ANALYSIS OF BALLISTIC PROPERTIES OF LAYERED TARGETS USING CAVITY EXPANSION MODEL

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Abstract. The investigations on the ballistic resistance of multi-layered targets on the basis of the cylindrical cavity expansion approach are announced. General statement of the problem, the mathematical model and some results for the target with large air gaps between the layers are represented.

2. Introduction. The review on sub-ordnance perforation of multi-layered targets can be found in the papers by Corbett et al. (1996) and Ben-Dor et al. (1997, 1998a, 1998b). Since a rigorous analytical theory is not feasible at present, qualitative laws obtained from approximate models can be useful for further investigation. The localized impactor-target interaction approach (see Zukas (1990), Bunimovich and Dubinsky (1995)) is applied by Ben-Dor et al. (1997, 1998a, 1998b). The announced current research is based on the more physically justified cylindrical cavity expansion approach (see Forrestal (1981), Sagomonian (1988)).

2. Mathematical model and statement of the problem. The normal penetration of a rigid sharp striker into a ductile target with a finite thickness with air gaps is considered. The impactor is assumed to be a body of revolution. The basic notations are showed in Fig. 1. The coordinate \( h \), the depth of penetration, is defined as the distance between the nose of the impactor and the upper surface of the target. The coordinates \( \xi \) and \( x \) are associated with the target and the impactor, respectively. Only the nose of the impactor with the length \( L \) interacts with the target, and the impactor may have also the cylindrical part with the length \( \ell \); \( r_0 \) is the impactor’s base radii. Assume that the target consists of \( N \) plates with the thicknesses \( b_1, b_2, b_3, \ldots, b_N \) made, generally, from different materials where the \( i \)-th plate is located between the sections \( \xi = \xi_i \) and \( \xi = \xi_i + b_i \), \( \xi_1 = 0 \), \( i = 1, 2, \ldots, N \) and the total thickness of the target is \( b = \xi_N + b_N \). The part of the lateral surface of the impactor between the cross-sections \( x = x_1 \) and \( x = x_2 \) (see Fig. 1) interacts with some layers of the target or is in contact with some air gaps where

\[
x_1(h) = \begin{cases} 
0 & \text{if } 0 \leq h \leq b \\
h - b & \text{if } b \leq h \leq b + L 
\end{cases}, \quad x_2(h) = \begin{cases} 
0 & \text{if } 0 \leq h \leq L \\
L & \text{if } b \leq h \leq b + L 
\end{cases} \tag{1}
\]

We assume that the impactor-target interaction can be described for every layer by the cylindrical cavity expansion approximation \([7-8]\). Then the solution of the problem of hole’s expansion from the time \( t=0 \) is represented for the considered multi-layered target in the following form:
\[ p = a^{(2)}(\xi)\hat{R}^2 + a^{(1)}(\xi)\hat{R} + a^{(0)}(\xi) \]  

where \( R \) is radii of the hole, \( p \) is the pressure applied in the normal direction at the part of the impactor’s surface,

\[
\begin{align*}
| a^{(j)}(\xi) & \text{ if } \xi_i \leq \xi \leq \xi_i + b_i, \ i = 1,...,N \\
0 & \text{ if } \xi_i + b_i \leq \xi \leq \xi_{i+1}, \ i = 1,...,N - 1
\end{align*}
\]

coefficients \( a^{(j)}(j = 0,1,2) \) depend on the properties of the material of the layer with the number \( i \). Since \( r = \Phi(x) \) is the equation of the surface of the impactor, then

\[ R = \Phi(h - \xi), \quad \dot{R} = \Phi'(h)\dot{h}, \quad \ddot{R} = \Phi''(h)\dot{h}^2 + \Phi'(h)\ddot{h}, \quad x = h - \xi\]  

and Eq. (2) can be written as follows:

\[ p = a^{(2)}(h - x)\Phi'\dot{h}^2 + a^{(1)}(h - x)(\Phi\Phi''\dot{h}^2 + \Phi\Phi'\ddot{h}) + a^{(0)}(h - x) \]

