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The Estimation of Systematic Risk under Differentiated Risk Aversion: A Mean-Extended Gini Approach

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Abstract. This paper examines a mean-Gini model of systematic risk estimation that resolves some econometric problems with mean-variance beta estimation and allows for heterogeneous risk aversion across investors. Using the mean-extended Gini (*MEG*) model, we estimate systematic risks for different degrees of risk aversion. *MEG* betas are shown to be instrumental variable estimators that provide econometric solutions to biases generated by the estimation of mean-variance (*MV*) betas. When security returns are not normally distributed, *MEG* betas are proved to differ from *MV* betas. We design an econometric test that assesses whether these differences are significant. As an application using daily returns, we estimate *MEG* and *MV* betas for U.S. securities.

Key words: Beta, mean-Gini, normality test, instrumental variable estimation

1. Introduction

As a measure of systematic risk, beta has dominated the world of finance since its inception in the sixties. Typically, under mean-variance (*MV*), beta is estimated using ordinary least-squares (*OLS*). Notwithstanding its widespread application, there are numerous problems related to the use of beta in the estimation of systematic risk (its nonstationarity over time (Kim, 1993), to name only one).

In this paper, we address and quantify two specific problems associated with mean-variance betas. The first deals with econometric biases that may arise in estimating betas; the second with the implications inherent in assuming normally distributed returns. Both problems are critical in the estimation of betas because they can bias or invalidate the evaluation of systematic risk, thereby rendering investor portfolios inoptimal.

To solve these problems, we propose the mean-extended Gini (*MEG*) model as an alternative to the *MV* beta. We demonstrate the econometric advantages of using *MEG* in beta estimation. When the market model used to estimate systematic risk is misspecified, *MV* betas may be biased. *MEG* betas, because they are instrumental variables estimators, provide an econometric solution to the specification bias.

The mean-Gini (*MG*) approach in finance has been proven to be a practical alternative to *MV* modeling. By using necessary and sufficient conditions for stochastic dominance, Yitzhaki (1982) shows *MG* to be compatible with expected utility maximization. Hence, *MG* analysis provides a consistent alternative to *MV* modeling whenever investments fail to be normally distributed or when investor utility is not quadratic. The *MG* approach to finance is used by Bey and Howe (1984) in portfolio analysis, by Okunev (1988) and Pink (1988) to rate mutual fund performance; by Shalit and Yitzhaki (1989) to derive optimum portfolio selection; by Cheung *et al.* (1990) to examine the hedging effectiveness of options and futures; and by Carroll, Thistle, and Wei (1992) to test whether *MV CAPM* is robust with respect to nonnormality.

Mean-extended Gini (*MEG*) modeling is an extension made by Yitzhaki (1983) to parametrize risk. The Gini coefficient is extended into a family of dispersion measures that differ from each other by the degree of risk aversion. Shalit and Yitzhaki (1984) show that different *CAPMs* can be developed to account for risk aversion where systematic risk varies according to investor risk preferences. This key feature is not possible in *MV* modeling of financial markets.

We also present the theoretical justification for using the *MEG* model in systematic risk analysis. Unless probability distributions of security returns are normal, betas obtained under the *MEG* model will be different from *MV* betas. When returns are not normally distributed, mean and variance are not sufficient parameters to describe the utility function, and *MV* betas will not be valid unless investors have quadratic utility. When investors and mutual fund managers rank securities according to estimated systematic risk using the available *MV* betas, and then follow such a classification to construct portfolios, risk aversion is ignored, and so the choices may be biased.

Our objective is to use the *MEG* model to estimate systematic risks for a sample of U.S. securities and test whether *MEG* betas are statistically different from *MV* betas for various degrees of risk aversion. Application of a model that considers different attitudes toward risk can produce different rankings for the same securities. For example, securities "*aggressive*" with respect to the market portfolio according to one risk aversion specification could be "*defensive*" under another. As we have no reason to think that risk aversion characteristics are identical among all individuals, we are compelled to believe that investors using *MV* betas might make sub-optimal decisions as betas estimated with different risk aversion coefficients yield different portfolio rankings. Such a problem would be eliminated by using *MEG* betas.

We address the problem analytically, statistically, and empirically and show the relevance of the *MEG* approach. Our estimation is that between 1985 and 1993, investors using *MV* betas misevaluated systematic risk for 20% of U.S. traded securities, incurring substantial costs by holding sub-efficient portfolios.

2. Estimating systematic risk

The basic market model of finance is often expressed as a linear relationship between security returns and the market portfolio. For security i, this requirement implies the existence of a *true* β that links security returns X_i with market returns M as:

$$X_i = \alpha_i + \beta_i M + \epsilon_i, \tag{1}$$

where ϵ_i is a disturbance variable. The market model has been tested extensively since its origin in the mid-sixties. Among the most notable specification tests are those of Fama *et al.* (1969) and Black *et al.* (1972), who then use the estimated betas to analyze, across firms, the linear relation between average return and systematic risk.

Estimation of $\beta_i s$ assumes that M and ϵ_i are normally distributed and statistically independent from each other. Under these conditions, the ordinary least-squares (*OLS*) estimator of β results in the mean-variance beta, also known as the *MV* systematic risk. Statistically speaking, the *OLS* estimator is a consistent estimator of β if the conditional expectation of ϵ_i given M is zero, and it is efficient if the disturbance variable is homoscedastic, i.e., $Var(\epsilon_i/M) = \sigma_i^2 I$. Under these conditions, *OLS* provides the best unbiased estimator and the most powerful tests.

Financial data, however, present major econometric problems that might violate these conditions as disturbances may be correlated with the market portfolio leading to biased estimators. If disturbances are heteroscedastic, generalized least-squares (*GLS*) should be used; otherwise tests based on *OLS* estimators may be less powerful because variance estimates of *OLS* estimators are biased. Hence the second-pass regression testing the *CAPM* will exhibit greater inconsistencies because of errors-in-variables problems (Litzenberger and Ramaswamy (1979)). These concerns about bias in systematic risk are also expressed by Brenner (1977) in an efficient market hypothesis test and Dimson (1979) in the case of infrequent trading.

Biased *OLS* estimators have compelled analysts to choose other estimation methods from a variety of econometric solutions. One preferred approach (see Rosenberg and Marathe (1979) for an example) is the *instrumental variables method*, which yields consistent estimators for β_i.

The ideal instrumental variable (*IV*) is chosen to be highly correlated with M but not with ϵ_i. A variable that would satisfy these criteria is the cumulative probability distribution for M, for by its very nature it is correlated with the variate and less dependent on the error term. This approach was first suggested by Durbin (1954) to solve the errors-in-variables problem. Here, as an instrumental variable, we use the computed cumulative probability distribution for M, $F_M(M)$, defined as the rank of market returns divided by the number of observations. The rank of market returns is a vector of integers obtained by sorting the sample in ascending order and using the ordinal position as the rank for each observation of M.

For a sample of n observations, the *IV* estimator for β_i, becomes:

$$\hat{\beta}_i^{IV} = \frac{\sum_{t=1}^{n} [F_M(m_t) - \frac{1}{2}]\,[x_{i,t} - \bar{x}_i]}{\sum_{t=1}^{n} [F_M(m_t) - \frac{1}{2}]\,[m_t - \bar{m}]} \tag{2}$$

where $F_M(m_t)$ is the rank of the market return given the observation m_t, $\bar{x}_i$ is the average return on security i, and $\bar{m}$ is the average market return. For the entire population of the returns, this estimator becomes:

$$\hat{\beta}_i^{IV} = \frac{cov\,[F_M(M), X_i\,]}{cov\,[F_M(M), M\,]} \tag{3}$$

where *cov* is the covariance function. Not by coincidence, this outcome for β is exactly the mean-Gini (*MG*) beta derived by Shalit and Yitzhaki (1984), where the Gini is a measure of dispersion and risk similar to the standard deviation.

In summary, when *M* and ϵ_i are not independent, biased *MV* betas will be obtained under *OLS* estimation, but *MG* betas will be consistent estimators for β. This result is also found by Carroll *et al.* (1992). Furthermore, we can extend the econometric procedure by using as *IV* other increasing monotonic transformations of $F_M(M)$, obtaining alternative consistent estimators for β that are sensitive to the choice of monotonic transformation. For example, if we use as an *IV* the ranking function $-\,[1 - F_M(M)\,]^{1-\nu}$, where $\nu > 1$, the procedure yields mean-extended Gini (*MEG*) betas are consistent estimators for β. These betas are dependent upon the power parameter ν, which is considered a coefficient for risk aversion.

