Advertising and Economic Welfare: Comment

By Oded Hochman and Israel Luski*

In a recent note, Len Nichols (1985), using the advertising model developed by George Stigler and Gary Becker (S-B) in their seminal work (1977), evaluates the efficiency of a market economy with advertising. However, his analysis is not entirely accurate. In this paper we remedy these inaccuracies and present a complete and correct analysis of the efficiency conditions in S-B's economy with advertising and identify the market allocations which fulfill these conditions.

In addition, we analyze S-B's model to derive results, beyond what Nichols and S-B did, and thus bring forward more fully the economy's market behavior.

The main results of our comment: A perfectly competitive commodity market with advertising constitutes a first-best solution and the monopolistic competition allocation is inefficient.

We show that a positive relationship exists between the firm's level of advertising, the firm's product output, and the firm's commodity output. This relationship is determined entirely by production characteristics, and has nothing to do with demand. Changes in demand affect advertising only due to changing the equilibrium output and with it the quantity of advertising.

We show, also, that in a perfectly competitive commodity market where the price of the commodity is constant the price of the product observable in the market may vary from firm to firm. The larger the firm, the higher its product price, its level of advertising, and its product output. These distinctive market features, which until now were used to characterize monopolistic competition, are shown here to typify perfect competition in the commodity market, which is also Pareto efficient.

Furthermore we show that the more concentrated a competitive industry is and the larger the firms in it—the more advertising is used and output produced per firm, than would have been in the case of firms in a less concentrated industry. Less product and more advertising per unit of commodity are used and the product prices are higher in a more concentrated industry even though it is perfectly competitive and operates efficiently.

Thus Stigler and Becker have unknowingly provided economic theory with a model which restores the efficiency properties to the invisible hand of retail markets, which this invisible hand was believed not to possess, since the 1930s—a decade during which Edward Chamberlin and Joan Robinson developed the theory of monopolistic competition to explain the operation of exactly such retail markets with advertising.

Analysis of the efficiency conditions and market allocations follows this section. Then we deal with the characteristics of different market allocations.

I. Efficiency and Market Allocations

We follow S-B's model and notations. We assume a population with homogeneous tastes, that is, all individual households have the same utility function $u(z, y)$, where $z$ is a commodity consumed by the household and produced by it via a household production function using as inputs the product $x$, purchased in the market, and advertising $A$ provided to all consumers by the producers of the particular brand of $x$ being consumed. Let $x$ and $z$ designate the quantities of the product and the commodity, respectively, consumed by a household and $X$ and $Z$ the quantities produced by an individual firm. The quantity of advertising $A$ produced by the firm is also the quantity consumed by each of its customers. Thus, as in S-B, equation (15), the household production function

*Department of Economics and The Monaster Economic Research Center, Ben-Gurion University, Beer-Sheva, Israel.
is

\[ z = g(A)x. \]

There are several brands of \( z \) in the market, the product \( x \) of each brand being produced by its own firm which also purchases the required advertising and provides it free of charge to its own consumers. However, the consumers do not prefer one brand of \( z \) over another, all other conditions being identical. The variable \( y \) stands for a second consumption good provided by a competitive industry and consumed without advertising.

Consider two customers, \( i \) and \( k \), of a given brand of \( z \). Then, as in the neoclassical welfare literature,

\[ RCS_{z,y}^i = RCS_{z,y}^k, \quad \text{for all } i \neq k \]

is a Pareto efficiency condition. Here \( RCS_{z,y}^i \) stands for the rate of substitution in consumption between \( z \) and \( y \) by household \( i \). If (2) is not fulfilled, the two individuals could exchange \( z \) with \( y \) between themselves, and by doing so raise the utility levels of both. Therefore (2) is necessary for Pareto efficiency. Since the two individuals consume the same brand, both are exposed to the same advertising and a transfer of \( z \) from one to the other involves only a transfer of \( x \). Hence (2) also implies

\[ RCS_{x,y}^i = RCS_{x,y}^k. \]

Indeed (3) follows from (2) upon noting that advertising \( A \) is the same for the two individuals, and that

\[ \frac{\partial u}{\partial x} = g(A)\left(\frac{\partial u}{\partial z}\right). \]

It should be noted again that relations (2) and (3) are efficiency conditions only if the two individuals are consuming the same brand of \( z \). If they are consuming different brands, no such substitution in the margin of \( z \) by \( y \) can take place.

