Optimization of Reinforced Concrete Panels with Rear Face Steel Liner under Impact Loading*

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Abstract: We investigate the effect of steel plate lining at the rear face of the reinforced concrete panel on the ballistic properties using the semi-empirical model. We determine the structure of the two-layered shield that provides the maximum critical velocity for a given area density and depends on the mechanical properties of concrete and characteristics of the reinforcement. It is found that the optimal shield with lining has a considerable advantage over the shield without lining.

Keywords: Concrete; Impact; Lining; Optimization; Perforation; Structure.

INTRODUCTION

Numerous engineering models were suggested to describe impact of rigid penetrators with concrete shields (see, e.g., overviews in Adeli and Amin, 1985; Bangash, 2009; Ben-Dor et al., 2006; Brown, 1986; Bulson, 1997; Corbett et al., 1996; Guirgis and Guirguis, 2009; Kennedy, 1976; Li et al., 2005). However, there are only a few publications that considered analytical models for predicting the effect of a steel liner on the ballistic properties of the shield although experimental and numerical investigations (Barr et al., 1983; Hashimoto et al., 2005;
Kojima, 1991; Koshika et al., 1993; Ohno et al., 1994; Tsubota et al., 1993; Walter and Wolde-Tinsae, 1984) have demonstrated that the effect can be significant. Hashimoto et al. (2005) suggested a semi-empirical relationship between the bulging height of the steel plate lining and impact velocity. Walter and Wolde-Tinsae (1984) and Ohno et al. (1994) proposed to assess the effect of the liner as an additional concrete slab thickness. The most comprehensive procedure, including the design formulas and recommendations about the range of their applicability, was developed by Barr (1990). In the present study, we employ these formulas for optimization of a steel liner at the rear face of the concrete plate. To the best of our knowledge, this problem was not considered before in the literature.

MATHEMATICAL MODEL AND STATEMENT OF PROBLEM

Consider normal penetration of a rigid flat-faced striker (a body of revolution) into a shield consisting of a reinforced concrete plate and a steel plate on its distal face. The notations are shown in Fig. 1. The ballistic limit velocity (BLV), or the critical velocity, is defined as the minimum velocity required for perforation of the shield. Following Barr (1990), we use the following formula for the BLV, $v_{bl}$:

$$v_{bl} = kb^{(1)4/3} \sqrt{B + 0.3},$$

(1)

Figure 1. Schematic view of the reinforced concrete panel with a rear face steel liner and notations.
where

\[ B = r + \frac{100b^{(2)}}{b^{(1)}}, \quad r = \frac{100a}{b^{(1)}}, \quad \alpha = \frac{a_1}{3c_1} + \frac{2a_2}{3c_2}, \quad k = 1.3\rho_{\text{conc}}f_{cy}^{1/2}\left(\frac{p}{\pi m}\right)^{2/3}, \]

(2)

\(a_1\) (\(a_2\)) is the cross section area of a single front (rear) rebar, \(c_1\) (\(c_2\)) is a front (rear) rebar spacing, \(f_{cy}\) is a characteristic compressive strength of concrete measured for 150 mm diameter, with 300 mm long cylinders, \(\rho_{\text{conc}}\) is density of concrete, \(m\) is mass of impactor, \(p\) is perimeter of the impactor, \(b^{(1)}\) is thickness of reinforced concrete plate, and \(b^{(2)}\) is steel plate thickness. Hereafter we use SI units.

Barr (1990) presents the ranges of parameters where Eq. (1) is valid. The following constraints are important for our analysis:

\[ 0 \leq r \leq 0.75, \quad 1.2 \leq B \leq 4.3, \]

(3)

where the second restriction is omitted when \(b^{(2)} = 0\).

The volume of steel in the reinforced concrete plate, \(V_{\text{steel}}^{(1)}\), can be calculated as follows:

\[ V_{\text{steel}}^{(1)} = \sum_{i=1}^{2} \left( \frac{l}{c_i} a_iw + \frac{w}{c_i} a_il \right) = 2lw \sum_{i=1}^{2} \left( \frac{a_i}{c_i} \right), \]

(4)

where \(l\) and \(w\) are the length and the width of the shield, correspondingly. Then formulas for the volume of concrete in the reinforced concrete plate, \(V_{\text{conc}}^{(1)}\), and for the area density of the shield, \(A\), read:

\[ V_{\text{conc}}^{(1)} = lw b^{(1)} - V_{\text{steel}}^{(1)}, \]

(5)

\[ A = \frac{m_{\text{sh}}}{lw} = \frac{\rho_{\text{steel}}(V_{\text{steel}}^{(1)} + V_{\text{steel}}^{(2)}) + \rho_{\text{conc}} V_{\text{conc}}^{(1)}}{lw} \]

\[ = \rho_{\text{conc}} b^{(1)} + \rho_{\text{steel}} b^{(2)} + \xi, \]

(6)

where

\[ \xi = 2(\rho_{\text{steel}} - \rho_{\text{conc}}) \sum_{i=1}^{2} \left( \frac{a_i}{c_i} \right), \]

(7)

\(V_{\text{steel}}^{(2)} = lw b^{(2)}\) is the volume of the steel plate and \(\rho_{\text{steel}}\) is density of steel.

