Moduli stabilization, SUSY breaking and Cosmology

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hep-th/0205042
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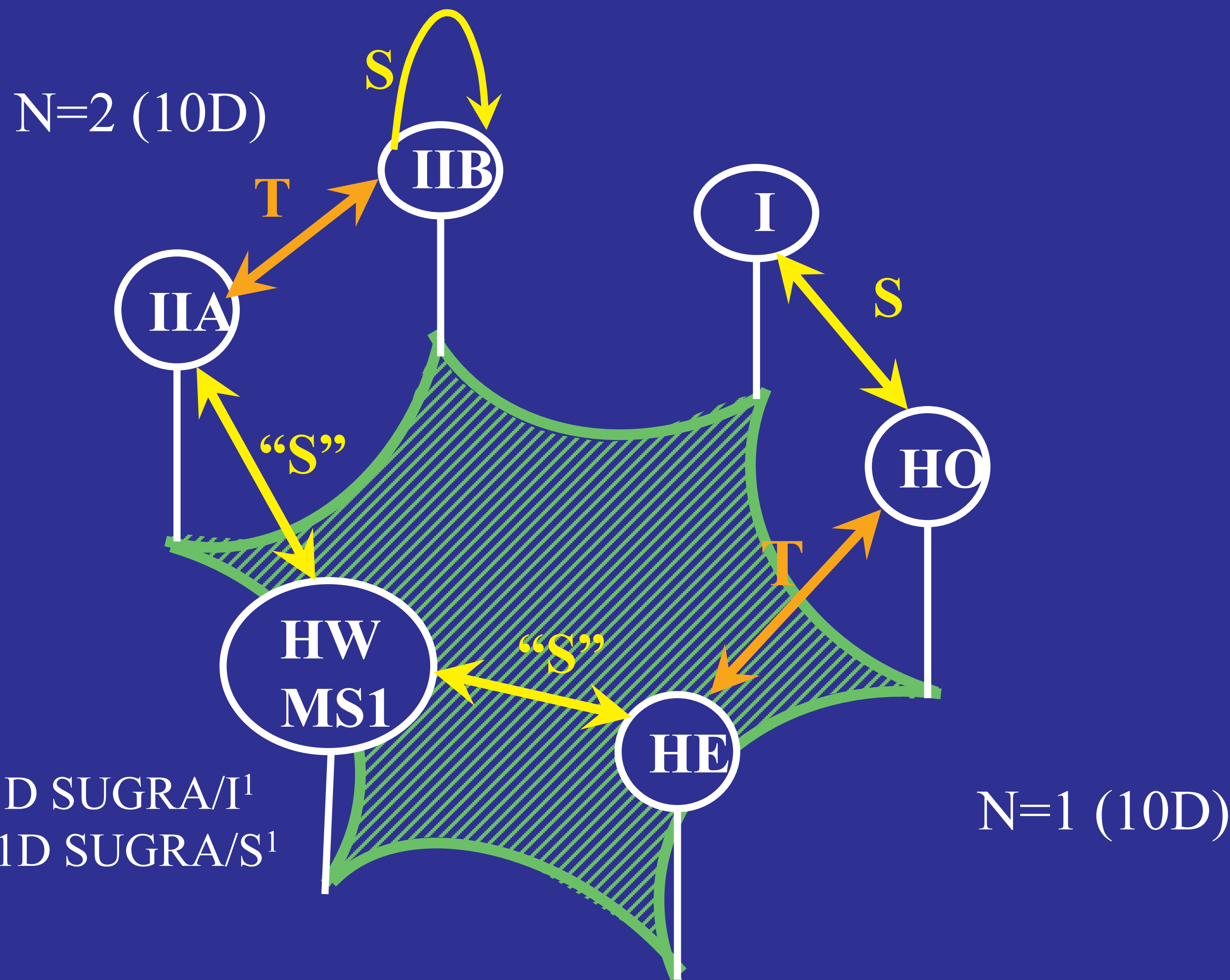
with S. de Alwis, E. Novak

