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Glass phases of flux lattices in layered superconductors

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Abstract. – We study a flux lattice which is parallel to superconducting layers, allowing for dislocations and for disorder of both short wavelength and long wavelength. We find that the long-wavelength disorder of strength $\tilde{\Delta}$ has a significant effect on the phase diagram —it produces a first-order transition within the Bragg glass phase and leads to melting at strong $\tilde{\Delta}$. This then allows a Friedel scenario of 2D superconductivity.

The phenomena of melting of the flux lattice and the influence of disorder are of considerable interest [1]. Weakly disordered samples reveal ordered flux arrays [2], [3], consistent with the "Bragg glass" phase of an elastic dislocation-free theory [4]-[6]. The Bragg glass phase exhibits algebraic decay of the translational order and the existence of divergent Bragg peaks. It was argued [5], [6] that the Bragg glass phase is stable against formation of dislocations in a finite range of the phase diagram.

Melting has been observed [2], [3], [7]-[10] as either a transition into a flux liquid phase or into a glass phase with a higher critical current. The latter transition occurs at low temperatures and is therefore driven by disorder. A model for melting, allowing for both disorder and dislocations [6], [11], was recently studied. The model considers flux lines parallel to and confined between superconducting layers and allows for dislocations. This model was studied without disorder [12] leading to flux melting at a critical temperature $T_{\rm c}$ which is about a factor ~ 2 from the solution of the more fundamental model in terms of superconducting phases [13], [14] (the latter model allows also for flux loops and overhangs). Since disorder has drastic effects on melting, we expect that the simplified model in terms of flux displacements yields an adequate description of the phase diagram.

The solution of Carpentier, Le-Doussal and Giamarchi (CLG) [11] has shown explicitly that short-wavelength disorder combined with dislocations leads to melting at a finite value of the disorder strength. CLG have used Replica Symmetry Breaking (RSB) methods as well as Renormalization Group (RG). They have also shown [11], [15] that this melting is compatible with a Lindemann criterion.

In the present work we allow for an additional term in the CLG model. This term is generated by RG within the CLG model and leads to significant effects on the phase diagram. Using RSB methods, we show that long-wavelength disorder of strength $\tilde{\Delta}$ leads to a first-order transition within the Bragg glass phase and as $\tilde{\Delta}$ increases it leads to melting. We find that the $\tilde{\Delta}$-induced melting is inconsistent with a universal Lindemann criterion. Finally we consider the quest for the Friedel scenario [16] in which a layered superconductor becomes a set of decoupled two-dimensional (2D) superconductors. This scenario fails in pure superconductors [17], [18], but is possible with some constraints in parallel fields [14] and in special models [19]. With disorder which affects interlayer coupling the Friedel scenario becomes feasible in the presence of a melted flux array.

The model [6], [11], [12] consists of layers with interlayer spacing l where modulation in the flux line density couples to a random potential. We consider a Hamiltonian with two types of random potentials,

$$H = \int \mathrm{d}^2 r \sum_i \left[\frac{c}{2}(\nabla \Phi_i(\mathbf{r}))^2 - \eta_i(\mathbf{r})\nabla\Phi_i(\mathbf{r}) - \mu\cos(\Phi_i(\mathbf{r}) - \Phi_{i+1}(\mathbf{r})) - \right.$$
$$\left. - 2\mathrm{Re}(\zeta_i(\mathbf{r})e^{i\Phi_i(\mathbf{r})}) \right] \tag{1}$$

with Gaussian disorder correlations $\langle\zeta_i(\mathbf{r})\zeta_j(\mathbf{r}')\rangle = 4Tg\delta_{i,j}\delta(\mathbf{r}-\mathbf{r}')$ and $\langle\eta_i(\mathbf{r})\eta_j(\mathbf{r}')\rangle = T\Delta\delta_{i,j}\delta(\mathbf{r}-\mathbf{r}')$, where T is the temperature. Here $\Phi_i(\mathbf{r})$ stands for in-plane displacement of the vortex line in the i-th layer, c is an in-plane elastic constant, g measures disorder with Fourier component $\approx 2\pi/a$ where a is the flux periodicity parallel to the layers, while Δ measures long-wavelength disorder [5]. For point defects with scale much shorter than a (*e.g.*, oxygen vacancies) we expect comparable magnitudes of Δ and ga^2. The μ term is the coupling between layers which allows for dislocations. Melting is achieved when the renormalized value of μ vanishes, *i.e.* melting is here a 3D-2D transition in which interlayer correlations are lost while each layer maintains its correlations as an elastic medium.

