Improved Florence model and optimization of two-component armor against single impact or two impacts

G. Ben-Dor, A. Dubinsky, T. Elperin *

Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, P.O. Box 653, Beer-Sheva 84105, Israel

Available online 20 February 2008

Abstract

A new version of the Florence model taking into account the ballistic resistance of the ceramic plate in a two-component armor is suggested, verified and used for armor optimization. The direct problems (maximization of the ballistic limit velocity for a given areal density (AD) or a total thickness (TT) of the armor) and the inverse problems (minimizations of AD or TT for a given impact velocity) are investigated in the case of a single impact. We also formulated and studied the armor optimization problems with a repeated impact. Numerical results are presented for aluminum/alumina armor.

Keywords: Impact; Penetration; Two-component armor; Ceramics; Optimization

1. Introduction

Two-component composite armors consisting of a ceramic front plate and a ductile back plate attract considerable interest because of their high ballistic performance and light weight. Although numerous analytical models have been suggested for analyzing perforation of two-component ceramic shields [1–11], Florence model [12] was found to be the most suitable for solving problems associated with armor optimization. Some early numerical results obtained with this approach have been presented in [12,13]. In the second study, the original model was slightly reworked and this modified version was later used for armor optimization against normal impact. Hetherington [14] investigated analytically the problem of determining the structure of the two-component armor with a given areal density (AD) that provides the maximum ballistic limit velocity (BLV). He suggested an approximate expression for the optimum value of the ratio of the front plate width to the back plate width. Wang and Lu [15] investigated a similar problem where the total thickness (TT) of the armor rather than the areal density (AD) was a given. Ben-Dor et al. [16–18] investigated problems of determining the armor with the maximum BLV for a given AD and the armor with the minimum AD for a given impact velocity. In these studies, using appropriate dimensionless variables, the solutions of the optimization problems for an arbitrary two-component composite armor have been determined in an analytical form. Shi and Grow [19] investigated the problem of two-component armor optimal whereby both, the TT and the AD, are used as the constraints. Fawaz et al. [20] studied the problems in a generalized formulation when the armour materials are employed as the unknown design parameters. Some data relating to the comparison of the results of the armor optimization based on the Florence model with numerical simulations and experiments can be found in [21–23]. Studies [24,25] investigated a modified Florence’s model for the case of an oblique impact.

In Florence model, the impactor was modeled as a short cylindrical rod that strikes the ceramic plate (Fig. 1). The ceramic plate breaks progressively into a cone of fractured material. The impact energy is transferred to the back plate which is deformed like a uniform membrane.
The simplifying assumptions that allowed deriving an analytical expression for the BLV can be summarized as follows [13]: (i) the diameter of the circular area at the back plate over which the momentum is distributed is equal to the base diameter of the fracture conoid in the ceramic facing, and the angle of the conoid is chosen to be equal to 63°; (ii) the deformation history of the back plate may be modeled by the motion of a membrane clamped around the perimeter of the base of the fractured conoid, and the initial conditions for the membrane’s motion are determined by the condition imposed by the projectile’s impact and by the conservation of momentum within the projectile/shield system; (iii) failure occurs when the maximum tensile strain in the membrane reaches the ultimate breaking strain of the back plate.

A significant imperfection of the Florence’s model which has been repeatedly mentioned in the literature (see, e.g., [10,26]) is that this model does not take into account mechanical properties of the ceramics and energy dissipation in the course of ceramics fracture. In this study, we propose a modification of the Florence model which is free of this shortcoming. Using the suggested model, we have solved the typical problems of two-component armor optimization against a single impact and new problems taking into account the possibility of the repeated impacts which cannot be considered on the basis of original Florence model.

2. Mathematical model

As a basis, we use the intermediate result from [12] determining the minimal momentum, \( I \), that must be applied to the system including impactor-conoid-membrane in order to break the membrane at the stage of motion of the system as a whole:

\[
I = \sqrt{\pi \beta_2 a^2 b_2 (M_0 + M_1 + M_2)}, \quad \beta_2 = \varepsilon v_0 / 0.91, \quad a = R + b_1 \tan \phi, \tag{1}
\]

where \( M \) is a mass, \( R \) is a projectile’s radius, \( b \) is a plate’s thickness, \( \sigma \) is the ultimate tensile strength, \( \varepsilon \) is the breaking strain, subscripts 0, 1 and 2 refer to projectile, ceramic plate and the part of the back plate under the conoid, respectively. The model is written in a slightly generalized form: an arbitrary angle \( \phi \) is introduced instead of the Florence model value 63° (Fig. 1).