Let \( P_x \) be the market price of \( x \) and \( \pi_z \) the derived price of \( z \). Then

\[ P_x \cdot x = \pi_z \cdot z = \pi_z g(A) \cdot x, \]

which in turn implies

\[ \pi_z = P_x / g(A). \]

In competition, utility-maximizing individuals equate marginal rate of substitution in consumption to the price ratio. Thus,

\[ \frac{\partial u}{\partial x} / \left(\frac{\partial u}{\partial y}\right) = \frac{P_x}{P_y}, \]

and since \( P_x/P_y \) is identical to all individuals purchasing the same brand, (7) implies (3), and with it (2).

Consider now the production of \( z \). The quantity of \( Z \) produced by the firm can be changed in the margin either by changing \( X \) or by changing \( A \). Let \( L \) be a production factor, say labor, used in the production of both \( A \) and \( X \). Then, to increase \( Z \) we can allocate an additional unit of \( L \) either to increase the production of \( X \) or to raise the level of \( A \). If we allocated an additional unit of \( L \) to the production of \( A \), and keep \( X \) unchanged, the increase in \( Z \), which can be termed the marginal product of labor in the production of \( z \) through \( A \), is

\[ (dz/dL)_{dA=0} = MP_L(A) \cdot g'(A) \cdot X, \]

where \( MP_L(A) \) stands for the marginal product of \( L \) in the production of \( A \). If we allocated an additional unit of \( L \) to the production of \( X \), and kept \( A \) unchanged, the increase in \( Z \), termed the marginal product of \( L \) in the production of \( Z \) through \( X \), will be

\[ (dz/dL)_{dX=0} = g(A) \cdot MP_L(X). \]

A Pareto efficiency condition is, that the firms in the economy operate so that the marginal products of \( L \) in the production of \( z \) through \( x \) and through \( A \), are the same. In mathematical notation the above condition is

\[ (dz/dL)_{dA=0} = (dz/dL)_{dX=0}. \]

\[ \text{Equations (8a) and (8b) are obtained from equation (1) by differentiation.} \]

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and after substituting (8a) and (8b) into (9) we obtain

\[ g(A)MP_L(X) = MP_L(A)g'(A) - X. \]  

That (9) and therefore (10) are efficiency conditions we prove by showing that if equality does not hold in (9) and (10) an inefficiency exists. Let us assume, as a working hypothesis, that inequality holds in (9) and (10), say the left-hand side (LHS) of (9) and (10) is larger than their respective right-hand sides (RHS). Then by shifting a unit of \( L \) from the production of \( A \) to the production of \( x \), we change the total amount of \( z \) by the (positive by assumption) difference between the LHS and the RHS of (10) and thus increase the total amount of \( z \) without employing any additional inputs. This in turn implies that when inequality holds in (10) (and in (9)), we are not at Pareto optimum, which proves that equality in (9) and (10) is a necessary condition for such an optimum.

The left- and right-hand sides of (9) and (10) can now be termed the marginal product of \( L \) in the production of \( z \) of a given firm. Let \( W \) be the price of the production factor \( L \). By substituting in the RHS of equation (18') in S-B (p. 86), \( W/MPL(X) \) for \( MC(X) \), and then multiplying the resulting RHS and the middle term in the chain (18') there by \( MPL(X) \) we obtain

\[ P_xMP_L(X)(1 + 1/\varepsilon_x) = W, \]  

where \( \varepsilon_x \) is the demand elasticity of \( z \) and \( x \).

In much the same way we substitute \( W/MP_L(A) \) for \( P_A \) in (19') in (S-B) and multiply both sides of the equation by \( MP_L(A) \) to obtain

\[ \pi_xg'MP_L(A)X(1 + 1/\varepsilon_y) = W, \]  

since the LHS of both (11) and (12) equal \( W \) they also equal each other, hence

\[ P_xMP_L(X)(1 + 1/\varepsilon_y) = \pi_xg'X(1 + 1/\varepsilon_y)MP_L(A). \]

By utilizing \( \pi_x = P_x/g(A) \), eliminating terms from both sides of (11) and rearranging terms we get equation (10) of the text above.