It is important to note that the model eq. (1) assumes the presence of superconducting layers which do not allow displacements of the flux lines perpendicular to them. Thus, melting as found from eq. (1) is consistent at low temperatures where superconductivity in the layers is maintained, *i.e.* a Friedel scenario. In particular, in the pure system [12] of eq. (1) thermal fluctuations lead to melting at $T_\mathrm{c} = 4\pi c$; this, however, is inconsistent since the superconducting layers become normal [13], [14] at a lower temperature. We therefore look for a disorder-induced melting at temperatures $T \ll T_\mathrm{c}$, where eq. (1) is a consistent description.

Dimensional Imry-Ma arguments are useful to check the stability of an ordered phase which is a d-dimensional elastic medium. In a domain of size L the elastic energy is $\propto L^{d-2}$, the short-wavelength disorder (after averaging the square) is $\propto L^{d/2}$, while the long-wavelength disorder is $\propto L^{(d-2)/2}$. Thus short-wavelength disorder is relevant at $d < 4$, while the long-wavelength disorder is marginal only at $d = 2$, *i.e.* the latter is consistent with long-range order in $d = 3$.

To average over disorder we start with the replicated version of the Hamiltonian (eq. (1)) which includes all relevant terms generated by renormalization,

$$H = \int \mathrm{d}^2 r \left\{ \frac{c}{2}\sum_{i,a}[(\nabla\Phi_i^a(\mathbf{r}))^2 - \mu\cos(\Phi_i^a(\mathbf{r}) - \Phi_{i+1}^a(\mathbf{r}))] - \right.$$
$$- \sum_{i,a,b}\left[\frac{\Delta}{2}\nabla\Phi_i^a(\mathbf{r})\nabla\Phi_i^b(\mathbf{r}) + \gamma\cos(\Phi_i^a(\mathbf{r}) - \Phi_{i+1}^a(\mathbf{r}) - \Phi_i^b(\mathbf{r}) + \Phi_{i+1}^b(\mathbf{r})) + \right.$$
$$\left.\left. + g\cos(\Phi_i^a(\mathbf{r}) - \Phi_i^b(\mathbf{r}))\right]\right\}, \tag{2}$$

where $a = 1 \ldots n$ is the replica index. Note in particular the γ term which was not considered by GLC; this term is generated in second-order RG from the μ term and we include it here as an additional interaction parameter. Since it couples different replicas it can lead to RSB, *i.e.* this term leads to distinct phenomena and should be included in the full Hamiltonian.

We consider the variational free energy $F_{\text{var}} = F_0 + \langle H - H_0 \rangle$ with

$$H_0 = \frac{1}{2} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \int_{-\pi}^{\pi} \frac{\mathrm{d}q_z}{2\pi} G_{ab}^{-1}(\mathbf{q}, q_z) \Phi_i^a(\mathbf{q}, q_z) \Phi_i^b(-\mathbf{q}, -q_z) , \tag{3}$$

where the Greens' function $G_{ab}^{-1}(\mathbf{q}, q_z)$ is determined by an extremum condition of F_{var} and $\mathbf{q}, q_z$ are Fourier variables for $\mathbf{r}$ and i, respectively.

Defining the inverse Green's function in the form $G_{ab}^{-1}(\mathbf{q}, q_z) = \delta_{ab} G_0^{-1}(\mathbf{q}, q_z) - \sigma_{ab} - \Delta q^2$ with $\sum_a \sigma_{ab} = 0$ and $\sigma_{ab} = 2(1 - \cos q_z)\sigma_{ab}^{\gamma} + \sigma_{ab}^{g}$, we obtain the self-consistent equations in the form

$$G_0^{-1}(q, q_z) = cq^2 + 2\tilde{\mu}(1 - \cos q_z) , \tag{4}$$

$$\sigma_{ab}^{g} = 2g \exp\left[-T \sum_{\mathbf{q}, q_z} [G_{aa}(\mathbf{q}, q_z) - G_{ab}(\mathbf{q}, q_z)]\right] , \tag{5}$$

$$\sigma_{ab}^{\gamma} = \gamma \exp\left[-T \sum_{\mathbf{q}, q_z} (1 - \cos q_z)[G_{aa}(\mathbf{q}, q_z) - G_{ab}(\mathbf{q}, q_z)]\right] , \tag{6}$$

$$\tilde{\mu} = \mu \exp\left[-T \sum_{\mathbf{q}, q_z} (1 - \cos q_z) G_{aa}(\mathbf{q}, q_z)\right] . \tag{7}$$

When $\tilde{\mu}$, the renormalized coupling between layers, vanishes, it signals a 2D phase, *i.e.* correlations in the z-direction are lost and the flux lattice has melted.

