Effect of air gaps on ballistic resistance of targets for conical impactors

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Abstract

High velocity penetration of a rigid conical impactor into a ductile target with air gaps between the plates is studied using the cylindrical cavity expansion approximation describing impactor–target interaction. It is showed that the latter model predicts improvement of the ballistic performance of the target with the increase of air gaps. It is found analytically that the ballistic limit velocity of the target consisting of $N$ plates with a fixed total thickness with large air gaps increases with the increase of $N$. The conditions are discussed when the predicted effects can be most pronounced. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction

An updated review of investigations on subordnance penetration and perforation of multi-layered plates can be found in Ref. [1]. The important subject of the investigations in this field is the comparison between the resistance properties of targets with the same total thickness, depending on the layering, the sequence of the plates in the target, the widths of the air gaps between the layers and the shape of the impactor. Since the analysis of these problems is quite involved the latter studies employed experimental investigations and theoretical studies based on simplified models [2–10]. Nevertheless the effect of the air gaps on the ballistic resistance is still poorly understood. Since this effect can depend on the shape of the impactor it is expedient to develop models that allow to describe impactor–target interaction accounting for the impactor’s shape.

Semi-phenomenological localized interaction approach [11–13] was used in studying the influence of air gaps on the ballistic resistance in Ref. [4]. It was found that such models imply the independence of the ballistic limit velocity upon the sequence of the plates in the target and the widths of air gaps. The present study uses a more physically justified cylindrical cavity expansion approach [14–16] which implies, in a general case, more sophisticated impactor–target interaction models. Analyzed is the influence of air gaps on the ballistic resistance using analytical methods and numerical simulations. The obtained results can be used for further experimental investigations.

2. Model description

Consider a high speed normal penetration of a rigid impactor into a ductile target of a finite
the impactor’s base is a conical body of revolution. Only the nose of the impactor with the length \( L \) interacts with some layers of the target or is in a thickness with air gaps. The basic notations are shown in Fig. 1. The coordinate \( h \), the depth of penetration, is defined as the distance between the nose of the impactor and the upper surface of the target. The coordinate \( \xi \) is associated with the target. The axis \( x \) is associated with the impactor which is a conical body of revolution. Only the nose of the impactor with the length \( L \) interacts with the target, and the impactor may have also the cylindrical part with the length \( \ell \). The radius of the impactor’s base is \( r_0 \).

Assume that the target consists of \( N \) plates with the thicknesses \( b_1, b_2, b_3, \ldots, b_N \) made from the same material where the \( i \)th plate is located between the sections \( \xi = \xi_i \) and \( \xi = \xi_i + b_i \), \( \xi_i = 0, i = 1, 2, \ldots, N \). The thicknesses of the air gaps between the plates with the numbers \( i \) and \( i+1 \) is \( \delta_i = \xi_{i+1} - \xi_i - b_i \) \((i = 1, 2, \ldots, N-1)\) and the total thickness of the target is \( b = \sum b_i \). The part of the lateral surface of the impactor between the cross-sections \( x = x_1 \) and \( x = x_2 \) (see Fig. 1) interacts with some layers of the target or is in a contact with some air gaps where (see Fig. 2)

\[
x_1(h) = \begin{cases} 
0 & \text{if } 0 \leq h \leq b, \\
h - b & \text{if } b < h \leq b + L.
\end{cases}
\]

\[
x_2(h) = \begin{cases} 
h & \text{if } 0 \leq h \leq L, \\
L & \text{if } b < h \leq b + L.
\end{cases}
\]

Assume that the impactor–target interaction can be described by the cylindrical cavity expansion approximation, and the solution of the problem of hole expansion from the time \( t = 0 \) reads (for details see Refs. [14,15])

\[
p = a_2 \tilde{R}^2 + a_1 \tilde{R} \tilde{R} + a_0,
\]

where \( R \) is radii of the hole, \( p \) a pressure applied in the normal direction at the part of the impactor’s surface, coefficients \( a_j \) \((j = 0, 1, 2)\) depend on the properties of the material of the target. Using the technique described in Refs. [4,15] the equation of motion of the impactor can be written as follows:

\[
[A_0 + A_1 J_2(h)] \frac{dw}{dh} + [A_2 w + 1] J_1(h) = 0,
\]

where \( w = v^2 \), \( v \) is the impactor’s velocity and it is considered to be a function of the depth of penetration \( h \).

