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Shape optimization of high-speed penetrators: a review

Review Article

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Abstract: In spite of a large number of publications on shape optimization of penetrating projectiles there are no dedicated surveys of these studies. The goal of the present review is to close this gap. The review includes more than 50 studies published since 1980 and devoted to solving particular problems of shape optimization of high-speed penetrators. We analyze publications which employed analytical and numerical method for shape optimization of high-speed penetrators against concrete, metal, fiber-reinforced plastic laminate and soil shields. We present classification of the mathematical models used for describing interaction between a penetrator and a shield. The reviewed studies are summarized in the table where we display the following information: the model; indicate whether the model accounts for or neglects friction at the surface of penetrator; criterion for optimization (depth of penetration into a semi-infinite shield, ballistic limit velocity for a shield having a finite thickness, several criteria); class of considered shapes of penetrators (bodies of revolution, different classes of 3-D bodies, *etc.*); method of solution (analytical or numerical); in comments we present additional information on formulation of the optimization problem. The survey also includes discussion on certain methodological facets in formulating shape optimization problems for high-speed penetrators.

Keywords: Impact • Projectile • Share optimization • High-speed penetration

1. Introduction

In this review the notion of shape optimization of impactors refers to the choice of the best impactor among the infinite number of possible solutions. Although for brevity the term "shape optimization of impactor" is commonly used, it is actually implied that the problem of determining the optimal shape of the nose of the impactor is considered. The desired solution includes either a function that determines the shape of the projectile (in this case a variational problem must be solved) or geometrical parameters which determine the shape of the projectile among the considered types of bodies, *e.g.* ogive-shaped projectiles (in this case optimization problem for a function of one or several variables must be solved). We do not consider publications where on the basis of numerical or experimental investigations the projectile having the best performance characteristics is selected among several considered projectiles.

All results on shape optimization of penetrators are based on approximate models that describe penetrator-barrier interaction. In most of the cases these are localized interaction models (LIMs) [1]. LIMs are based upon the assumption that the drag force acting at the penetrating projectile can be determined by integrating local interac-

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tions at different locations at the surface of the projectile which depend on the instantaneous velocity of the projectile and on the angle between the local normal vector at the surface and the velocity vector. It is assumed also that (i) collision occurs in the direction normal to the front surface of the shield; (ii) penetrator has the axis of symmetry and, therefore, moves in the direction of this axis; (iii) penetrator is not deformed during penetration. Brief mathematical description of LIMs is given in Appendix. In spite of these and other simplifications, not only analytical but also numerical methods of solution must be used, particularly when the model accounts for friction at the surface of penetrator.

The natural criteria for shape optimization of impactors are either the maximum depth of penetration (DOP) into a semi-infinite shield (the depth inside a barrier at which penetrator is brought to rest) or the minimum ballistic limit velocity (BLV), *i.e.* the minimum impact velocity required for perforation, in the case of a shield having a finite thickness. In order to simplify the mathematical formulation of the problem, along with these criteria (particularly at the early stages of investigating this problem) the indirect criteria, which did not have a clear physical meaning, were used. Corresponding studies are also mentioned in this survey, and the methodological aspects of the problem are discussed in Section 4.

Solutions of the problems of shape optimization of penetrating projectiles can be interpreted in two different ways. The penetrator having an optimum shape maximizes the efficiency of the attack or, in contrast, determining ballistic parameters of the optimal penetrator allows optimizing a shield for the most unfavorable case.

2. Projectile shape optimization using the indirect criteria

Kucher [2] optimized the shape of penetrator using as the criterion the "dynamic work" from the theory by Thomson [3] for thin plates. Nixdorff [4] compared the efficiency of conical, different power-law and ogival heads and found the impactors that are superior to "Kucher's optimum head", which was determined by solving the corresponding variational problem. This paradox was explained by Ben-Dor *et al.* [5] with reference to the correct solution of the mathematically similar variational problem in hypersonic aerodynamics.

