Particle renormalizations in presence of dissipative environments

Victor Kagalovsky[1] and Baruch Horovitz[2]
[1] *Sami Shamoon College of Engineering, Beer-Sheva, 84100 Israel and*
[2] *Department of Physics, Ben Gurion university, Beer Sheva 84105 Israel*

A charged particle of mass M on a ring of radius R is coupled to a dirty metal environment. With Monte-Carlo methods we evaluate the curvature of the Aharonov-Bohm oscillations and find a quantum phase transition at a critical R_c. At low temperatures T the curvature has the form $1/M^*R^2$ with an R independent $M^* > M$ in the $R > R_c$ phase, while M^* rapidly approaches M in the $R < R_c$ phase. The approach to $T = 0$ defines diverging length scales $\sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx \frac{1}{4}$ in the large and small R phases, respectively.

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The problem of interference in presence of a dissipative environment is fundamental for a variety of experimental systems. Interference has been monitored by Aharonov-Bohm (AB) oscillations in mesoscopic rings[1,2,3] or in quantum Hall edge states[4] in presence of noise from gates or other metal surfaces . Cold atoms trapped by an atom chip are sensitive to the noise produced by the chip[5,6,7]. In particular giant Rydberg atoms are studied[8] whose huge electric dipole is highly susceptible to such noise.

An efficient tool for monitoring the effect of the environment, as proposed by Guinea[9], is to find the AB oscillation amplitude as function of the radius R of the ring. This amplitude is measured by the curvature[10,11,12] of the ground state energy E_0 at external flux $\phi_x = 0$, i.e. $1/M^*R^2 = \partial^2 E_0/\partial\phi_x^2|_0$, defining an effective mass M^*. For free particles of mass M this curvature is the mean level spacing $1/MR^2$. The particle can be coupled to a variety of environments, with three systems of particular interest: (i) a Caledeira-Legget (CL) bath[9], (ii) a charged particle in a dirty metal environment[9,13] and (iii) a particle with an electric dipole in a dirty metal environment[14]. System (i) has been studied with a large variety of methods, all showing that the AB amplitude is exponentially suppressed $\sim e^{-\pi^2\gamma R^2}$, i.e. a new length scale $\sim 1/\sqrt{\gamma}$ is generated by the coupling γ to the environment[9]. System (ii) has been studied by renormalization group (RG) methods[9,15] finding $M^* \sim R^\mu$ with a small μ, a Monte Carlo (MC) numerical method gave[13] $\mu = 1.8$ for strong coupling, while a variational scheme[14] gave $\mu = 0$. System (iii) was also studied within the variational scheme[14], leading to $\mu = 0$ as well.

In the present work we use MC methods to analyze mostly system (ii). We find that the energy cutoff used in a previous study[13] is insufficient and a higher cutoff ω_c is needed. The low T data shows a quantum critical point at R_c. At $R > R_c$ we find $M^* > M$ and R independent, i.e. $\mu = 0$, while at $R < R_c$ we find that M^* rapidly approaches M. We also find that a free particle with mass M^* reproduces at $R > R_c$ the finite T curvature at $T \ll \omega_c$. The approach to $T = 0$ defines diverging length scales $\sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx \frac{1}{4}$ in the large and small R phases, respectively. A related study shows that similar scales correspond to a dephasing process[16].

The time dependent angular position $\theta_m(\tau)$ of a particle on the ring has in general a winding number m so that $\theta_m(\tau) = \theta(\tau) + 2\pi mT\tau$ where $\theta(0) = \theta(1/T)$ has periodic boundary condition and T is the temperature. In presence of an external flux ϕ_x (in units of the flux quantum hc/e) the partition sum has the form

$$Z = \sum_m e^{2\pi i m\phi_x} \int \mathcal{D}\theta e^{-S^{(m)}}$$
$$S^{(m)} = \tfrac{1}{2}MR^2 \int_0^{1/T} \left(\frac{\partial\theta}{\partial\tau} + 2\pi mT\right)^2 d\tau +$$
$$\alpha \int_0^{1/T}\int_0^{1/T} \frac{\pi^2T^2K[\theta(\tau) - \theta(\tau') + 2\pi mT(\tau-\tau')]}{\sin^2\pi T(\tau-\tau')} \quad (1)$$

where the effect of environments, in each of the 3 cases, is[9,13,14]

$$\begin{aligned} K(z) = &\ \sin^2 z/2; & \alpha = \gamma R^2 \quad & \text{(i)} \\ = &\ 1 - [4r^2\sin^2\tfrac{z}{2} + 1]^{-1/2}; & \alpha = \tfrac{3}{8k_F^2 l^2} \quad & \text{(ii)} \\ = &\ 1 - [4r^2\sin^2\tfrac{z}{2} + 1]^{-3/2}; & \alpha = \tfrac{p^2}{e^2l^2}\tfrac{3}{8k_F^2l^2} \quad & \text{(iii)}\,. \end{aligned} \quad (2)$$

