OPTIMIZATION OF THE SHAPE OF A PENETRATOR TAKING INTO ACCOUNT PLUG FORMATION

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Abstract. New formulation of the problem of the impactor's nose geometry optimization is proposed when penetrator with a planar bluntness causes plug formation. This problem is studied for conical-shape impactors using simplified model for impactor-target interaction.

1. Introduction. There are only a few publications on impactor's shape optimization using analytical models for impactor-target interaction in which high speed normal penetration into a ductile target is considered. Three-dimensional impactors (Ostapenko et al., 1994, Ostapenko and Yakunina, 1999; Vedernikov and Shchepanovsky, 1995) and bodies of revolution (Jones et al., 1998; Jones and Rule, 2000) were considered applying the criterion of the maximum depth of penetration into a semi-infinite target while the resistance of the target at the initial stage of penetration was neglected. The latter factor was taken into account by Bunimovich and Dubinsky (1995) where the optimization of three-dimensional conical impactors was studied. The suggested methodology was further developed by Ben-Dor et al. (1997a, 1997b, 1999a) and applied to the targets with a finite thickness and layered targets. To the best of our knowledge the problem of impactor's shape optimization when the penetration is associated with plug formation was not considered in the literature. First results of our investigations on that subject are presented in this Note.

2. Mathematical formulation of the model. Consider a high speed normal penetration of a rigid body of revolution into a target with the thickness \( b \). The notations are shown in Figs.1-2. The coordinate \( h \), the depth of the penetration, is defined as the distance between the nose of the impactor and the front surface of the target. The coordinates \( x, \rho \) are associated with the impactor. The part of the lateral surface of the impactor between the cross-sections \( x = x_1 \) and \( x = x_2 \) interacts with the target where

\[
x_1(h) = \begin{cases} 
0 & \text{if } 0 \leq h \leq b \\
0 & \text{if } h - b \leq h \leq b + L \\
L & \text{if } L \leq h \leq b + L 
\end{cases} 
\]

\[
x_2(h) = \begin{cases} 
0 & \text{if } 0 \leq h \leq L \\
0 & \text{if } L \leq h \leq b + L 
\end{cases} 
\]

We assume that impactor-target interaction at a given location at the impactor's lateral surface which is in contact with the target is determined by the following
equation (for details see Recht, 1990; Bunimovich and Dubinsky, 1995; Ben-Dor et al., 1999b and references therein):

\[ d\vec{F} = \left[ a_1 \left( -\vec{v} \cdot \vec{n} \right)^2 \vec{v} + a_0 \right] \vec{n} dS \] (2)

where \( d\vec{F} \) is the force acting at the surface element \( dS \) of the impactor along the inner normal vector \( \vec{n} \) at a given location at the impactor’s surface, \( \vec{v} \) is an unit local velocity vector, the parameters \( a_1 > 0 \) depend upon the properties of the material of the target. Hereafter we assume that \( a_0 \) and \( a_1 \) are the distortion pressure and density of the material of the target, respectively.

Let the function \( \rho = \Phi(x) \) determines the generator of the impactor, its length \( L \) and base radius \( R = \Phi(L) \) are assumed to be given. Then the expression for the
drag force acting at its lateral surface $D_{\text{imp}}$ can be obtained from Eqs. (1) and (2) using formulae of differential geometry:

$$D_{\text{imp}} = 2\pi \int_{x_1(h)}^{x_2(h)} \left( a_1V^2 \frac{\Phi_x^2}{\Phi_x^2 + 1} + a_0 \right) \Phi \Phi_x dx, \quad \Phi_x = \frac{d\Phi}{dx}$$

(3)

Generally, the impactor can have a planar bluntness with a radius $r$. It is assumed (Sagomonian, 1988) that in this case the cylindrical plug with a radius $r$, height $b$ and mass $m_{pl} = \pi a_1 r^2 b$ is formed at the beginning of penetration that moves together with the impactor while $h \leq b$ (at the first stage of penetration, Fig. 1). The resistance force associated with the plug during this stage of the penetration is $D_{pl} = 2\pi a_3 (b - h)$ where $a_3$ is shear strength. When $h = b$ the impactor and the plug brake apart, and the impactor with the mass $m_{\text{imp}}$ moves under the action of the force $D_{\text{imp}}$ (the second stage of penetration, Fig. 2).

![Figure 2. The second stage of penetration.](image-url)
Thus the equations describing penetration can be written as follows:

\[ (m_{imp} + m_{pl})vv' + D_{imp}(h, v) + D_{pl}(h) = 0 \quad \text{(the first stage)} \] (4)

\[ m_{imp}vv' + D_{imp}(h, v) = 0 \quad \text{(the second stage)} \] (5)

where the transformation \( \frac{dv}{dt} = v \frac{dv}{dh} = vv' \) is used.

The ballistic limit velocity (BLV) \( v_\infty \) is defined as the initial velocity of the impactor required to emerge from the target with a zero velocity. Let \( v = \tilde{v}(h) \) is a solution of Eq. (5) with the initial condition \( v(b + L) = 0 \) and \( v = \tilde{v}(h) \) is a solution of Eq. (4) with the initial condition \( v(b) = \tilde{v}(b) \). Then BLV is \( v_\infty = \tilde{v}(0) \). After some algebra we obtain:

\[ v_\infty^2 = \int_{\bar{b}}^{\bar{b}+1} \psi_1(h)dh + \psi_1(b) \int_{\bar{b}}^{\bar{b}+1} \psi_2(h) - \psi_2(b) dh \] (6)

where \( (i = 1, 2) \)