We study the full RSB solution of the saddle-point equations (5)-(7). The method of RSB [20] employs a representation of hierarchical matrices such as $\sigma_{ab}^{g,\gamma}$ in terms of functions $\sigma_{g,\gamma}(u)$ with $0 < u < 1$.

We define two order parameters for RSB, $m(\mu) = \tilde{\mu} + u\sigma_{\gamma}(u) - \int_0^u \sigma_{\gamma}(v)\mathrm{d}v$ and $w(u) = [u\sigma_g(u) - \int_0^u \sigma_g(v)\mathrm{d}v]/2m(u)$. Using the inversion formula [20] for G_{ab}, integrating cq^2 up to a high cut-off Λ ($\Lambda \gg m(u)$, $m(u)w(u)$) and differentiating eqs. (5), (6) with respect to u, we obtain two coupled differential equations:

$$\begin{aligned} \frac{2\tilde{T}}{u} \frac{\mathrm{d}m}{\mathrm{d}u} &= \frac{\mathrm{d}}{\mathrm{d}u} \left[\frac{m\rho(1 + w + \rho)}{\rho(1 + w + \rho) + Q(w + \rho)} \right] , \\ \frac{\tilde{T}}{u} \frac{\mathrm{d}m}{\mathrm{d}u}(Q + w) &= \frac{\mathrm{d}}{\mathrm{d}u} \left[\frac{m\rho(Q + w)}{Q + \rho} \right] . \end{aligned} \tag{8}$$

Here $Q(u) = m(\mathrm{d}w/\mathrm{d}u)/(\mathrm{d}m/\mathrm{d}u)$, $\rho(u) = [w(u)(w(u) + 2)]^{1/2}$, and $\tilde{T} = T/T_{\mathrm{c}}$ with $T_{\mathrm{c}} = 4\pi c$.

A general solution of these equations is rather difficult, so at first we consider special limits. When $\gamma = 0$ we recover the CLG solution [11] which exhibits a 3D Bragg glass phase with $\langle [\Phi_i(r) - \Phi_i(0)]^2 \rangle \sim \ln r$, *i.e.* positional correlations decay algebraically and long-range order is weakly destroyed. The Bragg glass phase undergoes a continuous melting transition (for $\Delta = 0$) at $g/\mu = 2/e\tilde{T}$ as shown in the $\tilde{\Delta} = 0$ plane of fig. 1 (where $\tilde{\Delta} = T\Delta/4\pi c^2$); for $\Delta \neq 0$ the transition becomes first order. Thus, the Bragg glass phase, due to both disorder and dislocations, melts into a 2D phase with $\tilde{\mu} = 0$.

Consider next the case $g = 0$, hence $w(u) = 0$; the solution in this case is formally similar to that of a 2D disordered Josephson junction [21]. Equation (8) then yields $(1 - 2\tilde{T}/u)m'(u) = 0$,

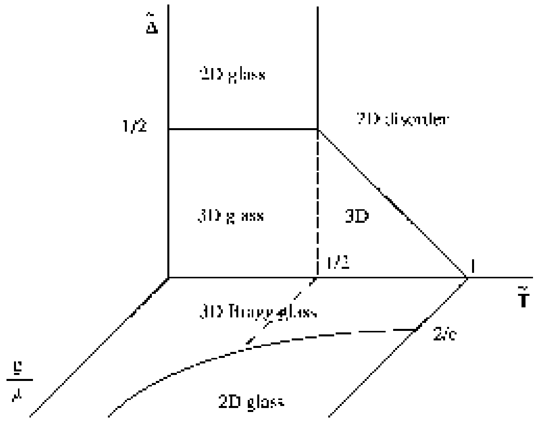

Fig. 1. – Phase diagram of layered flux lattices with disorder of short wavelength (with strength g) and of long wavelength (with strength $\tilde{\Delta}$). The dashed line is the line of first-order transition (discontinuity of the interlayer coupling and of the glass order); the spaced dashed line is its approximate extension to $g \neq 0$. The 3D glass regime at $g = 0$ has long-range order, while the 3D Bragg glass at $g \neq 0$ has algebraically decaying positional order.