Case (i) is the CL system where γ is the coupling to a harmonic oscillator bath; case (ii) is a charge coupled to a dirty metal where k_F is the Fermi wavevector, l is the mean free path in the metal, and $r = R/l$; case (iii) is an electric dipole of strength p coupled to a dirty metal. We note that at $r \lesssim 1$ the models (ii),(iii) correspond qualitatively to the microscopic model in describing the reduction of $K(z)$ with r.

We are interested in the effect of the environment on the visibility of quantum interference as measured by the particle. As a measure of this visibility we consider the curvature of the Aharonov-Bohm oscillations

$$\frac{1}{M^*(T)R^2} = \frac{\partial^2 F}{\partial\phi_x^2}|_{\phi_x=0} \quad (3)$$

where $F = -T\ln Z$. It is useful to consider a free particle

$\alpha = 0$, for which

$$\left(\frac{M}{M^*(T)}\right)_{\alpha=0} = 2\pi^2 t \sum_m m^2 e^{-\pi^2 m^2 t} / \sum_m e^{-\pi^2 m^2 t} \equiv f(t) \quad (4)$$

where $t = 2MR^2T$. This identifies the thermal length $L_T \sim 1/\sqrt{MT}$.

In the interacting system a high energy cutoff can be identified by considering $\tau \to \tau'$ (corresponding to high frequencies ω) so that expansion of K(z) and Fourier transform yield

$$S^{(m)} \to \tfrac{1}{2}\int \frac{d\omega}{2\pi}[MR^2\omega^2 + 2\pi\alpha K''(0)|\omega|]|\theta(\omega)|^2 + (2\pi m)^2[\tfrac{1}{2}MR^2T + \alpha K''(0)]\,. \quad (5)$$

The cutoff ω_c is identified when the kinetic $\sim \omega^2$ and $\sim |\omega|$ interaction terms are comparable, i.e.

$$\omega_c = \frac{2\pi\alpha K''(0)}{MR^2}\,. \quad (6)$$

This ω_c replaces a possibly higher environment cutoff, since significant renormalizations start only below ω_c where the linear $|\omega|$ dispersion leads to $\ln\omega$ terms in perturbation theory and to the need for either RG treatment, or an equivalent variational scheme[14]. Note that $K''(0) = \frac{1}{2}; r^2; 3r^2$ in the 3 models above, hence $\omega_c = \pi\gamma/M$ in case (i), while $\omega_c \sim \alpha/Ml^2$ in cases (ii) and (iii).

For the MC numerical method we need to discretize the time axis into a Trotter number N_T of segments, i.e. the time interval of each segment is $\Delta\tau = 1/(TN_T)$. The discrete action is

$$S^{(m)} = \tfrac{1}{2}[MR^2N_TT + \alpha K''(0)]\sum_n(\theta_{n+1} - \theta_n + \frac{2\pi m}{N_T})^2 + \frac{\alpha\pi^2}{N_T^2}\sum_{n\neq n'}\frac{K(\theta_n - \theta_{n'} + 2\pi m(n-n')/N_T)}{\sin^2(\pi(n-n')/N_T)}\,. \quad (7)$$

The $\frac{1}{2}\alpha K''(0)$ term comes from the $n = n'$ interaction term by expanding $K(z)$ around $z = 0$. A key issue in our MC study is the choice of energy cutoff $1/\Delta\tau$ and the corresponding Trotter number $N_T = 1/(T\Delta\tau)$. The correct choice is such that the free kinetic term dominates over the single $n = n'$ interaction term, i.e. $N_T \gtrsim \omega_c/T$; this corresponds to an energy cutoff of ω_c, as anticipated above. A previous MC study on the charge problem[13] has chosen N_T in the range $1/t$ to $4/t$, i.e. an energy cutoff of $\approx 1/MR^2$. For large r this cutoff is much smaller than ω_c and is therefore insufficient.

