NEW AREA RULES FOR PENETRATING IMPACTORS

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Summary—Area rules are established for a high-velocity normal penetration of a sharp and undeformable impactor into a ductile target under a condition of a localized impactor-target interaction. An impactor with a polygonal form of cross sections and a three-dimensional impactor of a close shape with the same distribution of the cross-sectional area along the longitudinal axis are considered. It is shown that under some additional requirements the difference in the ballistic limits of these two impactors is of the order of \( c^2 \) where \( c \) is of the order of the difference in their shapes. The established area rule is valid also for the drag force applied to flying projectiles under a wide range of conditions of high velocity flight.

Keywords: penetration, impact, projectile, target, localized interaction models.

NOTATIONS

\[ a_1, a_2, \ldots \] parameters characterizing properties of the material of the target
\[ h \] thickness of the target (Fig. 1)
\[ D, \bar{D} \] drag force
\[ g \] radius of a circle inscribed into the base of a reference impactor
\[ L \] distance from the front surface of the target to the impactor nose (Fig. 1)
\[ m \] the impactor’s mass
\[ n^x \] unit inner normal vector at the impactor’s surface
\[ r(x, \theta), r(x, \bar{\theta}) \] functions defining the shape of a reference impactor
\[ s(x) \] cross-sectional area of the impactor
\[ t \] time
\[ u \] cosines of the angle between \( n^x \) and \( (-v) \) at the impactor’s surface
\[ T_0 \] reference impactor with a polygonal cross section
\[ T_1 \] three-dimensional impactor
\[ v \] velocity of a surface element, velocity of the impactor
\[ v_{w0}, v_{w1} \] ballistic limit for the impactor \( T_0 \) and \( T_1 \), respectively
\[ v_o \] initial velocity of the impactor
\[ x, \rho, \theta \] cylindrical coordinates
\[ x_1(h), x_2(h) \] coordinates of cross sections of the impactor and the target (Fig. 1)
\[ \varepsilon \] a small parameter
\[ \Omega_0, \Omega_1, \hat{\Omega} \] functions determining the impactor-target interaction
\[ \Phi(x, \theta) \] function determining the impactor’s shape (Fig. 1)
\[ \theta_i, \gamma_i \] parameters determining the shape of the cross section of a reference impactor
\[ \tau^0 \] unit inner tangent vector at the impactor’s surface

Superscripts

\[ 0 \] unit vector

Subscripts

\[ x \] derivative with respect to \( x \)
\[ \theta \] derivative with respect to \( \theta \)

1. INTRODUCTION

Area rules determine the conditions when a difference in the magnitudes of some characteristics of a three-dimensional projectile and that of a reference projectile is of a higher order than the difference in their shapes. Area rules are used in gas dynamics where...
they have been established for drag coefficients of projectiles under different high velocity flight conditions provided a body of revolution is chosen as a reference projectile. It is assumed that both projectiles have the same length and distribution of cross-sectional area along the longitudinal axis and that the boundary contours are close in every cross section (the difference is of the order of ε). Then the difference in the drag forces acting on these projectiles is of the order of ε². Area rules were determined for specific ranges of flight conditions, for various integral characteristics (not only for drag force) including characteristics represented by functionals of a quite general form. Review of these results can be found in Refs [1, 2].

The significance and usefulness of the area rules is elucidated in the following. A reference impactor of simple shape is relatively straightforward to analyze. The method of "area rules" provides a means by which the behavior of a projectile with a more complex shape can be deduced from the behavior of the reference impactor. This can greatly simplify prediction procedures for complex projectiles and is of particular use in deducing the consequences of small geometrical changes for projectile performance.

Using the assumption of the localized projectile–medium interaction it was proved that area rules are valid also for the ballistic limit of penetrating impactors if a body of revolution is considered as a reference impactor (see Ref. [3]). In this study new area rules are found for penetrating and flying projectiles when a projectile with a polygonal cross section is chosen as a reference projectile.

2. DESCRIPTION OF THE MODEL

Consider a high-speed penetration of a sharp and undeformable impactor into a ductile target. Assume that the impactor–target interaction at a given point at the impactor’s surface which is in contact with the target, is given by the following relation:

\[
dF = (\Omega_p n^0 + \Omega_t \tau^0) \, dS,
\]

where

\[
\Omega_\alpha = \Omega_\alpha (a_1, a_2, \ldots, a_n; u, v), \quad \alpha = p, t,
\]

\[
\tau^0 = -\frac{v^0 + un^0}{\sqrt{1 - u^2}}, \quad u = -v^0 \cdot n^0,
\]

and \(dF\) is the force acting on the surface element \(dS\) of the impactor; \(\Omega_p = \Omega_t = 0\) if \(u \leq 0\). Hereafter, the parameters \(a_1, a_2, \ldots\) are not listed as arguments of the functions \(\Omega_\alpha\).