i.e. $m(u)$ is a one-step function, with the step at $u = 2\tilde{T}$. Since $u < 1$ the onset of this solution is at $\tilde{T} = 1/2$, *i.e.* at $T = T_c/2$ eqs. (6), (7) determine the jump in $[\sigma_\gamma](u)$ from zero ($u < 2\tilde{T}$) to a value σ_γ^0 at $2\tilde{T} < u < 1$, where

$$\frac{\tilde{\mu} + \sigma_\gamma^0}{\Lambda} = e^{-1}\left(4e\tilde{T}\frac{\gamma}{\Lambda}\right)^{1/(1-2\tilde{T})}, \tag{9}$$

$$\frac{\tilde{\mu}}{\Lambda} = e^{-1}\left[e^{\tilde{\Delta}+1/2}\left(4e\tilde{T}\frac{\gamma}{\Lambda}\right)^{-1/2}\frac{\mu}{\Lambda}\right]^{1/(1/2-\tilde{\Delta})}. \tag{10}$$

This solution is valid for $\tilde{\Delta} < 1/2$ and $\tilde{T} < 1/2$, since weak coupling, $\gamma \sim \mu \ll \Lambda$ (*i.e.* strong anisotropy), is assumed. Near the $\tilde{T} = 1/2$ transition $\tilde{\mu}$ is finite, while $\tilde{\mu} + \sigma_\gamma^0$ vanishes; thus $\sigma_\gamma^0 < 0$ is finite up to $\tilde{T} = 1/2$ and vanishes at $\tilde{T} > 1/2$, *i.e.* the transition is of first order. When $\tilde{\Delta} > \tilde{T}$ within this phase σ_γ^0 changes sign and becomes positive.

The phase at $\tilde{\Delta} < 1/2$ and $\tilde{T} < 1/2$ is an unusual coexistence phase —it has both long-range order ($\tilde{\mu} \neq 0$) and glass order ($\sigma_\gamma^0 \neq 0$). (As noted above, this is consistent with the Imry-Ma argument). At $\tilde{\Delta} = 1/2$, we find a disorder-driven transition where $\tilde{\mu}$ vanishes continuously, leading to a 2D glass phase at $\tilde{\Delta} > 1/2$.

We also note that a replica-symmetric solution is possible with $\tilde{\mu}/\Lambda \sim (\mu/\Lambda)^{1/(1-\tilde{T}-\tilde{\Delta})}$, *i.e* $\tilde{\mu} \neq 0$ for $\tilde{T} + \tilde{\Delta} < 1$, $\tilde{T} > 1/2$ as shown in fig. 1. Comparison with eqs. (14), (15) shows that $\tilde{\mu}$ is also discontinuous at the $\tilde{T} = 1/2$ transition.

Finally we consider the case where both g and Δ are finite. We can demonstrate the existence of a first-order transition at small g by showing a coexistence of two solutions. The first solution is an expansion near the $g = 0$ solution, *i.e.* $w(u) \ll 1$ with $[\sigma_g] = O(g)$, $[\sigma_\gamma] = O(1)$; for $u < \tilde{T}/2$ the solution for $[\sigma_g]$ is similar to the $\gamma = 0$ case, *i.e.* $[\sigma_g](u) \sim u^2$ for small u, consistent with Bragg glass correlations. This solution is valid (assuming $\mu \sim \gamma$) if

$$g/\Lambda \ll (\gamma/\Lambda)^{\frac{1-3\tilde{\Delta}+2\tilde{T}\tilde{\Delta}}{(1-2\tilde{T})(1-2\tilde{\Delta})}}, \tag{11}$$

i.e. for weak coupling $g/\Lambda, \gamma/\Lambda \ll 1$ this expansion breaks down close to the transitions at $\tilde{T} = 1/2$ and $\tilde{\Delta} = 1/2$. The second solution is an expansion around the $\gamma = 0$ solution with $[\sigma_\gamma](u) \ll \tilde{\mu}$. This leads to $[\sigma_g] = O(g^2)$, $[\sigma_\gamma] = O(g)$ and is valid for $\tilde{\Delta} < \tilde{T}$. Thus for small g there is a two-solution regime which implies a first-order transition at some $\tilde{T} \lesssim 1/2$. We indicate this transition by a spaced dashed line in fig. 1, though we do not know its precise location.

As shown in fig. 1, we find that the main feature of the CLG scenario is valid —for small disorder the Bragg glass is stable, while at large disorder, which can have either short- or long-wavelength, dislocations are enhanced by disorder and lead to melting.