Eqs. (1,3) identify $1/M^*(T)R^2 = 2\pi^2T\langle m^2\rangle|_{\phi_x=0}$ so that the MC evaluates the fluctuations in winding number $\langle m^2\rangle$ at external flux $\phi_x = 0$. The procedure is to start with some m, update θ_n at a time position n to θ'_n and accept or reject the change according to the MC rule with probability $\exp[S^{(m)}\{\theta_n\} - S^{(m)}\{\theta'_n\}]$. After the N_T points are successively updated, the winding number is

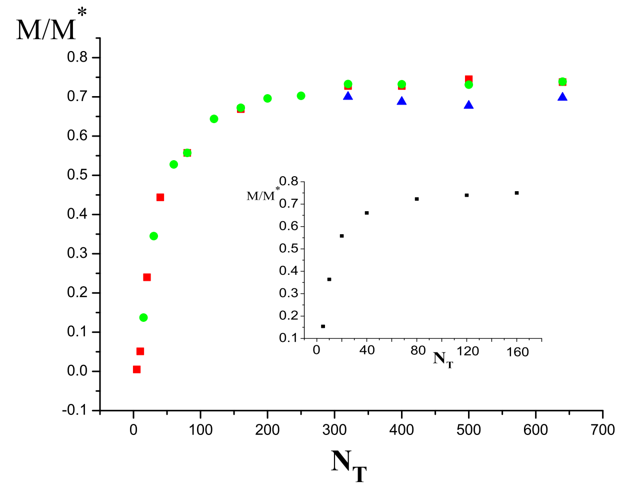

FIG. 1: Trotter number dependence of the effective mass for the dipole case with $r = 5, t = 0.2, \alpha = 0.02$, using (i) all N_T points in the double sum Eq. (7) – red squares, (ii) For points $|n - n'| > 0.03N_T$ sum is coarse grained (see text) – green circles, (iii) the whole sum is coarse grained – blue triangles. Inset: The charge case with $r = 5, t = 0.2, \alpha = 0.019$ using all N_T points in the sums.

shifted to $m' = m \pm 1$ and the shift is accepted or rejected with the probability $\exp[S^{(m)}\{\theta_n\} - S^{(m')}\{\theta_n\}]$. An update of θ_n is done randomly with a step size that produces an acceptance ratio of about 50%[11].

The inset in Fig. 1 shows the N_T dependence of M/M^* for the charge problem with $r = 5, t = 0.2, \alpha = 0.019$. A choice for N_T in the range $1/t - 4/t$ is clearly insufficient; saturation sets in around $N_T \approx 100$ which is of order of $\omega_c/T = 30$. In the following we choose our N_T, in the charge problem, to be $N_T = 40\alpha r^2/t = 10\omega_c/(\pi T)$, i.e. $N_T = 95$ for the inset parameters. For the dipole case, where ω_c is 3 times higher we choose $N_T = 120\alpha r^2/t = 10\omega_c/(\pi T)$. Fig. 1 shows that for $r = 5, t = 0.2, \alpha = 0.02$ (red squares) saturation indeed sets in near $N_T = 300$.

This high value of N_T restricts realistic MC studies. We have noticed, however, that this high N_T is necessary only in the vicinity of $n = n'$ in the double sum of (7), where the summand is rapidly varying. Hence the double sum is taken over all points only in the vicinity of the singularity, i.e. for $|n - n'| < 0.03N_T$. For points that are further separated we coarse grain the sum with fewer points, corresponding to an effective $N_T = 1/t$.

The results of this procedure are shown by the green circles in Fig. 1, and are in agreement with the full calculation that includes all N_T points. The double sum has then $\approx \frac{1}{2}10^{-3}N_T^2 + \frac{1}{2}t^{-2}$ terms, much less then the $\frac{1}{2}N_T^2$ terms of the full calculation. We also show data where the double sum is coarse grained at all points, including those near $n = n'$, by blue triangles. Here the double sum

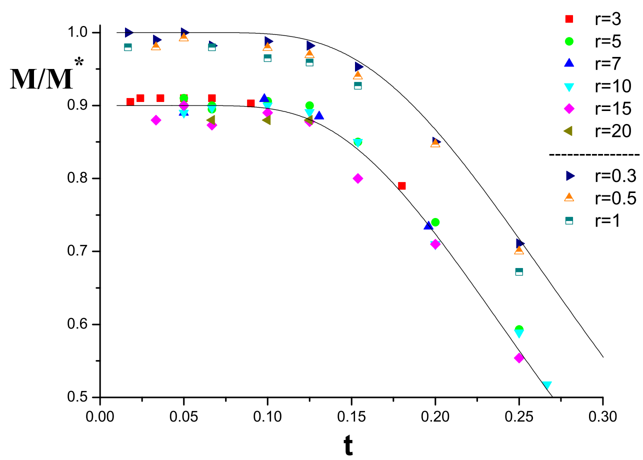

FIG. 2: AB curvature as function of reduced temperature with $\alpha = 0.019$. All $r \geq 3$ vlaues fit the renormalized form $0.9f(t/0.9)$ – the lower curve. At $r \leq 1$ the data approaches $f(t)$ of a free particles – the upper curve.