\[
J_\mu(h) = \int_{\hat{x}_1(h)}^{\hat{x}_2(h)} \tilde{x}^\mu \delta(h - \tilde{x}) \, d\tilde{x}, \quad \mu = 1, 2,
\]

\[
A_0 = \frac{M}{4\pi a_0 (\beta + k) L^3} = \frac{\gamma \beta (1 + \tilde{\gamma})}{12a_0 (\beta + k)} > 0,
\]

\[
A_1 = \frac{a_1 \beta^2}{2a_0} > 0, \quad A_2 = \frac{a_2 \beta^2}{a_0} > 0,
\]

\[
\hat{x}_1(h) = \begin{cases} 
0 & \text{if } 0 \leq h \leq b, \\
h - b & \text{if } b \leq h \leq b + 1,
\end{cases}
\]

\[
\hat{x}_2(h) = \begin{cases} 
h & \text{if } 0 \leq h \leq 1, \\
1 & \text{if } 1 \leq h \leq b + 1,
\end{cases}
\]

where \( M \) is the mass of the impactor, \( \gamma \) is its density, \( \beta = r_0 / L = \tan \vartheta, \vartheta \) is the half angle of the apex of the cone. \( k \) the friction coefficient, \( \tilde{\gamma} = y / L \) for any variable or parameter \( y \), and \( \delta(\xi) = 1 \) if \( \xi_1 \leq \xi \leq \xi_i + b_i \) for \( 1 \leq i \leq N \) and \( \delta = 0 \) otherwise.

The ballistic limit velocity \( v^* \) is defined as the initial velocity of the impactor required for it to emerge from the target with a zero residual velocity. The equation for \( v^* \) can be determined from solution of Eq. (3):

\[
\int_0^{v^*} \frac{dw}{A_2 w + 1} = \int_0^{h_1} \frac{J_1(z)}{A_0 + A_1 J_2(z)} \, dz.
\]

Then

\[
v^* = w^* = \frac{\exp(B \Psi) - 1}{A_2}, \quad B = \frac{A_2}{A_1} = 2 \frac{a_2}{a_1}.
\]
\[ \Psi(A; \bar{b}_1, \ldots, \bar{b}_N; A_1, \ldots, A_{N-1}) = \int_0^{b_1+1} \frac{J_1(z)}{A + J_2(z)} \, \text{dz}, \]
\[ A = \frac{A_0}{A_1} = \frac{\gamma(1 + 3\ell)}{6\alpha_1\beta(\beta + k)}. \tag{9} \]

3. Analytical consideration

For further analysis it is convenient to represent the integrals in Eq. (4) as follows.

\[ J_\mu(\bar{x}) = \sum_{i=1}^N \int_{\bar{x}_0(\bar{x} - \bar{x}_i)}^{\bar{x}_0(\bar{x} - \bar{x}_{i+1})} \tilde{x}^\mu \, \text{d}\tilde{x} \]
\[ = \sum_{i=1}^N J_{\mu,i}(\bar{x}), \quad \mu = 1, 2, \tag{10} \]

where

\[ J_{\mu,i}(\bar{x}) = \frac{1}{\mu + 1} \sum_{i=1}^N \{ \tilde{x}_0(\bar{x} - \tilde{x}_i) \}^{\mu+1} \]
\[ - \tilde{x}_0(\bar{x} - \tilde{x}_i - \bar{x}_i) \} \{ \tilde{x}_0(\bar{x} - \tilde{x}_i) \}^{\mu+1} \}, \]
\[ \mu = 1, 2, \quad i = 1, \ldots, N, \tag{11} \]

\[ \tilde{x}_0(\bar{x}) = \begin{cases} 0 & \text{if } \bar{x} < 0, \\ \bar{x} & \text{if } 0 \leq \bar{x} \leq 1, \\ 1 & \text{if } \bar{x} > 1. \end{cases} \tag{12} \]