The fact that equation (10) follows from equation (18') and (19') in S-B for all values of \( \varepsilon_y \) implies first that the allocation of resources by the firm to \( x \) and \( A \) in the production of \( z \) is efficient whether the firm operates in competitive or monopolized markets. Second, that the quantity of advertising used in the production of \( z \) is determined solely by production conditions and is not effected by demand elasticity beyond the effect of demand on the quantity of \( z \) produced. Thus a given quantity of \( z \) will be produced by given quantities of \( A \) and \( x \) regardless of demand elasticity.²

We have shown above that condition (10), being a production efficiency condition within the firm, holds under any market arrangement as long as the individual firm is maximizing profits.

Another necessary efficiency condition is one requiring equality between the rate of production transformation (RPT) of the commodity \( z \) and the good \( y \), and the rate of substitution in consumption (RCS) of the same two goods. In equational form this condition is as follows.

\[ RCS = \frac{\partial u(z, Y)}{\partial z} / \frac{\partial u(z, y)}{\partial y} = \frac{MP_L(y)}{g(A)MP_L(X)} = RPT. \]

It should be noted that the above relation is restricted to customers and production of a given brand of \( z \). The rationale for relation

²In their paper Stigler and Becker comment that “... the optimal level of advertising would be positively related to the commodity elasticity” (p. 86). Obviously, demand elasticity has no direct effect on advertising and it only effects advertising through its effect on the quantity of \( Z \) produced.

By equating the RHS of equation (7) in Nichols to zero, and performing proper substitutions, we again obtain equation (10) of the above text. Thus Nichols' interpretation of his equation (7) and its efficiency implications are not valid.
(14) is that, if (14) does not hold but rather an inequality holds in (14), we could by shifting factors from the production of \( z \) to the production of \( y \) (or vice versa) substitute \( z \) with \( y \) through production so that the added gain in utility due to the additional quantity of the increased good exceeds the loss in utility due to the reduction in quantity of the second good.\(^3\)

Returning to the market allocation, since \( \frac{\partial u}{\partial x} = (\frac{\partial u}{\partial z}) \cdot g(A) \), from (7) we obtain

\[
\frac{\partial u}{\partial x} = \frac{(\partial u/\partial z) g(A)}{P_x} = \frac{P_x}{P_y},
\]

hence

\[
\frac{\partial u}{\partial z} = \frac{P_x}{P_y} = \frac{\pi_z}{\pi_y}.
\]

Since \( y \) is produced by a competitive industry the following holds,

\[
\frac{P_y MP_L(y)}{W} = W.
\]

By substituting, for \( W \) in (16), the LHS of (11), then substituting \( \pi_y g(A) \) for \( P_x \), and rearranging terms we obtain

\[
\frac{\pi_z}{\pi_y} = \frac{MP_L(y)}{g(A) MP_L(x)(1+1/\epsilon_{\pi_z})}.
\]

By substituting (17) in (15), we obtain that in the market the following holds,

\[
\frac{MP_L(y)}{g(A) MP_L(x)(1+1/\epsilon_{\pi_z})} = \frac{(\partial u/\partial z)}{(\partial u/\partial y)}.
\]

Upon comparing (18) with the efficiency condition (14) we learn that the market solution fulfills this efficiency requirement if and only if,\(^4\)

\[
\epsilon_{\pi_z} = \epsilon_{\pi_y} = -\infty,
\]

which is a necessary condition for the efficiency of a market allocation.\(^5\)

Thus, as in traditional economic theory, efficiency is attained only if producers are product price takers, and this time, commodity price takers as well.

A situation in which customers are linked to a brand and cannot change it costlessly may occur for example, when different brands are produced in different geographical areas, or when changing brands involves a costly adjustment process of facilities, knowledge, habits, etc. In these situations, as in classical microeconomic theory, efficiency is unlikely to occur since demand elasticity facing producers is finite, and (19) is not fulfilled.

In many cases, however, changing brands by customers is easy and involves little or no cost, like changing the brands of a soft drink, changing the make of a car, or changing an airline, etc. When individuals can move freely from one brand of \( z \) to another, the price of \( z \) facing producers is fixed, and the demand elasticity facing each producer is \( -\infty \). Furthermore, in this case all producers are facing the same commodity price \( \pi_z \).

It should be noted that the same commodity price to two firms does not necessarily imply the same product price. The proof and implications of this fact are discussed fully below.

