Josephson Glass and Decoupling of Flux Lattices in Layered Superconductors

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Phase transitions of a flux lattice in layered superconductors with magnetic field perpendicular to the layers and in the presence of disorder are studied. We find that the Josephson coupling between layers leads to a strongly pinned Josephson glass (JG) phase at low temperatures and fields. The JG phase is bounded by a decoupling transition line and a depinning transition line. These lines cross and form a multicritical point where four phases meet. The phase diagram accounts for unusual data on Bi$_2$Sr$_2$CaCu$_2$O$_{8+}$ such as the “second peak” transition and the recently observed depinning transitions.

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The phase diagram of layered superconductors in a magnetic field $B$ perpendicular to the layers is of considerable interest in view of recent experiments on high temperature superconductors [1]. A first order transition in YBa$_2$Cu$_3$O$_7$ (YBCO) and in Bi$_2$Sr$_2$CaCu$_2$O$_8$ (BSCCO) has been interpreted as a melting transition of the flux lattice. This first order transition terminates at a multicritical point, which for BSCCO [2,3] is at $B_0 \approx 300-10^3$ G and $T_0 \approx 40-50$ K, while for YBCO [4] it is at $B_0 \approx 2-10$ T and $T_0 \approx 60-80$ K, depending on disorder and oxygen concentration. The multicritical point also terminates a “second peak” transition [1–4] which is manifested by a sharp increase in magnetization; the transition line at “second peak” transition [1–4] which is manifested by $T_d$ and $T_g$ point, which for BSCCO [2,3] is at $B_0 \approx 300-10^3$ G and $T_0 \approx 40-50$ K, while for YBCO [4] it is at $B_0 \approx 2-10$ T and $T_0 \approx 60-80$ K, depending on disorder and oxygen concentration. The multicritical point also terminates a “second peak” transition [1–4] which is manifested by a sharp increase in magnetization; the transition line at $B = B_0$ and $T < T_0$ is weakly $T$ dependent. Neutron scattering and $\mu$SR (muon spin rotation) data [1,5] show that positional correlations of the flux lattice are significantly reduced near these phase boundaries, except, however, near the multicritical point where a reentrant behavior is observed [6]. Recent data on Nd$_{1.85}$Ce$_{0.15}$CuO$_{4-\delta}$ (NCCO) has also shown a second peak transition extending up to the superconducting transition at $T_c \approx 23$ K with no apparent multicritical point [7].

In a recent remarkable experiment Fuchs et al. [8] have shown that the phase diagram of BSCCO is much more elaborate. They show that the spatial distribution of an external current exhibits a transition from bulk pinning to surface pinning of vortices with most of the current flowing at the sample edges. This depinning line crosses the multicritical point and its temperature is almost $B$ independent at $B < B_0$. The depinning transition correlates with anomalies in vibrating reed experiments [9] and in magnetization [10]. Thus there are four phase transition lines which emanate from the multicritical point at $B_0$, $T_0$: the first order line, the second peak line, and depinning lines for both $B < B_0$ and $B > B_0$.

An extensive theoretical effort has been devoted to understanding the field-temperature ($B$-$T$) phase diagram [11] in the presence of disorder. In particular, it was proposed that at low $T$ and $B$ a Bragg glass is stable [12,13], exhibiting algebraic decay of translational order and divergent Bragg peaks [14]. Melting is expected to occur by thermal or disorder induced dislocations, as indeed demonstrated for fields parallel to the layers [15,16].

The flux lattice can undergo a transition which is unique to layered superconductors, i.e., a decoupling transition [17,18]. In this transition the Josephson coupling between layers vanishes while the lattice can be maintained by the electromagnetic (EM) coupling between layers. A disorder induced decoupling was also proposed as a crossover phenomenon [19].