In the case of a single plate of thickness \( b_1 = b \)

\[ \Psi(A, \bar{b}) = \int_0^{b_1+1} \frac{J_{1,1}(z)}{A + J_{2,1}(z)} \, \text{dz} = \frac{3}{2} g^2 G(A, \bar{b}), \tag{13} \]

where

\[ G(A, t) = \begin{cases} \chi(g) - \chi((1 - t)g) & \text{if } t \leq 1, \\ \chi(g) + (t - 1)g & \text{if } t > 1, \end{cases} \]
\[ g = (3A + 1)^{-1/3} < 1, \tag{14} \]

\[ \chi(z) = \int_0^z \frac{\text{dz}}{1 - z^3} = \frac{1}{\sqrt{3}} \tan^{-1}\left( \frac{2z + 1}{\sqrt{3}} \right) \]
\[ - \frac{1}{6} \ln \left( \frac{z - 1}{z^2 + z + 1} \right) - \frac{\pi}{6\sqrt{3}}. \tag{15} \]

A monolithic target of thickness \( b_2 = b_1 + b_2 \)

+ \cdots + b_N and \( N \) plates in contact with thicknesses \( b_1, b_2, \ldots, b_N \) are equivalent in the framework of the considered model. Therefore,

\[ \Psi(A; \bar{b}_1, \bar{b}_2, \ldots, \bar{b}_N; 0, 0, \ldots, 0) = \Psi(A, \bar{b}_2) \]
\[ = \frac{3}{2} g^2 G(A, \bar{b}_2) \tag{16} \]

In the range \( \bar{A}_i \geq 1(A_i \geq L) \), i.e., when impactor perforates the plates sequentially, the ballistic limit velocity is independent on the widths of the air gaps. In this case

\[ \Psi(A; \bar{b}_1, \bar{b}_2, \ldots, \bar{b}_N; 1, 1, \ldots, 1) \]
\[ = \sum_{i=1}^N \int_{\bar{x}_i}^{\bar{x}_{i+1}} \frac{J_{1,i}(z)}{A + J_{2,i}(z)} \, \text{dz} \]
\[ = \sum_{i=1}^N \int_0^{\bar{x}_i + \bar{x}_{i+1}} \frac{J_{1,i}(z)}{A + J_{2,i}(z + \bar{x}_i)} \, \text{dz} \]
\[ = \sum_{i=1}^N \Psi(A, \bar{b}_i) = \frac{3}{2} g^2 \sum_{i=1}^N G(A, \bar{b}_i). \tag{17} \]

It is interesting that, for large air gaps, the ballistic limit velocity does not depend on the order of the plates in the target.

The difference in the ballistic limit velocities will be studied for the above limiting cases, i.e., determine the sign of the difference \( v'_i - v''_i \) where \( v'_i \) and \( v''_i \) are the ballistic limit velocities for a target having plates in contact and for a target with large air gaps, respectively. Eqs. (8), (16) and (17) imply that sign \( (v'_i - v''_i) = \text{sign} (\eta) \), where

\[ \eta = G(A, \bar{b}_2) - \sum_{i=1}^N G(A, \bar{b}_i). \tag{18} \]

It will be proved that \( \eta \) is always negative, i.e., \( \eta < 0 \). In the proof, use will be made of property of a concave function \( f(x) \), namely, the function \( F(x) = f(x + d) - f(x) \) is an increasing function if \( d > 0 \).

Case 1: \( \bar{b}_2 \leq 1 \).

In this case clearly all \( \bar{b}_i < 1 \). Then using the above property of the function \( \chi \) it can be written:
Then there always exist an index $i = i_*$ and a thickness $\bar{b}_{i_*} < b_{i_*}$ such that
\begin{equation}
\bar{s}_{i_*-1} + \bar{b}_{i_*} = 1
\end{equation}
and the following estimate is valid.
\begin{align*}
\eta_0 > & \sum_{i=1}^{i_*-1} \{ \chi_0(g) - \chi_0[g(1 - \bar{b}_i)] \} \\
& + \chi_0(g) - \chi_0[g(1 - \bar{b}_{i_*})] \\
& + \chi_0[g(1 - \bar{b}_{i_*})] - \chi_0[g(1 - \bar{s}_{i_*-1})] \\
& + \chi_0[g(1 - \bar{s}_{i_*-1})] - \chi_0(0) = \chi_0(g).
\end{align*}

Then $\eta < 0$.