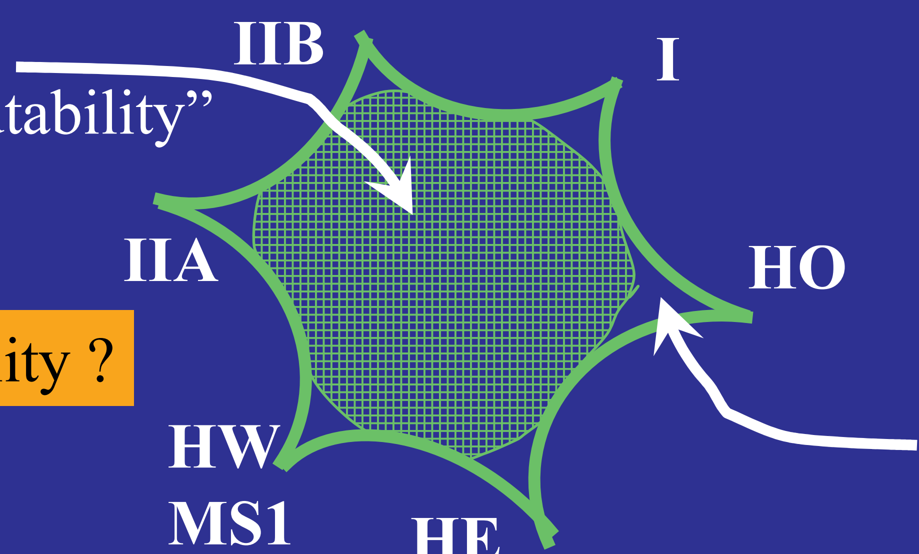
- Moduli space of effective theories of strings
- Outer region of moduli space: problems!
- "central" region:
  - stabilization
  - interesting cosmology

String Theories and 11D SUGRA



HW=11D SUGRA/I^1
MS1=11D SUGRA/S^1

String Moduli Space



String universality ?

