Optimization of two component ceramic armor for a given impact velocity

G. Ben-Dor a, A. Dubinsky a, T. Elperin a,*, N. Frage b

a Department of Mechanical Engineering, The Pearlstone Center for Aeronautical Engineering Studies, Ben-Gurion University of the Negev, P.O. Box 653, Beer-Sheva 84105, Israel
b Department of Materials Engineering, Ben-Gurion University of the Negev, P.O. Box 653, Beer-Sheva 84105, Israel

Abstract

Using Florence’s model a problem of two-component ceramic-faced lightweight armors design against ballistic impact is solved. Approximate analytical formulas are derived for areal density and thicknesses of the plates of the optimal armor as functions of parameters determining the properties of the materials of the armor components, cross-section and mass of an impactor, and of the expected impact velocity. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Ballistic perforation of multi-layered plates has been a subject of intensive research during recent years since non monolithic configurations are considered feasible for designing shields or elements of the shields. Simplified analytical models were derived and used for the analysis and optimization of the shields consisting of the layers manufactured from different materials, e.g., ductile multi-layered shields [1–7], aluminum/Lexan combinations [8], ceramic-faced armors [9–19]. Since the model [10] provides a relatively simple expression for the ballistic limit velocity, it is widely used for the investigation of the ballistic properties of two-component ceramic armors [11–13,15,16]. The model [10] was used to determine the armor with the minimum areal density with an example for the numerical calculation of a ceramic/aluminum shield [10]. Similar calculations for ceramic/GFRP (glassfibre reinforced plastic) shield can be found also in [11]. The problem of designing an armor with the maximum ballistic limit velocity was investigated analytically in the case when the areal density is given in [14] and in the case when the total thickness of the armor is given in [17].

In this study the problem of designing an armor with the minimum areal density which was formulated in [10], is completely investigated for an arbitrary two-component armor, and the approximate analytical solution is found.

2. Model description

Consider a normal impact by a rigid projectile on a two-layer composite armor consisting of a ceramic front plate and a ductile back plate. In this study, the following model is employed:

$$v_c^2 = \frac{x_0 \sigma_2 \sigma_2 z (p_1 h_1 + p_1 h_2) z + m}{0.91 m^2},$$

$$z = \pi (R + 2h_1)^2,$$

where $x_0$ is the initial velocity of the projectile, $\sigma_2$ is the tensile strength of the armor, $z$ is the effective length of the armor, $p_1$ is the density of the ceramic, $h_1$ and $h_2$ are the thicknesses of the ceramic and the back plate, respectively, and $m$ is the mass of the projectile.
where $v_*$ is the ballistic limit velocity, $m$ the projectile’s mass, $R$ the projectile’s radius, $h_1$, $h_2$ are the plate’s thicknesses, $\sigma$ the ultimate tensile strength, $\varepsilon$ is the breaking strain, $\rho$ is the density, subscripts 1 and 2 refer to a ceramic plate and a back plate, respectively.

For $\alpha = 1$ Eq. (1) describes the model suggested in [10] as it is re-worked in [11]. We generalized slightly this model introducing a coefficient $\alpha$ which can be determined using the available experimental data in order to increase the accuracy of the predictions.

Expressions for the areal density $A$ of the armor reads

$$A = \rho_1 h_1 + \rho_2 h_2. \tag{2}$$

The objective of the present study is to find the thicknesses of the plates $h_1$, $h_2$ which provide the minimum areal density of the armor $A$ for a given ballistic limit velocity $v_*$. Introduce the following dimensionless variables

$$\hat{h}_i = \frac{h_i}{R}, \quad \hat{\rho}_i = \frac{\pi R^3 \rho_i}{m}, \quad i = 1, 2;$$

$$\hat{w} = v_* \sqrt{\frac{0.91 \rho_*}{2 \varepsilon \sigma_2}}, \quad \hat{A} = \frac{\pi R^2 A}{m}. \tag{3}$$