\[ \psi_i(h) = \exp \left[ \frac{4 \tau^2}{\beta_2 + (2 - i) \beta_1 \tilde{r}} \Theta(h) \right], \quad \Theta(h) = \int_0^h (\xi) d\xi, \] (7)

\[ g_i(h) = 2 \frac{\beta_0 \tau I(h) + 2(2 - i) \beta_1 \tilde{r} (b - \tilde{r})}{\tau \left[ \beta_2 + (2 - i) \tilde{r} \right]^2} \quad \beta_0 = \frac{a_0}{a_1}, \quad \beta_1 = \frac{a_3}{a_1}, \quad \beta_2 = \frac{m_{imp}}{\pi R^2 L a_1} \] (8)

\[ I(h) = \Phi^2 \left[ \bar{x}_2(h) - \Phi \bar{x}_1(h) \right], \quad J(h) = \int_{\bar{x}_1(h)}^{\bar{x}_2(h)} \frac{\Phi \Phi^3}{\Phi_x^2 + 1} d\bar{x} \] (9)

\[ \bar{x} = \frac{x}{L}, \quad \Phi = \frac{\Phi}{R}, \quad \Phi_x = \frac{d\Phi}{dx}, \quad \bar{h} = \frac{h}{L}, \quad \bar{b} = \frac{b}{L}, \quad \bar{r} = \frac{r}{R}, \quad \tau = \frac{\tau}{L} \] (10)

and functions \( \bar{x}_1(h) \) are given by Eq. (1) after changing \( x, h, b, L \) by \( \bar{x}, \bar{h}, \bar{b}, \bar{L} \), respectively.

Generally, the problem is to find the impactor with the shape such that its BLV is minimum. However, even the relatively simple model used in this study yields quite
involved formulas when Eq. (6) is employed as the optimization criterion. In this study we consider the problem for the conical impactors.

3. Optimal conical impactors. In this case \( \Phi = \bar{r} + (1 - \bar{r})\bar{x} \) and the integrals I and J can be represented as follows:

\[
I(\bar{h}) = (1 - \bar{r})K(\bar{h}), \quad J(\bar{h}) = \mu K(\bar{h}), \quad \mu = \frac{1}{2} \frac{(1 - \bar{r})^3}{\tau^2 (1 - \bar{r})^2 + 1}
\]  
(11)

where \( K(\bar{h}) = 2\bar{r}\left[\bar{x}_2(\bar{h}) - \bar{x}_1(\bar{h})\right] + (1 - \bar{r})\left[\bar{x}_2(\bar{h}) - \bar{x}_1(\bar{h})\right] \)

Using Eq. (1) rewritten in dimensionless variables we obtain:

\[
\Theta(\bar{h}) = \mu\left[2\bar{r}\left[M_{2,1}(\bar{h}) - M_{1,1}(\bar{h})\right] + (1 - \bar{r})\left[M_{2,2}(\bar{h}) - M_{1,2}(\bar{h})\right]\right]
\]  
(12)

where

\[
M_{i,j}(\bar{h}) = \int_{0}^{\bar{h}} x_j(\xi)d\xi
\]  
(13)

\[
M_{1,j} = \begin{cases} 
0 & \text{if } \bar{h} \leq \bar{b} \\
\frac{1}{j+1}(\bar{h} - \bar{b})^{j+1} & \text{if } \bar{h} > \bar{b}
\end{cases}, \quad M_{2,j} = \begin{cases} 
\frac{1}{j+1}\bar{h}^{j+1} & \text{if } \bar{h} \leq 1 \\
\bar{h}^{j+1} & \text{if } \bar{h} > 1
\end{cases}
\]  
(14)

Formulas for BLV of a cylinder, \( v_{\text{cyl}} \), and a sharp cone, \( v_{\text{sh}} \), can be obtained from the above expression setting \( \bar{r} = 1 \) and \( \bar{r} = 0 \), respectively:

\[
v_{\text{cyl}}^2 = \frac{2\beta_1\bar{b}^2}{\tau(\beta_2 + \bar{b})} = \frac{2\pi R a_3 b^2}{m_{\text{imp}} + \pi R^2 b a_1}
\]  
(15)

\[
v_{\text{sh}}^2 = \frac{\beta_0}{\lambda} \left[ \exp(2\lambda\bar{b}/\beta_2) - 1 \right] = \frac{a_0}{\bar{a}^2 \lambda^2} \left[ \exp\left(2\pi a_1 b R^2 \lambda / m_{\text{imp}}\right) - 1 \right]
\]  
(16)

where \( \lambda = \frac{\tau^2}{\tau^2 + 1} \).
In the case of a conical impactor the problem is reduced to finding the optimal radius of planar bluntness. Since function \( v_*(\bar{r}) \) eluded analyzing using analytical methods, we applied computer simulation. The result is as follows.

The minimum value of BLV is attained at one of two boundaries of the closed interval \([0,1]\), namely, when \( \bar{r} = 0 \) (the sharp cone) or when \( \bar{r} = 1 \) (the cylinder). Thus, if \( v_{cyl}^2 < v_{sh}^2 \) then the cylinder is the impactor with minimum BLV. If \( v_{cyl}^2 > v_{sh}^2 \) then the sharp cone is the optimal impactor, and if \( v_{cyl}^2 = v_{sh}^2 \) then both these shapes provide the minimum BLV. Using Eqs. (15) and (16) the condition \( v_{cyl}^2 < v_{sh}^2 \) can be rewritten as:

\[
\frac{a_4 b}{a_0 R} < (\alpha + 1) \chi(2\alpha \lambda) \tag{17}
\]

where \( \alpha = \frac{\pi a_4 R^2 b}{m_{imp}} \), \( \chi(z) = \frac{1}{z} \{ \exp(z) - 1 \} \).

REFERENCES