Our result is best expressed as a question: To what extent is the *MV* systematic risk, obtained through *OLS*, analytically or statistically different from the various instrumental variables βs obtained via the *MG* and the *MEG* models? First we address the analytical issue, and then proceed to the question of econometric testing.

3. The mean-extended Gini CAPM

Here we establish the conditions under which *MV* systematic risk differs from *MEG* betas and develops the appropriate *CAPMs*. The extended Gini coefficient (Yitzhaki, 1983) is a measure of dispersion that weighs the investor's relative preference to various ranges of the probability distribution of returns, thus serving as a measure of risk aversion. Its use in financial theory and portfolio analysis is proposed by Shalit and Yitzhaki (1984), who summarize the basic properties of the extended Gini coefficient and its relation to stochastic dominance and systematic risk.

The simple Gini coefficient is defined as the expected absolute difference between all possible realization pairs of a random variable.[1] In finance, it is more convenient to use the formula that expresses the Gini as twice the covariance between the returns *X* and their cumulative probability distribution *F(X)*:

$$\Gamma_\chi = 2\,cov\,[\,X, F(X)\,]. \tag{4}$$

Equation (4) is easy to evaluate when the rank of the random variable is used as the cumulative distribution estimate. After the observations are sorted in ascending order, the covariance between the random variable and its rank is computed.

Use of the Gini has several advantages over the variance as a measure of dispersion for risk and portfolio analysis. The first advantage is rooted in the existence of mean-Gini *necessary* conditions to stochastic dominance. Second, *MG sufficient* conditions also exist for all cumulative probability distributions that intersect at most once. Therefore, *MG* analysis is consistent with expected utility maximization in cases where *MV* fails.

Third, *MG* analysis can be extended by expressing the Gini as a measure of dispersion that takes into account the investor's preference toward risk. Depending on their risk preferences, different individuals will attach different weights to various portions of the return probability distribution. Highly risk-averse investors will be more concerned about lower payoff realizations than risk-neutral investors.

To characterize that aversion, the method imputes more weight to the worst outcomes of the returns distribution. The extended Gini coefficient is defined by:

$$\Gamma(\nu) = \mu_x - a - \int_a^b [1 - F(x)]^\nu dx \quad \textit{for finite } a, \tag{5}$$

where ν is the power coefficient expressing the relative weight given to various segments of the probability distribution. More conveniently, the covariance formulation of the extended Gini coefficient is used:

$$\Gamma(\nu) = -\nu\, cov\, \{X, [1 - F(X)]^{\nu-1}\}. \tag{6}$$

As an investor applies a larger ν (i.e., has greater risk aversion), the lower portions of the distribution become relatively more important. The parameter ν ranges from 1 to infinity. A ν of 1 represents the coefficient for a risk-neutral investor, in which case Equation (6) equals zero, implying that the investor is interested only in the expected value of *X*. A $\nu = \infty$ represents the weight for a max-min investor who wants to avoid the worst possible outcome.

The Gini and the extended Gini coefficients can be used in portfolio analysis to select optimal portfolios and derive Capital Asset Pricing Models for various νs. The investors' problem, given the risk coefficient ν, is to minimize the extended Gini of portfolios subject to the budget constraint and a required expected return.

In a market with homogeneous risk-averse investors with identical ν and identical investment opportunities, a pricing equilibrium for each security is established as:

$$\mu_i = r_f + (\mu_M - r_f)\beta_i(\nu)\,, \tag{7}$$

where μ_i is the expected return on the security *i*, r_f is the risk-free rate, and μ_M is the expected return on the market portfolio. $\beta_i(\mu)$ is the extended Gini beta defined as:

$$\beta_i(\nu) = \frac{-\nu \, cov\,\{X,[1 - F_M(M)]^{\nu-1}\}}{-\nu \, cov\,\{M,[1 - F_M(M)]^{\nu-1}\}}. \tag{8}$$

Equation (7) is the well-known *CAPM* formula, adjusted for various $\beta_i(\nu)$ obtained for a specific ν. Given a value of ν, the equilibrium relationship between expected returns and systematic risk holds for the group of risk-averse investors who have that specific value of ν. The *MEG* model assumes that all investors have the same risk aversion expressed by ν. In principle, however, one is bound to obtain different betas for different νs. The question is how substantially these betas differ from each other, and how these differences account for biases that investors are prey to in computing the systematic risk of various securities.

We address the first part of the question by exploring the differences in β according to the probability distribution and the ν coefficient. As Nair (1936) shows, the simple Gini coefficient $\Gamma(2)$ becomes $\sigma/\sqrt{\pi}$ when the random variable is normally distributed with mean μ and standard deviation σ. Furthermore, as Schechtman and Yitzhaki (1987) have shown, when X and M have a bivariate normal distribution, the Gini correlation coefficient shown below becomes the standard (Pearson) coefficient of correlation between X and M:

$$\omega_{X,M} \equiv \frac{cov\,[M,F_X(X)]}{cov\,[M,F_M(M)]} = \rho_{X,M}, \tag{9}$$

where $\omega_{X,M}$ is the Gini correlation and $\rho_{X,M}$ is the standard coefficient of correlation. Therefore by Nair and Equation (9), when security returns are normally distributed, betas obtained for $\nu = 2$ are equivalent to mean-variance betas.

For *extended* Gini coefficients (i.e., $\nu > 2$), this result is shown as follows: When X and M are normally distributed bivariates, then (e.g., see DeGroot (1989)):

$$E(X \mid M) = \mu_X + \rho_{X,M}(M - \mu_M)\frac{\sigma_X}{\sigma_M}, \tag{10}$$

Substituting Equation (10) into Equation (8) yields:

$$\beta(\nu) = \frac{cov\,\{X, [1 - F_M(M)]^{\nu-1}\}}{cov\,\{M, [1 - F_M(M)]^{\nu-1}\}} = \frac{cov\,\{[\mu_X + \rho_{X,M}(M - \mu_M)\frac{\sigma_X}{\sigma_M}], [1 - F_M(M)]^{\nu}}{cov\,\{M, [1 - F_M(M)]^{\nu-1}\}} \tag{11}$$

Given a standard normal variable $Z = (M - \mu_M)/\sigma_M$ with mean 0 and variance 1, Equation (11) becomes:

$$\frac{\rho_{X,M}\,\sigma_X\, cov\,\{Z, [1 - F_Z(Z)]^{\nu-1}\}}{cov\,\{M, [1 - F_M(M)]^{\nu-1}\}} = \frac{\rho_{X,M}\,\sigma_X\, cov\,\{Z, [1 - F_Z(Z)]^{\nu-1}\}}{\sigma_M\, cov\,\{Z, [1 - F_Z(Z)]^{\nu-1}\}} = \frac{\rho_{X,M}\,\sigma_X}{\sigma_M} = \beta_{MV} \tag{12}$$

Hence, if returns are normally distributed, all betas obtained under the various ν's converge to the betas derived using the *MV* approach.

To conclude, *MEG* betas are *consistent* measures of systematic risk since the technique is *compatible* with expected utility maximization. In circumstances where *MV* analysis does not fail, as with normally distributed returns, *MEG* betas converge to *MV* betas. Hence *MV* systematic risk can be considered as a special case of *MEG* betas. Thus, whenever feasible, the analyst should choose *MEG* betas over *MV* betas because they are less deceptive.

4. Econometric procedures and testing

Although one expects *MV* betas to be identical to *MEG* betas when the underlying returns are normally distributed, the converse is not necessarily true. To test whether these betas differ, we use Hausman's (1978) specification test for non-tested models. Designed to examine an hypothesis in terms of model inconsistency, this test runs an efficient estimator, such as *OLS*, against a less efficient but consistent estimator such as *IV*.

To implement the test, we consider two hypotheses: The null, H_0, where M and ϵ_i are independent, and the alternative, H_1, where M and ϵ_i are not independent. Obtained through *OLS*, β_{MV} under H_0 is a consistent and efficient estimator of β, whereas it is not consistent under H_1. On the other hand, the *IV* estimator, $\beta(\nu)$, is consistent under *both* H_0 and H_1, although it is not efficient under H_0. Hausman shows that testing the difference between the betas is appropriate in testing the specification of the model. We use this approach to examine the equivalence of the betas and to show to what extent *MV* betas can lead to unbiased estimators.