Then Eqs. (1) and (2) can be rewritten, respectively, as follows:

$$\hat{w}^2 = \hat{\rho}_2 \hat{h}_2 z \left[ (\hat{\rho}_1 \hat{h}_1 + \hat{\rho}_2 \hat{h}_2) z + 1 \right], \tag{4}$$

$$\hat{A} = \hat{\rho}_1 \hat{h}_1 + \hat{\rho}_2 \hat{h}_2, \tag{5}$$

where

$$z = \frac{\sigma}{\pi R^2} = \left( 1 + 2 \hat{h}_1 \right)^2. \tag{6}$$

Eq. (4) is a quadratic equation with respect to $\hat{\rho}_2 \hat{h}_2$. Solving this equation and substituting the obtained expression for $\hat{\rho}_2 \hat{h}_2$ into Eq. (5) yields

$$\hat{A} \left( \hat{h}_1, \hat{\rho}_1, \hat{w} \right) = \frac{\hat{\rho}_1 \hat{h}_1 z - 1 + \sqrt{ \left( \hat{\rho}_1 \hat{h}_1 z + 1 \right)^2 + 4 \hat{w}^2}}{2z}. \tag{7}$$

Thus the problem is reduced to finding a positive $\hat{h}_1$ that provides the minimum $\hat{A}$. The value $\hat{h}_2$ can be then found from Eq. (5).

It is important to emphasize that the dimensionless areal density $\hat{A}$ is a function of one variable $\hat{h}_1$ and depends on only two parameters, $\hat{\rho}_1$ and $\hat{w}$. If

$$\hat{h}_{1\text{opt}} = \varphi_1 (\hat{\rho}_1, \hat{w}) \tag{8}$$

provides the minimum $\hat{A}$, then the dimensionless minimum areal density $\hat{A}_{\text{opt}}$ and the optimal ratio of the areal density of the second plate to the areal density of the first plate are also functions of $\hat{\rho}_1$ and $\hat{w}$:

$$\hat{A}_{\text{opt}} = \hat{A} \left[ \varphi_1 (\hat{\rho}_1, \hat{w}), \hat{\rho}_1, \hat{w} \right] = \varphi_A (\hat{\rho}_1, \hat{w}), \tag{9}$$

$$\hat{\rho}_2 \hat{h}_2^{\text{opt}} = \frac{\rho_2 h_2^{\text{opt}}}{\rho_1 h_1^{\text{opt}}} = \frac{\hat{A}_{\text{opt}}}{\hat{A}^{\text{opt}}} - 1 = \frac{\varphi_A (\hat{\rho}_1, \hat{w})}{\varphi_1 (\hat{\rho}_1, \hat{w})} - 1 = \varphi_0 (\hat{\rho}_1, \hat{w}) \tag{10}$$

and

$$\hat{\rho}_2 h_2^{\text{opt}} = \hat{A}_{\text{opt}} - \hat{\rho}_2 h_2^{\text{opt}} = \varphi_A (\hat{\rho}_1, \hat{w}) - \hat{\rho}_1 \varphi_1 (\hat{\rho}_1, \hat{w}) = \varphi_2 (\hat{\rho}_1, \hat{w}). \tag{11}$$

The above property of the model, i.e., the existence of only two independent dimensionless parameters, allows us to investigate the problem completely and for a general case, namely, for arbitrary combination of materials of the plates. The results for a given materials can be determined using the solution obtained in dimensionless form:

$$h_{1\text{opt}} = R \varphi_1 (\hat{\rho}_1, \hat{w}),$$

$$h_{2\text{opt}} = \frac{m}{\pi R^2 \rho_2} \varphi_2 (\hat{\rho}_1, \hat{w}),$$

$$A_{\text{opt}} = \frac{m}{\pi R^2 \rho_2} \varphi_A (\hat{\rho}_1, \hat{w}), \tag{12}$$

where superscript opt indicates the corresponding optimal parameters, $\hat{\rho}$ and $\hat{w}$ are determined by Eq. (3).