The theory of Daemen et al. [18] employed the method of self-consistent harmonic approximation (SCHA) to find the decoupling temperature $T_d(B)$. The SCHA leads to a conceptual difficulty since it predicts that the Josephson coupling vanishes for all purposes at $T > T_d$. Koshelev [20] has shown that above some critical temperature the Josephson critical current vanishes; however, a finite Josephson coupling is maintained and in fact accounts for the experimentally observed plasma resonance. Thus the decoupling transition, as found by SCHA, needs to be reinterpreted.

In the present work we consider low temperature phases, i.e., below the melting temperature $T_m$ of the flux lattice, and study (i) the decoupling phase transition in a renormalization group (RG) framework and (ii) effects of disorder by employing replica symmetry breaking (RSB) methods. We find a glass phase transition $T_g$ such that for $T < T_g$ strong pinning is expected. The lines $T_d$ and $T_g$ cross and lead to four distinct phases which meet at one point in the $B$-$T$ phase diagram, remarkably close to the experimental phase diagram [1–4,8,10].

Consider a flux lattice with an equilibrium position of the $l$th flux line at $\mathbf{R}_l$. The flux line is composed of a sequence of singular points, or “pancake” vortices, whose positions at the $n$th layer can fluctuate to $\mathbf{R}_l + \mathbf{u}_l^n$. Consider the transverse part of $\mathbf{u}_l^n$ with the Fourier transform $u_T(\mathbf{q}, k)$, where $\mathbf{q}, k$ are wave vectors parallel and perpendicular to the layers, respectively. The elastic energy due to EM coupling is given by

$$\mathcal{H}_{EM} = \frac{1}{2} \sum_{\mathbf{q}, k} (d_{22})^2 \left[ c_{ss}^0 (\mathbf{q}, k) + c_{44}^0 (\mathbf{q}, k) k_z^2 \right] |u_T(\mathbf{q}, k)|^2, \quad (1)$$
where the flux line density is $1/a^2$, $d$ is the spacing between layers, $q$ is within the Brillouin zone [of area $(2\pi/a)^2$], $|k| < \pi/d$, and $k_z = (2/d) \sin(kd/2)$. The shear and tilt moduli are given (for $a \gg d$) by [21]

$$c_{66} = \frac{\pi T}{16a^2d^4}$$

and

$$c_{44}^0(k) = \left[\frac{\pi}{8a^2d^2k_z^2}\right] \ln(1 + a^2k_z^2/4\pi),$$

where $\tau = \phi^2_d/4\pi^2\lambda_{ab}^2$ sets the energy scale and $\lambda_{ab}$ is the magnetic penetration length parallel to the layers; $\tau = 10^4$–$10^9$ K for YBCO or BSCCO parameters [1].

Note that the strong dispersion of $c_{44}^0(k)$ increases the large factor $(a/d)^2$ when $k$ varies from $k \ll 1/a$ to $k \sim k < \pi/d$.

The Josephson phase between the layers $n$ and $n + 1$ at position $r$ in the layer involves contributions from a nonsingular component and from singular vortex terms.

The singular phase around a vortex at position $R_i + u_i^n = \alpha(r - R_i)$, where $\alpha(r) = \text{atan}(y/x)$ with $r = (x, y)$. We assume that all vortices belong to the flux lines, i.e., there are no free pancake antipancreate $(\rho \Psi)$ pairs which appear as relevant fluctuations only above $T_m$. The effect of the nonsingular component is a negligible $1/T$ term in the RG equation [22] while expansion of the interlayer phase difference $\alpha(r - R_i - u_i^n) - \alpha(r - R_i - u_i^{n+1})$ yields for the Josephson phase $b_n(r) = \sum_i(u_i^n - u_i^{n+1})V\alpha(r - R_i)$. The Hamiltonian is then

$$H = H_{EM} - (J/\xi_0^2) \sum_n \frac{d^2r}{d^2r} \cos b_n(r),$$

where $J$ is the interlayer Josephson coupling and $\xi_0$ is the coherence length, serving as a short distance cutoff. Since $V\alpha \sim 1/r$ decays slowly, even if $u_i^n$ are small contributions of many vortices which move in phase ($q \rightarrow 0$) leads to a divergent response of $b_n(r)$. In Fourier space, the relevant $b(q, k)$ fluctuations involve $q \ll 1/a$, where $b(q, k) = 2\pi id(e^{ikd} - 1)u_T(q, k)/q$, i.e., enhanced $q \rightarrow 0$ fluctuations.