The first studies of shape optimization of penetrators against non-thin shields also used indirect optimization criteria. Using the previously developed disks model, Yankelevsky [6] optimized the shape of a projectile (a body of revolution) penetrating into soil by minimizing the instantaneous resistance force. The optimal shape was found to be determined by a single parameter that depends on the velocity, deceleration of the impactor and the properties of the medium. In the analytical studies [7–12] and numerical study [13] the shapes of the bodies with a minimum drag were determined using two-term localized interaction model (model using the notations specified below). Baranov and Lopa [14] and Baranov *et al.* [15, 16] minimized impactor drag force assuming that the normal stress at the surface of the penetrator is proportional to the normal component of the local velocity and neglecting friction. Khromov [17, 18] used similar model for determining the impactor with the minimum drag force among the truncated conical nosed impactors, and the model with a constant friction for numerical investigation of the variational problem for bodies of revolution.

3. Projectile shape optimization using the direct criteria

Table 1 includes the studies which used the "technologically" meaningful criterion for optimization, namely the maximum DOP or the minimum BLV. In the column «Model» the unified notations for the models are used.

In order to classify the models used for shape optimization of penetrators against non-composite shields, it is convenient to denote them as p & q models, where p and q are associated with the sub-models used at the first stage of penetration and at the second stage of penetration, respectively. In the existing models, it is assumed that at the first stage of penetration the resistance force is a function of the instantaneous distance between the nose (leading edge) of the impactor and the front surface of the shield, h. Hereafter the following notations are used: $p = 1$ if the resistance force is proportional to h and $p = 2$ if the dependence between the resistance force and h is linear with non-zero constant term. At the second stage of penetration it is assumed that the resistance force is, generally, a quadratic function of the instantaneous velocity of impactor. The latter assertion is implied by the adopted LIM with the quadratic dependence between the local normal stress and local normal velocity of impactor. In the case when all three terms in this quadratic polynomial are present, $q = 3$, and $q = 2$ if the linear term is missing.

Two-stage models are used to describe penetration into semi-infinite concrete shields whereby the first stage (cratering) is realized for $0 \leq h \leq h_0$ while the second stage (tunneling) comprises the subsequent motion of the impactor ($h > h_0$), where h_0 depends on the length of the nose of impactor. One-stage 0 & 2 and 0 & 3 models

Table 1. Optimization using criteria of maximum DOP and minimum BLV.