The model given by Eqns (1)–(3) belongs to a class of localized interaction models (see, e.g. Ref. [1]) whereby the integral effect of the interaction between a host medium and the moving projectile is described as a superposition of the independent local interactions of the projectile’s surface elements with the host medium. Every localized interaction model is determined by the local geometric and kinematic parameters of the surface element (primarily, by the local velocity and the angle between the velocity vector and unit normal vector to the projectile surface at this location) as well as by some global parameters which account for the integral characteristics of the medium.

Various specific localized interaction models have been suggested for modeling penetration of rigid and sharp impactors into ductile targets. The description and analysis of these models can be found in Refs. [4–7]. Studies reported in Refs. [8–10] provide some examples of their use in investigating the effect of the projectile shape on the penetration. Analysis presented in this study is based on the general localized interaction models given by Eqns (1)–(3) without specifying a particular form of the functions \(\Omega_p\) and \(\Omega_t\).

The total force \(F\) is determined by integrating the local force over the contact surface \(\sigma\):

\[
F = \int_{\sigma} (\Omega_p n^0 + \Omega_t \tau^0) \, dS.
\]
Equations (3) and (4) yield the expression for the drag force $D$:

$$D = F \cdot (-v^0) = \iint_{\sigma} \Omega_0(u, v) \, dS,$$

(5)

where

$$\Omega_0(u, v) = u\Omega_{\rho}(u, v) + \sqrt{1 - u^2}\Omega_{\theta}(u, v).$$

(6)

All the notations are presented in Fig. 1. Let the three-dimensional impactor with a shape given (in cylindrical coordinates $x, \rho, \theta$) by equation

$$\rho = \Phi(x, \theta)$$

(7)

move in the target with the velocity directed along the $x$ axis, i.e.,

$$v^0 = -x^0.$$

(8)

We consider a normal impact of a sharp and rigid impactor and assume that its shape is such that the total force is directed along the $x$-axis. Projectiles with an axis of symmetry provide examples of such impactors. The part of the lateral surface of the projectile between the cross sections $x = x_1$ and $x = x_2$ interacts with the barrier

$$x_1(h) = \begin{cases} 0 & \text{if } 0 \leq h \leq b, \\ h - b & \text{if } b \leq h \leq b + L, \end{cases}$$

$$x_2(h) = \begin{cases} h & \text{if } 0 \leq h \leq L, \\ L & \text{if } L \leq h \leq b + L. \end{cases}$$

(9)

(10)

Using standard formulae of differential geometry

$$n^0 \cdot x^0 = u = A/B, \quad A = \Phi \Phi_x, \quad B = \sqrt{\Phi^2(\Phi_x^2 + 1) + \Phi^2_{\theta}}, \quad dS = B \, dx \, d\theta.$$

(11)

Equation (5) can be rewritten as follows:

$$D[\Phi(x, \theta); h, v] = \int_{x_1(h)}^{x_2(h)} \int_{0}^{2\pi} \Omega(u, v) A \, dx \, d\theta,$$

(12)

Fig. 1. Coordinates and notations.
where
\[ \Omega(u, v) = \Omega_0(u, v)/u. \] (13)

The equation of motion of the impactor reads
\[ m \frac{d^2 h}{dt^2} = -D \left[ \Phi(x, \theta) ; h, \frac{dh}{dt} \right]. \] (14)

After a change of variables, \((dh/dt)^2 = W(h)\), it can be rewritten as
\[ \frac{m}{2} \frac{dW}{dh} = -\tilde{D}[\Phi(x, \theta) ; h, W], \] (15)

where
\[ \tilde{D}[\Phi(x, \theta) ; h, W] = D[\Phi(x, \theta) ; h, \sqrt{W}]. \] (16)

It is assumed that the target has been perforated if the impactor emerges from it with a zero velocity, i.e.,
\[ W(b + L) = 0. \] (17)

The solution of Eqn (15) with the boundary condition given by Eqn (17) describes the impactor's motion for the case of perforation with ballistic limit velocity. The formula for the ballistic limit \(v_*\) reads
\[ v_* = \sqrt{W(0)}. \] (18)

### 3. Reference Impactor

Along with a three-dimensional impactor \(T_1\) consider a reference impactor \(T_0\) having a polygonal cross section which are geometrically similar to each other. It is assumed that both impactors have the same length \(L\) and area \(s(x)\) at every cross section \(0 \leq x \leq L\), and density. Therefore they have identical volume and mass.

