Note

Method of basic impactors for simplified modeling of penetration

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Abstract

We suggest an engineering method that allows using the known (e.g., from the experiments) data on the depth of penetration of several high-speed impactors for predicting the depth of penetration for the impactors having different shapes. The suggested method is based on the conjecture that a complete description of the impactor–shield interaction model for determining the depth of penetration is redundant.

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1. Introduction

The method of invariant relationships (for a systematized description see [1]) can be applied for calculating aerodynamic characteristics (AC) when AC is expressed as an integral depending on the projectile's shape. This method allows determining the ACs of projectiles having different shape using the known (e.g., measured) ACs for several basic projectiles. The essence of the method is that it is based on the assumption about the local character of projectile–host fluid interaction [1,2] but does not require specifying the model and its parameters. In this study, the method of invariant relationships is modified for applying to problems of high-speed penetration. It is assumed that the impactors are rigid, non-eroding body penetrating normally into a shield with the same mechanical properties and the surface effects are neglected. Impactor–shield interaction model is the same for all impactors and is characterized by a linear dependence between the resistance force and the squared velocity of the impactor.

2. Statement of the problem

Consider \( n \) impactors, \( \Xi_1, \Xi_2, ..., \Xi_n \), having the shape of bodies of revolution. The generatrix of the nose of the \( i \)th impactor is determined in the coordinates \( x_iO_iy_i \) associated with the impactor, by the equation \( y_i = \Phi_i(x_i) \), where the origin of the coordinates coincides with forward point of the impactor, axis \( O_i x_i \) is directed along the axis of the impactor, and axis \( O_i y_i \) is perpendicular to the axis \( O_i x_i \). We assume that (i) only the nose of the impactor with the length \( L_i \) interacts with a host medium when the impactor penetrates into the shield and (ii) the following properties of the functions \( \Phi_i \) are valid

\[
\begin{align*}
\Phi_i(0) &= 0, & \Phi_i'(0) &
\geq 0, & \Phi_i''(x_i) &< 0, & i = 1, 2, ..., n, & 0 \leq x_i \leq L_i \\
\Phi_i'(0) &= 1/u, & \Phi_i'(L_i) &= 1/u, & i = 1, 2, ..., n
\end{align*}
\]
where \(u_1\) and \(u_\infty\) are the cotangents of the angle between the tangent to the generatrix and the axis of a projectile at the leading and trailing edges of the projectile, respectively. Each impactor is considered as a rigid body penetrating normally into a shield with the same mechanical properties along the axis \(h\), where the coordinate \(h\), the instantaneous location of the penetrator, is defined as the distance between the leading edge of the impactor and the front surface of the shield. The effects associated with the stage of the incomplete immersion of the impactor’s nose in the shield are neglected for semi-infinite shield or averaged localized interaction models are employed for shield with a finite thickness (for details see [2], pp. 52–55).

We assume that impactor–shield interaction at a given location at the impactor’s surface which is in contact with the shield is determined by the following equation:

\[
dF_i^l = [\Omega(c^{(1)}, c^{(2)}; \cdots); \mathbf{v}] v^2 + a|\mathbf{p}|^2 dS_i, \quad \mathbf{v}_i = \cos^{-1}(-\mathbf{v}_i \cdot \mathbf{n}_i)
\]

where \(dF_i^l\) is the force acting at the surface element \(dS_i\) of the impactor along the inner normal vector \(\mathbf{n}_i\) at a given location at the impactor’s surface, \(\mathbf{v}_i\) is an unit local velocity vector, \(\mathbf{n}_i\) is the tangent between the vector \(\mathbf{r}_i\) and the vector \((-\mathbf{v}_i)\), function \(\Omega > 0\) determines the particular model for impactor–shield interaction, the parameters \(a, c^{(1)}, c^{(2)}\) depend upon the properties of the material of the shield and the magnitude of the parameter \(\alpha\) is assumed to be known (hereafter we will not write these parameter as the arguments of the function \(\Omega\)).