Reference	Model	Friction	Criterion	Shapes of impactors	Method of solution	Comments
Yankelevsky and Gluck [23]	The disk model	–	Max. DOP	Ogive-nosed impactors	N	For details, see main text
Aptukov and Pozdeev [26]	0 & 2	–	Max. DOP	Bodies of revolution	A	For details, see main text
Bondarchuk *et al.* [27]	0 & 2	–	Max. DOP	A class of 3-D bodies	N	
Ostapenko *et al.* [28]	0 & 3	Constant μ_{fr}	Max. DOP	A class of 3-D bodies	N	Different sets of conditions
Bunimovich and Dubinsky [29]	0 & 2	–	Max. DOP	Sharp 3-D conical bodies	A	
Vedernikov and Shchepanovsly [30], Vedernikov *et al.* [31]	0 & 2	Constant μ_{fr}	Max. DOP	Classes of 3-D bodies	N	
Ben-Dor *et al.* [32]	0 & 2*	–	Max. DOP, Min. BLV	Sharp 3-D bodies: cones, slender bodies, pyramids	A	Mass of monolithic impactor is given. Layered shield
Ostapenko [12]	0 & 2	Constant μ_{fr}	Max. DOP	Slender 3-D bodies with self-similar cross sections	A	
Ostapenko and Yakunina [33]	0 & 2	Different models	Max. DOP	Slender 3-D bodies with self-similar cross sections	A	
Ben-Dor *et al.* [34]	0 & 2*	–	Min. BLV	Truncated cones	N	
Jones and Rule [35]	0 & 2	Constant μ_{fr}	Max. DOP	Bodies of revolution	N	
Rule and Jones [36]	0 & 2	Constant τ_{fr}	Max. DOP	Bodies of revolution	N	
Ben-Dor *et al.* [5]	0 & 2	–	Max. DOP, Min. BLV	3-D bodies	A	For details, see main text
Ben-Dor *et al.* [1]	1 & 2	–	Max. DOP	Different classes of bodies of revolution	N	
Ben-Dor *et al.* [19]	See main text*	–	Min. BLV	Bodies of revolution	A	FRP laminates
Ben-Dor *et al.* [20]	See main text*	–	Max. DOP	Bodies of revolution	N	FRP laminates
Ben-Dor *et al.* [21]	See main text*	–	Max. DOP. Min. BLV	A class of 3-D bodies	A	FRP laminates
Yakunina [37–40]	0 & 2	Different models	Max. DOP	3-D bodies	A	
Ben-Dor *et al.* [41]	1 & 2	–	Max. DOP	Bodies of revolution	A	
Ben-Dor *et al.* [42]	0 & 2*	Constant μ_{fr}	Max. DOP	Bodies of revolution	N	
Ben-Dor *et al.* [1, 43]	0 & 3, 1 & 3	–	Max. DOP	Bodies of revolution	A	
Ben-Dor *et al.* [44]	0 & 2	Variable μ_{fr}	Max. DOP	Bodies of revolution	N	
Ben-Dor *et al.* [45]	0 & 2*	–	Min. BLV	Bodies of revolution	N	Taking into account plug formation
Banichuk and Ivanova [46], Banichuk *et al.* [47]	1 & 3	–	Max. DOP	Bodies of revolution	N	Mass of monolithic impactor or mass of shell is given
Banichuk and Ivanova [48], Banichuk *et al.* [49]	1 & 2	–	Max. DOP	Sharp or truncated pyramids	A	Mass of monolithic impactor is given
Banichuk *et al.* [50]	1 & 2	–	Max. DOP	Sharp pyramids	A	Mass of monolithic impactor or mass of shell is given
Banichuk *et al.* [51]	1 & 2	–	Max. DOP	Bodies of revolution	A	Mass of monolithic impactor or mass of shell is given
Banichuk *et al.* [51]	1 & 3	–	Max. DOP	Bodies of revolution	N	Mass of monolithic impactor is given
Ben-Dor *et al.* [52]	2 & 3, 2 & 2	–	Max. DOP	Bodies of revolution	N	
Ben-Dor *et al.* [22]	See main text*	–	Min. BLV	Truncated cones	N	FRP laminates
Ragnedda and Serra [53]	1 & 3	–	Max. DOP	Special class of bodies	N	
Banichuk *et al.* [54, 55]	1 & 2	–	Max. DOP, min. mass of shell, min. of volume	Bodies of revolution	A	Multi-objective optimization

are most often used for penetration into metals and soils. Coefficients in the formulas for the resistance force depend on the mechanical properties of the material of the shield and on the shape of impactor. The latter dependence allows formulating the problem of shape optimization of penetrating impactors.

In the case of fiber-reinforce plastic (FRP) laminates, Ben-Dor *et al.* [19–22] used a model that is based on a linear dependence between the normal stress and the normal component of the impact velocity [19–21] and the normal component of the instantaneous velocity of impactor [22].

In the case of semi-infinite shield (the optimization criterion is the maximum DOP) the asterisk in the column «Model» indicates the "exact model" which takes into account only partial immersion of the projectile in a shield at the initial stage of penetration, while the absence of the asterisk indicates that the model neglects incomplete immersion of the penetrator into the shield at the initial stage of penetration. In the latter case the model is called an "averaged model". In shape optimization of projectiles penetrating into finite- thickness shields (optimization criterion is the BLV) the "averaged models" are not used; in this case incomplete immersion of the impactor into the shield at the initial and final stages of penetration is taken into account. Yankelevsky and Gluck [23] use the model suggested by Yankelevsky and Adin [24] which falls outside the scope of the above classification.

Symbol "—" in column «Friction» indicates that friction is not taken into account, while "constant μ_{fr} " or "constant τ_{fr}" indicate that either friction coefficient or friction force per unit surface area are considered constant and non zero; "variable μ_{fr}" indicates using the model with friction coefficient depending on sliding velocity, and "different models" indicates that analysis is conducted using different friction models (either a model with a constant μ_{fr} and the model suggested in [25]).

In column «Shapes of impactors» we indicate which shapes of the impactors are considered for optimization. Depending on the considered shape either a variational problem or optimization problem for the function of several variables is solved. The typical situation is solving a variational problem in the class of bodies of revolution (indicated as «Bodies of revolution»). Notation «A class (classes) of 3-D bodies» indicates that bodies with complicated shapes are considered.

In the column «Method of solution» we indicate the general characteristic of the method of solution: "A" when analytical method is used, and "N" if the problem is solved numerically.