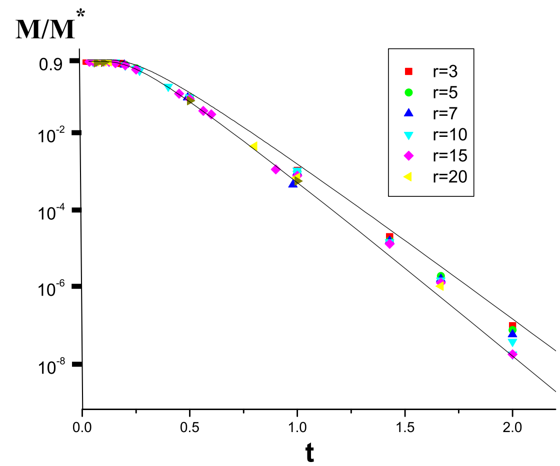

FIG. 3: AB curvature including high temperatures with $\alpha = 0.019$. All data fall in between the upper line $f(t)$ and the lower line $0.9f(t/0.9)$.

has only $\frac{1}{2}t^{-2}$ terms; this data has significant deviations from the full calculation.

We proceed to present our results on $M/M^*(T)$. At low temperatures we evaluate $\langle m^2\rangle$, and the average involves typically many values of m. To estimate errors we evaluate the correlation function for a given run and deduce a correlation length ξ. We discard the initial 10^4 MC iterations and then evaluate the standard deviation σ of the average data; the error is then[17] $\sigma\sqrt{2\xi+1}$.

We typically find a short correlation length of a few units and we run till an error of $\sim 2\%$ is achieved; the number of iterations is then $\approx (1-2)\cdot 10^5$, where each iteration is an update of N_T values of the θ_n. At high temperatures $t > 1$, where $M/M^* \lesssim 10^{-3}$, the probability of $m \neq 0$ becomes extremely small so that just $m = \pm 1$ determine the outcome[11]. Hence we evaluate $\langle m^2\rangle = 2\langle e^{S_1-S_0}\rangle_0$, averaging with e^{-S_0}. In this method we find a rather long correlation length of $\sim 10^3$, yet there is no need to vary m and a 2% accuracy can be achieved after $\approx (1-2)\cdot 10^5$ iterations.

In Fig. 2 we show our data at low temperatures, $t < 0.3$. We discuss first the data for $r \geq 3$, where we observe saturation at $M/M^* \approx 0.9$, independent of r. The possible dependence of $M^*(r)$ at $T = 0$ has been of interest as a means of monitoring anomalies in the ground state[9,13]. Previous studies proposed $M^* \sim r^\mu$ with either[9,15] a small μ or[13] $\mu = 1.8$ for $\alpha r > 1$ or[14] $\mu = 0$. Our MC shows that $\mu \lesssim 0.05$ and is consistent with the $\mu = 0$ prediction[14]. In view of the saturation of M/M^* at $3 < r < 20$ we expect this saturation to persist at higher r, though the numerics cannot exclude other assymptotic forms. The more likely $\mu = 0$ result shows that the AB curvature $\sim 1/R^2$ is the same as for free particles, i.e. the ground state has no anomaly. Furthermore, Fig. 2 shows that M^* determines the finite temperature behavior, as long as $T \ll \omega_c$. Thus if we replace $M \to M^* = M/0.9$ in Eq. (4) we obtain the lower curve $0.9f(t/0.9)$ in Fig. 2 which is a good fit to the data. The thermal length is then $L_T \sim 1/\sqrt{M^*T}$.

We turn now to discuss the $r \leq 1$ data in Fig. 2 which shows at low t saturation near $M^* = M$. In fact, in the narrow range of $1 < r < 3$ we find a continuous change of M/M^* with a critical point at $r_c \approx 2.5$. A possible interpretation is that at low r the level spacing $1/2MR^2$ becomes larger than ω_c leading to weaker renormalization at $r \lesssim 1/\sqrt{4\pi\alpha}$. This, however, does not account for a phase transition; furthermore, our preliminary data at higher α indicates a higher r_c.