**Case 2:** $\forall \bar{b}_i \geq 1$.

Clearly $\bar{b}_x > 1$. Then
\begin{align*}
\eta &= \chi(g) + g(\bar{b}_x - 1) - \sum_{i=1}^{N} \{ \chi(g) - \chi[g(1 - \bar{b}_i)] \} \\
&= (N-1)[g - \chi(g)] - (N-1)\chi_0(g) < 0,
\end{align*}
where
\begin{equation}
\chi_0(z) = \int_0^z \frac{s^3}{1 - s^3} \, ds.
\end{equation}

**Case 3:** $\forall \bar{b}_i \leq 0$ and $\bar{b}_x > 1$.

\begin{align*}
\eta &= \chi(g) + g(\bar{b}_x - 1) - \sum_{i=1}^{N} \{ \chi(g) \\
&- \chi[g(1 - \bar{b}_i)] \} = \chi_0(g) - \eta_0,
\end{align*}
where
\begin{equation}
\eta_0 = \sum_{i=1}^{N} \{ \chi_0(g) - \chi_0[g(1 - \bar{b}_i)] \}.
\end{equation}

Consider all possible cases.

If $\bar{b}_x^m \geq 1$ then the above analysis of Case 3 applied to the first $m$ plates implies that $\eta_i < 0$. The analysis similar to that of Case 2 as applied to plates with the numbers $m+1, \ldots, N$ implies the following estimate
\begin{equation}
\eta_2 = -(N-m)\chi_0(g) < 0,
\end{equation}
and $\eta < 0$ if $\bar{b}_x^m \geq 1$.

Consider the last case when $\bar{b}_x^m < 1$. Since

\begin{align*}
\eta &= \chi(g) + g(\bar{b}_x - 1) - \sum_{i=1}^{m} \{ \chi(g) - \chi[g(1 - \bar{b}_i)] \} \\
&= \sum_{i=1}^{i_*-1} \{ \chi_0[g(1 - \bar{s}_{i_*-1})] - \chi_0[g(1 - \bar{b}_{i_*})] \} \\
&+ \chi_0[g(1 - \bar{b}_{i_*})] - \chi_0(g) \\
&+ \chi_0[g(1 - \bar{s}_{i_*-1})] - \chi_0(0) = \chi_0(g).
\end{align*}
\[
\frac{d\Omega}{du} = \frac{g}{u} \left[ \chi'(g - gu) - \chi'(z_0) \right] < 0. \tag{39}
\]

Thus, \( \Omega \) decreases with increasing \( u \) and increases with increasing \( N \). Taking into account Eqs. (8), (17), (35) and (36), it can be concluded that the ballistic limit velocity increases with increasing number of plates if their total thickness in a spaced target is the same. Since
\[
\lim_{u \to 0} \Omega(u) = \lim_{u \to 0} \frac{1}{u} \int_{g - gu}^{g} \frac{dz}{1 - z^3} = \frac{g}{1 - g^3}, \tag{40}
\]
the ballistic limit velocity tends to the finite value when the number of plates approaches infinity.

5. Numerical analysis

In order to study the influence of air gaps with the width in the range between 0 and \( L \) a numerical simulation was performed. Numerical simulations also allow us to estimate the order of the difference in the ballistic limit velocities for a target having plates in contact and a target with air gaps.