In column "Comments" we indicate additional information about the formulation of the problem, in particular, requirements for the considered bodies. Some clarification is needed here.

The typical situation is when it is assumed that a projectile has a nose (its shape is to be determined) and a cylindrical part. The length of a cylindrical part is varied in order to keep the mass of the projectile constant. This constant is assumed to be given, and it appears in the equation of motion of the projectile and in the formula for optimization criterion. In this case the mass of projectile is not considered as a constraint for optimization of the nose shape and is not mentioned in the column "Comments". The situation is quite different when the shape of the whole impactor has to be determined and there is no provision for the presence of the cylindrical part. The latter approach is employed in [46–51]. Here either the mass of the monolithic projectile or the mass of the thin shell of a hollow striker are the constraints in the optimization problem and are indicated in the "Comments" column.

In the considered variational problems the size of the projectile (the total length or the length of the nose, shank diameter for bodies of revolution) is assumed to be given. For 3-D impactors it is a shank area that is commonly assumed to be given. In the case of conical 3-D impactors with a given length the latter constraint is equivalent to prescribing the mass of the projectile provided the mass is distributed uniformly over the volume of a striker.

Quite unusual approach is used in [26]. The authors considered the minimax problem for determining the shape of the projectile that penetrates to the maximum depth under the most unfavorable distribution of the mechanical properties along the depth of a shield with a given areal density. The two-term model without friction was used with a linear relationship between the parameters of the model.

4. Some methodological remarks

Ben-Dor *et al.* [5] proved that in the case of 0 & 2 "averaged" model without friction, the problems of determining the maximum DOP, the minimum BLV and the minimum drag for constant penetration velocity for 3-D bodies with a given shank area are reduced to minimization of the same functional, and the same variational problem is encountered in hypersonic aerodynamics for drag coefficient which is calculated using the Newton's model. For the particular case of sharp bodies of revolution, the similarity between drag minimization for constant velocity and maximization of the DOP was noticed in [35]. In [41] it is demonstrated that maximization of the DOP in the cases of 1 & 2 "averaged" model without friction is to the maximization of the same functional. Therefore, solution of the inadequate on the physical grounds problem of minimizing drag for projectile motion with constant velocity using 0 & 2 and 1 & 2 "averaged" models without friction, yields the shape of the striker that penetrates to the maximum depth. Due to this similarity (that is violated when friction is taken into account), for solving penetration problems using 0 & 2 and 1 & 2 "averaged" models without friction it is sufficient to refer to the solutions obtained earlier in hypersonic aerodynamics (many of these solutions can be found in [56]). In particular, the solution obtained by Newton [57] for drag minimization of a body of revolution using 0 & 2 model, is valid for the maximization of the DOP using 0 & 2 or 1 & 2 "averaged" models without friction. It is important to note that correct formulation of the problem of shape optimization of penetrating impactors must include provision for a striker with a flat bluntness. In most of the cases, it is precisely the strikers having the flat bluntness that are the optimal ones.

Examining solutions of shape optimization problems, in the class of bodies of revolution for high-speed projectiles penetrating into semi-infinite shields, leads to the conclusions that (i) the optimal projectiles have a flat bluntness and (ii) the maximum DOP of the optimal penetrator is close to the maximum DOP of the optimum truncated cone-nosed impactors. In view of the approximate nature of the employed models, the latter assertions imply that optimal truncated cone-nosed projectiles or truncated-cone like penetrators show considerable promise against shields manufactured from different materials.

Analysis based on the application of approximate models for projectile-shield interaction shows that resistance force for star-shaped impactors with plane fins tends to zero (and, consequently, the depth of penetration tends to infinity) when the number of fins grows. Since this shape and similar star-shape solutions are of no practical importance, the optimization problem in the traditional formulation is ill-posed. Typical method, that allows avoiding this problem, is to consider optimization for a certain given classes of projectile shapes. For comparison of the ballistic properties of 3-D impactors and projectiles having a traditional shape (body of revolution) it is important to choose correctly the requirements and constraints for the compared strikers as has been emphasized, *e.g.* in [58].