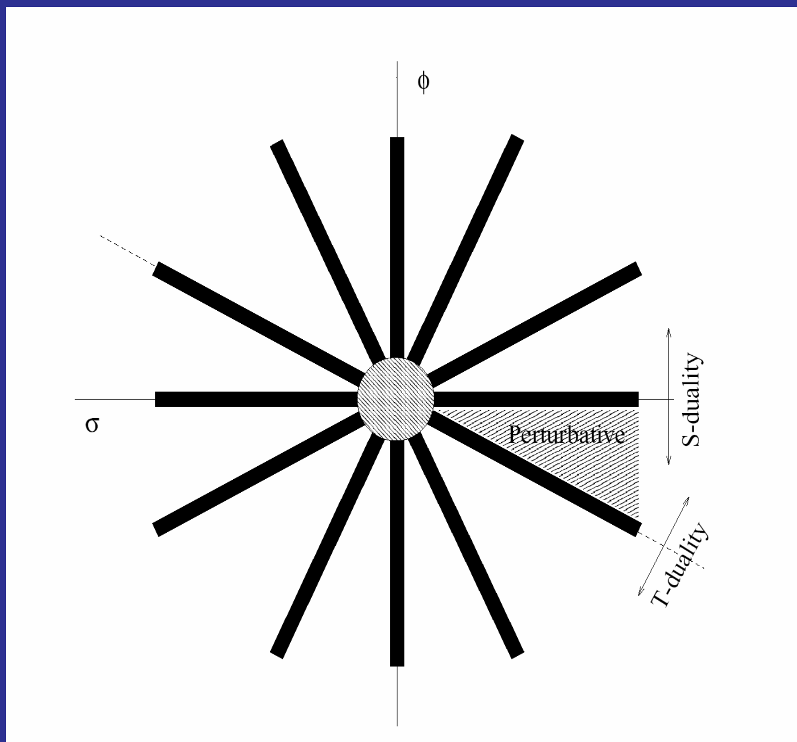
<u>Requirements</u>

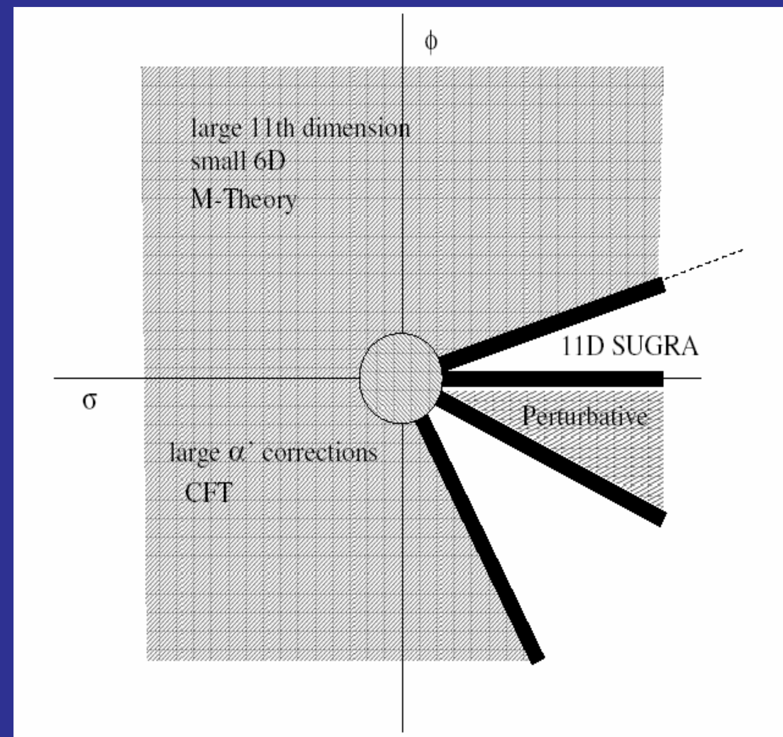
- D=4
- N=1 SUSY ➔ N=0
- $CC<(m_{3/2})^4$
- SM (will not discuss)
- Volume/Coupling moduli
  T S

Perturbative theories =
<u>phenomenological disaster</u>

- **SUSY+msless moduli**
- **Gravity ≠ Einstein's**
- **Cosmology**

Cosmological moduli space





$$ds^2_{10} = G_{MN}dx^M dx^N \;=\; e^{\frac{1}{2\sqrt{3}}\sigma+\frac{1}{2}\phi}(e^{-\frac{2}{\sqrt{3}}\sigma}g_{\mu\nu}dx^\mu dx^\nu + g^c_{mn}dy^m dy^n)$$

$$ds^2_{11} = e^{\frac{1}{2\sqrt{3}}\sigma-\frac{1}{6}\phi}(e^{-\frac{2}{\sqrt{3}}\sigma}g_{\mu\nu}dx^\mu dx^\nu + g^c_{mn}dy^m dy^n) + e^{\frac{4}{3}\phi}dz^2$$

“Lifting Moduli”

- Perturbative
  - Compactifications
  - Brane Worlds
- Non-Perturbative
  - SNP = Brane instantons
  - Field-Theoretic, e.g., gaugino-condensation
- Generic Problems
  - **P**ractical **C**osmological **C**onstant **P**roblem
  - Runaway potentials (not solved by duality)

BPS Brane-instanton SNP's

From hep-th/0002087

Euclidean wrapped branes

Potential V~e$^{-\text{action}}$

Complete under duality

$$D5 : V \sim e^{-\left[e^{\frac{\sqrt{3}}{2}\sigma + \frac{1}{2}\phi}\right]}$$

$$D1 : V \sim e^{-\left[e^{\frac{1}{2\sqrt{3}}\sigma - \frac{1}{2}\phi}\right]}$$

	Type I		HO	
	brane	action	brane	action
	$D5$	$\frac{1}{g_I}\left(\frac{R}{l_I}\right)^6$	$F5$	$\frac{1}{g_{HO}^2}\left(\frac{R}{l_{HO}}\right)^6$
	$D1$	$\frac{1}{g_I}\left(\frac{R}{l_I}\right)^2$	$F1$	$\left(\frac{R}{l_{HO}}\right)^2$
	HE		HW	
	brane	action	brane	action
	$F5$	$\frac{1}{g_{HE}^2}\left(\frac{R}{l_{HE}}\right)^6$	$M5$ transverse	$\left(\frac{R}{l_{11}}\right)^6$
	$F1$	$\left(\frac{R}{l_{HE}}\right)^2$	$M2$ longitudinal	$\left(\frac{R}{l_{11}}\right)^2\frac{\rho}{l_{11}}$
	IIA		MS1	
	brane	action	brane	action
	$D0$	$\frac{1}{g_{IIA}}\frac{R}{l_{IIA}}$	KK graviton	$\frac{R}{\rho}$
	$F1$	$\left(\frac{R}{l_{IIA}}\right)^2$	$M2$ longitudinal	$\left(\frac{R}{l_{11}}\right)^2\frac{\rho}{l_{11}}$
	IIA		IIB	
	brane	action	brane	action
L	$D0$	$\frac{1}{g_{IIA}}\frac{R_{IIA}}{l_{II}}$	$D-1$	$\frac{1}{g_{IIB}}$
T	$D0$	$\frac{1}{g_{IIA}}\frac{R}{l_{II}}$	$D1$	$\frac{RR_{IIB}}{g_{IIB}l_{II}^2}$
L	$F1$	$\frac{RR_{IIA}}{l_{II}^2}$	KK MM	$\frac{R}{R_{IIB}}$
T	$F1$	$\frac{R^2}{l_{II}^2}$	$F1$	$\frac{R^2}{l_{II}^2}$