The test determines the statistical significance of the difference in betas. Let $\hat{q} = \beta(\nu) - \beta_{MV}$. Hausman proves that the variance of is equal to the variance of $\beta(\nu)$ minus the variance of β_{MV}. With $\hat{V}(\hat{q})$ as a consistent estimator of that variance, one can establish that the following m statistic has a χ^2 distribution with 1 degree of freedom:

$$m = \frac{\hat{q}^2}{\hat{V}(\hat{q})}, \tag{13}$$

In the case of *OLS vs. IV*, the variance estimator $\hat{V}(\hat{q})$ is shown to be:

$$\hat{V}(\hat{q}) = \hat{V}(\beta_{MV})\,\frac{1 - \rho^2}{\rho^2}, \tag{14}$$

where ρ^2 is the squared correlation between the market return and the instrumental variable, which is here the appropriate rank function $-[1 - F_M(M)]^{\nu-1}$. Using the *m* statistic, we can test H_0 against H_1 and establish whether the difference between *MV* betas and *MEG* betas is significant, and whether the *MEG* model provides superior econometric results to *MV*.

Normality plays a leading role in statistical testing and finance. Assuming normality allows application of the most powerful tests in econometrics and produces conclusions of mean-variance efficiency in finance. Testing for normality is therefore crucial in asset pricing, for, as Affleck-Graves and McDonald (1989) note, not all tests are robust in the presence of non-normalities. Non-normality is also a necessary but not a sufficient condition for systematic risks to differ according to various degrees of risk aversion, so we use several procedures to test the sample.

Fama (1965) uses the Studentized Range test to show that daily security returns do not follow a normal distribution. We use the χ^2 test of goodness-of-fit, the Royston (1982) procedure to the Shapiro-Wilk (1965) test, the Kolgomorov distance test, and the D'Agostino (1971) statistic to reach Fama's conclusion. We present only the results obtained using the D'Agostino statistic as the test compares the standard deviation of a distribution with its Gini's mean difference.

The D statistic is defined as:

$$D = \frac{\Gamma_X}{2\,S_X},$$

where Γ_X is the sample's Gini and S_X is the sample's standard deviation. D'Agostino shows that:

$$\frac{\sqrt{n}(D - 0.282095\,)}{0.029986}$$

is asymptotically distributed as a normal N(0,1) variable and can serve as an omnibus normality test for large samples. This test is appropriate for detecting deviations from normality resulting from skewness or kurtosis. We will also use the standard methods to check the persistence of these moments over time to ascertain whether deviations from normality is a continuing phenomenon.

5. Empirical evidence

5.1 The data

To validate the results, we use two sets of daily returns data. The first is for 1,590 firms that have no missing daily returns from January 2, 1985, through December 31, 1987, in the Center for Research in Security Prices (*CRSP*) daily file. Our analysis covers three different time periods, each with 201 observations:

Period I: January 2, 1985, through October 17, 1985;

Period II: February 5, 1986, through November 19, 1986;

Period III: March 18, 1987, through December 31, 1987.

Second, to support the results for a longer range period, we use data from 1,140 firms with no missing daily returns from January 2, 1985 through December 31, 1993, as provided in the *CRSP* daily file. This gives us nine annual periods, each with the same number of observations as the number of trading days in the year.

For each of the three time periods and the nine annual periods, we estimate *MV* and *MEG* betas for all the securities in the sample and perform Hausman's specification test and D'Agostino's normality test. Ten values for ν [1.5, 2, 2.5, 3, 4, 6, 8, 10, 15, 20] are chosen arbitrarily to represent a large variation in risk aversion.[2] For $\nu = 2$, the results become the standard *MG* beta.

5.2 The basic results

As the results are voluminous, in table 1 we present selected estimation results for a small number of blue chip stocks for Periods I, II and III. In all cases, *MEG* betas are compared to *MV* betas. Hausman's *m* statistic (shown in parentheses below each mean-extended Gini beta estimator) indicates whether the *MEG* beta is significantly different from the *MV* beta.

Table 1 shows that the variability of *MEG* betas and their difference from *MV* beta changes from firm to firm. For *American Express*, for example, the *MEG* betas in Period I range from 1.250 for $\nu = 20$ to 2.10 for $\nu = 1.5$, while the *MV* beta is 1.885. The Hausman test shows that for the values of $\nu = 2$, 2.5, and 3, β_{MV} is not significantly different from $\beta(\nu)$ (*m* statistic less than 1.32).[3] For $\nu = 1.5$, however, the difference between $\beta(\nu = 1.5)$ and β_{MV} is statistically significant at the 1% level. For $\nu \geq 4$, the difference is significant at the 5% level.

Suppose an investor is more risk-averse than implied by the *MG* ($\nu = 2$) model and has a $\nu \geq 4$. In the case of investment in *American Express* stock, the use of *MEG* betas instead of *MV* beta for this investor would improve the estimation of the true systematic risk. For this stock, more risk-averse individuals with $\nu \geq 6$ would overestimate beta if they used β_{MV} instead of $\beta(\nu)$. Indeed, for high risk aversion, the more appropriate beta is 1.516, 1.422, 1.361, or 1.280, according to the corresponding ν, rather than the 1.885 reached by using mean-variance.

Not all stocks in the sample have *MEG* betas that are significantly different from the *MV* beta. *Coca Cola* stock is one example in Period I. For *Pfizer* in Period I, the differences in betas do not necessarily increase with ν. On the contrary, for this firm, $\beta(\nu = 2.5) = 1.487$ exhibits the largest significant difference from $\beta_{MV} = 1.677$. On the other hand, for *USX* stock, the difference in betas can be substantial and highly significant.

Period II was considerably less volatile. Comparison of its results with those in Period I shows that only minor changes exist in the various betas. The first security, *American*

Table 1. Mean-variance and mean-Gini betas for select securities. (Daily returns).*