Standard RG proceeds [22] by integrating high $q$ components leading to a new cutoff $\xi > \xi_0$ and a $\xi$ dependent coupling $J(\xi)$. The significant softening of $c_{44}^0(k)$ at $k \gg 1/a$ implies that the $k$ integration is dominated by $k \approx \pi/d$ so that the resulting $c_{66}^0[q^2/(k_z^2)] [b(q, k)]^2$ term from Eq. (1) can be replaced by an upper cutoff on the $q$ integration, $q_a = 2\ln^{1/2}(a/d)/\lambda_{ab}$. To first order in $J/T$ we obtain $J(\xi) \sim (q_a^{2\xi_0}(\xi - t)/T)$, where $t = T/T_d$ and the decoupling temperature (similar to the SCHA result [18]) is

$$T_d = 4q_a^{2\xi_0} \left[\frac{dk}{c_{44}^0(k)}\right]^{-1} \approx \frac{\tau_a^2 \ln(a/d)}{4\pi^2 \lambda_{ab}^2}.$$  

Thus for $T > T_d$ $J(\xi)$ vanishes on long scales ($\xi \rightarrow \infty$). Second order RG results in renormalization of $c_{44}^0$ and in generation of Josephson coupling between next nearest neighbors [22]. The second order terms enhance $T_d$ by a factor $(1 - \gamma J/T)^{-1}$, where $\gamma$ is a nonuniversal parameter.

The RG process shows that the decoupling transition is manifested only on long scales. Thus the thermal average of the local observable $\langle \cos b_n(r) \rangle$ remains finite at $T > T_d$, as in the $J/T$ expansion [20]. This, however, does not imply long range order in $\cos b_n(r)$ — the same $J/T$ expansion yields a power law decay for $\langle \cos b_n(r) \cos b_n(0) \rangle$ correlation, as also obtained from RG.

A hallmark feature of two-dimensional superconductivity is the ln $\rho$ dependence of a $p\bar{\Psi}$ interaction on their separation $\rho$, leading to a power law $J$-$V$ relation [23]. Probing this feature, a high temperature expansion of Eq. (2) with an added $p\bar{\Psi}$ pair leads to an effective free energy to order $J^2$

$$F_{p\bar{\Psi}}(\rho) \sim (J^2/T) \int d^2r d^2r \left| r - r' \right|^{-4l} \times \left[ 1 - \cos\left[\alpha_0(\rho, r) - \alpha_0(\rho, r') \right] \right],$$

where $\alpha_0(\rho, r) = \alpha(\rho) - \alpha(r)$. Equation (4) can be shown to be bounded by a $\sim \ln^2 \rho$ term, supporting a nonlinear $J$-$V$ relation at $T > T_d$. In contrast, at $T < T_d$ the $p\bar{\Psi}$ interaction increases as $\sim \rho$ leading to a finite critical current. Thus the decoupling transition is manifested by the change in correlation function, vanishing of the Josephson critical current [20] and by nonlinear $J$-$V$ relation.