In order to elucidate the analysis based on the dimensionless variables we will refer (where it is possible) to the special kind of the armor which will be referred to as a “basic armor” (BA). To this end, select the ceramic/GFRP armor, and use the experimental data in [11] for perforation of the
The approach suggested in \[11–13\] is based on the assumption that the momentum transferred from projectile to the armor is independent on the impact velocity, while the model in \[10\] is used for calculating the ballistic limit velocity. This approach implies the following correlation between the impact velocity \(v_{imp}\), the residual velocity \(v_{res}\) and the ballistic limit velocity \(v_*\):

\[
\frac{v_{imp}}{v_*} - \frac{v_{res}}{v_*} = 1. \tag{13}
\]

If the experimental data on \(v_{imp}\) and \(v_{res}\) are available, Eq. (13) can be used for finding the best value of the parameter \(\alpha\) in Eq. (1) and for determining the corresponding ballistic limit velocities. Certainly, it is assumed that \(\alpha\) is constant for a given combination of the materials of the plates. For the data presented in \[11\], it is found that \(\alpha = 0.90\). Comparison of Eq. (13) with the experimental data is showed in Fig. 1 where solid circles correspond to the experimental \(v_{imp}\) and \(v_{res}\) \[11\] together with the calculating \(v_*\) using Eq. (1).

### 3. Optimal armor

Now we will analyze the dependence on \(A\) on \(h_1\). Eqs. (6) and (7) imply that \(A(0) = (q - 1)/2\) where

\[
q = \sqrt{4\hat{v}^2 + 1} > 0. \tag{14}
\]

The value \(A(0)\) is independent on \(h_1\) When \(h_1 \to \infty\) function \(A(h_1)\) attains the asymptotic value \(A = \hat{\rho}_1 \hat{h}_1\). The derivative

\[
\hat{A}' = \frac{dA}{dh_1} = \frac{\hat{\rho}_1 + 4 + \frac{\hat{\rho}_1 - 4 - 16\hat{w}^2}{\sqrt{1 + 4\hat{w}^2}}}{q}
\]

is too involved for the complete analytical analysis. Consider its behavior for \(h_1 = 0\)

\[
2\hat{A}'(0) = -4q^2 + (\hat{\rho}_1 + 4)q + \hat{\rho}_1 \tag{16}
\]

The positive solution of the equation

\[
-4q^2 + (\hat{\rho}_1 + 4)q + \hat{\rho}_1 = 0 \tag{17}
\]

Thus

\[
\hat{A}'(0) \leq 0 \quad \text{if} \quad q \geq q_0 \tag{18}
\]

and

\[
\hat{A}'(0) \geq 0 \quad \text{if} \quad q \leq q_0 \tag{19}
\]

The condition given by Eq. (19) is satisfied only for very small ballistic limit velocities (e.g., for BA for \(v_* \leq 17 \text{ m/s}\)). Therefore, consider the case when a condition expressed by Eq. (18) is satisfied.

The typical behavior of function \(A(h_1)\) is showed in Fig. 2 for \(\hat{\rho}_1 = 0.06\) and different \(\hat{w}\) where solid circles are plotted using experiments \[11\]. The minimum of function \(A(h_1)\) provides the solution of the problem. However, the variation is quite small in the neighborhood of the minimum (see also \[11\]). This means that the thickness of the
ceramic plate may be changed in the vicinity of the optimal value without considerable loss in areal density.