In Fig. 3 we show our $r \geq 3$ data up to $t = 2$. The data falls in between two lines: $0.9f(t/0.9)$ and $f(t)$. The lower curve $0.9f(t/0.9)$ corresponds to the renormalized system and fits data with $T \ll \omega_c$, i.e. $t \ll 4\pi\alpha r^2$. For a fixed t as r decreases T approaches ω_c and the data approaches the upper curve which is the unrenormalized free particle form $f(t)$.

We therefore parameterize our data by a function $x(r,t)$ such that $M/M^* = f(tx)/x$. In the $r \geq r_c$ phase we expect $x(r,t)$ to increase towards $x = 1/0.9$ as T decreases while for $r \leq r_c$ we expect the opposite trend, i.e. x decreases into a weakly renormalized value near $x = 1$ as T decreases. Fig. 4 shows that $x(r,t)$ has a scaling form that is markedly distinct in the two phases. In the $r > r_c$ phase the scaling variable is t/r while in the in the $r < r_c$ it is r^2t. The scaling for $r \geq r_c$ is consistent with a high t expansion[13] that yields $x = 1 + 4\alpha r/(\pi t)$, though we have not tested scaling with α.

Temperature affects both renormalizations: due to the

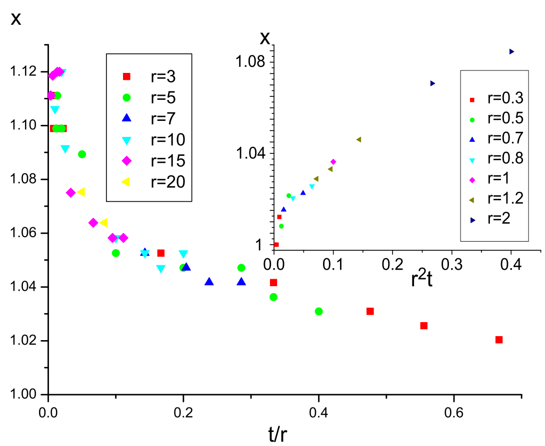

FIG. 4: Scaling of the x variable in $M/M^* = f(tx)/x$. Note the distinct scaling forms of the large and small r phases.

bath, controlled by T/ω_c, and due to the free particle spectra, controlled by $t = 2MR^2T$ via Eq. (4). To focus on the bath renormalization, it is useful to study scaling of r with t fixed, rather than with T fixed. In both phases, increasing r at a fixed t leads to a larger M^*, which is expected since more degrees of freedom become coupled. However, since the scaling variables are t/r and r^2t in the two phases, a remarkable result follows that the t dependence is opposite. In particular, at $T = 0$ the renormalization parameter x is maximal for $r > r_c$ while it is minimal (near $x = 1$) for $r < r_c$. We can interpret r_c as an unstable fixed point, with RG flow for $r > r_c$ to $r \to \infty$ leading to fully renormalized M^*, while for $r < r_c$ the flow is to $r \to 0$ (possibly α is also renormalized at very low t) leading to a weakly renormalized M^* at $T = 0$.

To support our claim for a phase transition we note that a $T = 0$ perturbation[13] in α can be extended to finite small t,

$$M/M^* = 1 - 2\alpha \sum_{n=1} a_n + 4t\alpha \sum_{n=1} a_n/n^2 \qquad (8)$$

where $K(z) = \sum_{n=1} a_n \sin^2(nz/2)$. At $T = 0$ Eq. (8) shows a crossover of M/M^* from 1 to $1 - 4\alpha$ at large r, similar to the MC data. However, M/M^* approaches its $T = 0$ value from above at all values of r, inconsistent with our data at low r.

Recalling that $t = 2MR^2T$, we conclude that the data shows length scales $r_M \sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx \frac{1}{4}$ in the large and small r phases, respectively. At scales $r > r_M$ the system approaches its $T = 0$ fixed point, which depends on the initial r. Similar length scales, with the same exponents as above, were recently identified as dephasing lengths[16].

In conclusion, we have found an unexpected phase transition between two phases of model (ii) with distinct T dependence and renormalization properties. The $r > r_c$ ground state corresponds to an r independent M^* (at least for $r < 20$) while at $r < r_c$ M^* approaches rapidly M as r decreases. The approach to these ground states is via distinct scaling laws, corresponding to length scales that diverge as $\sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx \frac{1}{4}$ in the two phases, respectively.

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