Cavitation phenomenon was observed during motion of particularly shaped blunt bodies in some types of soils (*e.g.* sand) in certain range of velocity [59–62] whereby the cavity is formed, and the projectile moves inside the cavity without contact of its lateral surface with a barrier. This phenomenon results in (i) sharp reduction of drag and (ii) independence of drag on the projectile shape as long as the projectile remains inside the cavity. In this context it seems worthwhile to consider shape optimization of the projectiles taking into account the cavitation phenomenon. In particular, it is conceivable to suggest that using the projectile with a jet thruster [63, 64] for maintaining stable cavitation regime of penetration can result in considerable increase of the DOP.

5. Concluding remarks

It must be noted that there are no published studies that compare the obtained optimal solutions with experimental data and with the "exact" numerical simulations. The reason is that specially planned experiment and "exact" numerical calculations are needed for this purpose, because it is necessary to meet the same requirements for the compared shapes as in the theoretical investigations (the same mass, the diameter and the length of the impactor's nose, *etc*.). One of the goals of this review is to draw attention to this problem.

It would be unwarranted and inappropriately optimistic to expect that the obtained solutions of the shape optimization problems can be used directly for selecting the shape of penetrators. Similarly to other situations, the solutions obtained on the basis of approximate models may be used only for elucidating tendencies which must be taken into account in planning experiments and "exact" numerical calculations for design of penetrating projectiles.

Appendix A: Localized interaction models

Localized interaction model (LIM) for translational motion of penetrator is determined by the following formula [1]:

$$d\vec{F} = \begin{cases} [\Omega_n(u,v)\vec{n}^0 + \Omega_\tau(u,v)\vec{\tau}^0]ds & if \quad u_* < u < 1 \\ \Omega_n(1,v)\vec{n}^0 ds & if \quad u = 1 \\ 0 & if \quad u \le u_*, \end{cases} \tag{A1}$$

where

$$\begin{aligned} \vec{\tau}^0 &= -(\vec{v}^0 + u\cdot\vec{n}^0)/\sqrt{1-u^2}, \\ u &= -(\vec{v}^0 \cdot n^0) = cos\upsilon, \end{aligned} \tag{A2}$$

$d\vec{F}$ is the force acting at the surface element dS of the projectile that is in contact with the shield, $\vec{n}^0$ and $\vec{\tau}^0$ are the inner normal and tangent unit vectors at a given location on the projectile surface, respectively, $\vec{v}^0$ is an unit vector of the translational velocity of the projectile, $\vec{v}$, υ is the angle between the vector $\vec{n}^0$ and the vector $(-\vec{v}^0)$. The non-negative functions Ω_n and Ω_τ (normal stress and tangential stress, respectively) determine the model of the projectile-shield interaction (variables that characterize material properties of the shield are not indicated as parameters). Parameter u_* $(0 \le u_* < 1)$ determines the maximum magnitude of the angle υ, $\upsilon_* = cos^{-1}u_*$, whereby the impactor still interacts with the shield. It is assumed that for $\upsilon = \upsilon_*$ the projectile looses contact with

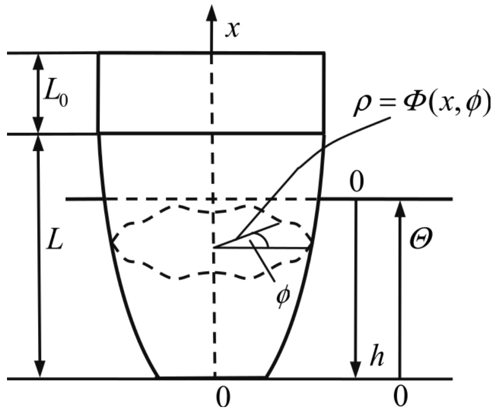


Figure 1. The notations.

a shield and a cavity is formed (it is commonly assumed that $u_* = 0$).

