Optimization of the nose shape of an impactor against a semi-infinite FRP laminate

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Abstract

A model is proposed to describe the penetration of a monolithic semi-infinite FRP laminate struck transversely by a rigid projectile with an arbitrary shape. The shape of the impactor that penetrates at a given depth of penetration (DOP) with the minimum impact velocity is found by the use of a numerical procedure for solving a corresponding non-classical variational problem. It is shown that the optimum shape depends on the given DOP which is assumed to be larger than the length of the nose of the impactor. For a relatively small DOP, the optimum impactor is a sharp, awl-shaped body. With increasing DOP, the optimal nose geometry of the impactor varies and becomes close to a blunt cone. It is proved analytically that a flat-nosed cylinder requires the maximum impact velocity in order to penetrate at a given DOP. Comparison of different projectile head shapes for penetration into a semi-infinite target is performed. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Some recent results and reviews on shape optimization of impactors penetrating into homogeneous targets can be found in [1,2], and shape optimization problems for ductile multi-layered targets are studied in [3,4]. A semi-empirical model for predicting the penetration and perforation of monolithic fibre-reinforced plastic (FRP) laminates struck normally by projectiles with several kinds of nose shapes is proposed in [5,6]. This model is generalized in [7] for projectiles with an arbitrary shape, and the generalized model is applied to impactor shape optimization for the case of perforation of a finite-thickness FRP laminate by analytical methods. In this study this approach is developed for a semi-infinite target.

2. Description of the model

Consider normal penetration of a rigid striker (a body of revolution) into a FRP laminate with a finite thickness. The notations are shown in Fig. 1. The coordinate $h$, a current depth of the penetration, is defined as the distance between the nose of the impactor and the front surface of the target. The cylindrical coordinates $x$, $\rho$, $\theta$ are associated with the impactor. The part of the lateral surface of the impactor between the cross-sections $x = 0$ and $x = x_*$ interacts with the target where (see Fig. 2)

$$x_*(h) = \begin{cases} h & \text{if } 0 \leq h \leq L \\ L & \text{if } h \geq L \end{cases}$$ (1)

Since the local impactor-target interaction model with a zero tangent component of the force is considered, the presence of the cylindrical part at the rear side of the impactor’s nose does not influence the drag force.

Our investigation is based on the model that was proposed in [5,6] for the impactors which had the shape of bodies of revolution. In [5,6] it is assumed that the local pressure $\sigma$ which is applied normally to the surface of the projectile can be represented as $\sigma = \sigma_\ell + \sigma_d$, where $\sigma_\ell$ is the cohesive quasi-static resistive pressure due to the elastic–plastic deformations of the laminate material and $\sigma_d$ is the dynamic resistive pressure arising from the velocity effects. It is assumed also that $\sigma_\ell$ is equal to the quasi-static linear elastic limit $\sigma_\ell$ in through-thickness compression of the FRP laminates and that $\sigma_d$ is given by $\sigma_d = \beta \sqrt{r/\sigma_\ell} v_{\text{imp}} \sigma_\ell$, where $\gamma$ is the density of the...
material of the shield, \( v_{\text{imp}} \) is the impact (initial) velocity of the impactor. The parameter \( \beta \) is a constant which is determined empirically and it depend on the shape of the impactor. In particular, \( \beta = 2 \sin \alpha \) for conical-nosed impactor [5], where \( \alpha \) is the half angle of the apex of the cone. Thus the model suggested in [5,6] can be represented as:

\[
\sigma = \sigma_c + \sqrt{\gamma \sigma_c} \beta v_{\text{imp}}
\]  

(2)