The assumption of free mobility of customers from one brand to another leads us to the inter-firm, intra-industry efficiency condition

\[
MP_L(Z_i) = MP_L(Z_k),
\]

keeping in mind, that from (10), \( MP_L(Z_i) \) is

\(^3\)This argument is identical to the accepted argument concerning the necessity for efficiency, of having equality between RCS and RPT. This can be found in any welfare economics textbook. Therefore, no further elaboration on this topic is offered.

\(^4\)See S-B, p. 86, fn. 15, for proof that \( \epsilon_{\pi_z} = \epsilon_{\pi_y} \).

\(^5\)This condition has not been observed by Nichols to be consistent with efficiency. Instead he states that efficiency is attained when the market for \( x \) is not perfectly competitive, that is, \( \epsilon_{x} > -\infty \) (\( E_x, \pi \) in Nichols' notation, see bottom p. 216, ibid., Case I)
uniquely defined for each brand \( i \) and that \( i, k, i^* k \) vary over all possible brands of the commodity \( z \), each produced by a different firm.

The rationale of (20) is as follows: Suppose that, instead of equality, inequality holds in the above equation (20). Then we could shift a customer from the brand with low-marginal product to a brand with high-marginal product and shift production factors in the same direction so that the production of the commodity by the low-marginal product firm should decline by exactly the amount consumed by the shifted customer. Then the high-marginal product firm will produce, by using the added production factors, additional quantity of its brand of the commodity, which is, because of its higher MPL, larger than the quantity reduced from the output of the low-marginal product firm. Therefore, the utility of the shifted customer will increase due to the shift, while the utility of all other customers in the economy remain unchanged. This implies that an allocation in which inequality holds in (20) can be improved and hence is not optimal. This, in turn, implies that equality in (20) is a necessary condition for Pareto efficiency.

For condition (20) to hold in a market allocation what is needed is for the firm to be a commodity price taker, (which is also required for the fulfillment of (14)), and for the commodity price \( \pi_z \) to be fixed and equal to all firms. This is proved by substituting in equation (17) \( e_{m} = -\infty \) and noting that \( \pi_z \) is the same for all \( z \)-producing firms, equation (20) now follows immediately from (17). This in turn implies \( g(A_i)MP_i(X_i) = g(A_k)MP_k(X_k) \), for all \( i \neq k \), which in turn implies (20).

The existence of many firms in the economy, each producing the same commodity and competing among themselves for the same customers indeed guarantees the same commodity price to all firms, and by it guarantees efficiency.

In summary in this section we have derived a full set of independent efficiency conditions\(^6\) of S-B's economy, this being equations (2), (10), (14), (20). These conditions are fulfilled if and only if the economy, including the commodity market, is perfectly competitive, that is, in the advertised industry all firms are commodity price takers. When firms are facing finite demand elasticities in some of the good and/or in commodity markets, efficiency is not attained.\(^7\)

II. Characterization of Solutions

First, we consider two firms of our advertised industry, operating in the same competitive market, having different production functions and therefore each producing different output levels. Both firms are facing the same commodity price, still the larger of the two firms (the one producing more \( x \)) will provide more advertising to its customers, a unit of its commodity will consist of more advertising and less of the product and its product price will be higher.

To prove the above assertion, let \( F_1 \) and \( F_2 \) be the two production functions of the two above-mentioned firms. Without loss of exhaust a mutually independent set of necessary conditions to the general optimization problem of S-B's economy. Thus they constitute a full set of first-best conditions.

\(^6\)Nichols' Case I (ibid., p. 216) cannot exist. On one hand he assumes that \( e_{m} = -\infty \) and on the other hand he assumes that the demand price \( \pi_z \) (\( P_z \) in his notation) fulfills \( \partial \pi_z / \partial A = 0 \). These two assumptions contradict each other. To prove that, we carry out the following derivation.

\[
\partial \pi_z / \partial A = \left( d\pi_z / dz \right) \left( dz / dA \right) = \left( d\pi_z / dz \right) g'(A)x = \frac{(\pi_z g'(A)x)}{[e_{m}(g(A))]}.
\]

It should be noted that \( e_{m} = e_{m} \); see fn. 4. Since \( P_z \) and \( g'(A) \) are all finite, it follows that \( \partial \pi_z / \partial A \) can vanish if and only if \( e_{m} \) is infinite. Thus Nichols' Cases I and II cannot exist. Therefore, out of the four market situations Nichols mentions only two exist. Thus Nichols' Cases III and IV exist. Indeed, these are the two cases investigated by S-B and in our paper. Thus, only Cases III and IV of Nichols really exist, yet he does not study them and instead concentrates on the nonexisting cases.
generality, assume that the marginal product of firm 1 is larger than that of firm 2, when both are producing the same quantity of \( X \); that is, \( MP_1(X) > MP_2(X) \), for all positive \( X \).