FIRM	Mean	β_{MV}	$\beta_{\nu=1.5}$	$\beta_{\nu=2}$	$\beta_{\nu=2.5}$	$\beta_{\nu=3}$	$\beta_{\nu=4}$	$\beta_{\nu=6}$	$\beta_{\nu=8}$	$\beta_{\nu=10}$	$\beta_{\nu=15}$	$\beta_{\nu=20}$
Period I (January 2, 1985 to October 17, 1985)												
AMERICAN	.00093	1.885	2.010	1.934	1.855	1.784	1.669	1.516	1.422	1.361	1.280	1.250
EXPRESS			(7.58)	(.796)	(.178)	(1.33)	(3.58)	(5.74)	(6.29)	(6.26)	(5.46)	(4.55)
COCA COLA	.00088	.635	.683	.661	.632	.604	.561	.513	.488	.474	.459	.466
			(1.97)	(.400)	(.003)	(.210)	(.712)	(1.07)	(1.081)	(1.02)	(.788)	(.549)
EASTMAN	-.00020	1.000	.979	.926	.901	.890	.890	.909	.928	.944	.972	.992
KODAK			(.345)	(2.97)	(3.13)	(2.55)	(1.51)	(.569)	(.246)	(.115)	(.019)	(.001)
FED NAT	.00153	2.497	2.599	2.548	2.495	2.454	2.405	2.365	2.338	2.306	2.218	2.144
MORTGAGE			(2.11)	(.368)	(.000)	(.098)	(.265)	(.301)	(.308)	(.345)	(.483)	(.584)
GOODYEAR	.00032	1.337	1.291	1.279	1.284	1.294	1.318	1.371	1.425	1.477	1.588	1.673
			(1.33)	(1.44)	(.704)	(.312)	(.035)	(.063)	(.299)	(.587)	(1.24)	(1.67)
HEWLETT	-.00040	1.933	2.001	1.966	1.938	1.916	1.878	1.808	1.742	1.684	1.568	1.481
PACKARD			(1.42)	(.221)	(.003)	(.025)	(.146)	(.411)	(.671)	(.889)	(1.25)	(1.44)
JP MORGAN	.00165	1.747	1.795	1.666	1.580	1.522	1.452	1.395	1.382	1.384	1.395	1.391
			(1.32)	(2.48)	(.620)	(7.59)	(7.68)	(.599)	(4.49)	(3.45)	(2.13)	(1.64)
PFIZER	.00056	1.677	1.654	1.552	1.487	1.448	1.413	1.425	1.469	1.517	1.641	1.768
			(.240)	(4.79)	(6.42)	(6.30)	(4.89)	(2.44)	(1.17)	(.534)	(.017)	(.087)
PHILIP MORRIS	-.00009	1.066	1.123	1.128	1.123	1.118	1.112	1.111	1.108	1.096	1.031	.938
			(2.41)	(1.96)	(.959)	(.529)	(.250)	(.132)	(.078)	(.031)	(.027)	(.277)
RJR NABISCO	-.00020	1.364	1.395	1.419	1.440	1.458	1.487	1.507	1.490	1.452	1.324	1.192
			(.475)	(1.07)	(1.18)	(1.24)	(.124)	(.919)	(.498)	(.189)	(.027)	(.361)
USX	.00103	1.056	1.048	.877	.779	.714	.635	.564	.541	.538	.556	.580
			(.037)	(14.0)	(19.6)	(20.1)	(17.9)	(13.4)	(10.3)	(8.08)	(4.92)	(3.36)
Period II (February 5, 1986 through November, 19 1986)												
AMERICAN	-.00001	1.585	1.698	1.663	1.634	1.613	1.589	1.573	1.566	1.557	1.513	1.461
EXPRESS			(1.47)	(.907)	(.409)	(.142)	(.003)	(.020)	(.040)	(.082)	(.392)	(.951)
COCA COLA	.00137	1.744	1.762	1.739	1.719	1.705	1.688	1.680	1.682	1.684	1.681	1.669
			(.031)	(.003)	(.081)	(.212)	(.425)	(.478)	(.376)	(.300)	(.254)	(.289)
EASTMAN	.00171	1.075	1.003	1.078	1.123	1.150	1.178	1.199	1.211	1.222	1.240	1.242
KODAK			(.501)	(.001)	(.327)	(.823)	(1.50)	(1.85)	(1.89)	(1.89)	(1.77)	(1.47)
FED NAT	.00126	1.986	2.284	2.176	2.095	2.037	1.965	1.900	1.860	1.822	1.728	1.643
MORTGAGE			(6.20)	(3.30)	(1.22)	(.277)	(.041)	(.630)	(1.15)	(1.68)	(3.12)	(4.41)
GOODYEAR	.00151	1.304	1.099	1.149	1.182	1.218	1.291	1.402	1.466	1.499	1.531	1.545
			(2.77)	(2.06)	(1.41)	(.729)	(.018)	(.789)	(1.81)	(2.26)	(2.28)	(2.06)
HEWLETT	.00033	1.635	1.756	1.696	1.656	1.634	1.620	1.639	1.662	1.675	1.666	1.620
PACKARD			(1.02)	(.333)	(.044)	(.000)	(.022)	(.001)	(.052)	(.100)	(.044)	(.009)

Table 1.(Continued)

FIRM	Mean	β_{MV}	$\beta_{\nu=1.5}$	$\beta_{\nu=2}$	$\beta_{\nu=2.5}$	$\beta_{\nu=3}$	$\beta_{\nu=4}$	$\beta_{\nu=6}$	$\beta_{\nu=8}$	$\beta_{\nu=10}$	$\beta_{\nu=15}$	$\beta_{\nu=20}$
JP MORGAN	.00154	1.327	1.563	1.575	1.562	1.541	1.497	1.423	1.365	1.315	1.203	1.105
			(9.20)	(13.3)	(13.3)	(11.4)	(6.96)	(1.88)	(.249)	(.022)	(1.71)	(4.41)
PFIZER	.00117	1.530	1.558	1.561	1.556	1.553	1.553	1.563	1.575	1.582	1.587	1.582
			(.112)	(.173)	(.144)	(.115)	(.108)	(.195)	(.295)	(.348)	(.313)	(.205)
PHILIP MORRIS	.00199	1.589	1.578	1.535	1.512	1.499	1.492	1.507	1.533	1.555	1.582	1.583
			(.012)	(.356)	(.814)	(1.13)	(1.29)	(.771)	(.307)	(.098)	(.003)	(.002)
RJR NABISCO	.00204	1.531	1.482	1.507	1.541	1.569	1.610	1.650	.663	.664	.638	.592
			(.179)	(.057)	(.009)	(.162)	(.677)	(1.29)	(1.35)	(1.18)	(.566)	(.150)
TEXAS UTILITIES	.00064	.904	.954	.943	.935	.929	.920	.904	.888	.873	.838	.809
			(.624)	(.491)	(.356)	(.242)	(.091)	(.000)	(.069)	(.227)	(.747)	(1.24)
USX	.00021	.841	1.007	.968	.941	.918	.876	.806	.751	.709	.641	.599
			(.995)	(.763)	(.522)	(.318)	(.065)	(.054)	(.305)	(.566)	(.968)	(1.15)
Period III (March 18, 1987 through December 31, 1987)												
AMERICAN EXPRESS	-.00190	1.535	1.601	1.595	1.582	1.568	1.545	1.514	1.495	1.482	1.464	1.457
			(.389)	(.318)	(.204)	(.107)	(.011)	(.049)	(.204)	(.388)	(.783)	(1.01)
COCA COLA	-.00040	1.058	1.199	1.193	1.172	1.146	1.096	1.020	.969	.933	.882	.857
		(1.39)	(1.26)	(.927)	(.580)	(.117)	(.143)	(.842)	(1.76)	(3.92)	(5.43)	
EASTMAN KODAK	.00060	1.113	1.235	1.236	1.215	1.187	1.131	1.038	.973	.929	.870	.849
			(.840)	(.835)	(.593)	(.330)	(.021)	(.432)	(1.65)	(3.07)	(5.97)	(7.52)
FED NATL MORTGAGE	-.00150	1.310	1.623	1.620	1.602	1.579	1.534	1.464	1.414	1.375	1.299	1.241
			(10.0)	(9.66)	(8.89)	(7.92)	(6.02)	(3.27)	(1.65)	(.691)	(.022)	(.931)
GOODYEAR	.00130	1.582	1.642	1.605	1.573	1.547	1.508	1.462	1.438	1.423	1.410	1.410
			(.452)	(.070)	(.008)	(.155)	(.775)	(2.35)	(3.78)	(4.89)	(6.47)	(6.92)
HEWLETT PACKARD	.00041	1.153	1.438	1.475	1.465	1.446	1.409	1.354	1.320	1.299	1.272	1.262
			(7.64)	(9.63)	(9.41)	(8.72)	(7.26)	(5.14)	(3.92)	(3.18)	(2.37)	(2.13)
JP MORGAN	-.00020	1.168	1.252	1.226	1.193	1.161	1.108	1.031	.977	.938	.876	.846
			(.321)	(.150)	(.029)	(.002)	(.194)	(1.16)	(2.48)	(3.88)	(6.98)	(9.04)
PFIZER	-.00180	1.046	1.089	1.123	1.146	1.161	1.176	1.175	1.159	1.138	1.089	1.051
			(.236)	(.742)	(1.30)	(1.81)	(2.52)	(2.85)	(2.40)	(1.72)	(.409)	(.005)
PHILIP MORRIS	.00043	.958	1.271	1.294	1.284	1.264	1.218	1.141	1.084	1.039	.963	.917
			(18.2)	(20.6)	(20.1)	(18.6)	(14.7)	(8.38)	(4.32)	(1.92)	(.007)	(.588)
RJR NABISCO	-.00080	.735	1.165	1.208	1.201	1.180	1.137	1.072	1.028	.993	.928	.879
			(23.8)	(28.4)	(28.6)	(27.4)	(24.5)	(19.8)	(16.4)	(13.7)	(8.51)	(5.05)
USX	.00159	1.593	1.477	1.452	1.435	1.418	1.386	1.335	1.300	1.276	1.250	1.249
			(.825)	(1.20)	(1.57)	(2.01)	(3.06)	(5.46)	(7.78)	(9.73)	(12.8)	(13.71)

* Hausman's *m*, that tests the hypothesis that the MEG betas are different from the MV betas is in parentheses under the MEG betas. *m* has a χ^2 distribution with 1 degree of freedom.