These analytic results for melting allow us to test the Lindemann criterion, which is of common use [1]. For the $\gamma = 0$ case, CLG consider a Lindemann criterion of the form [11], [15] $\langle [\Phi_{i+1}(\mathbf{r}) - \Phi_i(\mathbf{r})]^2 \rangle = c_{\mathrm{L}}^2$, with average done in the elastic limit, *i.e.* the cosine of the μ term in eq. (1) is expanded. This criterion leads [11] to a reasonable value of $c_{\mathrm{L}} \lesssim 1$. For the $g = 0$ case an elastic limit leads to an expansion of both the μ and γ terms in eq. (2), so that RSB is not induced. Since long-range order is present, the Lindemann criterion is $\langle \Phi_i^2(r) \rangle = c_{\mathrm{L}}^2$; however, the replica-symmetric solution yields $\langle \Phi_i^2(r) \rangle = (\tilde{\Delta} + \tilde{T}) \ln(\Lambda/\mu)$, *i.e.* at melting $c_{\mathrm{L}}^2 \approx \ln(\Lambda/\mu)$; since Λ/μ depends on the anisotropy of the system, the Lindemann number c_{L} is non-universal.

In order to relate the phase diagram to the actual magnetic field B we need to identify c by the elastic constants [1], [22] which are dispersive, $c_{44}^{\|} \approx c_{11}^{\|} = (B^2/4\pi)/(1 + \lambda_{\mathrm{c}}^2 q^2 + \lambda^2 q_z^2)$; here λ, λ_{c} are penetration lengths and $\epsilon = \lambda/\lambda_{\mathrm{c}} < 1$ is the anisotropy ($c_{44}^{\|}$ has a smaller second term which is neglected here). The behavior near melting is dominated by $q \to 0$ and $q_z \approx 1/l$ (recall that 3D-2D melting involves only the interlayer periodicity l rather than the periodicity a parallel to the layers). The lattice periodicities satisfy $l = a\epsilon$ if $l > d$ for weak fields, *i.e.* $B = \phi_0/a^2\epsilon < \phi_0\epsilon/d^2$ (d is the spacing of the superconducting layers, which is the lower bound on the interlayer spacing l of the flux lattice, and ϕ_0 is the flux quantum), or $l = d$ for strong fields, $B = \phi_0/ad > \phi_0\epsilon/d^2$. By rescaling the x, z coordinates, we identify

$$\begin{aligned} 4\pi c &\approx a\phi_0^2/(4\pi^2\lambda\lambda_c) \sim B^{-1/2}\,, && B < \phi_0\epsilon/d^2\,, \\ 4\pi c &\approx d\phi_0^2/(4\pi^2\lambda^2)\,, && B > \phi_0\epsilon/d^2\,. \end{aligned} \tag{12}$$

T_{c} $(= 4\pi c)$ is smallest for large fields, but even then the onset of 2D superconductivity [13], [14] is below T_{c}, *i.e.* T_{c} is not a consistent description of 2D-3D melting. This reflects the result of the more fundamental model (in terms of superconducting phases) for which the Friedel scenario, in the pure case, usually fails. Thus we focus on the disorder-induced melting at $T \ll T_{\mathrm{c}}$ which is also relevant to experimental data.

Since Δ couples to $\nabla\Phi_i(r)$ [5], it is B-independent so that $\tilde{\Delta} = \Delta/4\pi c^2 \sim B$ for small fields and $\tilde{\Delta} \sim$ constant for strong fields. Thus the Δ-induced melting can be induced by increasing the magnetic field if $\tilde{\Delta} = 1/2$ is achieved for weak fields. On the other hand, for the g-induced melting [5] $g \sim 1/a^2$, and by using the c_{66} elastic constant [23] we identify $\mu = a^2\phi_0 B\epsilon^3/[4\pi^2 l(8\pi\lambda)^2]$. For weak fields the melting temperature is $T_{\mathrm{m}}^{-1} = eg/(8\pi c\mu) \sim B^{3/2}$, while for strong fields $T_{\mathrm{m}}^{-1} \sim B^3$. In this case melting is induced by increasing B for both weak or strong fields.

In conclusion we have shown that a new interaction term, generated by RG, leads to a significant role of the long-wavelength disorder. This interaction extends the CLG results to the more complex phase diagram of fig. 1. We find that the Bragg glass is stable for weak disorder of either short or long wavelength. The long-wavelength disorder induces a first-order transition within the Bragg glass phase; it also leads to melting which is inconsistent with a Lindemann criterion. The phase diagram demonstrates a Friedel scenario [16], *i.e.* the melting transition decouples the layers, while the latter maintain 2D superconductivity, at least at low temperatures and weak intralayer disorder. We propose that experiments with parallel magnetic fields can test the present theory of melting as well as test the possibility of 2D superconductivity.

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