We proceed now to study effects of disorder. Since $T < T_m$ we assume small fluctuations $|u_i^n| \ll a$. Consider a short range pinning potential $U_{pin}(r)$ which couples to the vortex shape function $p(r)$ as $\int d^2r \sum_n U_{pin}(r) p(r - R_i - u_i^n)$. Expansion in $u_i^n$ and averaging $U_{pin}(r)$ by the replica method [24] leads to the replicated Hamiltonian,

$$\frac{H_r}{T} = \frac{1}{2} \sum_{q, k, \alpha, \beta} \left[ c(k)q^2\delta_{\alpha, \beta} - s_0 q^2/k_z^2 \right] b^\alpha(q, k)b^\beta(q, k) - \frac{J}{T\xi_0^2} \sum_{n, \alpha} \int d^2r \cos b_n^\alpha(r) - \frac{\nu}{\xi_0^2} \sum_{n, \alpha, \beta} \int d^2r \cos[b_n^\alpha(r) - b_n^\beta(r)],$$

where $\alpha, \beta$ are replica indices, $c(k) = (a^2/2\pi d^2)c_{44}^0(k)/T$, and $s_0 = \overline{U}a^2d/(4\pi d^2T)^2$ with $\overline{U}$ an average of the pinning potential. In Eq. (5) the $c_{44}^0$ term has been replaced by a cutoff $q_a$ on $q$ integrations, as above. The inter-replica Josephson coupling, i.e., the $\nu$ term in Eq. (5), is generated from the $J$ term in second order RG. It is essential to keep the $\nu$ term from the start since it couples different replica indices and can lead to distinct physics by RSB [16,25].

Note that the more general form of the disorder term is $12-14 \cos(Q \cdot (u_n^\alpha - u_n^\beta))$, where $Q$ is a reciprocal lattice vector; expansion of this cosine leads to the $s_0$ term in Eq. (5). The cosine form is essential for deriving the Bragg glass properties of the flux lattice, i.e., the $1/r$ decay of the displacement correlation at distances $r > \ell$. The domain size $\ell$, over which the flux lattice is well correlated will be of significance below.
The RSB method [24] proceeds by employing a variational free energy $\mathcal{F}_{\text{var}} = \mathcal{F}_0 + \langle \mathcal{H} - \mathcal{H}_0 \rangle$ with $\mathcal{F}_0$ the free energy of $\mathcal{H}_0 = \sum_{q,k,a,\beta} G_{a,\beta}(q,k) \times b^a(q,k)b^{\beta}(q,k)$ and $G_{a,\beta}(q,k)$ is determined by an extremum condition on $\mathcal{F}_{\text{var}}$. This yields

$$G^{-1}_{a,\beta}(q,k) = [c(k)q^2 + z] \delta_{a,\beta},$$

(6a)

$$z = \left( J/2T \xi_0^2 d \right) \exp \left[ -\frac{1}{\xi_0} \sum_{q,k} G_{a,a}(q,k) \right],$$

(6b)

$$\sigma_{a,\beta} = \left( \nu/\xi_0^2 d \right) \left[ \exp(-\frac{1}{2}B_{a,\beta}) - \delta_{a,\beta} \sum_{\gamma} \exp(-\frac{1}{2}B_{a,\gamma}) \right],$$

(6c)

where $B_{a,\beta} = 2 \sum_{q,k}[G_{a,a}(q,k) - G_{a,\beta}(q,k)]$ and $z$ is a renormalized Josephson coupling. The method of RSB [24] represents a hierarchy of matrices such as $\sigma_{a,\beta} B_{a,\beta}$ in terms of functions $\sigma(u), B(u)$, respectively, with $0 < u < 1$. The amount by which the replica symmetry is broken is measured by a glass order parameter $\Delta(u) = u\sigma(u) - \int_0^1 \sigma(v) dv$. Using standard methods [24], we find that for $u < 2\nu$ while $\Delta(u) = \Delta_0$ for $2\nu < u$, where

$$(z + \Delta_0)/\Delta_c = \left[ 2\nu/\xi_0^2 \Delta_0 \right]^{(1-2\nu)/1(1-2\nu)}$$