Express, shows that nothing would have been gained by using the *MEG* approach because the betas are not significantly different from *MV* beta. The opposite is true, however, for *J P Morgan*, where using $\beta(\nu = 3)$, for example, provides an estimator for beta that is statistically different from β_{MV} and also more stable for the two periods.

Period III covers the period of October 1987 crash. Results here show the clear advantage of the *MEG* approach in a more turbulent period. Investors with differentiated risk aversion in this case would have been clearly mistaken about the true beta had they used *MV* beta. For example, highly risk-averse investors ($\nu > 10$) investing in *Coca Cola, Eastman Kodak, Goodyear*, and *USX* would have overestimated the aggressiveness of their investments. On the other hand, moderately risk-averse individuals using *MV* beta would have underevaluated the systematic risk of *Federal National Mortgage* and *Hewlett-Packard*.

The case of *Philip Morris* stock further demonstrates the advantage of *MEG* beta. For $\nu \leq 10$, the differences between $\beta(\nu)$ and β_{MV} show high statistical significance; hence less risk-averse investors underevaluated the true beta by using *MV*. *Philip Morris* results in Periods I and II show that *MEG* betas are basically similar for the three periods, and therefore relatively more stable than the *MV* beta. The same conclusion applies for other stocks, such as *RJR Nabisco*.

5.3 Testing normality and differentiated systematic risk

We examined the entire sample of securities to assess the importance of normality in using the *MEG* approach *vs.* the *MV* model for beta estimation. For each period and each ν, we computed the percentage of securities of the total sample of 1,590 that have a different beta according to Hausman's *m* test and are normal or not-normal according to D'Agostino's *D* test. Table 2 shows the results.

The first three columns of table 2 show the percentages of securities where $\beta(\nu)$ is different from β_{MV} at 1%, 5%, and 10% significance levels according to the *m* test for securities that test normal under the *D* test. The next three columns show the percentages of securities with different β at 1%, 5%, and 10% significance levels for the securities that are not normally distributed.

For example, for $\nu = 4$ in Period I, only 0.69% of the securities have $\beta(\nu = 4)$ different from β_{MV} at the 10% significance level and tested normally distributed at the same 10% level. For the same group of ν, 14.65% of the securities had different betas and tested not normal.

Also note that for any given ν in Period I, there is at least a 10.88% chance that $\beta(\nu)$ is different from β_{MV} (for $\nu = 1.5$). Further, there is a 33.33% chance (10% significance level) that the betas would be different for at least one ν, as shown by the last row. In the more volatile Period III these results are amplified. There is at least a 31.95% chance of difference for any given ν ($\nu = 10$), and a 65.47% chance of different betas for at least one ν.

We interpret these results to indicate that in a moderately stable period like Period I, for any given investor with a particular ν, there is a chance between 11% and 15% that use

Table 2. Percentages of securities with $\beta(\nu)$ significantly different from β_{MV}, in the class of statistically "normal" and "non-normal" probability distributions.

	"Normally Distributed"			"Non-Normally Distributed"		
Significance Level	1%	5%	10%	1%	5%	10%
Period I						
For $\beta(\nu = 1.5)$	0.13%	0.19%	0.25%	1.07%	5.53%	10.88%
For $\beta(\nu = 2)$	0.25%	0.50%	0.50%	1.64%	8.11%	13.58%
For $\beta(\nu = 2.5)$	0.19%	0.69%	0.75%	2.45%	8.30%	14.78%
For $\beta(\nu = 3)$	0.25%	0.63%	0.88%	2.20%	8.24%	14.72%
For $\beta(\nu = 4)$	0.19%	0.63%	0.69%	2.26%	8.18%	14.65%
For $\beta(\nu = 6)$	0.19%	0.63%	0.69%	1.82%	7.11%	14.03%
For $\beta(\nu = 8)$	0.19%	0.63%	0.69%	1.45%	6.98%	13.58%
For $\beta(\nu = 10)$	0.06%	0.57%	0.63%	1.38%	6.98%	13.08%
For $\beta(\nu = 15)$	0.06%	0.31%	0.38%	1.70%	6.79%	12.39%
For $\beta(\nu = 20)$	0.13%	0.31%	0.25%	1.45%	6.73%	11.95%
For at least 1 ν	0.63%	1.32%	1.32%	5.60%	21.32%	33.33%
Period II						
For $\beta(\nu = 1.5)$	0.31%	0.50%	0.75%	2.52%	8.24%	13.71%
For $\beta(\nu = 2)$	0.25%	0.38%	0.82%	2.39%	7.67%	13.14%
For $\beta(\nu = 2.5)$	0.19%	0.44%	0.63%	2.26%	6.86%	11.82%
For $\beta(\nu = 3)$	0.25%	0.57%	0.63%	1.95%	6.48%	11.51%
For $\beta(\nu = 4)$	0.25%	0.69%	0.50%	1.19%	6.23%	11.82%
For $\beta(\nu = 6)$	0.13%	0.57%	0.38%	0.88%	5.66%	11.57%
For $\beta(\nu = 8)$	0.25%	0.38%	0.50%	0.75%	6.16%	11.38%
For $\beta(\nu = 10)$	0.19%	0.25%	0.57%	0.88%	6.48%	12.39%
For $\beta(\nu = 15)$	0.13%	0.25%	0.44%	1.32%	7.11%	11.95%
For $\beta(\nu = 20)$	0.13%	0.50%	0.63%	1.51%	6.92%	12.89%
For at least 1 ν	0.69%	1.38%	1.51%	5.47%	19.94%	32.89%
Period III						
For $\beta(\nu = 1.5)$	0.13%	0.06%	0.00%	14.65%	26.79%	35.03%
For $\beta(\nu = 2)$	0.13%	0.06%	0.00%	17.17%	28.87%	36.79%
For $\beta(\nu = 2.5)$	0.13%	0.06%	0.00%	16.73%	28.74%	36.42%
For $\beta(\nu = 3)$	0.13%	0.06%	0.00%	16.42%	28.36%	36.35%
For $\beta(\nu = 4)$	0.13%	0.06%	0.00%	14.97%	26.48%	35.53%
For $\beta(\nu = 6)$	0.13%	0.06%	0.00%	12.39%	24.59%	33.33%
For $\beta(\nu = 8)$	0.06%	0.06%	0.00%	11.51%	24.03%	32.33%
For $\beta(\nu = 10)$	0.06%	0.06%	0.00%	10.38%	24.03%	31.95%
For $\beta(\nu = 15)$	0.06%	0.06%	0.00%	12.39%	24.91%	34.40%
For $\beta(\nu = 20)$	0.00%	0.06%	0.00%	13.02%	27.23%	35.97%
For at least 1 ν	0.13%	0.06%	0.00%	32.33%	52.77%	65.47%

of $\beta(\nu)$ rather than β_{MV} would result in a better systematic risk estimate and therefore superior performance. More significantly, there is about a 33% chance that *at least one* investor would achieve superior performance.

In especially volatile periods, such as Period III, for any given investor there is at least a 32% chance of superior performance using *MEG* betas. Indeed, given a particular ν such as $\nu = 6$, an individual investor would find 33% of the firms with *MEG* betas different from their *MV* betas. Furthermore for the same period, there is a 65% chance that *at least one* investor would benefit by estimating systematic risk using *MEG*.

Table 3 provides normality results. In Period I, 94.65% of the securities test non-normal at the 10% significance level. For the very volatile Period III, this increases to 100%. Therefore we find, as have previous researchers, that securities returns are overwhelmingly not normal. In fact, non-normality is so prevalent that any use of β_{MV} is likely to lead to biased estimates of systematic risk, a good case for use of *MEG* betas.

5.4 Empirical results for annual periods

To confirm the findings for longer time periods and reduce any concerns that the general results are firms for the years 1985 through 1993. In table 4, we first present, as an example, the *MV* and *MEG* betas for three securities over the nine year period.

For 1985, 1986, and 1987, the results are not the same as those in table 1 because the number of observations is now the number of trading days during the year. For the three firms, the variability and the non-sationarity of the betas over the years are notable. It is not surprising that the *MEG* betas do not follow the *MV* betas (and vice versa). According to Hausman's *m* statistic, the β_{MEG} are statistically different from β_{MV} for only a small number of ν and years.[4] This difference, when statistically significant, however, can be very important. For example, *Federal National Mortgage*, in 1987 for low risk-averse investors and in 1992 for high risk-averse investors, exhibits $\beta(\nu)$ that are 33% greater than β_{MV}. The same can be said for *Hewlett-Packard* where the $\beta(\nu)$ in 1987, 1988, and 1991 are statistically different from β_{MV}.

