statistic replaces the variance in an array of decisions-under-risk problems without the inconsistency that may be found in mean-variance modeling.[3] The mean-Gini model therefore serves to develop efficient input utilization rates to control externalities and to select optimal price or quantity control policies that are free of the problems posed by mean-variance analysis.

The plan of the paper is as follows: First, the mean-Gini approach is used to analyze production under uncertainty and risk aversion. Second, the model is extended to account for detrimental externalities. Next, I propose internalization solutions and discuss policy implications and recommendations.

2. The Mean-Gini Approach to Production under Uncertainty

To present the production-under-uncertainty model, consider a representative risk-averse producer whose primary goal is to maximize expected utility of profit, where profit is defined as the difference between revenue and the costs of production factors. For simplicity, I use a production function with two sets of inputs and a random variable:

$$y = f(X, N, \varepsilon), \tag{1}$$

where f is increasing and concave, X is an aggregate set of all production factors except for fertilizer that is treated as a single factor, and N is the fertilizer input. The random variable ε is bounded from below ($\varepsilon \geq a$) and has a finite expectation, $E(\varepsilon) = \mu$. In a narrow sense, variate ε can be viewed as rainfall that affects the production level. Output increases with the amount of precipitation up to a certain level (both drought and heavy rain are detrimental to agricultural production). The farmer's problem is to choose those values of X and N that maximize expected utility of profit:

$$EU(\pi) = EU(py - wX - cN), \tag{2}$$

where π is profit, p the output price, w the unit cost of factor X, and c the unit cost of fertilizer.

First-order conditions for maximization are attained when

$$w = p\left\{f_X(X, N, \mu) + \frac{cov[U', f_X(X, N, \varepsilon)]}{EU'}\right\}, \quad (3)$$

$$c = p\left\{f_N(X, N, \mu) + \frac{cov[U', f_N(X, N, \varepsilon)]}{EU'}\right\}, \quad (4)$$

where $f_X(\cdot)$ is the partial derivative of f with respect to X, given the variate ε or its expected value μ, $f_N(\cdot)$ is the partial derivative with respect to N, *cov* is the covariance function, and U' is the first derivative of U.

Sufficient conditions for maximum expected utility of profit are satisfied when the production function is concave and the producer is risk-averse (see Rothschild and Stiglitz, 1971). Risk aversion implies a negative covariance between marginal utility U' and the value of marginal product $pf_X(X, N, \varepsilon)$. The necessary conditions (3) and (4) show that, under risk aversion, efficient input levels are lower than the level of inputs under the growing conditions at expected value levels as expressed by $f(X, N, \mu)$. This result is the basic increasing risk theorem first shown by Sandmo (1971),[4] which states, where there is risk aversion, a competitive firm produces less under uncertainty than it will under certainty at the expected value level.

To solve the set of Equations (3) and (4) that determine the efficient input level, a utility function must be specified. The analyst wanting to obtain quantitative results is thus bound to transform the problem into a mean-variance model by considering quadratic utility and/or assume that the random variable ε is normally distributed.[5] This approach has been criticized by many investigators (e.g., Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970) because of the limited assumptions under which it applies. That is, the quadratic utility condition is not appropriate because it implies increasing absolute risk aversion and eventually negative marginal utility. Assuming normal distribution is deficient as well, for the random variable ε in many applications is bounded from below and cannot be limited by an assumption of normality.

An alternative way to circumvent the difficulties involved in solving Equations (3) and (4) is to assume an exponential utility function and a well-defined probability distribution thereby expressing expected utility as a moment-generating function (via the Laplace transform).[6] This approach requires one to assume a specific probability distribution of the random variable and constant relative risk aversion utility functions.[7]

The approach here is to use the mean-Gini analysis as a two-parameter model for expected utility maximization. Mean-Gini has the advantage of following second-degree stochastic dominance without limiting the instances in which expected utility maximization for risk-averse individuals is valid. Developed by Yitzhaki (1982), the method was applied to finance theory by Shalit and Yitzhaki (1984) as a useful alternative to mean-variance analysis. Yaari (1986) supported the approach in developing an axiomatic theory of individual risk-averse behavior without the implied assumption of decreasing marginal utility of income. Mean-Gini analysis also can be extended to account for differentiated risk aversion, as in Yitzhaki (1983). In agricultural economics, the model was used by Buccola and Subaei (1984) to choose among alternative cooperative pooling rules.