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\[
dF_i^l = [\Omega(u_i) v^2 + a|\mathbf{p}|^2 dS_i, \quad u_i \leq u_i \leq u_\infty,
\]

where \(\Omega(u_i) = \Omega(\tan^{-1}(u_i)), \quad \mathbf{n}_i = \tan^{-1}(u_i)\), Eq. (4) comprises essentially all existing phenomenological models (see, e.g., overview in [2]). The most widely used models are described in the form of Eq. (4) with \(\Omega(u_i) = \beta^{(i)} \sin^2 \mathbf{v}_i = \beta^{(i)} u_i^2/(u_i^2 + 1)\).

The total force \(F_i\) is determined by integrating the local force over the impactor–shield contact surface. The drag force, \(D_i\), is determined as \((-\mathbf{v}_i \cdot \mathbf{F}_i\)). Integration of the equation of motion of the impactor along the axis \(u\) allows to obtain the relationships for the depth of penetration (DOP), \(H_i\), for a semi-infinite shield and the ballistic limit velocity (BLV), \(V_i\), for a shield with a finite thickness [2]:

\[
H_i = \varphi(l_i); \quad \varphi(l_i) = \frac{m_i}{4\pi l_i} \ln \left(1 + \frac{2\pi (\nu_i^{\text{imp}})^2}{a\sigma^*} l_i \right)
\]

\[
V_i = \psi(l_i); \quad \psi(l_i) = \frac{\sqrt{a\sigma^*}}{2\pi l_i} \exp \left(\frac{4\pi b_i}{m_i l_i} - 1\right)
\]

where

\[
i_i = \int_0^{l_i} \omega(u_i) \Phi_i \Psi_i d\xi_i, \quad \sigma_i^* = 2\pi \int_0^{l_i} \Phi_i \Psi_i d\xi_i = \pi \Phi_i^2(l_i), \quad u_i = 1/\Phi_i^2(x_i)
\]

\[m_i, \nu_i^{\text{imp}}, \sigma_i\] are the mass of the impactor, its impact velocity and area of the shank, respectively, \(b_i\) the thickness of the finite shield. The DOP is defined as the depth of penetration at which the velocity of the impactor vanishes while the BLV is the minimal initial (impact) velocity of the impactor required to perforate the shield.

We consider the following problem concerning the semi-infinite shields (the problem for shields with finite thicknesses can be considered in a similar manner). Suppose that the DOPs of the impactors \(\Xi_1, \Xi_2, \ldots, \Xi_n\) (the values \(H_1, H_2, \ldots, H_n\)) are known. Consider an impactor \(\Xi_0\) for which the above penetration model is valid, i.e., in the corresponding relationships the subscript \(i\) is replaced by a subscript \(0\). Our goal is to determine a set of the impactors \(\Xi_0\) such that their DOPs will be determined on the basis of characteristics \(H_1, H_2, \ldots, H_n\) independently of the particular impactor–host medium interaction model (function \(\omega\)). Concerning the shields with finite thicknesses the BLVs must be considered instead of the DOPs.

### 3. Solution of the problem

The main idea of the suggested method is illustrated in Fig. 1a and b where tilde denotes some value of the corresponding variable. The equation of the generatrix of the impactor is written in a parametric form whereby the variable \(u_i\) which varies in the same range for all impactors is selected as a parameter. The latter allows (after transformation of the variables, \(x_i \rightarrow u_i\)) representing all integrals \(I_i\), as the integrals with the same lower and upper limits for the same integration variable \(u\). Mathematically this approach can be realized as follows.

Choosing \(u_i = 1/\Phi_i(x_i)\) as a parameter, the equation of the impactor’s generatrix can be written in a parametric form as follows:

\[
x_i = x_i(u_i), \quad y_i = y_i(u_i), \quad u_i = \dot{x}_i(u_i)/\dot{y}_i(u_i), \quad \dot{x}_i = dx_i/du_i, \quad \dot{y}_i = dy_i/du_i
\]