The entire sample of 1,140 firms is summarized in table 5, which presents for each year the percentages of securities that have a different $\beta(\nu)$ according to Hausman's *m* test for at least one ν and are normal or not-normal according to D'Agostino's *D* test. At a 5% significance level, combining normal or not-normal, more than 20% of the firms had different *MEG* betas in all years except 1993. In 1987, 1988, and 1989, it was more than 30% of the firms.

This is further supported by the normality test shown in table 6. In relatively stable years, normality is rejected for 83% of the firms at the 1% significance level. For turbulent years, it is rejected for more than 90% of the firms. Hence, *MV* betas will be estimated optimally in all those cases. The longer time period analysis confirms the basic results that, for any time period, there is a 30% chance that at least one investor would benefit using $\beta(\nu)$, as those are shown to differ statistically from β_{MV}.

Table 3. Percentages of "non-normally" distributed securities. (Three periods).

"Non-Normally Distributed"			
Significance Level	1%	5%	10%
Period I			
	85.22%	91.64%	94.65%
Period II			
	88.24%	94.40%	95.91%
Period III			
	99.81%	99.94%	100.0%

The question remains whether the distributional deviations from normality persist or not from one year to another (e.g. from an estimation period to a portfolio holding period).[5] Singleton and Wingender (1986) show that skewness does not persist over time implying that" ... investment strategies based on selecting skewed stock are likely to fail." Our interest in the skewness persistence issue resides in validating the *MEG* estimation of systematic risk over time. To check for skewness and kurtosis, we use the standard

Table 4. Mean-variance and mean-Gini betas for three securities over 9 years. (Daily returns).*

YEAR	β_{MV}	$\beta_{\nu=1.5}$	$\beta_{\nu=2}$	$\beta_{\nu=2.5}$	$\beta_{\nu=3}$	$\beta_{\nu=4}$	$\beta_{\nu=6}$	$\beta_{\nu=8}$	$\beta_{\nu=10}$	$\beta_{\nu=15}$	$\beta_{\nu=20}$
AMERICAN EXPRESS											
1985	1.892	**1.989**	1.948	1.896	1.844	1.751	**1.610**	**1.516**	**1.451**	**1.361**	**1.324**
		(7.03)	(1.58)	(0.00)	(0.43)	(2.11)	**(4.54)**	**(5.60)**	**(5.94)**	**(5.59)**	**(4.79)**
1986	1.494	1.574	1.539	1.515	1.498	1.480	1.472	1.469	1.463	1.428	1.383
		(1.15)	(0.50)	(0.12)	(0.01)	(0.04)	(0.09)	(0.09)	(0.11)	(0.36)	(0.83)
1987	1.853	1.536	1.541	1.539	1.534	1.525	1.512	1.500	1.490	1.473	1.464
		(0.02)	(0.04)	(0.03)	(0.02)	(0.00)	(0.01)	(0.06)	(0.14)	(0.39)	(0.58)
1988	1.974	1.536	1.541	1.539	1.534	1.525	1.512	1.500	1.490	1.473	1.464
		(2.88)	(2.55)	(1.71)	(1.02)	(0.28)	(0.00)	(0.10)	(0.23)	(0.45)	(0.53)
1989	2.065	2.036	1.964	1.922	1.897	**1.869**	**1.846**	**1.837**	**1.837**	**1.855**	**1.883**
		(0.06)	(0.79)	(1.68)	(2.40)	**(3.25)**	**(3.79)**	**(3.75)**	**(3.46)**	**(2.45)**	**(1.62)**
1990	1.960	2.081	2.020	1.981	1.955	1.922	1.886	1.868	1.856	1.838	1.822
		(1.65)	(0.46)	(0.06)	(0.00)	(0.17)	(0.54)	(0.71)	(0.76)	(0.76)	(0.78)
1991	1.902	1.941	1.934	1.921	1.906	1.876	1.823	1.779	1.745	1.706	1.712
		(0.27)	(0.13)	(0.03)	(0.00)	(0.04)	(0.22)	(0.41)	(0.56)	(0.63)	(0.48)
1992	1.225	1.277	1.322	1.343	1.348	1.337	1.304	1.278	1.256	1.202	1.152
		(0.46)	(1.93)	(2.47)	(2.14)	(1.21)	(0.35)	(0.11)	(0.03)	(0.01)	(0.09)
1993	1.246	1.013	1.107	1.167	1.209	1.266	1.337	1.398	1.457	1.578	1.650
		(2.19)	(1.11)	(0.41)	(0.09)	(0.03)	(0.39)	(0.86)	(1.37)	(2.41)	(2.84)
FED NAT MORTGAGE											
1985	2.520	2.600	2.609	2.593	2.573	2.543	2.513	2.489	2.460	2.377	2.302
		(1.91)	(1.57)	(0.58)	(0.20)	(0.02)	(0.00)	(0.02)	(0.04)	(0.16)	(0.28)
1986	2.061	**2.288**	2.203	2.135	2.085	2.024	1.975	1.952	1.934	1.890	1.844
		(4.68)	(2.47)	(0.75)	(0.08)	(0.16)	(0.69)	(0.87)	(0.98)	(1.29)	(1.64)
1987	1.264	**1.622**	**1.614**	**1.592**	**1.568**	**1.523**	1.458	1.414	1.381	1.320	1.271
		(10.7)	**(9.99)**	**(8.82)**	**(7.60)**	**(5.51)**	(2.93)	(1.55)	(0.75)	(0.01)	(0.34)
1988	1.464	1.622	1.614	1.592	1.568	1.523	1.458	1.414	1.381	1.320	1.271
		(0.44)	(1.51)	(2.31)	(2.71)	(2.68)	(1.70)	(0.94)	(0.51)	(0.12)	(0.03)
1989	2.230	2.370	2.308	2.267	2.242	2.221	2.226	2.245	2.264	2.308	2.344
		(0.80)	(0.04)	(0.13)	(0.59)	(1.48)	(1.99)	(1.79)	(1.51)	(1.06)	(0.86)
1990	2.356	2.581	2.581	2.572	2.559	2.528	2.455	2.379	2.304	**2.140**	**2.009**
		(4.76)	(2.84)	(1.16)	(0.29)	(0.07)	(1.29)	(2.29)	(2.82)	**(3.17)**	**(3.17)**
1991	1.995	2.001	2.011	2.022	2.029	2.035	2.034	2.024	2.009	1.976	1.953
		(0.01)	(0.06)	(0.13)	(0.17)	(0.17)	(0.11)	(0.04)	(0.01)	(0.01)	(0.05)
1992	1.260	1.190	1.279	1.336	1.379	**1.440**	**1.515**	**1.558**	**1.582**	**1.602**	1.590
		(1.20)	(0.10)	(1.50)	(2.92)	**(4.53)**	**(5.41)**	**(5.31)**	**(4.93)**	**(3.77)**	(2.72)
1993	1.775	1.850	1.791	1.752	1.727	1.701	1.697	1.717	1.740	1.779	1.791
		(0.47)	(0.03)	(0.07)	(0.32)	(0.71)	(0.59)	(0.26)	(0.08)	(0.00)	(0.01)

(Continued)

Table 4. (Continued)