Gini's mean difference is a measure of dispersion defined as the expected absolute difference between two random select realizations of the variate ε:

$$\Gamma = \frac{1}{2} E \, |\varepsilon_1 - \varepsilon_2|. \qquad (5)$$

where ε_1 and ε_2 are realizations of ε. This definition can be somewhat cumbersome to use but the following formulations of the Gini are more accessible to the practitioner.

First, the formula of Equation (6) is of special interest because it expresses the Gini coefficient as a function of the mean and the cumulative probability distribution and therefore can be used to establish necessary and sufficient conditions for stochastic dominance:

$$\Gamma = \mu - a - \int_a^{\infty} [1 - H_\varepsilon(e)]^2 de, \qquad (6)$$

where a is the lower bound of the probability distribution H_ε.[8]

Second, as shown by Kendall and Stuart (1977), the Gini can be expressed as twice the covariance between the variate ε and its cumulative probability distribution H_ε, which simplifies its computation:

$$\Gamma = 2 \, cov(\varepsilon, H_\varepsilon). \qquad (7)$$

Hence, calculating the Gini coefficient of a random variable involves two basic steps that are as easy as computing the sample variance. For a sample of discrete observations, one first ranks the data set in ascending order. Then, one computes the covariance between the ranked variable and the vector of natural integers. To obtain the Gini, divide by the number of observations and multiply the result by 2.[9]

The mean-Gini approach calls for the use of the Gini coefficient instead of the standard deviation to derive the efficient set of uncertain prospects. The set is constructed so that no feasible prospect is included unless that prospect fulfills the requirements of having either a lower dispersion measure for a given mean or a higher mean for a given dispersion.

In the optimal selection process, the Gini is preferable to the variance because it encompasses the two propositions asserting necessary and sufficient conditions for *second-degree stochastic dominance* (SSD) as shown by Yitzhaki (1982). Let ε_1 and ε_2 be two random variables with means μ_1 and μ_2 and Ginis Γ_1 and Γ_2 respectively, then:

PROPOSITION 1. $\mu_1 \geq \mu_2$ *and* $\mu_1 - \Gamma_1 \geq \mu_2 - \Gamma_2$ *are necessary conditions* *to SSD for* ε_1 *to dominate* ε_2. *For cumulative distributions that intersect at most once,*[10] $\mu_1 = \mu_2$ *and* $\mu_1 - \Gamma_1 \geq \mu_2 - \Gamma_2$ *are sufficient conditions to SSD, for* ε_1 *to dominate* ε_2.[11]

The proposition was proved by Yitzhaki (1982).

The necessary conditions for SSD rules hold for *any* probability distribution. They therefore provide a reliable two-parameter method that can be used to discard the stochastically inferior possibilities from the efficient set. The sufficient conditions hold for cumulative distributions that intersect *at most once*. Families of such distributions are the normal, lognormal, uniform, exponential, beta, chi-square, and gamma distributions. The sufficient conditions are also valid for a wide range of discrete probability distributions.

Mean-Gini analysis allows determination of the efficient use of inputs in production under uncertainty. Although mean-Gini necessary conditions for stochastic dominance can be established for any kind of distribution, the only prerequisite for ensuring sufficient conditions is the assumption that cumulative distributions intersect at most once. Application calls for finding the set of efficient realizations in the mean-Gini space and then, according to Proposition 1, discarding from the efficient set the outcomes for which the Gini index is greater than the mean. This is equivalent to finding the set of outcomes that maximizes the expected profit less its Gini.

In the case of the problem of efficient allocation of fertilizer, the mean and the Gini of profit are dependent upon the specification of the random variable in the production function. Since the analysis can be performed without loss of generality with an additive variate and yield identical results, I use a multiplicative specification of the random variable to simplify the algebra.[12] With this assumption, the expression of expected profit is

$$E\pi = pf(X, N) \cdot \mu - wX - cN \tag{8}$$

and the Gini of profit is[13]

$$\Gamma_\pi = pf(X, N) \cdot \Gamma_\varepsilon. \tag{9}$$

Solving the following first-order conditions for maximizing $E\pi - \Gamma_\pi$ will yield a solution that also maximizes expected utility of profit:

$$w = pf_X(X, N) \cdot (\mu - \Gamma_\varepsilon) \tag{10.a}$$

$$c = pf_N(X, N) \cdot (\mu - \Gamma_\varepsilon) \tag{10.b}$$

Conditions (10a and b) are similar to conditions (3) and (4), yet free of the concomitant problems related to utility measurement as the mean-Gini approach provides necessary and sufficient conditions for second-degree stochastic dominance. Here, the two measures needed to describe the random variable are sufficient to portray the demand level for the farmer's production factors. To determine the level of input use, the value of marginal product is set equal to the factor cost divided by the difference between the variate's mean and its Gini. Decreasing marginal productivity of X and N implies that the use of inputs decreases by virtue of an increase in the level of uncertainty as represented by the Gini.

Under production uncertainty and risk aversion, efficient fertilizer application rates can be designed using production parameters, input and output prices, and weather statistics. If actual fertilizer recommendations disregard the uncertainty involved in the production process, whether intentionally or not, the application rate will exceed that suggested by the conditions (10). The consequence of overapplication is presumably trivial when excess fertilizer *does not* harm the environment. Allocation of production resources due to uncertainty factors will nevertheless be non-Pareto efficient, although its costs will be incurred only by the agricultural sector. In the specific case of water pollution however, the external effects can be substantial and harmful to the economy. Hence, the search for social optimality must also include the parties affected by the externalities.

3. A Basic Model of Stochastic Externalities

Let us consider externalities generated by a less-than-efficient use of fertilizer in a stochastic environment. Under certainty, proper fertilization techniques would prevent fertilizer waste, and excess fertilizer would not leach into groundwater. Under adverse and uncertain growing conditions, however, some fertilizer remains unused by the plant and thus leads to pollution. At times of severe drought, fertilizer applied at the beginning of the growing season contaminates the post-season water supply because the soil lacks sufficient moisture to allow a timed dissolution of the fertilizer, making it impossible for the plant to extract the necessary nutrients. Unused excess fertilizer in this case will attain the watershed with the new season. Times of very heavy rainfall are similarly difficult; then fertilizer may be washed away from the crop field and reach the aquifer more rapidly. Hence, fertilizer applications that in normal rain seasons are efficient from the grower's point of view can, in extreme weather conditions, generate potentially substantial environmental damages.

The damage function (11) also assumes a multiplicative random variable:

$$Z = g(N)\cdot\eta, \tag{11}$$

where Z is a water quality index showing the level of fertilizer contamination (e.g., the concentration of nitrogen-nitrate in water supplies). I assume that $g(N)$ is increasing and convex because as greater quantities of fertilizer are applied, the marginal damage to water quality increases as a result of the fertilizer's inefficient use. The random variable η in (11) is related to rain, with high values in periods of drought and flood and very small values in periods of normal rainfall, and expresses the damage to water quality when fertilizer is not taken up by the crop because of adverse weather conditions. At the variate's mean θ the damage results only from the deterministic inefficiency of fertilizer application.

Knowing the damage function allows for derivation of the socially optimal solution to fertilizer allocation for individual farming units. To provide solutions that internalize the external diseconomies created by individual fertilizer applications, I set the problem up in terms of a central agency that maximizes a well-defined social welfare function of the utilities of producers and consumers subject to the damage function (11). The solution provides social optimal policy guidelines. For example, if the damage level Z is measured in terms of currency, the central agency might impose Pigovian taxes to correct the fertilizer externalities.

Baumol and Oates (1988) show that corrective taxes would be difficult to implement and suggest the standards and charges approach as an alternative. Their idea is establishment of a given environmental standard for the maximum acceptable damage level and imposition of corrective charges to achieve this standard. This is the approach taken here because, as with other potential health hazards, there are national health standards for nitrate concentration in drinking water.