According to the method of tangent cones [8,9] which is used in [7] to generalize the model [5,6] the pressure at some point of interaction between the impactor (a body of revolution) and a host medium is assumed equal to the pressure at the surface of the tangent cone this point. Thus, the parameter \( \beta \) in Eq. (2) must be replaced by \( \frac{2(v^0 \cdot \vec{n})}{2 \cos \nu} \) where \( \vec{n} \) is the inner normal vector at a given location at the impactor’s surface, \( v^0 \) is a unit vector along the corresponding axis, \( \nu \) is the angle between the vector \( \vec{v}^0 \) and the vector \( \vec{v}^0 \). Then the force \( d\vec{F} \) acting at the surface element \( ds \) of the impactor is as follows:

\[
d\vec{F} = [\sigma_c + 2 \sqrt{\gamma \sigma_c} v_{\text{imp}}] \vec{n}^0 ds
\]  

(3)

The proposed model coincides with the model suggested in [5,6] for cones and flat-faced (cylindrical) projectiles. Therefore, we can refer to [5,6] where it is shown that this model is in good agreement with the experimental results for these nose shapes. Comparison between the prediction using the proposed model (for perforation of a finite-thickness shield, and the experimental data for hemispherical-ended impactors is performed in [7]. This analysis shows that the theoretical predictions are in good agreement with the experimental data. Thus, there is reason to believe that the proposed model can be used for determining the optimal nose geometry of an impactor.

The total force \( \vec{F} \) is determined by integrating the local force over the impactor-shield contact surface. Using formulae of differential geometry the following expression for the drag force \( D \) can be derived:

\[
D(h, v_{\text{imp}}) = \pi \left[ r_0^2 + (R^2 - r_0^2) \delta(h) \right] \left( \sigma_c + 2 \sqrt{\gamma \sigma_c} v_{\text{imp}} \right)
\]

\[
+ 2\pi \left. \left( \sigma_c + 2 \sqrt{\gamma \sigma_c} v_{\text{imp}} \frac{\dot{y}}{\sqrt{\dot{y}^2 + 1}} \right) \right| \left[ \begin{array}{c} x_{*}(h) \\ \dot{x} \end{array} \right] \dot{x} dh
\]

(4)

where all additional notations are shown at Fig. 1, the function \( \rho = y(x) \) determines the projectile’s shape, \( \frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} \),

\[
\delta(h) = \begin{cases} 
0 & \text{if } 0 \leq h \leq L \\
1 & \text{if } h > L 
\end{cases}
\]

(5)

The equation of motion of the impactor with the mass \( m \) reads:

\[
m \frac{dv}{dh} + D(h, v_{\text{imp}}) = 0
\]

(6)

where \( v \) is the velocity of the impactor.

3. Solution for DOP and its properties

DOP, \( h_* \), is defined as the value of the coordinate \( h \) where the velocity of the impactor is 0. Then integrating Eq. (6) yields the relationship between impact velocity \( v_{\text{imp}} \) and DOP \( h_* \):

\[
m \int_0^{v_{\text{imp}}} v dh + \int D(h, v_{\text{imp}}) dh = 0
\]

(7)

Hereafter the case \( h_* > L \) is considered. Since for any function \( \Psi(x) \) the following property is valid (see Fig. 2)

\[
h_* = \left[ \int_0^{x_*} \Psi(x) dx \right] = \left[ \int_0^L \Psi(x) dx \right] \left[ \int_{h_*}^L dh \right] = (h_* - x)\Psi(x) dx
\]

(8)
Eq. (7) yields:
\[
\frac{1}{2} \frac{m v_{\text{imp}}^2}{\sigma} - \pi \left[ r_0^2 + (R^2 - r_0^2)(h_s - L) \right] \left( \sigma_e + 2 \sqrt{\gamma \sigma_e} \ v_{\text{imp}} \right) - \frac{2\pi}{\sigma_e} \int_0^L \left[ \sigma_e + 2 \sqrt{\gamma \sigma_e} \ v_{\text{imp}} - \frac{y}{\sqrt{y^2 + 1}} \right] (h_s - x) y \, dx = 0 \tag{9}
\]

It is convenient to use dimensionless variables; hereafter symbol $\bar{z}$ for any variable $z$ denotes the ratio $z/L$.