The shape of the impactor \(T_0\) is defined by a function \(R(x)\) determining its longitudinal contour, and by the radius of the inscribed circle \(r\) and angles \(\theta_i (i = 1, 2, \ldots, N)\) determining the form of a polygon with \(N\) sides at section \(x = L\) (see Fig. 2). The equation of the surface of the impactor \(T_0\) can be written as
\[ \rho = r(x, \theta) = \begin{cases} r_1(x, \theta) & \text{if } \gamma_0 \leq \theta \leq \gamma_1, \\ \cdots & \cdots \\ r_i(x, \theta) & \text{if } \gamma_{i-1} \leq \theta \leq \gamma_i, \\ \cdots & \cdots \\ r_N(x, \theta) & \text{if } \gamma_{N-1} \leq \theta \leq \gamma_N, \end{cases} \] (19)

where
\[ r_i(x, \theta) = gR(x)\sec(\theta - \theta_i), \quad i = 1, 2, \ldots, N, \quad R(L) = 1, \] (20)
\[ \gamma_i = (\theta_i + \theta_{i+1})/2, \quad i = 1, 2, \ldots, N - 1, \] (21)
\[ \gamma_0 = -\pi + (\theta_1 + \theta_N)/2, \quad \gamma_N = \pi + (\theta_1 + \theta_N)/2. \] (22)

Assume that the shapes of the impactors are close in the sense that
\[ \Phi(x, \theta) = r(x, \theta) + \varepsilon \xi(x, \theta) \] (23)

where \(\xi(x, \theta)\) is some function and \(\varepsilon\) is a small parameter. Using Eqns (19)-(23) the equality of the cross-sectional areas of the projectiles \(T_0\) and \(T_1\) can be written, for some \(x\), as follows
\[ \frac{1}{2} \sum_{i=1}^{N} \gamma_i \int_{\gamma_{i-1}}^{\gamma_i} \left[ gR(x)\sec(\theta - \theta_i) + \varepsilon \xi \right]^2 d\theta = \frac{1}{2} \sum_{i=1}^{\gamma_N} \left[ gR(x)\sec(\theta - \theta_i) \right]^2 d\theta. \] (24)
Equation (24) yields the following estimate:
\[
\delta(x) = \sum_{i=1}^{N} \int_{\gamma_{i-1}}^{\gamma_{i}} \xi(x, \theta) \sec(\theta - \theta_{i}) \, d\theta = -\frac{\varepsilon}{2} \sum_{i=1}^{N} \int_{\gamma_{i-1}}^{\gamma_{i}} \xi^{2} \, d\theta \propto \varepsilon,
\]
which will be used subsequently.

4. AREA RULE

Using the perturbation technique (see, e.g., Ref. [11]) the solution of Eqns (15) and (17) can be represented in the form of a series of the small parameter \( \varepsilon \)
\[
W(h) = W_{0}(h) + W_{1}(h)\varepsilon + O(\varepsilon^{2}).
\]

Let us now determine the first and the second terms in this series. Substituting \( \Phi(x, \theta) \) from Eqn (23) and \( W(h) \) from Eqn (26), Eqn (15) can be rewritten as
\[
\frac{m}{2} \left[ \frac{dW_{0}}{dh} + \frac{dW_{1}}{dh} \varepsilon + O(\varepsilon^{2}) \right] = -\hat{D} [r(x, \theta) + \varepsilon \xi(x, \theta)h, W_{0} + W_{1}\varepsilon + O(\varepsilon^{2})].
\]

The boundary conditions for the functions \( W_{0} \) and \( W_{1} \) are determined from Eqns (17) and (26):
\[
W_{0}(L + b) = 0,
\]
\[
W_{1}(L + b) = 0.
\]