For the sake of simplicity, assume identical farmers with similar risk preferences. The central agency that regulates the aquifer wants to establish fertilization techniques that will control water quality standards. To internalize the damages caused by fertilization, the agency uses the representative farmer's utility function as a vehicle to establish and value the specific policy. Let $Z°$ be the acceptable standard for water contamination as a result of agricultural practices. In a sense, society's welfare is already accounted for because the standard $Z°$ results from a social welfare maximization process. The environmental standard is basically exogenous in the model at hand. The agency's problem is to maximize the representative farmer's expected utility subject to the water quality standards:

$$\max EU(py - wX - cN) \tag{12}$$

$$\text{subject to } Z° \geq g(N)\cdot\eta, \quad \text{where } y = f(X, N)\cdot\varepsilon. \tag{13}$$

As the random variable η is not bounded from above, satisfying constraint (13) for all possible realizations gives the result that no fertilizer would be used, which is a meaningless solution. Another approach is to look at the expected contamination level and express the constraint as:

$$Z° \geq g(N)\cdot\theta, \quad \text{where } \theta = E(\eta). \tag{14}$$

This assumption recognizes that, while assuming risk aversion in profit, production models ignore the risk preference with respect to pollution, resulting in a biased fertilizer policy. Alternatively, as suggested by Lichtenberg and Zilberman (1988), one can set a probabilistic safety margin on the pollution level such that

$$Pr[g(N)\cdot\eta \leq Z^\circ] = 1 - P \tag{15}$$

where P is the probability that the contamination level will exceed the imposed standard. The decision problem then can be solved by assuming a specific probability distribution.

To solve the problem I choose to internalize, through the utility function, the standard constraint by including it in the conditional profit, thus attributing to the pollution standard the same type of risk preferences. Let λ be the Lagrangian multiplier of the minimum standard constraint and define conditional profit subject to this constraint as:

$$pf(X, N)\cdot\varepsilon - wX - cN + \lambda[Z^\circ - g(N)\cdot\eta].$$

The central agency decision problem is now to maximize the expected utility of the conditional profit. The first-order conditions for a social optimum are:

$$w = pf_X\cdot\left[\mu + \frac{cov(U', \varepsilon)}{EU'}\right], \tag{16}$$

$$c = pf_N\cdot\left[\mu + \frac{cov(U', \varepsilon)}{EU'}\right] - \lambda g_N\cdot\left[\theta + \frac{cov(U', \eta)}{EU'}\right]. \tag{17}$$

Condition (16) is essentially identical to condition (3), but for a different input level X because the level of fertilizer application is reduced according to condition (17). Indeed, this condition shows that, *ceteris paribus*, for positive λ and g_N, the average amount of fertilizer applied declines because of internalizing the external diseconomies. This outcome holds regardless of the type of uncertainty involved in the production and pollution processes. Because cov (U', ε) is negative for risk-averse individuals, and cov (U', η) is positive,[14] internalization of externalities under uncertainty reduces the overall use of fertilizer, as one would expect.

Conditions (16) and (17) require a utility function specification to formulate policy recommendations, which can be a complex process whenever producers have different utilities. The mean-Gini approach to risk analysis, however, overcomes this difficulty, allowing the choice of stochastic dominance policies that will be utility-free and based on the random variables statistics.

As mean-Gini necessary conditions for stochastic dominance are possible for any type of distribution, one needs to assume that the parametric cumulative distributions of variates ε and η also satisfy the requirements for the sufficient conditions as stated in Proposition 1. The stochastic dominant solution, that which is preferred by all risk-averse producers subject to the pollution standard Z°, is the solution * that satisfies the conditions:

$$E\pi^* = E\pi_i \quad \text{for all feasible solutions } i,$$

$$E\pi^* - \Gamma^* \geq E\pi_i - \Gamma_i \quad \text{for all feasible solutions } i,$$

where the expected conditional profit is:

$$E\pi^* = pf(N, X)\cdot\mu - wX - cN + \lambda[Z^\circ - g(N)\cdot\theta],$$

the Gini of the conditional profit is

$$\Gamma^* = pf(N, X)\cdot\Gamma_\varepsilon - \lambda g(N)\cdot\Gamma_\eta,$$ [15]

and i is the index of alternative decision-making policies.