(7)

with the cutoff $\Delta_c = c(\pi/d)q^2_a$. Thus a solution with $\Delta_0 = 0$ is possible only if $t < 1/2$. To solve for $z$ in Eq. (6b) we need the diagonal part,

$$\sum_{q,k} G_{a,a}(q,k) = \ln(2\nu t v/z\xi_0^2 d),$$

(8)

where $I(z) = \int d^2 k dk/\{k_i^2 c(k)\} [c(k)q^2 + z]$ and $I'(z) = dI(z)/dz$. Formally $I(z)$ diverges at $k = 0$: this divergence can be traced back to our assumption that the cos$\{Q \cdot (\mathbf{u}_{i,n} - \mathbf{u}_{i,n+1})\}$ term is expanded into the $s_0$ term in Eq. (5). Retaining this cosine leads to Imry-Ma type domains of correlated $\mathbf{u}_i^n$ whose size perpendicular to the layers is $\xi_z$. Within this domain the $\mathbf{u}_i^n$ expansion is valid so that $\pi/\xi_z$ serves as a lower cutoff in the $k$ integration. More formally, keeping the cos$\{Q \cdot (\mathbf{u}_{i,n} - \mathbf{u}_{i,n+1})\}$ term replaces $s_0$ in Eq. (6a) by a matrix $\sigma_{a,\beta}^{(1)}$ which corresponds to an RSB function $\sigma_1(u)$. This leads to an additional glass order parameter $\Delta_1(u) = u\sigma_1(u) - \int_0^u \sigma_2(v) dv$ and the divergent $I(z)$ is replaced by a term in Eq. (8) of the form

$$\sum_{q,k} \frac{1}{c(k)q^2 + z} \int_0^1 \frac{dv}{v^2} \frac{\Delta_1(u)q^2/k^2}{c(k)q^2 + z + \Delta_1(u) + \Delta_1(u)q^2/k^2},$$

(9)

which converges at $k \rightarrow 0$. The general solution for both $\Delta(u)$ and $\Delta_1(u)$ involves a rather difficult set of two coupled differential equations. For $J = 0$ the Bragg glass solution is [12,14] $\Delta_1(u) \sim u^2$ for $u < u_c$ and $\Delta_1(u) = \Delta_1(u_c)$ for $u_c < u < 1$, where $u_c \sim s_0$ is small. Thus for $t = T/T_d \gg u_c$ the structure of $\Delta_1(u)$ at small $u$ should not be affected by $\Delta(u)$ with its step at $u = 2\nu$. The scale at which the $k$ divergence is cut off is at $k < \Delta_1(u_c)/\xi_0/\Delta_c = 1/\xi_z$.

\[ z/\Delta_c = e^{-1}[J/2T \xi_0^2 d \Delta_1]^{1/(1-2\nu)} \]

(10)

Comparing Eqs. (7) and (10) shows that $\Delta_0$ vanishes at $s = t$ (up to a nonuniversal $\sim 1/\ln v$ term) and formally there is a solution with $\Delta_0 = 0$ when $s < t$. However, the average distribution [24] of $[b(q,k)]^2$ is $\sim \exp[-(b(q,k)]/G_{a,a}(q,k)])$ is acceptable only if $G_{a,a}(q,k) > 0$. This is a thermodynamic stability criterion and for our solution it reduces to $\Delta_0 > 0$. Thus the regime where both $\Delta_0$ are finite is limited to $s < t$ and $t < s$; we term this regime the Josephson glass (JG) phase. The glass parameter vanishes (continuously) at $s = t$ while the Josephson coupling vanishes (continuously) at $s = 1/\xi_z$. For $s > 1/\xi_z$ and $t < 1/\xi_z$ the solution is $z = 0$ while $\Delta_0 = 0$ satisfies Eq. (7), i.e., it is a decoupled glass phase. Finally, for $\Delta_0 = 0$ a replica symmetric solution is valid at $s < t < 1$ with

$$z/\Delta_c \approx (J/2T \xi_0^2 d \Delta_1)^{1/(1-2\nu)}.$$  

(11)

Thus $s + t = 1$ for $s < 1/\xi_z$ defines a decoupling transition.