A Taylor series expansion of the right-hand side of Eqn (27) with respect to \( \varepsilon \) can be written as
\[
\hat{D} [r(x, \theta) + \varepsilon \xi(x, \theta)h, W_{0} + W_{1}\varepsilon + O(\varepsilon^{2})] = \hat{D} [r(x, \theta)h, W_{0}] + (\hat{cD}/\hat{c}\varepsilon)_{\varepsilon=0} + O(\varepsilon^{2}).
\]

Equating the coefficients near \( \varepsilon^{0} \) in both sides of Eqn (27) with the right-hand side given by Eqn (30) we arrive at the equation for the function \( W_{0} \):
\[
\frac{m}{2} \frac{dW_{0}}{dh} = -\hat{D} [r(x, \theta)h, W_{0}].
\]
The solution of Eqn (31) with the boundary condition (28) describes the motion of the impactor \( T_0 \) for the case of target perforation with the ballistic limit velocity \( v_{*0} \).

More sophisticated analysis is required to derive the equation for \( W_1 \). Let us represent the function \( \bar{D} \) as the following sum:

\[
\bar{D} \[ \Phi(x, \theta); h, W \] = \sum_{i=1}^{N} \bar{D}_i \[ r_i(x, \theta) + \varepsilon \xi(x, \theta); h, W \],
\]

where

\[
\bar{D}_i \[ r_i(x, \theta) + \varepsilon \xi(x, \theta); h, W \] = \int_{x_{i-1}}^{x_{i+1}} \int_{\gamma_{i-1}}^{\gamma_{i+1}} \tilde{\Omega}(u_i, W) A_i \, dx \, d\theta,
\]

\[
\tilde{\Omega}(u, W) = \Omega(u, \sqrt{W})
\]

variables \( u_i \) and \( A_i \) have the same meaning as \( u \) and \( A \) in Eqn (11), respectively, whereas the subscript indicates the range of the angle \( \theta \in [\gamma_{i-1}, \gamma_i] \). Using Eqns (19) and (20) we arrive at the following expressions for \( u_i \) and \( A_i \):

\[
A_i = \left[ gR \sec(\theta - \theta_i) + \varepsilon \xi \right] \times \left[ gR \sec(\theta - \theta_i) + \varepsilon \xi \right],
\]

\[
B_i = \left[ \left[ gR \sec(\theta - \theta_i) + \varepsilon \xi \right]^2 \times \left[ \left[ gR \sec(\theta - \theta_i) + \varepsilon \xi \right]^2 + 1 \right] \right. + \left. \left[ gR \sin(\theta - \theta_i) \sec^2(\theta - \theta_i) + \varepsilon \xi \right] \right]^{1/2},
\]

\[
u_i = A_i / B_i.
\]

Using Eqns (32)–(34) the derivative \( \partial \bar{D} / \partial \varepsilon \) can be represented as the sum

\[
\frac{\partial \bar{D}}{\partial \varepsilon} = \sum_{i=1}^{N} \frac{\partial \bar{D}_i}{\partial \varepsilon}
\]

where

\[
\frac{\partial \bar{D}_i}{\partial \varepsilon} = \int_{x_{i-1}}^{x_{i+1}} \int_{\gamma_{i-1}}^{\gamma_{i+1}} \frac{\partial \tilde{\Omega}(u_i, W)}{\partial \varepsilon} \frac{\partial u_i}{\partial \varepsilon} A_i + \tilde{\Omega}(u_i, W) \frac{\partial A_i}{\partial \varepsilon} + \frac{\partial \tilde{\Omega}(u_i, W)}{\partial W} \frac{\partial W}{\partial \varepsilon} A_i \, dx \, d\theta.
\]

Equations (35)–(37) allow to determine the derivatives \( \partial u_i / \partial \varepsilon \) and \( \partial A_i / \partial \varepsilon \). Equation (26) implies that \( \partial W / \partial \varepsilon = W_1 \). After transformations, Eqn (38), for \( \varepsilon = 0 \), can be written as

\[
(\partial \bar{D} / \partial \varepsilon)|_{\varepsilon = 0} = I - J + K,
\]

where

\[
I = \int_{x_{i-1}}^{x_{i+1}} E(x, W_0) \delta_x \, dx, \quad J = \int_{x_{i-1}}^{x_{i+1}} G(x, W_0) \Psi(x) \, dx, \quad K = \beta(h, W_0) W_1,
\]

\[
G(x, W_0) = \frac{\tilde{\Omega}(u, W_0) u^3}{gR_x}, \quad \tilde{\Omega}(u, W_0) = \frac{gR_x}{(g^2 R_x^2 + 1)^{1/2}},
\]