This is essentially equivalent to choosing values of X and N that maximize $\pi^* - \Gamma^*$ leading to the first-order conditions:

$$w = pf_X\cdot(\mu - \Gamma_\varepsilon) \tag{18}$$

$$c = pf_N\cdot(\mu - \Gamma_\varepsilon) - \lambda g_N\cdot(\theta - \Gamma_\eta) \tag{19}$$

and $\quad Z^\circ = g(N)\cdot\theta.$

These conditions are analogous to conditions (16) and (17). Still, they are free of the utility measurement problems discussed above. By comparing those conditions with equations (10a and b), where externalities are not internalized, one obtains the first result that, unless $\lambda = 0$ or $g_N = 0$, the relative use of fertilizer will decrease. Indeed from Equations (10a and b), the input use ratio is

$$\frac{c}{w} = \frac{f_N}{f_X}, \tag{20}$$

while from conditions (18) and (19) it is

$$\frac{c}{w} = \frac{f_N}{f_X} - \frac{\lambda g_N\cdot(\theta - \Gamma_\eta)}{pf_X\cdot(\mu - \Gamma_\varepsilon)}. \tag{21}$$

For constant input prices, equation (20) is identical to (21) only if $\lambda = 0$ or $g_N = 0$. For $\lambda > 0$ or $g_N > 0$, the marginal product of fertilizer in (21) must be greater than f_N in (20), implying that less fertilizer is used when the externality is internalized, or, alternatively, more inputs X are used to compensate for the condition in (21). This ratio condition also results in accounting for uncertainty once λ is computed: Indeed, λ is the charge imposed on the user of fertilizer for polluting the aquifer at the rate g_N. This value can be obtained only by solving the set of first-order conditions (18), (19) and (11) for a given Z°, which solution requires the knowledge of production parameters, prices, and uncertainty statistics.

4. Generalization and Alternative Solutions to Stochastic Externalities

The model I have developed deals mainly with the pollution problem of a representative farmer. Imposing charges according to a particular damage assessment is, in theory, easily accomplished. In this instance, as Griffin and

Bromley (1982) show in the specific case of a single profit-maximizing farmer, there is no advantage to the type of policy (price incentives or direct controls) being chosen. Shortle and Dunn (1986), however, allowing for a differential information structure about farming operations between the producer and the central planner, show that a tax-cum-subsidy policy on fertilizer is superior to all alternative strategies.

Inclusion of several risk-averse producers makes the problem more pragmatic but at the same time more complex to solve. First, groundwater contamination from nitrogen fertilization is a non-point source externality that cannot be directly monitored. Second, because of the number of producers, emissions monitoring is quite difficult, implying that regulations cannot be enforced and charges cannot be assessed. A solution would be indirect controls on pollution levels in the form of regulation of fertilizer use, either by input taxes/subsidies or by quotas. When they relax the assumption of a single expected profit-maximizer, Shortle and Dunn find that no policy would achieve the first best optimum. They prove that, for a single farmer maximizing a mean-variance utility of profit, an incentive policy on fertilizer would outperform the quantity policy.

To address the question of non-point pollution by multiple expected utility maximizers, consider a set of incentive rules for non-fertilizer use as Griffin and Bromley propose and let σ be the per unit incentive for the non-use of fertilizer up to the predetermined limit N_j°. If there are J potential polluters (i.e., the number of producers), the regulating agency determines the maximal amount of fertilizer N_j° for each farming unit j such that

$$Z^\circ \geq \sum_{j=1}^{J} g(N_j^\circ)\cdot\eta \tag{22}$$

Equation (22) does not address the equity problem, although it is an important factor in the distribution of allowances N_j°, which are assumed to be determined on a per area basis. The farmer's problem is reformulated as maximizing expected utility of the conditional profit function that includes the incentive for non-fertilizer use:

$$pf(N_j, X_j)\cdot\varepsilon - wX_j - cN_j + \sigma(N_j^\circ - N_j). \tag{23}$$

Instead of expected utility maximization, mean-Gini analysis is used to obtain the first-order conditions for mean-Gini profit maximization:

$$w = pf_{X_j}\cdot(\mu - \Gamma_\varepsilon) \tag{24}$$

$$c + \sigma = pf_{N_j}\cdot(\mu - \Gamma_\varepsilon). \tag{25}$$

These conditions are identical to Equations (10a) and (10b) except for the price of fertilizer as shown in Equation (25). To be optimal as in conditions (18) and (19), it is necessary for the central agency to establish the correct incentive price for non-use of fertilizer as follows:

$$\sigma = \lambda g_{N_j} \cdot (\theta - \Gamma_\varepsilon). \tag{26}$$

This incentive will not be identical for all producers depending as it does upon their individual marginal propensity to pollute. Still, it must take into account the statistics of the random variable η. The same type of result is obtained whenever a tax is added to the price of fertilizer instead of using an incentive for non-application.