YEAR	β_{MV}	$\beta_{\nu=1.5}$	$\beta_{\nu=2}$	$\beta_{\nu=2.5}$	$\beta_{\nu=3}$	$\beta_{\nu=4}$	$\beta_{\nu=6}$	$\beta_{\nu=8}$	$\beta_{\nu=10}$	$\beta_{\nu=15}$	$\beta_{\nu=20}$
HEWLETT-PACKARD											
1985	1.923	1.968	1.984	1.990	1.988	1.971	1.922	1.872	1.825	1.722	1.635
		(0.93)	(1.16)	(0.76)	(0.47)	(0.15)	(0.00)	(0.06)	(0.18)	(0.48)	(0.74)
1986	1.580	1.685	1.622	1.579	1.554	1.537	1.553	1.578	1.595	1.598	1.563
		(1.23)	(0.26)	(0.00)	(0.11)	(0.28)	(0.08)	(0.00)	(0.02)	(0.02)	(0.01)
1987	1.277	**1.532**	**1.567**	**1.549**	**1.520**	**1.466**	**1.394**	**1.353**	1.327	1.292	1.276
		(11.9)	**(14.2)**	**(13.2)**	**(11.6)**	**(8.81)**	**(5.49)**	**(3.87)**	(2.96)	(1.88)	(1.46)
1988	1.704	1.532	1.567	1.549	1.520	1.466	1.394	**1.353**	**1.327**	**1.292**	**1.276**
		(2.32)	(0.85)	(0.26)	(0.02)	(0.18)	(1.47)	**(3.02)**	**(4.43)**	**(7.17)**	**(9.03)**
1989	1.817	2.054	1.959	1.896	1.853	1.796	1.720	1.659	1.608	**1.521**	**1.478**
		2.53)	(1.08)	(0.36)	(0.08)	(0.02)	(0.50)	(1.23)	(2.00)	**(3.36)**	**(3.87)**
1990	1.643	1.835	1.769	1.727	1.695	1.645	1.571	1.521	1.486	1.432	1.390
		(3.60)	(1.78)	(0.82)	(0.31)	(0.00)	(0.44)	(1.06)	(1.50)	(2.00)	(2.28)
1991	1.587	**1.714**	**1.761**	**1.786**	**1.791**	1.762	1.645	1.523	1.417	1.224	**1.104**
		(3.17)	**(4.12)**	**(3.91)**	**(3.22)**	(1.69)	(0.13)	(0.11)	(0.69)	(2.30)	**(3.31)**
1992	1.568	1.686	1.663	1.620	1.574	1.495	1.394	1.337	1.302	1.248	1.213
		(1.16)	(0.91)	(0.23)	(0.00)	(0.25)	(0.86)	(1.09)	(1.15)	(1.12)	(1.07)
1993	1.811	1.788	1.874	1.923	1.953	1.985	1.993	1.971	1.939	1.863	1.814
		(0.02)	(0.19)	(0.69)	(1.15)	(1.59)	(1.33)	(0.81)	(0.43)	(0.05)	(0.00)

* Hausman's *m*, that tests the hypothesis that the MEG betas are different from the MV betas is in parentheses under the MEG betas. The bold faced figures indicate the MEG betas significantly different from the MV beta at the 5% significance level.

statistics being, for the skewness, the third central moment divided by the cube of the standard deviation, and for the kurtosis, the fourth central moment over the squared variance minus 3.

In the spirit of Singleton and Wingender, we check whether a security skewed one year persists to be skewed the next year. The results are reported on table 7. For each year, we calculate the percentage of securities whose skewness and kurtosis parameters exceed the critical values.[6] Then the following year, we check whether skewness persists or reverses

Table 5. Percentages of securities with $\beta(\nu)$ significantly different from β_{MV}, for at least one ν in the class of "normal" and "non-normal" probability distributions for years 1985–1993, 1140 firms.

		"Normally Distributed"			"Non-Normally Distributed"		
Significance Level		1%	5%	10%	1%	5%	10%
Year							
1985	0.96%	2.19%	2.89%	6.32%	18.77%	32.02%	
1986	0.96%	1.40%	2.02%	5.26%	19.39%	32.63%	
1987	0.09%	0.00%	0.00%	34.21%	56.05%	67.89%	
1988	1.23%	1.49%	1.40%	11.14%	30.61%	44.74%	
1989	1.67%	1.93%	2.02%	11.32%	28.33%	42.11%	
1990	1.05%	2.37%	2.72%	8.77%	25.18%	38.60%	
1991	1.23%	2.02%	2.63%	5.96%	20.61%	33.07%	
1992	1.40%	2.89%	3.07%	5.18%	17.37%	30.00%	
1993	0.61%	1.84%	2.98%	4.47%	14.91%	26.32%	

Table 6. Percentages of "normally" vs "non-normally" distributed securities. (1140 firms over 9 years).

	"Non-Normally Distributed"		
Significance Level	1%	5%	10%
Year			
1985	82.54%	87.98%	90.96%
1986	86.93%	91.49%	93.33%
1987	99.91%	100.00%	100.00%
1988	93.07%	96.67%	97.72%
1989	91.93%	95.61%	97.02%
1990	86.23%	91.32%	93.95%
1991	84.30%	89.30%	91.67%
1992	83.33%	88.25%	92.28%
1993	83.95%	88.60%	91.14%

itself to the opposite sign. The percentages reporting persistence and negatively skewed securities, reinforcing the results obtained by the normality D'Agostino *D* test. Indeed, the total percentage of skewed stocks exceed 50% at the 1% significance level, and 60% at the 5% significance level. Now it one looks at the number of securities that remain skewed the following year, the same picture is obtained. Apart from year 1987, positively skewed securities persist to be positively skewed (more than 50% at the 1% and 5% significance level) and a very small proportion reverses to negative skewness. In 1987, because of the Crash, a larger proportion of stocks appears to be negatively skewed and this modifies the persistence results for 1986 and 1988.

By valuing the kurtosis of the stocks one notices that only a small number (around 20%) has kurtosis consistent with that of a normal distribution. Of those normal stocks approximately 30% remain normal the following year. By looking at the kurtosis we confirm again that most stocks are not normally distributed and remain so over time.

Our results based on daily returns and one-year test periods support some of the conclusions obtained by Singleton and Wingender for individual simple stock returns. Using monthly data and five-year test periods, Singleton and Wingender report a 30 to 40% chance for positive skewness to persist, whereas for portfolio returns, the skewness persistence is reduced to around 10%. The skewness and kurtosis statistics further strengthen the hypothesis that most securities are not normally distributed and some efficiency is to be gained by estimating systematic risk using *MEG* methods.

5.5 Ranking securities with respect to differentiated systematic risk

Taking into account investor risk aversion, how significant are the differences in ranking securities according to the various systematic risks? If *MEG* betas produce substantially different rankings of securities from rankings obtained using *MV* beta, the importance of *MEG* in constructing portfolios becomes further strengthened. Our test ranks securities in ascending order of beta for a given ν and compares the deciles of the different rankings with the deciles obtained by ranking securities in ascending order of the *MV* beta. If a

Table 7. Percentage of securities that exhibit skewness and kurtosis in year t and conditional percentage of skewness and kurtosis persistence for year $t+1$.

Significance Level 1%								
				Skewness			Kurtosis	
Year	Positive this year	Positive next year	Negative next year	Negative this year	Negative next year	Positive next year	Kurtosis this year	Kurtosis next year
1985	0.4877	0.4496	0.0845	0.0956	0.2294	0.2477	0.1982	0.2832
1986	0.3798	0.2610	0.4711	0.1035	0.5508	0.1271	0.1395	0.0189
1987	0.1868	0.5305	0.0563	0.5360	0.1162	0.4403	0.0070	0.7500
1988	0.4781	0.5229	0.1064	0.0921	0.2762	0.2667	0.1246	0.3239
1989	0.4202	0.4718	0.0939	0.1430	0.1534	0.2761	0.1368	0.2885
1990	0.3868	0.6463	0.0544	0.1114	0.0472	0.5748	0.1693	0.3212
1991	0.5491	0.5367	0.0783	0.0596	0.2647	0.3676	0.1912	0.3761
1992	0.4482	0.4599	0.0783	0.1088	0.2661	0.2419	0.2140	0.2910
1993	0.3640	------	------	0.1368	------	------	0.1886	------
Significance level 5%								
				Skewness			Kurtosis	
Year	Positive this year	Positive next year	Negative next year	Negative this year	Negative next year	Positive next year	Kurtosis this year	Kurtosis next year
1985	0.5833	0.5459	0.1113	0.1254	0.2797	0.3077	0.1088	0.2016
1986	0.4711	0.2663	0.5158	0.1447	0.5939	0.1818	0.0667	0.0395
1987	0.2149	0.6531	0.0571	0.5816	0.1508	0.5385	0.0026	0.6667
1988	0.5693	0.5609	0.1479	0.1175	0.2910	0.3284	0.0544	0.2581
1989	0.4956	0.5575	0.1274	0.1798	0.2049	0.3902	0.0693	0.2532
1990	0.4807	0.7208	0.0748	0.1465	0.0719	0.6766	0.0842	0.2292
1991	0.6719	0.6188	0.1097	0.0842	0.2500	0.3854	0.1044	0.2353
1992	0.5544	0.5316	0.1187	0.1298	0.3041	0.3041	0.1219	0.1871
1993	0.4553	------	------	0.1693	------	------	0.1035	------

security remains in the same decile, then estimating beta according to *MEG* will not provide additional information to the investor. If the number of firms that move one decile up or down in substantial, however, investors would be better off estimating beta using their appropriate *MEG* model. In addition, if firms move up or down by more than one decile, the *MV* systematic risk would hinder investors' ability to build portfolios that diversify risk.