Outer Region

Moduli – chiral superfields of N=1 SUGRA,

N=1 SUGRA

$$V = e^{-K}\left(F_i K^{i\bar{j}} F_{\bar{j}} - 3|W|^2\right)$$

$K=K(S,S^*)$, $W=W(S)$

$$F_S = \partial_S W + K_S W$$

Pert. Kahler $K=-ln(S+S^*)$

$$V(S,S^*) = (S+S^*)F(S,S^*)F^*(S,S^*) - \frac{3}{S+S^*}W(S)W^*(S^*)$$

Steep potentials

$$\frac{|(S+S^*)\partial_S^{(n+1)}W|}{|\partial_S^n W|} \gg 1, \quad n=0,1,2,3.$$

$W(S)=\Sigma_i e^{-\beta_i S}$, with Re $\beta_i \gg 1$,

e.g: $ReS = e^{-2\phi}\sqrt{\det G_c}$

$\Lambda<0$

Outer Region Stabilization?

$$\partial_S V = (S+S^*)\partial_S^2 W(S) F^* - \frac{2}{S+S^*} F W^*,$$

$$\partial_{S^*} V = (S+S^*)\partial_{S^*}^2 W^*(S^*) F^* - \frac{2}{S+S^*} F^* W.$$

Extremum:

$$(S+S^*)^2 \partial_S^2 W(S) F^* = 2 F W^*$$

$$F(S,S^*) = \partial_S W(S) - [1/(S+S^*)] W(S)$$

Two types:

(ii) $F \neq 0 \quad \text{and} \quad (S+S^*)^2 \partial_S^2 W(S) F^* = 2 F W^*$

(i) $F = 0.$

Min?, Max?, Saddle?

$$H = \frac{\partial^2 V}{\partial S_R \partial S_R} \frac{\partial^2 V}{\partial S_I \partial S_I} - \left(\frac{\partial^2 V}{\partial S_R \partial S_I} \right)^2$$

$$= -4(\partial_{SS} V \partial_{S^*S^*} V - \partial_{SS^*} V \partial_{S^*S} V).$$

Case (i)

$$H = 4\left((S+S^*)|\partial_S^2 W|^2 - \frac{2}{(S+S^*)^3}|W|^2 \right)^2 - \frac{4}{(S+S^*)^2}|\partial_S^2 W|^2 |W|^2.$$

$$\partial^2_{S_R S_R} V \approx 2(S+S^*)|\partial_S^2 W|^2 > 0.$$

Case (i) is a minimum

Case (ii)

$$H \approx -4(S+S^*)^2 |\partial_S^3 W|^2 |F|^2,$$

Case (ii) is a saddle point

In general, max or saddle, but never min !

Outer Region Cosmology:Slow-Roll?

• Without a potential: 4D, 5D, 10D, 11D : "fast-roll"

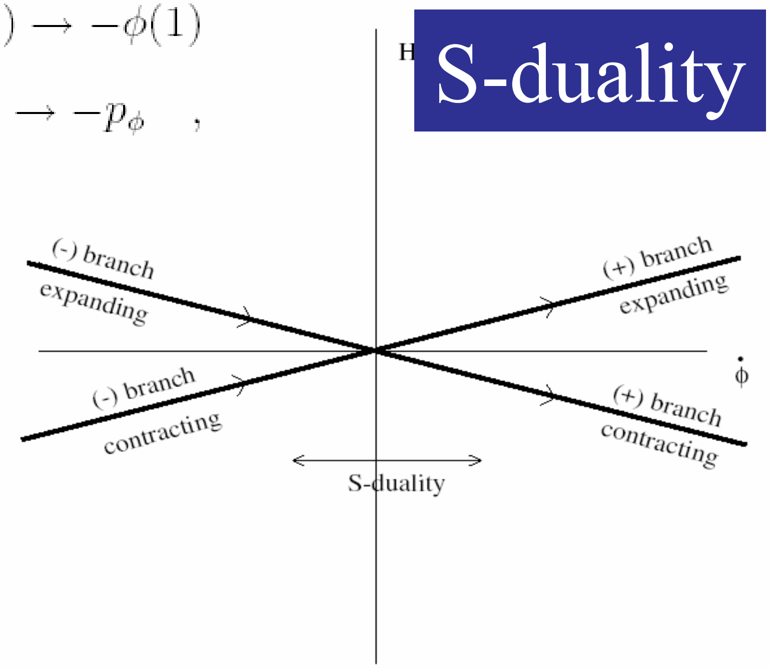
$$a(t) = a(1)|t|^{\frac{1}{3}}$$
$$\phi(t) = \phi(1) + p_\phi ln|t|$$
$$\sigma(t) = \sigma(1) + p_\sigma ln|t|$$