The problem is to determine the thicknesses of the reinforced concrete and a steel plate providing the maximum BLV of the shield for given area density taking into account constraints in Eq. (3). It is assumed that \(a_1, a_2, c_1, c_2, f_{cy}, \rho_{\text{conc}}, \rho_{\text{steel}}, m, p,\) and \(A\) are known.
Using dimensionless variables, the mathematical formulation of the problem is as follows:

\[ W_{bl} = \frac{V_{bl}}{100 k^2 b^{5/3}} = \bar{b}^{(1)5/3} \left( \bar{\alpha} + \bar{b}^{(2)} + 0.003 \bar{b}^{(1)} \right) \rightarrow \max \]  

(8)

taking into account the constraints:

\[ \bar{b}^{(1)} \geq 0, \quad \bar{b}^{(2)} \geq 0, \quad \bar{b}^{(1)} + \bar{b}^{(2)} = 1, \]

\[ \bar{b}^{(1)} \geq (400/3) \bar{x}, \quad 0.012 \bar{b}^{(1)} \leq \bar{x} + \bar{b}^{(2)} \leq 0.043 \bar{b}^{(1)}, \]

(9)

where

\[ \bar{b}^{(1)} = \frac{\bar{b}^{(1)}}{b}, \quad \bar{b}^{(2)} = \frac{\bar{b}^{(2)}}{b}, \quad \bar{x} = \frac{x}{b}, \quad b = \frac{A - \xi}{\rho_{conc}} > 0, \quad \bar{\theta} = \frac{\rho_{steel}}{\rho_{conc}} \]  

(10)

Hereafter we take into account that

\[ 3.2 \leq \bar{\theta} \leq 4.1. \]  

(11)

INVESTIGATION OF THE PROBLEM

Excluding the variable \( \bar{b}^{(2)} \),

\[ \bar{b}^{(2)} = (1 - \bar{b}^{(1)})/\bar{\theta}, \]  

(12)

we obtain the following optimization problem:

\[ F(\bar{b}^{(1)}) = \frac{\partial W_{bl}}{1 - 0.003 \bar{\theta}} = \bar{b}^{(1)5/3} (\bar{\eta} - \bar{b}^{(1)}) \rightarrow \max \]  

(13)

with the following constraints:

\[ (400/3) \bar{x} \leq \bar{b}^{(1)} \leq 1, \]  

\[ \bar{b}^{(1)}_* \leq \bar{b}^{(1)} \leq \bar{b}^{(1)}^{**}. \]

(14)

(15)

where

\[ \bar{\eta} = \frac{\bar{x} \bar{\theta} + 1}{1 - 0.003 \bar{\theta}}, \quad \bar{b}^{(1)} = \frac{\bar{x} \bar{\theta} + 1}{1 + 0.043 \bar{\theta}}, \quad \bar{b}^{(1)}^{*} = \frac{\bar{x} \bar{\theta} + 1}{1 + 0.012 \bar{\theta}}. \]  

(16)

Consider the behavior of the function \( F(\bar{b}^{(1)}) \). Since

\[ F'(\bar{b}^{(1)}) = \bar{b}^{(1)2/3} (5 \bar{\eta} - 8 \bar{b}^{(1)})/3, \]

(17)
function $F$ increases when $0 < \bar{b}^{(1)} < 0.625\bar{\theta}$ and decreases when $\bar{b}^{(1)} > 0.625\bar{\theta}$; that is, $\bar{b}^{(1)} = \bar{b}^{(1)}_{opt} = 0.625\bar{\theta}$ is the maximum of $F$.

If inequalities in Eqs. (14) and (15) contradict the solution, the considered problem does not exist. The latter occurs when

$$\bar{b}^{(1)}_{opt} > 1 \text{ or } \bar{b}^{(1)}_{opt} < (400/3)\bar{x}. \quad (18)$$

Since

$$\bar{b}^{(1)}_{opt} - 1 = \frac{(\bar{x} - 0.043)\bar{\theta}}{1 + 0.043\bar{\theta}}, \quad \bar{b}^{(1)}_{opt} - (400/3)\bar{x} = \frac{1 - \bar{x}(0.6\bar{\theta} + 400/3)}{1 + 0.012\bar{\theta}} \quad (19)$$

Equation (18) implies that

$$\bar{x} > 0.043 \quad (20)$$

$$\bar{x} > \bar{x}_{\text{opt}} \quad (21)$$

where

$$\bar{x}_{\text{opt}} = 1/(0.6\bar{\theta} + 400/3). \quad (22)$$

Since $1/(0.6\bar{\theta} + 400/3) < 0.043$, the constraint given by Eq. (20) can be ignored. Therefore the solution of the problem does not exist if Eq. (21) is valid. Otherwise, if

$$\bar{x} \leq \bar{x}_{\text{opt}} \quad (23)$$

the solution of the optimization problem can be found.