Florence [12] assumed that \( I \) equals to the momentum of the projectile that impacts the shield with the BLV, \( v_{bl} \). Substituting \( I = M_0 v_{bl} \) into Eq. (1) we can obtain the original Florence model in the form of the expression for the BLV:

\[
v_{bl} = \sqrt{\pi \beta_2 a^2 b_2 (M_0 + M_1 + M_2) / M_0^2}. \tag{2}
\]

The masses \( M_1 \) and \( M_2 \) can be expressed through the densities \( \gamma_1 \) and \( \gamma_2 \) and the volumes of the conoid and the cylindrical part of the back plate under the conoid:

\[
M_1 = 1/3 \pi \gamma_1 b_1 (R^2 + aR + a^2), \quad M_2 = \pi \gamma_2 b_2 a^2. \tag{3}
\]

Substituting the above formula for \( M_2 \) and the simplified formula for \( M_1 \), \( M_1 = \pi \gamma_1 b_1 a \) (the conoid is replaced by the cylinder with the same basis and the same height), into Eq. (2) we obtain the following version of the model [13]:

\[
v_{bl} = \sqrt{\pi \beta_2 a^2 b_2 [M_0 + \pi a^2 (\gamma_1 b_1 + \gamma_2 b_2)] / M_0^2}. \tag{4}
\]

Eq. (4) describes the model which is used for investigating optimization problems.

In this study we use the expressions for \( M_1 \) from Eqs. (3) and consider the process of penetration in more details.

Eq. (1) implies physically incorrect value \( v_{bl} = 0 \) for \( b_2 = 0 \) because of neglecting energy required for ceramic fracture. This can result in the erroneous solution of the optimization problems. Consequently this model must be improved.

Using momentum conservation at all the stages of penetration the momentum balance equation associated with the forming of conoid can be written as follows [27]:

\[
M_0 v_{bl} = I_{0-1} + \bar{I}, \quad I_{0-1} = (M_0 + M_1) v_{0-1}, \tag{5}
\]

where \( \bar{I} \) is the momentum transmitted to the ceramic plate due to shear strength of plate material, \( v_{0-1} \) and \( I_{0-1} \) are the velocity and the momentum of the impactor-conoid system after formation of conoid. Taking into account Eqs. (1) and (5) the momentum balance equation, \( I_{0-1} = I \), yields:

\[
v_{bl} = \left( \sqrt{\pi \beta_2 a^2 b_2 (M_0 + M_1 + M_2) + M_0 \bar{I}} / M_0^2 \right). \tag{6}
\]

Using the approach suggested in [28] (see also [29]) for the limiting case of ceramic plate (\( b_2 = 0 \)) we assume that formation of the conoid requires the following energy:

\[
E_1 = \beta_1 S_1, \quad S_1 = \pi b_1 (R + a) / \cos \phi, \tag{7}
\]

where \( \beta_1 \) is the fracture energy for transverse shearing per unit shear area and \( S_1 \) is the area of the lateral surface of the conoid. We slightly modified the model proposed in [28] by replacing the angle \( \phi = 45° \) by an arbitrary angle \( \phi \)
(Fig. 1). Substituting $v_{bl} = \bar{v}/M_0$ and $E_1$ from Eq. (7) into the expression $1/2M_0v_{bl}^2 = E_1$ we find that $\bar{v} = \sqrt{2\beta_1 S_1M_0}$ and 

$$v_{bl} = \sqrt{\pi\beta_1 a^2 b_2 (M_0 + M_1 + M_2)/M_0 + 2\beta_1 S_1 M_0}.$$  

(8) 

Substituting $M_1$ and $M_2$ from Eq. (3) we can rewrite Eq. (8) as follows: 

$$v_{bl} = f_1(b_1) + f_2(b_1, b_2),$$  

(9) where 

$$f_i = \theta_i Q_i, \quad \theta_i = \sqrt{\beta_i}, \quad i = 1, 2,$$

$$Q_1(b_1) = \sqrt{2\pi b_1 (2R + b_1 \tan \phi)/(M_0 \cos \phi)},$$

$$Q_2(b_1, b_2) = \sqrt{\pi a^2 b_2 [M_0 + 1/3\gamma_1 b_1 (R^2 + aR + a^2) + \gamma_2 b_2 a^2]/M_0}.$$  