5D – same solutions!

$$ds_5^2 = C_5^2|t|^{-\frac{1}{2\sqrt{3}}p_\sigma - \frac{1}{2}p_\phi}(-dt^2 + a^2(1)|t|^{\frac{2}{3}}dx^i dx^i) + C_5^{-4}|t|^{\frac{1}{\sqrt{3}}p_\sigma + p_\phi}dz^2$$
$$\sigma_5 = \frac{\sqrt{3}}{2}\sigma(1) - \frac{1}{2}\phi(1) + (\frac{\sqrt{3}}{2}p_\sigma - \frac{1}{2}p_\phi)\ln|t|$$
$$C_5 = e^{-\frac{1}{4\sqrt{3}}\sigma(1) - \frac{1}{4}\phi(1)} .$$

$$\phi(1) \to -\phi(1)$$
$$p_\phi \to -p_\phi \quad ,$$

S-duality



T-duality

$$p_\phi \to -\frac{1}{2}p_\phi - \frac{\sqrt{3}}{2}p_\sigma$$
$$p_\sigma \to -\frac{\sqrt{3}}{2}p_\phi + \frac{1}{2}p_\sigma \quad ,$$

- With a potential

Ansatz

$$a(t) = a(1)|t|^{p_a}$$
$$\psi_1(t) = \psi_1(1) + p_1 \ln|t|$$
$$\psi_2(t) = \psi_2(1) + p_2 \ln|t|$$

Solution

$$\gamma p_1 = -2$$
$$p_a^2 = \frac{1}{6}\left(\frac{1}{2}p_1^2 + \frac{1}{2}p_2^2 + Ae^{\gamma\psi_1(1)}\right)$$
$$p_a = \frac{1}{2}\left(\frac{1}{2}p_1^2 + \frac{1}{2}p_2^2\right)$$
$$-p_1 + 3p_a p_1 + A\gamma e^{\gamma\psi_1(1)} = 0$$
$$-p_2 + 3p_a p_2 = 0 \quad .$$

Use to find properties of solutions with real potential

$$V(\phi, \sigma) = Ae^{\alpha\sigma + \beta\phi}$$

$$\psi_1 = \frac{\alpha\sigma + \beta\phi}{\sqrt{\alpha^2+\beta^2}} \qquad \psi_2 = \frac{-\beta\sigma + \alpha\phi}{\sqrt{\alpha^2+\beta^2}}$$

$$\gamma = \sqrt{\alpha^2 + \beta^2}$$

$\Longrightarrow$ $$p_a = \frac{1}{\gamma^2}$$

realistic steep potential

$$V'/V \sim 1/g_s, 1/g_s^2, \; V'/V \sim R/l_s$$

No slow-roll for real steep potential

Central Region

Our proposal:

- Parametrization with D=4, N=1 SUGRA
- Stabilization by SNP effects @ string scale
- Continuously adjustable parameter
- SUSY breaking @ lower scale by FT effects
- PCCP o.k. after SUSY breaking

VADIM: CAN YOU HAVE A CONTINOUSLY ADJUSTABLE PARAMETER THAT IS NOT A MODULUS? ARE 2 AND 3 CONSISTENT
OFER: KACHRU ET AL CENTRAL REGION.

Stable SUSY breaking minimum

Two Moduli, S (susy breaking direction), T (orthogonal) , $m_{3/2}/M_P=\varepsilon \sim 10^{-16}$

(a) a minimum of the potential at (S_0, T_0): $\partial_S V|_{\min} = 0$, $\partial_T V|_{\min} = 0$

(b) with broken SUSY: $F_S|_{\min} \sim 0(\varepsilon)$

(c) and a small cosmological constant: $V|_{\min} < 0(\varepsilon^2)$

(d) in the central region: $ReS_0, ReT_0 \sim 1$.