If the inequality in Eq. (23) is satisfied the nonempty domain of the admissible values of $\bar{b}^{(1)}$ for which inequalities (14) and (15) are satisfied can be described as follows:

$$\bar{b}^{(1)}_{opt} \leq \bar{b}^{(1)} \leq \bar{b}^{(1)}_{max} \quad (24)$$

where

$$\bar{b}^{(1)}_{max} = \max ((400/3)\bar{x}, \bar{b}^{(1)}_{opt}), \quad \bar{b}^{(1)}_{max} = \min (1, \bar{b}^{(1)}_{opt}). \quad (25)$$

Since

$$\bar{b}^{(1)}_{opt} - \bar{b}^{(1)}_{opt} = \frac{(\bar{x}\bar{\theta} + 1)(3 - 0.239\bar{\theta})}{(1 - 0.003\bar{\theta})(1 + 0.043\bar{\theta})} > 0, \quad (26)$$

function $F(\bar{b}^{(1)})$ decreases in the domain determined by Eq. (15) and, consequently, it attains the maximum in the point $\bar{b}^{(1)} = \bar{b}^{(1)}_{opt} = \bar{b}^{(1)}_{opt}$.

$$\bar{b}^{(1)}_{opt} = \begin{cases} 
(400/3)\bar{x} & \text{if } (400/3)\bar{x} \geq \bar{b}^{(1)}_{opt} \\
\bar{b}^{(1)}_{opt} & \text{if } (400/3)\bar{x} \leq \bar{b}^{(1)}_{opt} 
\end{cases}. \quad (27)$$
After some algebra taking into account Eq. (23), the solution in Eq. (27) can be represented in the following form:

\[
\tilde{b}^{(1)}_{opt} = \begin{cases} 
(400/3)\bar{z} & \text{if } \bar{x}_{xx} \leq \bar{z} \leq \bar{x}_x \\
\tilde{b}^{(i)}_1 & \text{if } 0 < \bar{z} \leq \bar{x}_{xx} 
\end{cases},
\]

(28)

where

\[
\bar{x}_{xx} = 3/(14.2\tilde{\theta} + 400) < \bar{x}_x.
\]

(29)

Formula for the thickness of the steel plate corresponding to the optimal solution, \(\tilde{b}^{(2)}_{opt}\), reads (see Eq. (12)):

\[
\tilde{b}^{(2)}_{opt} = (1 - \tilde{b}^{(1)}_{opt})/\tilde{\theta}.
\]

(30)

It is convenient to use the ratio

\[
\delta = \frac{v_{hl}^{opt}}{v_{hl}^{0}} = \frac{F(\tilde{b}^{(1)}_{opt})}{F(1)} = \sqrt{\frac{\tilde{b}^{(1)5/3}_{opt}(\beta - \tilde{b}^{(1)}_{opt})}{\beta - 1}}
\]

(31)

for quantifying the advantage of the optimal two-layered shield, where \(v_{hl}^{0}\) is the BLV of the reinforced concrete plate with the same area density.

**Figure 2.** Normalized ballistic limit velocity of the optimal shield versus parameters \(\tilde{\theta}\) and \(\bar{z}\).
Figure 3. Steel liner width normalized by the concrete panel width of the optimal shield vs. parameters $\bar{\theta}$ and $\bar{\alpha}$.

Figure 4. Ballistic limit velocity of the shield with a given concrete panel width normalized by the ballistic limit velocity of the optimal shield for different values of parameter $\bar{z}$. 
Results of the investigation of the optimization problem are shown in Figs. 2 and 3 and demonstrate the effectiveness of rear lining using a steel plate. Inspection of Fig. 4 shows that the effect of the thickness of the steel plate on the BLV of the shield is large; that is, it is anticipated that optimization results in significant improvement of ballistic characteristics.

Barr (1990) claimed that the above model is valid for $45 < v_{bl} < 300$. At the same time, he recommended using a more accurate formula for the BLV if the BLV is larger than 70 m/s

$$\tilde{v}_{bl} = v_{bl}(1 + 0.000004v_{bl}^2), \quad (32)$$

where $v_{bl}$ is the BLV determined by Eq. (1). Since $\tilde{v}_{bl}$ increases as a function of $v_{bl}$, the obtained optimal solution remains valid when the criterion given by Eq. (32) is used. However, quantification of the advantage of the optimal solution is less transparent in the latter case because it depends not only on $\tilde{\theta}$ but also on $\tilde{x}$.

**CONCLUDING REMARKS**

Using the semi-empirical model we investigated the effect of lining with a steel plate on the rear face of the reinforced concrete panel on ballistic properties of the two-layer shield and found the structure of the shield that provides the minimal BLV for a given area density, depending on the mechanical properties of concrete and characteristics of the reinforcement. We demonstrated a considerable advantage of the optimal shield over the shield without lining. Clearly, the obtained theoretical predictions require experimental validation.

**REFERENCES**