If \( A \gg J_2(z) \), Eqs. (6) and (9) imply the following estimate (see also Fig. 2):
\[
\Psi(A; \bar{b}, \ldots, \bar{b}_n; A_1, \ldots, A_{N-1}) \approx \frac{1}{A} \int_{0}^{\bar{b}+1} J_1(z) \, dz
\]
\[
= \frac{1}{A} \int_{0}^{\bar{h}} \int_{\tilde{x}(\tilde{h})}^{\bar{x}(\tilde{h})} \delta(\tilde{h} - \bar{x}) \, d\tilde{x} \, d\tilde{h}
\]
\[
= \frac{1}{A} \int_{0}^{\bar{x}} d\tilde{x} \sum_{i=1}^{N} \left[ \tilde{x}_{i+1} - \tilde{x}_{i} \right] \int_{\tilde{x}_{i}}^{\tilde{x}_{i+1}} \delta(\tilde{x} - \bar{x}) d\tilde{x}
\]
\[
= \frac{1}{A} \int_{0}^{\bar{x}} d\tilde{x} \sum_{i=1}^{N} \left( \tilde{x}_{i+1} - \tilde{x}_{i} \right) d\tilde{h}
\]
i.e., the ballistic limit velocity is independent on the widths of air gaps. Since
\[
J_2(\bar{h}) \leq \int_{\bar{h}}^{\bar{x}(\tilde{h})} d\bar{x} \frac{\bar{x}}{\bar{x} - \bar{h}} = \int_{\bar{h}}^{\bar{x}(\tilde{h})} \bar{x}^2 \, d\bar{x} = \frac{1}{\bar{x}(\bar{h})} \left\{ \left[ \bar{x}(\bar{h}) \right]^3 - \left[ \bar{x}(\bar{h}) \right]^3 \right\} \leq \frac{1}{3}, \tag{42}
\]
the weak dependence of the ballistic limit velocity on the widths of air gaps may be expected for \( A > 1 \).
In the numerical simulations, use is made of the following simple model describing the elastic-plastic response of the ductile metal target \([15]\):

\[
a_0 = \tau(1 + \varepsilon_1), \quad a_1 = \frac{\rho}{2} \varepsilon_1, \quad a_2 = \frac{\rho}{2} \left[ \varepsilon_1 - \frac{\varepsilon_0}{1 + \varepsilon_0} \right],
\]

where

\[
\varepsilon_0 = \frac{E}{2\tau(1 + \nu)}, \quad \varepsilon_1 = \ln(1 + a_0),
\]

where \(\rho, E, \tau, \nu\) are the density, the Young’s modulus, the compressive shear strength, and the Poisson’s ratio, respectively.

Some characteristic results of the calculation are shown in Fig. 3 (for plates manufactured from aluminum alloy 7075-T6) and in Fig. 4 (for soft steel plates). The numerical values of the material properties are taken from Ref. \([13]\). Consider a rigid steel impactor with \(\ell = 0.5\). The parameter

\[
\varepsilon = \frac{v_0^r - v_0^a}{v_0^r}
\]

is applied as an estimation of the difference in the ballistic resistance where \(v_0^r\) and \(v_0^a\) are the ballistic limit velocities for a target having plates in contact and for a spaced target, respectively.

The results of the numerical simulation support the above theoretical findings for the limit widths of air gaps and allow to generalize these conclusions to the intermediate magnitudes of the air gaps widths, namely, the ballistic limit velocity increases if the widths of the air gaps between the plates increase.

The cavity expansion approximation is justified to a greater extent for slender impactors (conical impactors with small \(\vartheta\)). If \(A \gg 1\) for small \(\vartheta\), the influence of the air gaps is insignificant (Fig. 3). The air gaps affect more tangibly the ballistic limit velocity if \(A < 1\) for small \(\vartheta\) (Fig. 4). Eq. (9) shows that the latter is valid for relatively large values of coefficient \(a_1\) which, usually, is proportional to the density of the material of the target.

6. Concluding remarks

The effect of air gaps on the ballistic performance of the ductile spaced target penetrated by rigid, sharp, conical-nosed impactors is studied...
using analytical cylindrical cavity expansion models describing the interaction between a target and an impactor. It was found that the ballistic limit velocity of the target increases with the increase of the widths of the air gaps and increase of the number of the plates in the target while the total thickness of the target is kept constant. It is predicted that the increase in the ballistic limit velocity must be more pronounced with increase in the apex half angle of the nose cone and the density of the material of which the target is made. Certainly, the validity of these result for various conditions of penetration (different materials of the target, widths of the plates, half angles of the apex, etc.) must be confirmed experimentally.

References