Table 8 presents, for the three periods, the number of securities that change deciles when *MEG* betas are used as the ranking factor instead of *MV* betas. The first ten columns (Panel A) show the number of securities in a decile that change decile following the application of the *MEG* model. For the three data samples, the ranking of securities with respect to the *MV* beta is not maintained when using *MEG* beta, particularly for higher degrees of risk aversion ($\nu > 2$). Indeed, more than 50% of the securities change decile.

A claim might be made that the large number of securities changing deciles depends on the arbitrary boundaries of the deciles themselves. Therefore, a valid evaluation of the impact of *MEG* ranking is to count the securities that move more than one decile. This is shown in the nine columns in Panel B of table 8, which show the total number of securities that move 1, 2, 3, …, 9 deciles. For example, in the first period, for $\nu = 4$ a total of 854 out of 1,590 securities shift deciles (Panel A). The distribution of those shifts is given in Panel B. That is 655 securities that move one decile only, 148 securities that move two deciles, 35 securities that move three deciles, 9 securities that move four deciles, and so on. The sum of the ten Panel A columns equals the sum of the nine Panel B columns.

The number of securities that move more than one decile is around 20%. With the number of securities that move just one decile seen to be 40%, the results in table 8 show that the ranking of securities is substantially different when investors estimate systematic risk according to their particular risk aversion.

6. Conclusion

Choosing securities according to their risk and mean return is the essential challenge for investors who want to make investment decisions consistent with risk aversion. Therefore, ranking assets with respect to systematic risk has been standard practice for investment and porfolio analysis since the development of betas. As we have seen, however, the ranking of assets with respect to systematic risk also depends upon investors' *degree* of risk aversion. This is especially significant whenever one cannot assume normally distributed returns to ensure the validity of the *MV* model

Our study demonstrates how to incorporate risk aversion into the evaluation of systematic risk. We have shown the importance of the issue in terms of its effect in capital markets. *MEG* analysis in beta estimation only improves our understanding of systematic risk.

When investors use *MEG* betas to rank securities, they will always be at least as well off as if they had used *MV* betas. Much of the time they will be better off, particularly during periods of high volatility.

Table 8. Number of securities that move deciles when the beta ranking is done using MEG instead of MV.

Period 1				Panel A											Panel B					
Number of Securities in a Decile that Move at Least One Decile											Total Number of Securities Moving Deciles									
Decile #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
MV vs ν=1.5	11	32	49	51	51	56	57	36	23	11	377 366	11								
MV vs ν=2	17	45	67	74	70	69	66	56	40	16	520	489	27	3	1					
MV vs ν=2.5	26	67	83	88	81	83	79	72	54	23	656	579	64	9	2	2				
MV vs ν=3	32	76	95	98	100	95	90	80	62	27	755 642	83	23	2	3	2				
MV vs ν=4	40	83	105	112	114	108	98	92	71	31	854 655	148	35	9	4	0	2	1		
MV vs ν=6	54	92	121	117	129	114	111	108	88	40	974 628	251	58	21	10	2	2	2		
MV vs ν=8	64	99	120	124	135	118	121	112	89	46	1028	619	255	93	37	10	10	2	2	
Mv vs ν=10	70	105	128	126	132	126	124	120	93	51	1075	606	271	113	53	16	10	3	3	
Mv vs ν=15	81	118	136	128	133	127	126	128	98	54	1129	577	290	132	76	31	9	8	4	2
MV vs ν=20	84	115	136	133	136	130	125	134	112	63	1168	560	296	163	74	44	13	8	7	3
Period 2				Panel A											Panel B					
Number of Securities in a Decile that Move at Least One Decile											Total Number of Securities Moving Deciles									
Decile #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
MV vs ν=1.5	31	74	86	104	107	100	104	89	90	44	829 643	136	33	9	6	2				
MV vs ν=2	24	64	81	92	97	93	88	81	79	35	734 605	96	23	5	2	3				
MV vs n=2.5	23	56	72	88	92	92	93	81	69	32	698 586	86	18	5	2	1				
MV vs ν=3	23	57	75	87	86	87	83	73	70	33	674 567	82	18	5	1	1				
MV vs ν=4	25	59	78	87	88	89	83	77	67	29	682 569	85	22	3	3					
MV vs ν=6	27	56	76	94	97	86	87	88	78	37	726 595	105	18	5	1	0	2			
MV vs ν=8	29	61	84	100	99	93	93	91	85	38	773 613	133	17	5	2	1	2			
MV vs ν=10	32	69	98	102	98	99	93	89	86	37	795 608	149	27	4	3	2	2			
MV vs ν=15	37	79	97	97	105	110	104	90	90	43	852 630	160	44	11	2	3	2			
MV vs ν=20	43	86	100	103	108	117	112	94	94	47	904 636	185	56	16	7	3	1			

(Continued

Table 8. Continued.

Period 3	Panel A										Panel B									
Number of Securities in a Decile that Move at Least One Decile											Total Number of Securities Moving Deciles									
Decile #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
MV vs ν=1.5	42	84	109	113	126	125	124	113	112	60	1008	628	248	100	24	6	1	1		
MV vs ν=2	46	84	114	120	124	126	124	113	112	65	1028	603	271	111	34	6	3			
MV vs ν=2.5	44	83	111	119	123	125	119	112	115	66	1017	607	260	109	32	7	1	1		
MV vs ν=3	46	86	111	119	117	125	117	110	109	61	1001	603	260	99	32	5	1	1		
MV vs ν=4	43	81	105	117	115	125	122	113	110	54	985 618	259	80	21	5	1	1			
MV vs ν=6	43	75	96	115	115	118	116	103	99	47	927 622	228	53	15	6	2	1			
MV vs ν=8	42	73	94	112	115	118	118	100	95	43	910 639	202	50	12	4	2	1			
MV vs ν=10	42	72	96	109	111	115	115	100	91	40	891 632	198	43	12	2	3	1			
mv vs ν=15	44	77	94	103	110	114	109	92	92	40	875 641	177	40	11	1	3	2			
mv vs ν=20	42	75	92	103	111	113	104	94	85	38	857 620	187	35	8	3	2	2			

To the practitioner, the question remains as to how best to choose the coefficient of risk aversion ν to be used in the analysis. Two solutions are feasible. When capital markets are in equilibrium, this value can be estimated by comparing the market portfolio with the position obtained by optimizing the *MEG* portfolio, as is done by Shalit and Yitzhaki (1989). Here, the value of ν that brings the *MEG* portfolio closer to the market portfolio is to be used.

More pragmatically, the choice of ν can be secured by checking whether or not returns are normally distributed. If normality is rejected, the practitioner estimates *MEG* betas for several ν's together with the appropriate Hausman statistic to assess whether the *MEG* betas are significantly different from *MV* betas. In this reduced set of statistically different betas, the analyst can now choose an appropriate ν to fit one's sensitivity to risk.

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Notes

1. The various formulae of the Gini are provided in Shalit and Yitzhaki (1984).
2. The value for ν are chosen on the basis of previous research (Shalit and Yitzhaki (1989), Okunev (1988).
3. The critical values for χ^2 with 1 D.F. are 2.7 for a significance level of 10%, 3.84 for 5%, and 6.63 for 1%.
4. The significantly different betas are shown in bold in table 4.
5. We are grateful to an anonymous referee who pointed out the issue of skewness persistence when checking for deviations from normality.
6. The critical values are +0.360 and-0.360 for positive and negative skewness at the 1% significance level and 0.251 and -0.251 for positive and negative skewness at the 5% significance level. The critical values interval for kurtosis are (2.42, 3.87) at the 1% significance level and (2.55, 3.52) at the 5% significance level.

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