The quota system is an alternative solution to the problem of reducing groundwater pollution. Quotas would be assigned to the producers depending on size and area. Quotas would not be permitted to be traded unless the damage function $g(N_j)$ is not convex across all producers.[16] Quotas are determined by the regulating agency using condition (22). Maximization of the expected profit utility of the production unit will yield the first-order conditions:

$$w = pf_{X_j} \cdot (\mu - \Gamma_\varepsilon) \tag{24a}$$

$$c + \lambda_j = pf_{N_j} \cdot (\mu - \Gamma_\varepsilon), \tag{27}$$

where λ_j is the Lagrangian associated with the constant $N_j^\circ \geq N_j$. Again fertilizer use will be optimal from the central agency's point of view only if

$$\lambda_j = \lambda g_{N_j} \cdot (\theta - \Gamma_\eta) \quad \text{for all } j = i, \ldots, J. \tag{28}$$

The tax and quota policies use essentially the same information on the marginal effect of fertilizer, the marginal damage to groundwater, and the uncertainty statistics. Underlying the model is the assumption that the central agency and the producers share the same information. Furthermore, the optimal tax/incentive rates and the optimal quotas depend not only upon the mean of the variate but also upon the dispersion statistic expressed by the Gini. Hence, the two policies yield the same result when internalizing the production externalities under uncertainty.

When the perception of uncertainty differs between the producers and the central agency, this result eventually diverges for two reasons. The first has to do with information and the evaluation of uncertainty; producers can compute different dispersion statistics because they use a variety of information sources. The second is related to the perception of uncertainty that varies because of differentiated risk aversion among producers and planners. Indeed, producers exhibit different preferences toward the distribution of rainfall and its dispersion. In this paper, I express these divergences in uncertainty only by different Gini statistics.

Let us first address the problem of information and the evaluation of uncertainty and assume that the producer's Gini is smaller than the Gini computed by the central agency. Indeed, each production unit computes the Gini statistic according to its own observations sample, which is necessarily smaller than the central agency's sample, leading to this result. Hence the farmer views ε as less "random". The task here is to establish which type of regulation (price or quota) is more efficient in controlling externalities and reducing their damages. Let Γ_{ε_j} be the Gini statistic computed by the producers, and assume $\Gamma_{\varepsilon_j} < \Gamma_\varepsilon$. From equations (25), (26), and (19), one gets

$$c + \lambda g_{N_j} \cdot (\theta - \Gamma_\eta) = pf_{N_j} \cdot (\mu - \Gamma_{\varepsilon_j}) > pf_{N_j} \cdot (\mu - \Gamma_\varepsilon). \tag{29}$$

This implies that the N_j the farmer uses is larger than the amount of fertilizer that should have been used to internalize externalities. Under the quota regulation, N_j° is set to the maximal amount a producer can legally use as input. This main result is stated as follows:

RESULT 1. *When producers underestimate production uncertainty, they use more inputs. Hence, a quota policy on resource use will be more efficient than a tax/incentive policy to internalize the level of externality and reduce the level of inputs. This is true whenever the marginal pollution created by the use of input is increasing.*

Now consider the converse, where producers overestimate weather uncertainty, i.e., $\Gamma_{\varepsilon_j} < \Gamma_\varepsilon$. Following the tax/incentive regulation,

$$c + \lambda g_{N_j} \cdot (\theta - \Gamma_\eta) = pf_{N_j} \cdot (\mu - \Gamma_{\varepsilon_j}) < pf_{N_j} \cdot (\mu - \Gamma_\varepsilon), \tag{30}$$

implying that the producer uses less fertilizer than the quantity recommended by the central agency and thus will add less to the pollution level. The same is true for quota regulation because the production unit cannot be made to use all its input quota. This result is obtained:

RESULT 2. *When producers overestimate uncertainty, total use of inputs declines in response to regulation, regardless of quota or tax policy. The final allocation is not optimal, but the level of pollution caused by the externality is reduced.*