Now consider equations (10) and (20), both of which are fulfilled in perfect competition. Let \( MP(Z) \) be the common marginal product of the commodity \( Z \), then,

\[
(21) \quad MP(Z) = g(A_1)MP_1(X_1) = g(A_2)MP_2(X_2) = g'(A_1)X_1 = g'(A_2)X_2,
\]

where \( A_i \) and \( X_i \) are respectively the advertising and product of firm \( i \).

The solution to (21) must fulfill the conditions

\[
(22) \quad X_1 < X_2 \text{ and } A_1 < A_2,
\]

any other relations will lead to a contradiction. For example, suppose \( X_1 = X_2 \), then the last equality in the chain above implies that \( A_1 = A_2 \). This in turn implies that the second equality in the chain above cannot hold since we assumed \( MP_1(X_1) = MP_2(X_2) \) while \( g(A_1) = g(A_2) \), a contradiction.

Thus, if we assume\(^8\) \( F'' < 0 \) and \( g'' < 0 \), the two following relations must hold \( g'(A_1) > g'(A_2) \) and \( MP_1(X_1) > MP_2(X_2) \), which leads to (22). Thus a larger firm is producing more output as well as using more advertising. Therefore a unit of the commodity \( z \) produced by the larger firm contains less of the product and more advertising than a unit of commodity produced by a smaller firm (the amount of \( x \) per unit \( z \) for firm \( j \) is \( X_j/Z_j = 1/g(A_j) \)). The market price, \( P_x \), of the product of the larger firm is higher as well. To see that, note that \( P_x = \pi_z \cdot g(A) \), and since \( \pi_z \) is the same for all firms and \( g(A_1) < g(A_2) \) the result is straightforward.

Second, we consider our industry with different concentrations. We argue that the amount of concentration in a competitive industry also affects the ratios of \( X \) and \( A \) in the production of \( Z \). An industry with high concentration has few large firms instead of many small ones in an industry with low concentration. An increase in a firm's size implies both an increase in advertising by the firm and in the output level of the firm.\(^9\) This in turn leads to an increase in product prices and a decrease in quantity of product per unit commodity as shown above.

All of the above results characterize firms facing finite demand elasticities as well. The only certain way, therefore, to distinguish between a perfectly competitive industry and one engaged in monopolistic competition is to find out whether in each firm in the industry product price equals marginal cost of production. In the case of monopolistic competition this price is above marginal cost, and in perfectly competitive commodity market, product price equals marginal cost of production, even though both the product price and marginal cost may vary from one firm to another.

Since Chamberlin and Robinson, a market in which each firm sells its product at a difference price and uses advertising is believed to be engaged in monopolistic competition and therefore to operate inefficiently. Stigler and Becker (1977) provide us with a

\[\text{\textsuperscript{8}}\text{If any or both assumptions did not hold, then we would be in the realm of economies of scale, in which case efficiency and market conditions lead to a single-operating firm in the industry. The market allocation is a single-monopolistic firm which operates inefficiently.}\]

\[\text{\textsuperscript{9}}\text{We assume constant marginal product in advertising when factor price does not change (due to RCS in the advertising industry). Then differentiating (10) totally and rearranging terms we obtain}\]

\[
\frac{dA/dX}{g' \cdot MP(X) - MP(A) \cdot g'' \cdot X} > 0,
\]

where \( g \) is \( g(A) \) and \( g' = dg(A)/dA \) and \( g'' = d^2g(A)/dA^2 \). The inequality in the above equation follows when diminishing marginal product is assumed in the production of \( x \), \( d(MP_1(X))/dX < 0 \) and in the household production of \( z \) with respect to \( A \), \( g'(A) < 0 \).
model which utilizes the "new theory of consumption," advanced by both Becker and Kelvin Lancaster (1971) in the 1960s and 1970s to bring us back to the realm where the invisible hand of the market operates efficiently.

REFERENCES