\[
E(x, W_0) = G(x, W_0) / R_x + g \tilde{\Omega}(u, W_0),
\]

\[
\Psi(x) = \sum_{i=1}^{N} \int_{\gamma_{i-1}}^{\gamma_{i+1}} \frac{\partial \xi \sin(\theta - \theta_i) \, d\theta}{\partial \theta},
\]

\[
= 2 \left[ \sin \frac{\theta_{N+1} - \theta_1}{2} \xi(x, \gamma_0) + \sum_{i=1}^{N-1} \sin \frac{\theta_{i+1} - \theta_i}{2} \xi(x, \gamma_i) \right],
\]

\[
\beta(h, W_0) = 2 \int_{x_{i-1}}^{x_{i+1}} \frac{\tilde{\Omega}(u, W_0) R_x \delta}{\delta} \, dx, \quad \delta = \frac{g^2}{2} \sum_{i=1}^{N} \int_{\gamma_{i-1}}^{\gamma_{i+1}} \sec^2(\theta - \theta_i) \, d\theta,
\]

\[
\delta = \frac{g^2}{2} \sum_{i=1}^{N} \int_{\gamma_{i-1}}^{\gamma_{i+1}} \sec^2(\theta - \theta_i) \, d\theta,
\]
where $\delta$ is the area of the base of the body $T_0$. Integrating the expression for $I$ by parts we obtain

\[ I = E \left[ x_2(h), W_0 \right] \delta \left[ x_2(h) \right] - E \left[ x_1(h), W_0 \right] \delta \left[ x_1(h) \right] - \int_{x_1(h)}^{x_2(h)} E_2 \delta(x) \, dx. \]  

(46)

Equations (46) and (25) imply the following estimate:

\[ I = O(\varepsilon). \]  

(47)

Let us now consider the case when

\[ J = 0. \]  

(48)

(1) The requirement that for every $x$ the contour of the cross-section of the impactor $T_1$ passes through the apexes of the polygon at the cross section of the impactor $T_0$ implies that in Eqn (44) all $\xi(x, \gamma_i) = 0$. Therefore, $\Psi(x) = 0$ and Eqn (48) is valid.

(2) If the cross section of the impactor $T_0$ is a regular polygon, then

\[ \gamma_i = \gamma_0 + \frac{2\pi}{N} i, \quad \theta_i = \gamma_0 + \frac{\pi}{N} (2i - 1), \quad i = 1, \ldots, N \]  

(49)

and the function $\Psi(x)$ can be written as

\[ \Psi(x) = 2 \sin \frac{\pi}{N} \sum_{i=0}^{N-1} \xi(x, \gamma_i). \]  

(50)

Equation (50) implies the estimate given by Eqn (48) if

\[ \sum_{i=0}^{N-1} [\Phi(x, \gamma_i) - r(x, \gamma_i)] = 0. \]  

(51)

(3) If in Eqn (50) $N \to \infty$, then $\Psi(x) \to 0$ and the reference impactor is a body of revolution. The latter special case was analyzed in our previous study [3].

Therefore, Eqns (47) and (48) imply the estimate

\[ (\partial/D/\partial x)_{x=0} = \beta[h, W_0(h)] W_1 + O(\varepsilon). \]  

(52)

Equating the coefficients near $\varepsilon$ in both sides of Eqn (27) and using Eqns (30), (40), (47), and (48) we arrive at the equation for the function $W_1$:

\[ \frac{m}{2} \frac{dW_1}{dh} = - \beta[h, W_0(h)] W_1; \]  

(53)

with the boundary condition of Eqn (29). The only possible solution of this equation is

\[ W_1(h) = 0. \]  

(54)

Therefore,

\[ W(h) = W_0(h) + O(\varepsilon^2), \]  

(55)

\[ W(0) = W_0(0) + O(\varepsilon^2) \]  

(56)

and the estimate for the ballistic limit of the impactor $T_1$ is:

\[ v_* = \sqrt{v_{*0}^2 + O(\varepsilon^2)} = v_{*0} + O(\varepsilon^2), \]  

(57)

where the subscript zero denotes the ballistic limit of the impactor $T_0$.