The solution of the optimization problem in a graphical form is showed in Figs. 3 and 4. Useful conclusions about the properties of optimal armor can be made using Fig. 5 where the ratio of the areal densities of the plates in the optimal armor \( A_2/A_1 \) is plotted. Inspection of this figure shows that even for a relatively small \( \bar{w} \), the ratio \( A_2/A_1 \) is close to a constant value \( \approx 0.30 \) (for BA it corresponds to \( v_s = 400 \text{ m/s} \)). Thus, for \( \bar{w} \geq 4 \) the ratio of the thicknesses of the plates in the optimal armor is inversely proportional to the ratio of their densities:

\[
\frac{h_2^{opt}}{h_1^{opt}} \approx 0.3 \frac{\rho_1}{\rho_2}.
\]  

Fig. 2. Areal density of armor vs. thickness of ceramic plate; solid circles – experimental data from [11] combined with model given by Eq. (1).

Fig. 3. Areal density of optimal armor vs. ballistic limit velocity.

Fig. 4. Optimal thickness of ceramic plate vs. ballistic limit velocity.

Fig. 5. Optimal ratio of the areal densities of the plates vs. ballistic limit velocity.
The families of curves plotted in Figs. 3 and 5 can be approximated with the average accuracy of 3% in the range $0.04 \leq \bar{\rho}_1 \leq 0.1$, $1 \leq \bar{w} \leq 10$ as follows:

\[ \hat{A}_{opt} = \varphi_A(\bar{\rho}_1, \bar{w}) = (0.04 + 1.12\bar{\rho}_1)\bar{w}^{0.425}, \quad (21) \]

\[ \frac{\bar{\rho}_2\hat{h}_{opt}^2}{\hat{h}_{opt}^1} = \varphi_0(\bar{\rho}_1, \bar{w}) = 0.29 + (0.1 + \bar{\rho}_1)\bar{w}^{1.47}. \quad (22) \]

Using Eqs. (9)–(12) $\hat{h}_{opt}^1$ and $\hat{h}_{opt}^2$ can be expressed through functions $\varphi_A$ and $\varphi_0$:

\[ \hat{h}_{opt}^1 = \varphi_1(\bar{\rho}_1, \bar{w}) = \frac{\varphi_A(\bar{\rho}_1, \bar{w})}{\bar{\rho}_1 [\varphi_0(\bar{\rho}_1, \bar{w}) + 1]} \]

\[ = \frac{(0.04 + 1.12\bar{\rho}_1)\bar{w}^{1.895}}{\bar{\rho}_1 [\bar{\rho}_1 + 1.29\bar{w}^{1.47} + 0.1]}, \quad (23) \]

\[ \hat{h}_{opt}^2 = \frac{1}{\bar{\rho}_2} \varphi_2(\bar{\rho}_1, \bar{w}) = \frac{\varphi_A(\bar{\rho}_1, \bar{w})\varphi_0(\bar{\rho}_1, \bar{w})}{\bar{\rho}_2 [\varphi_0(\bar{\rho}_1, \bar{w}) + 1]} \quad (24) \]

Taking into account Eq. (12) the expressions for the solution given by Eqs. (21)–(24) provides characteristics of the optimal armor in term of the parameters determining the properties of the materials of the armor components, its cross-section area, mass of the impactor and the expected impact velocity.

4. Concluding remarks

It is showed that the solution of the optimization problem for a two-layer armor can be presented in terms of dimensionless variables whereby all the characteristics of the impactor and the armor are expressed as a function of two independent dimensionless parameters. The latter allows to present the solution of the optimization problem for an arbitrary two-component composite armor in analytical form which is convenient for practical applications.

The validity of the result obtained using the above simplified model must be investigated experimentally for different materials applied for armor component. Clearly, experimental procedure must be adjusted for testing the optimality of the obtained solution.

Investigations of Functionally Graded Composites (FGC) have received considerable attention during recent years (see, e.g., [20]). The results of our research in this field [21, 22] show that analysis of two-component materials can be useful for optimization of FGC.

Acknowledgements

We are grateful to Prof. A.L. Florence (International Stanford Research Institute) and Prof. R. Zaera (Carlos III University, Madrid) for providing us with their research reports and reprints of their papers. This study was partially supported by the Israel Ministry of Science (Grant No. 8450-1-98).

References