(10) 

3. Validation of the model. Optimal armor against a single impact 

In our calculations we use the experimental data [30] in the digital form from [31] for two-component armor consisting of the AD85 alumina ceramic front plate ($\gamma_1 = 3430$ kg/m$^3$) and the 6061-T6 aluminum rear plate ($\gamma_2 = 2765$ kg/m$^3$) penetrated by the conical projectile ($M_0 = 8.32$ g, $R = 3.81$ mm). 

Eqs. (9)-(10) imply the following relationship 

$$Y = \theta_2 X + \theta_1, \quad Y = v_{bl}/Q_1, \quad X = Q_2/Q_1,$$  

(11) 

which can be considered as a linear regression equation with the unknown coefficients $\theta_1$ and $\theta_2$. The set of experimental data [30] includes the value of the BLV, $v_{bl}$, for the shield consisting only of the front ceramic plate (without a rear plate) with the thickness $b_i^0$ which is of our special interest. Consequently, we require that the approximating line will pass through the point $X = X^{(1)} = 0, Y = Y^{(1)} = v_{bl}^0/Q_2(b^0)$. This condition implies that $\theta_1 = Y^{(1)}$ and only $\theta_2$ must be determined. A standard criterion of the quality of the approximation, the average relative error, $\varepsilon_{\text{avr}}$, can be used for determining $\theta_2$. Here we avoid a usual contradiction when the least squares method is used for approximating while the quality of approximation is accessed using another criterion, e.g., the average relative error. Then the problem is reduced to the following minimization problem: 

$$F(\theta_2) = n_{\text{avr}} = \sum_{i=2}^{n} \left| \frac{\theta_2 X^{(i)} + \theta_1 - Y^{(i)}}{Y^{(i)}} \right|$$

$$= \sum_{i=2}^{n} |e^{(i)}| \quad \theta_2 - c^{(i)} \to \text{min},$$  

(12) 

where 

$$c^{(i)} = |X^{(i)}/Y^{(i)}|, \quad c^{(i)} = (Y^{(i)} - Y^{(1)})/X^{(i)},$$

$$X^{(i)} \neq 0, \quad Y^{(i)} \neq 0, \quad i = 2, 3, \ldots, n.$$  

(13) 

In order to avoid rearranging superscripts we assume they are changed so that $c^{(2)} \leq c^{(3)} \leq \ldots \leq c^{(n-1)} \leq c^{(n)}$. Then for some $c^{(i)} \leq \theta_2 \leq c^{(j+1)}$ it follows that 

$$F(\theta_2) = \left[ \sum_{i=2}^{j} c^{(i)} - \sum_{i=j+1}^{n} c^{(i)} \right] \theta_2 + \left[ \sum_{i=j+1}^{n} c^{(i)} - \sum_{i=2}^{j} c^{(i)} \right].$$  

(14) i.e., $F(\theta_2)$ is a piecewise linear function. Consequently, taking into account that $F(\theta_2) \to +\infty$ for $\theta_2 \to \pm \infty$, the minimum $F(\theta_2)$ is located in one of the end points of the intervals, $F(c^{(2)})$, $F(c^{(3)})$, $\ldots$, $F(c^{(n)})$, and can be readily found by inspection. 

The results of calculations are showed in Fig. 2a–e for different values of the angle, $\phi$. Clearly, the scatter of the points around the optimum regression line strongly depends upon the selected value of the angle, $\phi$, and the Florence value $\phi \approx 63^\circ$ is not the optimal. The latter observation is expressed quantitatively in Fig. 3 which shows the minimal relative error of the approximations as a function of $\phi$. Consequently, this non-unique choice of $\phi$ which has been mentioned previously in the literature [24,32] is supported by our calculations. For the further analysis, we select the value $\phi \approx 45^\circ$ that is close to the optimum (see Fig. 3). In this case, $\theta_1 = 865(N/m)^{1/2}$, $\theta_2 = 15797$ m$^{-1/2}$ and $\varepsilon_{\text{avr}} = 7.4\%$. 