Then the force \( \overrightarrow{dF} \) acting at the surface element \( dS \) of the impactor at a given location at the surface of the impactor is \( (p\hat{n}^0 + k\hat{t}^0)dS \) where \( \hat{n}^0 \) and \( \hat{t}^0 \) are the local inner normal and local tangent unit vectors, respectively, \( k \) is the friction coefficient. The total force \( \overrightarrow{F} \) is determined by integrating the local force over the impactor-target contact surface \( S \). The drag force \( D \) is directed in the opposite direction to the projection of the total force on the direction of the motion of the impactor \( \hat{h}^0 \):

\[ D = -\hat{h}^0 \int_S \overrightarrow{dF} = \int_{x_i(h)}^{x_f(h)} \varphi(x)pdx = f_2(h)\dot{h}^2 + f_1(h)\dot{h} + f_0(h) \]  

where

\[ \varphi(x) = 2\pi\Phi(\Phi' + k), \quad f_\mu(h) = \int_{x_i(h)}^{x_f(h)} a^{(\mu)}(h - x)\varphi(x)(\Phi\Phi')^\mu dx, \ \mu = 0,1 \]

\[ f_2(h) = \int_{x_i(h)}^{x_f(h)} \varphi(x)\left[a^{(1)}(h - x)\Phi\Phi'' + a^{(2)}(h - x)\Phi'\right]dx \]

Thus, the equation of motion of the impactor of the mass \( M \), \( Md\dot{h}/dt^2 = -D \), can be rewritten as follows:

\[ \frac{dv^2}{dh} + F_2(h)v^2 + F_0(h) = 0 \]
where $v$ is velocity of the impactor and $F_p(h) = 2f_p(h)/[M + f_i(h)], \mu = 0.2$.

The ballistic limit velocity $\hat{v}$ is defined as the impact velocity of the impactor required to emerge from the target at the moment of perforation $h = b + L$ with a zero residual velocity. The value $\hat{v}$ can be determined from Eq.(9):

$$\hat{v}^2 = \int_0^{b+L} F_p(h) \cdot \exp\left[\int_0^b F_z(z)dz\right]dh$$

(10)

The above model is used as a basis of current investigation of influence of the order of the plates in the target, values of the air gaps and the shape of the impactor to ballistic resistance of the target.

3. The case of large air gaps. One illustrative result is given in this Note, namely, the case of conical-nose impactor and all $b_i \geq L$ is considered. Then, after transformations which are omitted, Eq. (10) imply:

$$\hat{v}^2 = \beta^{-2} \sum_{i=1}^{N} A_i (g_i - 1) \prod_{j=0}^{i-1} g_j, \ g_0 = 1$$

(11)

where

$$A_i = \frac{a_i(0)}{a_i(2)} \cdot g_i = \exp(3c a_i(2)) \int_0^{b+L} \frac{\{[x_2(h)]^2 - [x_1(h)]^2\} dh}{3M + c a_i(1) \{[x_2(h)]^3 - [x_1(h)]^3\}} > 0, \ i = 1, \ldots, N$$

(12)

and $c = 2\pi \beta^3 (\beta + k)$, $\beta = r_0/L = \tan \vartheta$, \( \vartheta \) is the half angle of the apex of the cone.

Consider the expressions for the square of the ballistic limit velocity of the target $\hat{v}_{m,m+1}^2$ when the plates are in the initial order and that $\hat{v}_{m+1,m}^2$ when the plates labeled by numbers $m$ and $m+1$ are interchanged:

$$\Delta \hat{v}^2 = \hat{v}_{m,m+1}^2 - \hat{v}_{m+1,m}^2 = -\beta^{-2} (g_m - 1)(g_{m+1} - 1) (\prod_{j=0}^{m-1} g_j)(A_m - A_{m+1})$$

(13)

Thus, we have proved that if two adjacent plates in a target with large air gaps are such that the value of the parameter $A = a^{(0)}/a^{(2)}$ for the first plate is larger than that for the second plate, the ballistic limit velocity of the target can be increased by interchanging these plates. It follows that the maximum ballistic limit of the target is achieved when the plates are arranged in the order of increasing values $A$. It is interesting that the parameter $a^{(1)}$ and the thicknesses of the plates do not influence on the optimal order of the layers.

The criteria $A$ is determined by special model. For instance, the simple model by Sagomonian (1988) describing the elastic-plastic response of the ductile metal target imply:
A = \frac{2\tau(1 + \varepsilon')(1 + \varepsilon)}{\rho(\varepsilon'(1 + \varepsilon) - \varepsilon)} , \quad \varepsilon = \frac{E}{2\tau(1 + \nu)} , \quad \varepsilon' = \xi n(1 + \varepsilon) \tag{14}

where \( \rho, E, \tau, \nu \) are density \( \left(\text{kg/m}^3\right) \), Young’s modulus \( \left(\text{N/m}^2\right) \), compressive shear strength \( \left(\text{N/m}^2\right) \) and Poisson’s ratio, respectively.

REFERENCES


Figure 1. Coordinates and notations.