In addition, for the SUSY preserving direction T we have

(e) $F_T|_{\min} = 0$.

$$V = e^{-K}\left(F_i K^{i\bar{j}} F_{\bar{j}} - 3|W|^2\right) \qquad (1)$$

$$F_S = \partial_S W + K_S W \; ; F_T = \partial_T W + K_T W. \qquad (2)$$

$$K^{S\bar{T}}|_{min} = 0. \qquad (3)$$

(b),(c),(e) & (2) → $W|_{\min},\ \partial_S W|_{\min},\ \partial_T W|_{\min} \sim O(\varepsilon)$ (4)

(a),(b),(e) & (2,3,4) →

$$\partial_T F_S|_{\min},\ \partial_S \partial_T W|_{\min},\ \partial_S F_T|_{\min} \sim O(\varepsilon) \qquad (5)$$

With more work

$$W|_{\min},\ \partial_S W|_{\min},\ \partial_T W|_{\min},\ \partial_S^2 W|_{\min},$$
$$\partial_{ST}^2 W|_{\min},\ \partial_S^3 W|_{\min} \sim O(\varepsilon).$$

- Higher derivatives in S (> 3) and T (> 1), & mixed derivatives of order > 2 generically O(1).
- In SUSY limit, in T direction, V is steep, all derivatives > 2 generically O(1) @ min. In S direction, potential is very flat around min.
- Masses of SUSY breaking S moduli o(ε) in general masses of T moduli O(1).

Simple example

$$W_{SNP} = a_4(S - \widetilde{S}_0)^4$$

$$\Delta W_{FT} = \sum b_i e^{-\beta_i S}$$

$$\Delta W_{FT} = \sum c_i e^{-\beta_i (S - S_0)}, \text{ so } c_i \sim O(\varepsilon)$$

$$W = a_4(S - S_0)^4 + a_3(S - S_0)^3 + a_2(S - S_0)^2 + a_1(S - S_0) + a_0$$

$$a_4 \sim O(1),\ a_3 = -\tfrac{1}{6} \sum c_i \beta_i^3,\ a_2 = \tfrac{1}{2} \sum c_i \beta_i^2,\ a_1 = -\sum c_i \beta_i$$

$$a_1 = (\pm\sqrt{3K_{S\bar{S}}} - K_S) a_0.$$

→Reasonable working models,

→Additional SUSY preserving Λ<0 minima!

Scales & Shape of Moduli Potential

- The width of the central region

In effective 4D theory:
kinetic terms multiplied by $M_S^8 V_6$ ($M_{11}^9 V_7$ in M).
Curvature term multiplied by same factors
"Calibrate" using 4D Newton's const. $8\pi G_N = m_p^{-2}$

$$\Gamma = \frac{1}{2}\int d^4x \left\{ m_p^2 \sqrt{-g} R + m_p^2 \partial\psi \partial\psi \right\}$$

➔Typical distances are $O(m_p)$

$$W \approx M_S^3 \Rightarrow V \approx M_S^6 / m_p^2 \equiv \Lambda^4$$

NO VOLUME FACTORS!!!

Banks

Numerical examples:

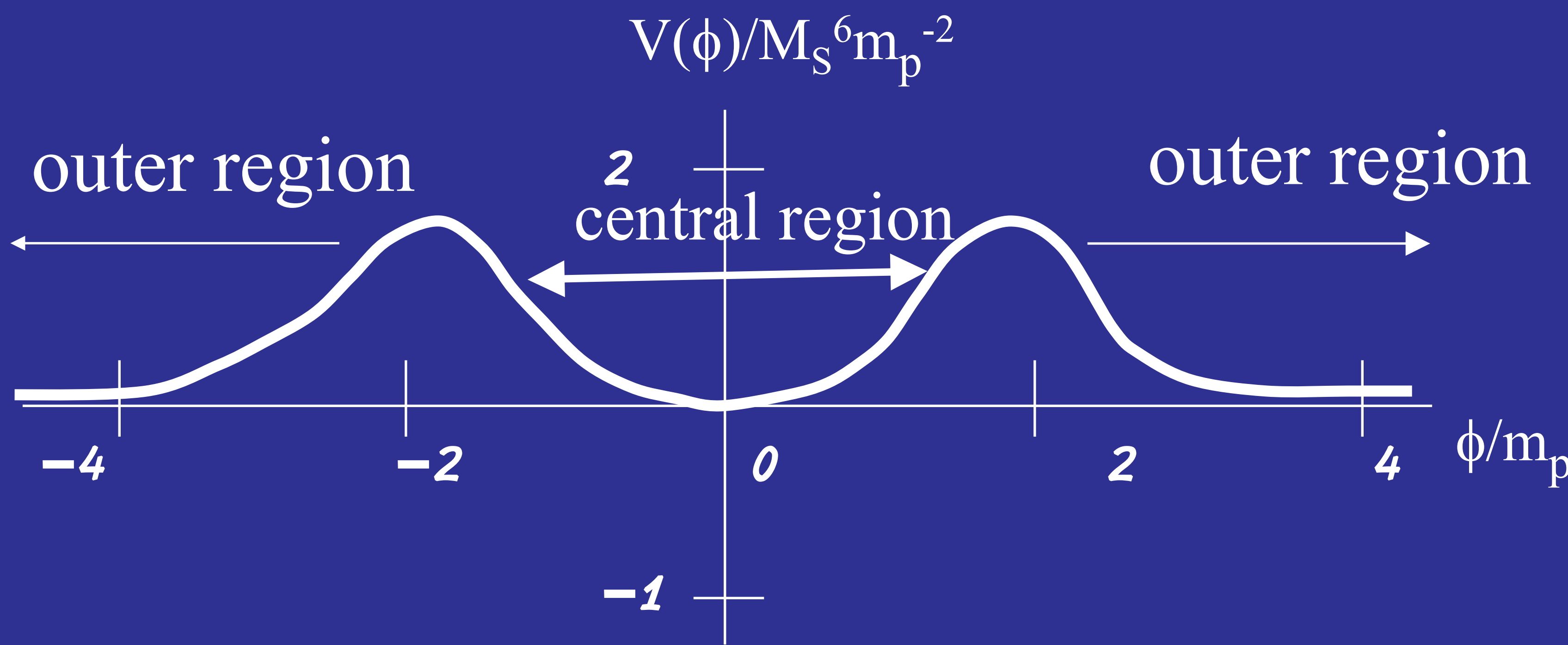
$$\Lambda_I = 8.6 \times 10^{16}\, GeV \left(\frac{\alpha_{YM}}{1/25}\right)^{3/4} g^{3/4}$$

$$\Lambda_{HW} = 7.6 \times 10^{16}\, GeV \left(\frac{\alpha_{YM}}{1/25}\right)^{-1/4} (4 M_{GUT} V_6^{1/6})^{-3/2}$$

$$\Gamma = \int d^4x \left\{ \tfrac{1}{2} m_p^2 \partial\psi \partial\psi - \Lambda^4 V(\psi) \right\}$$

$$= \int d^4x \left\{ \tfrac{1}{2} \partial\phi \partial\phi - \Lambda^4 V(\phi / m_p) \right\}$$