Consider the problem of determining $b_1$ and $b_2$ that provide the minimum BLV taking into account the following restrictions: 

$$b_1 \geq 0, \quad b_2 \geq 0,$$  

(15) 

$$x_1 b_1 + x_2 b_2 = \mu,$$  

(16) 

where $x_1$, $x_2$ and $\mu$ are considered to be known. Two sets of the coefficients in Eq. (16) have a clear meaning: (i) $x_2 = \gamma_2 (i = 1, 2)$, $\mu$ is the AD of the armor, and the problem is optimization of the BLV for the armor with a given AD, and (ii) $x_1 = 1 (i = 1, 2)$, $\mu$ is TT of the armor, and the problem is optimization of the BLV for the armor with a given TT, $b$. Let us introduce the new variable, $x = x_1 b_1/\mu$. After substituting $b_1 = \mu x/x_1$ and $b_2 = \mu (1-x)/x_2$ into the expression for the BLV, the optimization problem is reduced to the maximization of the function of one variable, $x$, subject to the linear constraints, $0 \leq x \leq 1$. Because the dependence of the BLV on the parameters is quite involved, the problem cannot be solved analytically while the numerical solution of the problem is straightforward. Consider the first problem when the AD of the armor is given. The BLV as a function of the normalized AD of the ceramic plate, $x = \overline{A_1} = \gamma_1 b_1/\gamma_2 (\gamma_1 b_1 + \gamma_2 b_2)$, is shown in Fig. 4a for different values of AD of the armor. Inspection of Fig. 4a shows that the maximum BLV is attained for $\overline{A_1} = A_{1}^{\text{opt}} \approx 0.75$, and is practically independent of the given AD of the armor. The optimum ratio of the thickness of the plates can be calculated using the following relationship: 

$$\frac{b_1}{b_2} = \frac{\gamma_2}{\gamma_1 - \gamma_1 A_{1}^{\text{opt}}}.$$  

(17) 

Substituting $\overline{A_1} = 0.75$ we obtain that in the considered case, $b_1^{\text{opt}}/b_2^{\text{opt}} \approx 2.4$ where $b_1^{\text{opt}}$ and $b_2^{\text{opt}}$ are the thicknesses.
corresponding to the solution of the optimization problem. The minimization criterion, i.e., the BLV as a function of the given AD, is shown in Fig. 4b. Since this function is increasing, the same curve is the solution of the inverse problem: minimizing AD for a given BLV. The latter statement can be easily proved by contradiction.

We have solved also several optimization problems similar to those considered above when TT of the armor, $b$, is considered as a criterion of optimization instead of the AD. The results are shown in Fig. 5a–b. In this case, $b_{\text{opt}}/b \approx 0.77$ (Fig. 5a) and hence, $b_{\text{opt}}/b_{\text{TT}} \approx 3.3$, practically independent of the given thickness of the armor.

The circles in Figs. 4b and 5b denote the experimental data from [30]. Special attention must be given to the fact that practically every experimental point corresponds to the value of the criterion which is worse than the optimum

![Figure 2](image-url)
value obtained as a result of optimization. The latter is an important argument in favor of that the proposed model is not only a good approximation for the experimental data, but it adequately describes the effect of the plates thicknesses on the armor’s BLV. Our results are in a good agreement with the numerical and experimental data [22] as well as with theoretical conclusions [16–18] that have been obtained using Florence model. The plausible explanation is that in the considered case, the competitive versions of the armor structure do not include the variants with small values of the thicknesses of the back plate for which the Florence model is known to provide a crude approximation.

The calculations show that the optimum is attained for the same ratio of plate’s thicknesses (plate’s ADs) both for the direct and for the inverse problems. In order to

![Diagram](image1)

Fig. 3. Relative error of approximation as a function of the angle of the fracture conoid.

![Diagram](image2)

Fig. 4. Optimization against a single impact taking into account the armor’s total AD; (a) The BLV as a function of the normalized AD of the ceramic plate and (b) results of solution of the direct and the inverse optimization problems.