Risk aversion is another vantage point from which to look at the differentiated perception of uncertainty. Individuals who more averse to risk than others lower the level of economic activity to avoid unwarranted losses. As shown by Sandmo (1971) and Booth (1990), higher risk aversion, expressed by the degree of concavity of the utility function, further reduces the use of inputs. With respect to the Gini, a new measure of aversion toward risk is necessary for use in this context. This is the extended Gini statistic, which was developed by Yitzhaki (1983) and applied to finance theory and portfolio selection by Shalit and Yitzhaki (1984, 1989). To apply this measure to the stochastic externalities problem, the extended Gini coefficient is defined similarly as in Equation (6):

$$\Gamma_\varepsilon(v) = \mu - a - \int_a^\infty [1 - H_\varepsilon(e)]^v de, \tag{31}$$

where v is the power parameter ($1 \leq v \leq \infty$). Using a notation similar to Equation (7), the extended Gini parameter can be expressed as:

$$\Gamma_\varepsilon(v) = -v\, cov[\varepsilon, (1 - H_\varepsilon)^{v-1}]. \tag{32}$$

The link between the extended Gini power parameter and risk aversion is explained as follows. For any value of v, Equation (32) displays a "weighted covariance" between the random variable and its cumulative distribution. As v rises, the weights assigned to the lower portions of the distribution increase. In the extreme case, where v approaches infinity, the extended Gini reflects the attitude toward risk of an individual who cares only about the lowest realization possible. This individual who characterizes the distribution solely by its lowest outcome is the *maximin individual*. If, on the other hand, v approaches 1, all weights are equal. This portrays a risk-neutral individual. Hence, if individual i is more risk-aversion than individual j, one can state that $v_i \geq v_j$. Therefore, because $H_\varepsilon \leq 1$, using definition (31) leads to the conclusion: $\Gamma(v_i) \geq \Gamma(v_j)$ if $v_i \geq v_j$.

From a policy perspective, knowing the intensity of risk aversion is essential, as higher risk aversion reduces inputs use and lessens pollution externalities, whereas risk neutrality acts the other way. Hence, if the risk aversion of the central agency differs from that of the producers, distinct policies will be socially preferable. Two cases are to be considered: The first, which is rarer, is when producers are less risk-averse than the central agency and the second, more common, is when they are more risk-averse. To quantify the differentiation of risk aversion, extended Ginis can now be used in conjunction with Equations (29) and (30) to yield the following results:

RESULT 3. *When producers are less risk-averse than the central agency, they use more inputs than optimally recommended. Hence, a quota policy is more efficient than a tax policy in reducing the level of damage caused by the externality.*

The converse is when producers are more risk-averse than the central agency. Under those conditions, the planner is indifferent between the two regulations as the level of detrimental externality is reduced because of higher risk-aversion. It is stated:

RESULT 4. *When producers are more risk-averse than the central agency, they use less fertilizer than the recommended quantity, and the level of the pollution externality declines, regardless of the quota or tax/subsidy regulation.*

5. Conclusion

The simple model of production under uncertainty has permitted the isolation of pertinent issues regarding policy recommendations that control externalities. When farmers tend to underestimate uncertainty, a regulation limiting quantity of fertilizer is superior to a price policy. Taylor (1975) obtained a different result when he proposed a market for rights to use fertilizer instead of a quantity restriction. The market-for-rights policy was almost identical to that of a per unit excise tax on fertilizer. This conclusion is based upon two considerations that must operate to ensure proper policy recommendations. The first is consistent estimation of the damage function in order to determine not only its convexity but also the degree of contamination. The second has to do with the assessment of uncertainty by individuals *vs.* the evaluation of risk by policy makers.

Uncertainty assessment is not addressed here, nor do we resolve the problem of whether fertilizer applications are reduced with increasing risk and risk aversion. It is possible to formulate a methodology whereby reduced fertilizer use decreases the incidence of risk. In that case, increasing fertilizer applications would augment the level of externalities with increasing uncertainty.

Some studies have suggested that the health effects of nitrate contamination of groundwater are not of great significance. The purpose of this paper is not to debate such an assessment but to assure that potential health risks are sufficiently significant to involve the policy maker. Intensified agricultural production and irrigation practices present the potential for increased nitrate contamination, and there is no indication that the demand for drinking water will drop. Political pressure may also cause strengthening of health standards, compelling the agricultural policy maker to learn the appropriate production parameters that would enable promulgation of the most efficient policy.