Since \(u_i\) varies in the range from \(u\) to \(u_\infty\), for all impactors, the variables in the integrals \(l_i\) in Eq. (7) can be replaced by the common variable \(u\)

\[
l_i = \int_{u_i}^{u_\infty} \omega(u) y_i(u) d\dot{y}_i(u) du = \frac{1}{2} \int_{u_i}^{u_\infty} \omega(u) d\dot{\gamma}_i^2(u) du, \quad i = 1, 2, \ldots, n
\]
Consider an impactor $\Xi_0$ with the generatrix determined by the equations $x_0 = x_0(u_0), y_0 = y_0(u_0), u_1 \leq u_0 \leq u_n$, where

$$y_0(u) = \xi(u), \quad x_0(u) = \int_{u_0}^{u} U \frac{d\xi(U)}{dU} dU = u\xi(u) - \int_{u_0}^{u} \xi(U)dU \quad (10)$$

$$\xi(u) = \sum_{i=1}^{n} \beta_i \gamma_i(u), \quad y_0(u_i) = 0, \quad x_0(u_0) = 0, \quad u = \frac{dx_0}{du} = \frac{dy_0}{dy_0}/du \quad (11)$$

$\beta_1, \beta_2, ..., \beta_n$ are some coefficients. Using Eqs. (10) and (11) the integral $I_0$ which is associated with the impactor $\Xi_0$ and is similar in form to the integrals in Eq. (9), can be written as follows:

$$I_0 = \frac{1}{2} \int_{u_0}^{u_n} \omega(u) \frac{dy_0^2(u)}{du} du = \frac{1}{2} \sum_{i=1}^{n} \beta_i \int_{u_0}^{u_n} \omega(u) \frac{dy_i^2(u)}{du} du \quad (12)$$

Eqs. (9) and (12) imply that the following relationship

$$I_0 = \sum_{i=1}^{n} \beta_i I_i \quad (13)$$

regardless of the choice of the function $\omega$. Using Eq. (10) the length of the impactor $\Xi_0$, $L_0$, its shank radius, $R_0$, and shank area, $\sigma_0$ can be found

$$L_0 = x_0(u_n), \quad R_0 = y_0(u_n), \quad \sigma_0 = \sum_{i=1}^{n} \beta_i \sigma_i \quad (14)$$

Suppose that the shapes of the impactors $\Xi_1, \Xi_2, ..., \Xi_n$ and the values $H_i, \nu_i^{\text{imp}}, m_i (i = 1, 2, ..., n)$ and $a$ are known. Then the following procedure allows determining the shape of the impactor $\Xi_0$ and its DOP, $H_0$, for given coefficients $\beta_1, \beta_2, ..., \beta_n.$

Step 1. Determining $I_i$ by solving the equations $\phi(I_i) = H_i$ ($i = 1, 2, ..., n$). Function $\phi(I_i)$ decreases in the range from $\phi = \mu = m_i (\nu_i^{\text{imp}})^2/(2\sigma_i)$ to 0 when $I_i$ increases from 0 to $+\infty$. Therefore, the equation $\phi(I_i) = H_i$ has a single root if $H_i \leq \mu$. If $H_i > \mu$ then these data are excluded from consideration. Note that several experiments with different $m_i$ and $\nu_i^{\text{imp}}$ for the same impactor can be used to obtain the averaged values of $I_i$ on the basis of several values of $H_i$.

Step 2. Formulating equations of the generatrices of the impactors $\Xi_1, \Xi_2, ..., \Xi_n$ in the parametric form and formulating the equation of the generatrix for the impactor $\Xi_0$ using Eqs. (10) and (11).

Step 3. Calculating integral $I_0$ using Eq. (13).

Step 4. Determining the DOP, $H_0$, using Eq. (5) for $i = 0$.

Varying coefficients $\beta_i$, the DOPs for a large number of various impactors with different shapes can be calculated.

4. Example

Consider as impactors $\Xi_1$ and $\Xi_2$ ($n = 2$) power–shaped bodies with the same shank radius, $R = R_1 = R_2$, and let us illustrate the most complicated Step 2 of the above procedure.