Thus the difference in the ballistic limits of the impactors $T_0$ and $T_1$ is of the order of $\varepsilon^2$, i.e., the area rule for the ballistic limit is proved.
5. SOME OTHER VERSIONS OF AREA RULES

Similar rules can be established for any other characteristics of the penetration process. Let the impactor $T_1$ start its motion in the target with an initial velocity $v_{in}$, i.e., the following boundary condition replaces Eqn (17):

$$W(0) = v_{in}^2.$$  \hspace{1cm} (58)

Then the boundary conditions for the functions $W_0(h)$ and $W_1(h)$ read:

$$W_0(0) = v_{in}^2,$$  \hspace{1cm} (59)

$$W_1(0) = 0.$$  \hspace{1cm} (60)

The boundary value problem given by Eqns (53) and (60) have the solution given by Eqn (54). Therefore, the following estimate is valid:

$$v(h) = v_0(h) + O(\varepsilon^2).$$  \hspace{1cm} (61)

Thus, if the initial velocities of the impactors $T_0$ and $T_1$ are the same, the difference in their velocities (including a residual velocity) at any depth $h \leq L + b$ is of the order of $\varepsilon^2$ if both impactors continue their motion in the target. The latter law holds if both impactors penetrate a semi-infinite target. The only difference is that in this case the functions $x_1(h)$ and $x_2(h)$ are determined by the following equations:

$$x_1(h) = 0,$$  \hspace{1cm} (62)

$$x_2(h) = \begin{cases} h & \text{if } 0 \leq h \leq L, \\ L & \text{if } h \leq L. \end{cases}$$  \hspace{1cm} (63)

Note that the above analysis does not allow to derive estimates for the difference in the maximum depth of penetration. However, the following property can be formulated: if the impactor $T_0$ terminated its motion in the target while the impactor $T_1$ continues to move, the velocity of the second impactor is of the order of $\varepsilon^2$.

Consider an impactor which is totally immersed into the target and moves with a constant velocity $\bar{W}$. Setting $x_1(h) = 0$, $x_2(h) = L$, and $W = \bar{W}$ we obtain that $\bar{D}$ is the drag force acting on an impactor moving at a zero angle of attack. Since $\bar{W}$ appears in the expression for the drag force $\bar{D}$ as a parameter, the third term in the integral in Eqn (39) vanishes and in Eqn (40) must be set $K = 0$.

Then Eqns (40), (47), and (48) and the expansion given by Eqn (30) yield a new area rule for the drag force acting on flying projectiles:

$$\bar{D}[\Phi(x, \theta)] = \bar{D}[r(x, \theta)] + O(\varepsilon^2).$$  \hspace{1cm} (64)

The latter area rule is valid under condition of localized projectile–host medium interaction for high-speed continuum, intermediate and free molecular flow regimes (see Ref. [1]).

It is interesting to note that if both impactors $T_0$ and $T_1$ have the same shape in the longitudinal cross section, i.e.

$$\Phi(x, \theta) = R(x) \theta(\theta),$$  \hspace{1cm} (65)

where $\theta(\theta)$ determines the cross-section contour of projectile $T_1$, the conditions of small deviation in forms of the projectiles and of equality of their cross-sectional areas must be valid for the bases of the projectiles only. For instance, if conical bodies are considered $R(x) = x$ and the body $T_0$ is a pyramid.

6. CONCLUSIONS

The motion of a three-dimensional rigid projectile in a target is analyzed when the interaction of the projectile's surface with the barrier can be described at each contact point
as a localized interaction. This general model covers a wide range of specific models for penetration of sharp and rigid impactors into ductile targets. It is assumed that the shape of the three-dimensional projectile is close to the shape of any projectile with polygonal cross sections of the same length allowing an inscribed circle. The difference in the shapes of both bodies is characterized by a small parameter $\varepsilon$ and some additional conditions of small deviations in their surfaces at the apexes of polygons are assumed. If the projectiles have the same distribution of the cross-sectional area along the longitudinal axis the following area rules are valid for penetrating impactors: the difference in the ballistic limits of the impactors is of the order of $\varepsilon^2$; the difference in the velocities of the impactors at a given depth is of the order of $\varepsilon^2$ if both impactors have the same initial velocity before the penetration. It is also shown that similar area rules for the ballistic limit of penetrating impactors are valid for flying projectiles under various conditions of high velocity flight.

The derived area rules can be validated by conducting an experimental study in which the penetration of a “reference” and three-dimensional impactors satisfying the above specified area rules conditions will be compared for different initial velocities, target materials and thicknesses. The results obtained in this study may provide motivation for such experimental investigations.

REFERENCES