Notes

[1] Work on this paper was carried out when visiting the University of Maryland. Financial aid for the work was provided by the USDA ERS-NRED under a cooperative agreement between the Department of Agricultural and Resource Economics, University of Maryland, and the USDA – Economic Research Service – Natural Resource Economics Division, I am grateful to John Miranowski and Darrell Hueth for that support. I am indebted as well to Lana Shalit, who helped me revise the paper.

[2] Using only the probability distributions, stochastic dominance allows for an ordering of alternatives that holds for *all* individuals regardless of their preferences, subject only to nondecreasing utility and risk-aversion. In this paper, I use second-degree stochastic dominance (SSD) that expresses the probabilistic conditions under which all risk-averse individuals prefer one risky asset to another. The conditions were developed by Hanoch and Levy (1969) and Rothschild and Stiglitz (1970). For an excellent recent survey on stochastic dominance, see Levy (1992).

[3] For example, take two risky investments with bounded and nonoverlapping distributions

with the mean *and* the variance of the first alternative smaller than the mean *and* the variance of the second alternative. Clearly, all maximizing expected utility individuals prefer the second alternative over the first one by stochastic dominance rules. However, according to the mean-variance model, the two investments are in the efficient set and cannot be differentiated. In general, mean-variance fails to pass the consistent test whenever prospects are not normally distributed. By using the mean-Gini criterion, we can discard the first alternative and obtain a consistent ranking of risky alternatives. See Hanoch and Levy (1969), Rothschild and Stiglitz (1970), Shalit and Yitzhaki (1984), and Levy (1992) for additional arguments and examples.

[4] This result was recently extended by Booth (1990).

[5] Chamberlain (1983) characterized the class of distributions for which mean-variance analysis is appropriate, a class which is much broader than the normal distribution.

[6] See Hammond (1974) and Keeny and Raiffa (1976) for the theoretical background, and Yassour, Zilberman, and Rausser (1981) for some applications.

[7] Collender and Chalfant (1986) using exponential utility maximization and empirical moment-generating functions have extended the approach to include nonparametric distributions.

[8] See Dorfman (1979) for the derivation of this equation and the relation between the Gini and the Lorenz curve of income distribution.

[9] The fastest way to compute the Gini with econometric and statistical packages is to regress the sorted variable on its rank. For normal probability distributions, the Gini is equal to the standard deviation divided by $\sqrt{\pi}$.

[10] Cumulative distributions are said to intersect if their graphs intersect; that is, if for every $\varepsilon_0 < \varepsilon_1$ such that $H_1(\varepsilon_1) > H_2(\varepsilon_1)$ and $H_1(\varepsilon_0) < H_2(\varepsilon_0)$, then $H_1(\varepsilon) < H_2(\varepsilon)$ for all $\varepsilon < \varepsilon_0$ and $H_1(\varepsilon) > H_2(\varepsilon)$ for all $\varepsilon > \varepsilon_1$.

[11] Sufficient conditions using extended Gini can also be established for distributions with non-equal means (Shalit and Yitzhaki (1984)).

[12] Readers of Just and Pope (1979) might object to use of this functional form, claiming that it fails to capture the variance-reducing nature of some inputs. In fact, the confounding issue in that paper is not with the production function specification but with specification of risk. Just and Pope, primarily interested in the econometric specification of production functions under risk, narrow the notion of risk to the variance of the random variable, a specification that is valid only under (i) normality of probability distributions or (ii) quadraticity of utilities. Although the first restriction is appropriate in econometric testing procedures, its validity is questionable when specifying random disturbances such as rainfall. The second restriction, moreover leads to increasing relative risk aversion and inconsistencies with respect to marginal utility.

[13] This result holds because $\Gamma(a\varepsilon) = |a|\Gamma$, for a scalar a.

[14] As η increases the level of pollution, utility is reduced and marginal utility increases.

[15] The Gini of a sum of random variables $k = l + m$ is equal to $\Gamma_k = 2cov[l + m, F(k)] = 2cov[l, F(k)] + 2cov[m, F(k)]$ where F is the ranking function. If the ranking function for k is identical to the ranking function for l and m, $\Gamma_k = \Gamma_l + \Gamma_m$. If the ranking functions are not identical the sum of the Ginis exceeds the Gini of a sum.

[16] This restriction is necessary because of the fact that quota transfers will increase the total pollution level without allowing for any kind of control.

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