Eq. (9) can be transformed and $v_{\text{imp}}$ can be expressed as follows:
\[
\sqrt{\frac{v}{\sigma_e}} v_{\text{imp}} = J \left( \bar{y}(\bar{x}) \right) = \varphi(eJ_1, eJ_2) \tag{10}
\]

where
\[
\varphi(u, w) = w + \sqrt{u + w^2}, \quad e = \frac{2\pi y L^3}{m}
\]

\[
J_1 = \psi + 2I_1, \quad I_1 = \int_0^1 \left( \bar{h}_s - \bar{x} \right) \bar{y} \left( \frac{\bar{y}}{\sqrt{\bar{y}^2 + 1}} \right)^{v-1} \, d\bar{x}, \quad v = 1, 2 \tag{12}
\]

\[
\psi = \bar{r}_0^2 \bar{h}_s + (r^2 - \bar{r}_0^2)(\bar{h}_s - 1), \quad \tau = R/L \tag{13}
\]

The generator of the impactor’s nose comprises, generally, two “vertical” intercepts of a straight line with equations $\bar{x} = 0$, $0 \leq \bar{y} \leq \bar{r}_0$ and $\bar{x} = 1$, $\bar{r}_1 \leq \bar{y} \leq \tau$, and the curve $\bar{y} = \bar{y}(\bar{x})$, $\bar{r}_0 \leq \bar{y} \leq \bar{r}_1$.

Let us prove that for penetrators with given $R$ and $L$ the maximum impact velocity is required for the impacator with a cylindrical shape in order to penetrate at a given DOP. Interchanging dependent and independent variables the function $\bar{x}(\bar{y})$ can be considered as a description of the generator for $0 \leq \bar{x} = \tau$. Thus for $v = 1, 2$:
\[
I_1 = K_1[ \bar{x}(\bar{y}), \bar{r}_0, \bar{r}_1 ], \quad \psi = K_1[ 0, 0, \bar{r}_0 ] + K_1[ 1, \bar{r}_1, \tau ] \tag{14}
\]

where
\[
K_1[ \bar{x}(\bar{y}), a, b ] = \frac{2}{a} \int_a^b \left( \frac{\bar{h}_s - \bar{z}}{\bar{z}^2 + 1} \right)^{v-1} \frac{d\bar{z}}{d\bar{y}}, \quad v = 1, 2
\]

Thus:
\[
J_1 = K_1[ \bar{x}(\bar{y}), 0, \tau ] \tag{16}
\]

whereas Eq. (12) implies that the “horizontal” straight segment at the generator with the equation $\bar{y} = \text{const}$ do not affect the integrals $I_1$ and $K_1$. Eqs. (15) and (16) imply that the maximum $J_1$ is attained for $\bar{x}(\bar{y}) = 0$, i.e. for the cylinder-shape nose, and this maximum value is $\bar{h}_s \bar{r}_0^2$, $\nu = 1, 2$. Since the maximum $J_1$ and the maximum $J_2$ are attained for the same function $\bar{x}(\bar{y}) = 0$, and $\varphi(u, w)$ is an increasing function of both arguments, the maximum impact velocity, $v_{\text{max}}$, is as follows:
\[
\sqrt{\frac{v}{\sigma_e}} v_{\text{max}} = \varphi_0(\bar{h}_s \bar{r}_0^2), \quad \varphi_0(u) = \varphi(u, u) = u + \sqrt{u + u^2} \tag{17}
\]

In the further analysis two more impactor’s shapes must be considered. The impactor with an equation of the nose’s generator $\bar{y} = 0$ if $0 \leq \bar{x} \leq 1$ and $\bar{x} = 1$ if $0 \leq \bar{y} \leq \tau$ will be termed hereafter the awl-shaped impactor. Eqs. (12) and (13) for $\bar{r}_0 = \bar{r}_1 = 0$, $\bar{y}(\bar{x}) = 0$ imply that $J_1 = J_2 = (\bar{h}_s - 1) \tau^2$ and
\[
\sqrt{\frac{v}{\sigma_e}} v_{\text{imp}} = \varphi_0 \left[ e(\bar{h}_s - 1) \tau^2 \right] \tag{18}
\]