Equations of the impactor’s generatrices can be written as follows:

$$y_i = \Phi_i(x_i) = R(x_i/L_i)^{\alpha_i}, \quad 0 < \alpha_i < 1, \quad i = 1, 2 \quad (15)$$
where \( z_i \) is given. Since \( \Phi_i(x_i) = z_i R x_i^{2 - 1} / L_i^2 \), Eq. (2) implies that \( u_* = 1 / \Phi_i(0) = 1 / \Phi_2(0) = 0 \) and

\[
u_\ast = 1 / \Phi_i(L_i) = L_i / z_i R, \quad i = 1, 2, \ldots, n \tag{16}\]

where Eq. (16) describes the requirement concerning the impactor's shapes, \( L_i / z_i R = L_2 / z_2 R \). Choosing \( u = 1 / \Phi_i(x_i) = L_i x_i^{2 - 2} / (z_i R) \) as a parameter and substituting \( L_i = z_i R u_\ast \), we obtain the equations of the impactor's generatrices in a parametric form:

\[
x_i(u) = z_i R u_\ast^{2/(2z_i-1)} u^{1/(1-z_i)}, \quad y_i(u) = R(u/u_\ast)^{x_i/(1-z_i)}, \quad 0 \leq u \leq u_\ast \tag{17}\]

In order to elucidate the obtained formulas, let us consider a special case with the parameters \( z_1 = 1/2, z_2 = 2/3 \). Then Eqs. (10) and (11) imply the following equations of the generatrix for the impactor \( X_0 \):

\[
\frac{3}{R u_\ast} x_0(u) = \eta(u) \left[ 2 \left( \frac{u}{u_\ast} \right)^2 - \frac{\beta_1}{\beta_2} \right] + \frac{\beta_3}{\beta_2}, \quad \frac{y_0(u)}{R} = \frac{u}{u_\ast} \eta(u) \tag{18}\]

where

\[
\eta(u) = \sqrt{\beta_1 + \beta_2 (u/u_\ast)^2}, \quad 0 \leq u \leq u_\ast \tag{19}\]

The length and the shank radius of the impactor \( X_0 \) are as follows:

\[
L_0 = x_0(u_\ast) = (1/3) R u_\ast \left( \sqrt{\beta_1 + \beta_2 [2 - (\beta_1 / \beta_2)] + \beta_3^2 / \beta_2} \right)
\]

\[
R_0 = y_0(u_\ast) = R \sqrt{\beta_1 + \beta_2} \tag{20}\]

Excluding \( u \) in the Eq. (18), the equation of the generatrix of the impactor \( X_0 \) can be written as a dependence between \( x_0 \) and \( y_0 \):

\[
3 x_0 \beta_2 / u_\ast = (\xi - 2 \beta_1) \left( \sqrt{\beta_1 + \xi} / 2 + \beta_3^2 / \beta_2 \right) \tag{21}\]

where

\[
\xi = \sqrt{\beta_1^2 + 4 \beta_2 y_0^2}, \quad \bar{x}_0 = x_0 / R, \quad \bar{y}_0 = y_0 / R, \quad 0 \leq \bar{y}_0 \leq \sqrt{\beta_1 + \beta_2} \tag{22}\]

If \( \beta_1 + \beta_2 = 1 \), then all the impactors, \( X_0, X_1, X_2 \), have the same shank radius. Versions of the shape of impactor \( X_0 \) for different \( \beta_1 \) and \( \beta_2 \) as well as the shape of the “basic” impactors \( X_1 \) and \( X_2 \) are shown in Fig. 2.

5. Concluding remarks

The suggested approach is especially useful when there are no alternative methods for estimating the DOP of an impactor using the available experimental data on the DOPs of the impactors having other shapes.

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Fig. 2. Shape of the impactors.
Along with the direct problem studied above, an inverse problem can be considered whereby determining the DOP of the impactor \( \Sigma_0 \) with a given shape is required. This problem has, generally, only an approximate solution and is reduced to optimal selection of the coefficients \( \beta_1, \beta_2, \ldots, \beta_n \).

Analysis of the accuracy and range of the applicability of the suggested method can be performed on the basis of experiments with the impactors having the shapes which satisfy the conditions given by Eq. (2).

References