![Diagram](image3)

Fig. 5. Optimization against a single impact taking into account the armor’s TT; (a) The BLV as a function of the normalized thickness of the ceramic plate and (b) results of solution of the direct and the inverse optimization problems.
and in Fig. 5 b. Since \( x^{\text{opt}} \) is possible. The problem can be formulated as follows.

\[
x = x^{\text{opt}} \text{ is a point of local minimum of } \psi(x, \mu), \text{ then }
\]

\[
\psi(x^{\text{opt}}, \mu) = 0, \quad \psi_{xx}(x^{\text{opt}}, \mu) > 0
\]

for any \( \mu \). The inverse problem can be formulated as minimization of \( \mu \) when the BLV, \( \psi_{bl} \), is given, i.e., the equation \( \psi(x, v) = \psi_{bl} \) is valid. Let us differentiate the latter equation considering \( \mu \) as a function of \( x \):

\[
\psi(x, v) + \psi_{x}(x, \mu)\mu'(x) = 0,
\]

\[
\psi_{xx}(x, \mu) + 2\psi_{x\mu}(x, \mu)\mu'(x) + \psi_{\mu}(x, \mu)[\mu'(x)]^2 + \psi_{\mu}(x, \mu)\mu''(x) = 0.
\]

Since \( \psi_{x}(x, \mu) > 0 \) and \( \psi_{\mu}(x^{\text{opt}}, \mu) = 0 \), the first equation in Eqs. (19) implies that \( \mu(x^{\text{opt}}) = 0 \). Taking into account Eq. (18), the second equation in Eqs. (19) implies that \( \mu(x^{\text{opt}}) = -\psi_{x}(x^{\text{opt}})/\psi_{\mu}(x^{\text{opt}}, \mu) > 0 \), where \( \mu \) is the solution of the equation \( \psi(x^{\text{opt}}, \mu) = \psi_{bl} \). Consequently, \( x = x^{\text{opt}} \) is the point of the local minimum that is the point of optimum of the inverse problem. Note that \( \psi(x^{\text{opt}}, \mu) = \psi_{bl} \) is the equation of the optimum curve in Fig. 4 b (\( \mu = A \)) and in Fig. 5 b (\( \mu = b \)). An interrelation between the solutions of the direct and the inverse problems can also be revealed in a more general case when \( x^{\text{opt}} \) in the direct problem is not a constant but investigating this special mathematical problem is beyond the scope of this study.

4. Optimization of the armor taking into account repeated impact

In contrast to the Florence model, the proposed model allows consideration of optimization problems for the case when the repeated impact at the same location of the shield is possible. The problem can be formulated as follows. Determine the thicknesses of the plates \( b_1 \) and \( b_2 \) with the minimum AD or TT that provide defense against a single or repeated impacts with the velocity \( v_{\text{imp}} \). It is assumed that the mass and the diameter of the impactor and mechanical properties of the materials of the plates are known.

There are two scenarios for perforation of the shield. The first scenario is realized when the armor is perforated as the result of the first impact. In this case, the impact velocity is larger than the BLV, i.e.,

\[
f_1(b_1) + f_2(b_1, b_2) \leq v_{\text{imp}},
\]

(20)

According to the second scenario, only the fracture conoid is formed after the first impact whereas the perforation occurs after the repeated impact. This scenario is described by the following inequalities:

\[
f_1(b_1) + f_2(b_1, b_2) > v_{\text{imp}},
\]

\[
f_1(b_1) \leq v_{\text{imp}}, \quad f_2(b_1, b_2) \leq v_{\text{imp}}.
\]

The domain ACBO (see Fig. 6a), determined by Eqs. (15) and (20), is located below the curve ACB with the equation \( b_2 = G(b_1) \), where

\[
G(b_1) = \left( \sqrt{\frac{v^2}{\xi_1} + 4\frac{\epsilon_{l}^2}{c_{s}^2} - \xi_1} \right) / (2\xi_2), \quad 0 \leq b_1 \leq b_1^\text{opt},
\]

(22)

\[
\xi_0 = M_0[\tan \theta_2 - f_1(b_1)]/\theta_2,
\]

\[
\xi_1 = M_0 + 1/3\pi v_1 b_1 (R^2 + aR + a^2), \quad \xi_2 = 4\pi^2 \gamma_2 a^4,
\]

(23)

\[
b_1^\text{opt} = \left( \frac{R^2}{\tan^2 \phi - \frac{M_0 \cos \phi \sin \phi}{\tan \phi} - \frac{R}{\tan \phi}} \right)^{\frac{1}{2}}.
\]

(24)

\( G(b_1) \) is the function which is obtained by solving the equation \( f_1(b_1) + f_2(b_1, b_2) = v_{\text{imp}} \) with respect to \( b_2 \) and \( b_1^\text{opt} \) is the solution of the equation \( f_1(b_1) + f_2(b_1, 0) = v_{\text{imp}} \) that is reduced to the equation \( f_1(b_1) = v_{\text{imp}} \) because \( f_2(b_1, 0) = 0 \).