Consider a projectile with a conical shape of the nose with a plane bluntness. Substituting the equation of the generator $\bar{y} = \bar{r}_0 + (\tau - \bar{r}_0) \bar{x}$ and $\bar{r}_1 = 0$ into Eqs. (12) and (13) we obtain:
\[
J_1 = \bar{r}_0^2 \bar{h}_s + \psi_0, \quad J_2 = \bar{r}_0^2 \bar{h}_s + \frac{\tau - \bar{r}_0}{(\tau - \bar{r}_0)^2 + 1} \psi_0 \tag{19}
\]

where
\[
\psi_0 = \frac{1}{3} (\tau - \bar{r}_0) \left( \bar{h}_s \bar{r}_0 + 3 \bar{h}_s \tau - \bar{r}_0 - 2 \tau \right) \tag{20}
\]

Eqs. (19) and (20) are valid also for sharp cones with $\bar{r}_0 = 0$. The impact velocity can be calculated using Eq. (12) with $J_1$ and $J_2$ given by Eqs. (19) and (20).

4. Optimization of the nose geometry

Consider the problem of minimizing the impact velocity $v_{\text{imp}}$ for the shield with known parameters determining its mechanical properties. The mass of the impactor, length and the base radius of the nose of the projectile are assumed to be given. The problem is reduced to the optimization of the functional $J$ in Eq. (10) whereas the solution $\bar{y}(\bar{x})$ must satisfy the conditions:
\[
\bar{y}(0) = 1, \quad \bar{y}(1) = \tau, \quad \bar{y}(\bar{x}) \geq 0, \quad 0 \leq \bar{x} \leq 1 \tag{21}
\]

Eq. (12) shows that optimal curve is independent of the parameter $\sigma_e$.

The optimal curve $\bar{y} = \bar{y}(\bar{x})$ can have “vertical” straight segments, particularly, $\bar{x} = 0$, $0 \leq \bar{y} \leq \bar{r}_0$ and
\[ \hat{x} = 1, \quad \hat{r}_1 \leq \hat{y} \leq \tau \] where \( \hat{r}_0 \) and \( \hat{r}_1 \) are unknowns. These straight segments can be forbidden by using some artificial conditions, e.g. additional condition \( \hat{y} \leq k \) with arbitrary constant \( k \) can be employed. However, we do not use this method in order to determine the lower bound of the criterion for comparing different nose shapes. The functional that describes the criterion of optimization is defined as a function of the integral functionals. In principle, such functional can be investigated analytically (see, e.g., [10]). However, the solution of the problem can not be obtained in a closed analytical form. Therefore it is appropriate to use a numerical approach. In the developed computer code we used a slightly modified method of local variations [11]. Some typical results for \( \tau = 1 \) are shown in Figs. 3 and 4.

Fig. 3 shows that the optimum shape depends on the given DOP. For relatively small DOP, the optimal impactor is a sharp awl-shaped body. With increasing the DOP, the optimal nose geometry of the impactor varies and becomes close to a blunt cone. Comparison of the required impact velocity for the optimal impactor, awl-shaped impactor, sharp cone impactor, and the optimal impactor among the conical strikers with a plane bluntness is presented in Fig. 4 as functions of DOP. The results of solving the optimization problem for the blunt conical impactors are shown in Fig. 5 where the optimal radius of bluntness of the cone as a function of DOP is presented.

5. Concluding remarks

Determining the optimal impactor’s nose geometry is not the only and the main goal of investigation of the
corresponding variational problems. “Theoretically optimal” solutions are determined usually using approximate models, and the obtained optimal shapes are often quite involved. However, such analysis allows to determine the optimal value of the criterion and to estimate (in the framework of the same model) the efficiency of different really admissible shapes and their closeness to the optimal ones. The latter can be very useful for further comprehensive theoretical and experimental investigations.

References