The phase diagram, Fig. 1, has four phases which all meet at a point defined as $B_0,T_0$. $B_0$ is determined by the disorder strength via $s = t$ while $T_0 = T_d(a = a_0)$ [Eq. (3)], where $a_0 = \phi_0/B_0$. Since $s$ increases with $B$ the $s = 1/\xi_z$ line defines a decoupling transition from a JG phase at low $B$ to a pinned glass phase (G) at high fields. It is remarkable that although the G phase is vanishing Josephson coupling ($z = 0$) the Josephson induced disorder [the $v$ term in Eq. (5)] is dominant in determining the glass nature of this phase. In fact, RG shows that $J$ first increases (scaling from $\xi_0$ to $1/a_0$), generating the $v$ term, and only at scales beyond $1/a_0$ $J$ decreases to zero.
The JG phase at $B < B_0$ undergoes another transition at $t = s$, i.e., at $T = T_0$ (up to $\ln B$ factors) into a phase with finite Josephson coupling while the glass parameter $\Delta_0$ vanishes. This Josephson (J) phase has, however, the Bragg glass type disorder. The condition that $\ell_z$ has negligible effects in the JG or G phases implies that the pinning effect from the Josephson induced $\Delta_0$ is much stronger than that associated with the Bragg glass. Thus the JG-J transition is a depinning transition, from strong to weak pinning. The G phase also undergoes a depinning transition into a decoupled phase (D) at $T = B_0 T_0 / B$; the D phase is a Bragg glass phase maintained by the interlayer EM coupling.

The J phase undergoes a decoupling transition at $B = 2 B_0 T_0 / (T + T_0)$, using $s = B / 2 B_0$. The J-D transition is continuous for small $J / T$; for higher $J / T$ the SCHA shows a first order transition [18].

We interpret the experimentally observed second peak phenomena [1–4,7] as the JG-G transition, i.e., a decoupling transition within the glass phase. While a decoupling scenario has been suggested as a crossover phenomenon [1,19], the present theory predicts a strict phase transition. The JG-G transition at $B = B_0$ is $T$ independent up to $T_0$ and $B_0$ decreases with impurity strength; both features are consistent with experimental data [1–3]. For NCCO [7] with its low $T_c = 23$ K a multicritical point with $T_0 < T_c$ is probably not realized.

Recent data [8–10] have shown an additional phase boundary in BSCCO, i.e., a depinning transition line which crosses the critical point $B_0, T_0$. Our result for the depinning temperature, $T = T_0$ at $B < B_0$ being $B$ independent (up to $\sim \ln B$ terms), is in accord with the data. At $B > B_0$ we expect the depinning line at $T = B_0 T_0 / B$, in qualitative agreement with a stronger $B$ dependence [10].

Neutron data [6] have shown a reentrant behavior in the $600 - 10^3$ G range with positional correlations increasing with temperature. This is consistent with our decoupled phase which is weakly pinned, leading to enhanced positional correlations. The reentrant behavior seems to extend to $B < B_0$ so that our J-D line may be the first order line, at least near $B_0$; at lower fields this decoupling line probably joins the melting line.

In conclusion, we have found a phase diagram which is remarkably close to the experimental one [1–4,8,10], having a multicritical point where four phases meet. Our theory provides a fundamental interpretation of both the second peak transition and the recently discovered depinning transition.

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FIG. 1. Phase diagram. Full lines are decoupling phase transitions where the Josephson coupling vanishes. Dashed lines are depinning phase transitions where the Josephson glass parameter vanishes. $B_0$ is determined by the disorder strength while $T_0 = \frac{1}{2} T_d (a = a_0)$ [Eq. (3)], where $a_0 = \phi_0 / B_0$. 