The domain AEDBC (see Fig. 6a), determined by Eqs. (15) and (21), is located between the curves AED and ACB and the line \( b_1 = b_1^\text{opt} \). The equation of the curve AED, \( b_2 = g(b_1) \), is obtained by solving the equation \( f_2(b_1, b_2) = v_{\text{imp}} \) with respect to \( b_2 \):

\[
g(b_1) = \left( \sqrt{\frac{v^2}{\xi_1} + 4\frac{\epsilon_{l}^2}{c_{s}^2} - \xi_1} \right) / (2\xi_2),
\]

(25)

\[
\tilde{\xi}_0 = M_0 v_{\text{imp}} / \theta_2, \quad 0 \leq b_1 \leq b_1^\text{opt}.
\]

Since \( f_1(b_1) > 0 \) for \( b_1 > 0 \) and \( f_2(b_1, 0) = 0 \), the curve AED is always located above the curve ACB when \( b_1 > 0 \) and the curves intersect in the point \( b_1 = 0 \).

Therefore the domain AEDBC includes the values of \( b_1 \) and \( b_2 \) for which the armor is perforated. Accordingly, the domain bounded by the curve AED, by the line DB, by the axis \( \theta_2 \) above the point \( A \) and by the axis \( \theta_1 \) to the right of the point \( B \) is the “not perforation domain” (NPD). The above optimization problem is to find \( b_1 \) and \( b_2 \) from the NPD that provide the minimum \( \mu = x_1 b_1 + x_2 b_2 \) (AD or TT) for a given BLV. Due to linearity of the optimization criterion, the outline of the solution can be readily illustrated geometrically.

Consider the family of lines \( x_1 b_1 + x_2 b_2 = \mu \), where \( \mu > 0 \) is a parameter (see Fig. 6b). The distance from the origin of coordinates to a line \( \mathbf{K}_1 \mathbf{K}_2 \), \( L_{OH} = \mu / \sqrt{x_1^2 + x_2^2} \), is proportional to \( \mu \). Our goal is to find such maximum \( \mu \) that at least one point at the corresponding line is located in the
Clearly, this point can belong only to the part of the border, ADB. Analysis shows that \( g(b_1) \) is a concave function. Consequently, one of two locations of the optimal line is possible: the location \( K_0 K_1 \) (the line is tangent to the curve \( b_2 = g(b_1) \) between the points \( A \) and \( D \) or passes through the point \( A \)) or the location \( K_{00} K_{01} \) (the line passes through the point \( B \)). This property of the optimal solution implies the simplest way of looking for the best parameter \( b_1 \): its choice must provide a minimum between \( l_0 \) and \( l_1 \), where

\[
l_0 = \frac{\alpha_1 b_1}{C_2} \]

\[
l_1 = \min_{0 < b_1 < \beta_1} \left( \frac{\alpha_1 b_1 + \alpha_2 g(b_1)}{C_2} \right) \]

and \( \mu_1 \) is determined by exhaustion.

The results of calculations are presented in Figs. 7 and 8 for the criterion of the minimum AD and of the minimum TT, correspondingly. If the BLV is less than some critical value, \( v_{bl0} \), then the optimal armor consists only of a ceramic plate. If \( v_{bl0} > v_{bl} \), then optimal armor includes two plates with the approximately constant ratio of the thicknesses. Figs. 7 and 8 show that the possibility of the repeated impact considerably increase the minimal AD and TT required for the defense against perforation.

**5. Concluding remarks**

We suggested and validated a new modification of the Florence model taking into account the ballistic resistance of the ceramic plate in a two-component armor and studied problems of armor optimization on the basis of this improved model. Direct problems (maximization of the BLV for a given AD or TT) and inverse problems (minimizations of AD or TT for a given impact velocity) were investigated in the case of a single impact. We studied armor optimization in a new formulation which allows a repeated impact. Numerical results are presented for aluminum/alumina armor. The proposed approach can be generalized to multi-component shields.

**References**