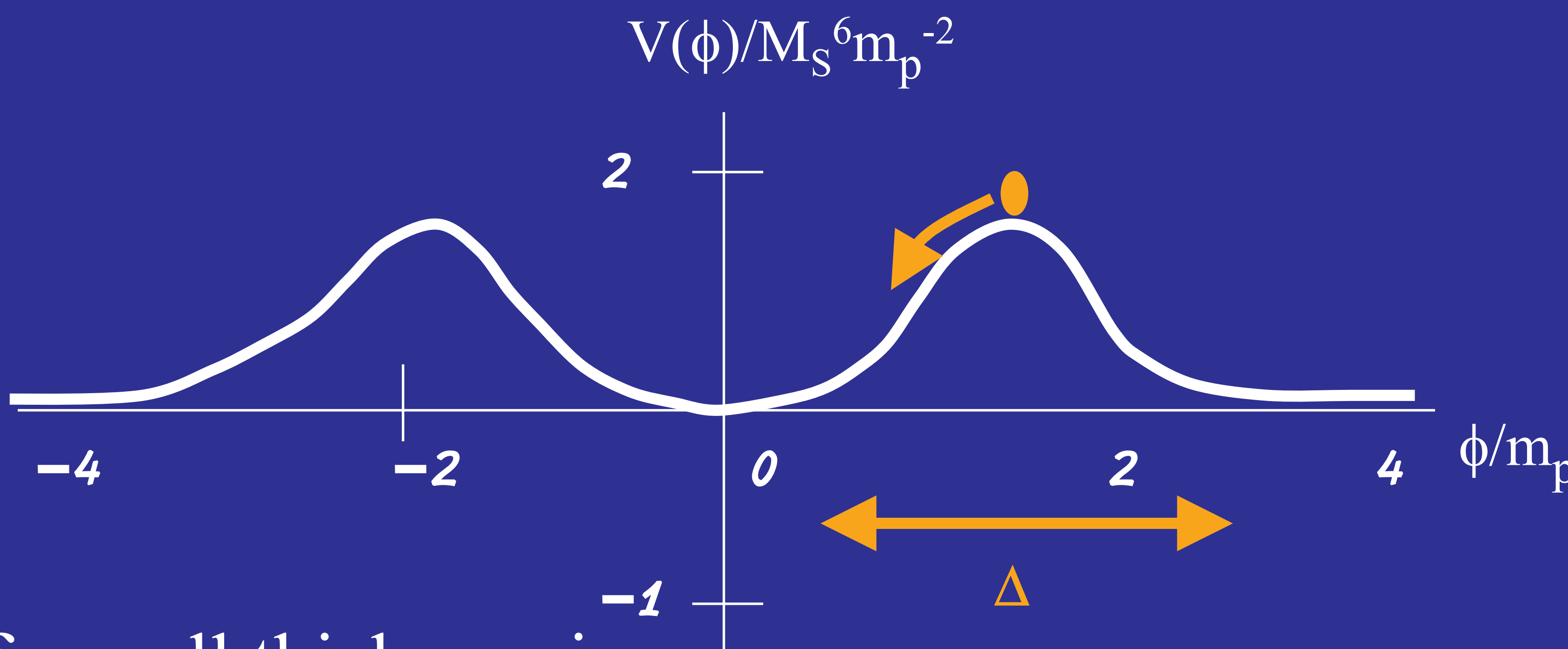
- The shape of the potential



zero CC min. & potential vanishes @ infinity ➔ intermediate max.

Inflation: constraints & predictions

- Topological inflation



δ – wall thickness in space

$(\Delta/\delta)^2 \sim \Lambda^4$

$H^2 \sim 1/3\ \Lambda^4/m_p^2$

Inflation ⇔ $\delta H > 1$ ⇔ $\Delta > m_p$

CMB anisotropies and the string scale

Slow-roll parameters $\varepsilon \sim 2\frac{m_p^2}{\mu^4}(\phi-\phi_{\max})^2, \eta \sim V''(\phi_{\max})$

The "small" parameter $\mu^2 = \frac{2m_p^2}{|V''(\phi_{\max})|}$

Number of efolds $N(\phi,\phi_{end}) = \frac{1}{\sqrt{2}m_p}\int_{\phi}^{\phi_{end}}\frac{1}{\sqrt{\varepsilon(\phi)}}d\phi$

Sufficient inflation $\phi_{init}-\phi_{\max} \cong \mu\, Exp\left(-\left[120(m_p/\mu)^2\right]\right)$

Qu. fluct. not too large $\phi_{init}-\phi_{\max} \succ \frac{H}{2\pi} \Rightarrow V''(\phi_{\max}) \prec 1/6$

$$\Lambda^2 \cong 6.5\times10^{16}\,GeV\left(|V''(\phi_{\max})|/2\right)^{1/4} Exp\left(-\left[25|V''(\phi_{\max})|\right]\right)$$

For consistency need |V''|~1/25

For our model

$$\varepsilon_{CMB} \propto (V'/V)^2 \sim 0$$
$$\eta_{CMB} \cong V''(\phi_{\max}) < 0$$

$$n_S = 1 - 4\varepsilon_{CMB} + 2\eta_{CMB}$$
$$r = 13.7\varepsilon_{CMB}$$

→→

$$n_S = .92 - .08(25|V''(\phi_{\max})| - 1)$$
$$r \cong 0$$

1/3 < 25|V''| < 3 →

$$.76 \prec n_S \prec .97$$
$$r \cong 0$$

✓ WMAP

If consistent:

$$M_S \cong 1.7\times 10^{17}\, GeV \left(\frac{P_\varsigma^{1/2}}{10^{-4}}\right)^{1/3} (1 - n_S)^{-1/6} e^{-\frac{25}{3}(1-n_S)}$$

Summary and Conclusions

- Stabilization and SUSY breaking
  - Outer regions = trouble
  - Central region: need new ideas and techniques
  - Prediction: “light” moduli
- Consistent cosmology:
  - Outer regions = trouble
  - Central region:
    - scaling arguments
    - Curvature of potential needs to be “smallish”
  - Predictions for CMB