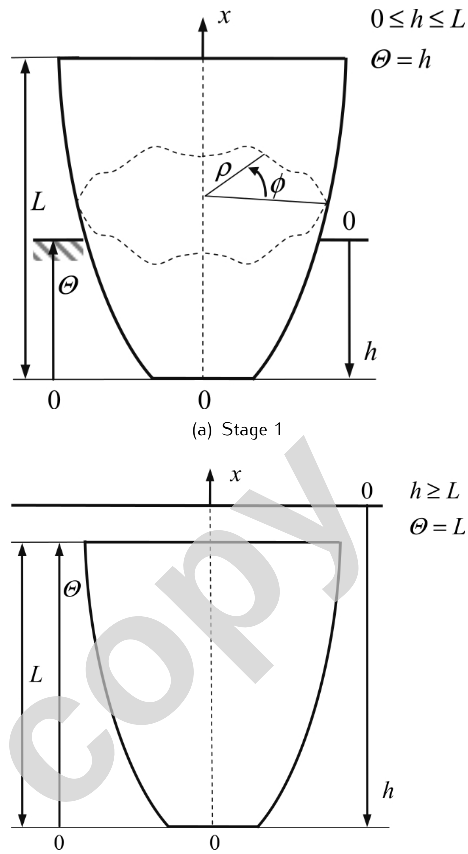
We consider normal penetration into a semi-infinite shield and use the following notations (see Fig. 1). The coordinate h, the instantaneous depth of penetration, is defined as the distance between the nose (leading edge) of the impactor and the front surface of the shield, and L is the length of the nose of the impactor. We take into account possible interaction between the shield and the lateral surface of the penetrator only for $0 \leq h \leq L$. The cylindrical coordinates x, ρ, ϕ are associated with the impactor, and the equation $\rho = \Phi(x, \phi)$, where Φ is some function, determines the shape of the impactor. We consider impactors with flat bluntness and a cylindrical part with the length L_0, and assume that this cylindrical part does not interact with the shield. In other words, all the above formulas describe the situation when the nose of the projectile is located between the cross-sections $x = 0$ and $x = L$.

The formalism of the description of the impactor-shield interaction surface is illustrated in Fig. 2a–b. Generally, two stages of penetration can be considered. At the first stage, entry into the shield, when $0 \leq h \leq L$ (Fig. 2a), the flat bluntness of the impactor (if any) and the part of its lateral surface between the cross-sections $x = 0$ and $x = h$ interact with the shield. The second stage, when $x \geq L$ (Fig. 2b), *i.e.*, motion inside the shield, is characterized by full immersion of the flat bluntness and of the lateral surface of the impactor into the shield.

Therefore, the moving contact area of the impactor-shield interaction can be described as (see Fig. 3) $0 \leq x \leq \Theta(h)$, where

$$\Theta(h) = \begin{cases} h & if \quad 0 \leq h \leq L \\ L & if \quad h \geq L \end{cases} \tag{A3}$$

The instantaneous resultant force acting at the projectile is determined by integrating $d\vec{F}$ over the surface of the



(a) Stage 1

(b) Stage 2

Figure 2. Two stages of penetration into a semi-infinite shield.

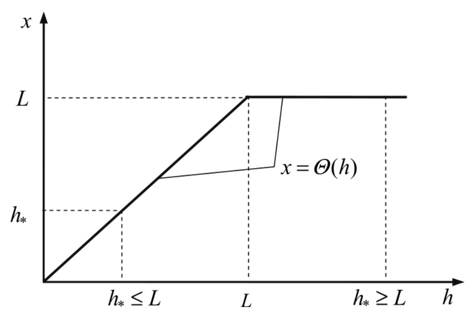


Figure 3. Penetration into a semi-infinite shield. Description of the contact area of impactor shield interaction.

projectile-shield contact at the same instant, S, and the resistance force, D, is given by the following formula:

$$D(h, v) = (-\vec{v}^0) \cdot \int\int_S d\vec{F} = \frac{1}{2}\Omega_n(1, \vec{v}) \int_0^{2\pi} \Phi^2(0, \phi) d\phi + \int_0^{\Theta(h)} dx \int_0^{2\pi} \Omega_0(u(x, \phi), v) u_0(x, \phi) d\phi \tag{A4}$$

where

$$\Omega_0(u,v) = u\Omega_n(u,v) + \sqrt{1-u^2}\Omega_\tau(u,v), \tag{A5}$$

$$u(x,\phi) = \frac{\Phi\Phi_x}{u_0(x,\phi)}, \quad u_0(x,\phi) = \sqrt{\Phi^2(\Phi_x^2+1)+\Phi_\phi}. \tag{A6}$$

Friction between the impactor and the shield is usually taken into account as follows:

$$\Omega_\tau = \mu_{fr}\Omega_n, \tag{A7}$$

where μ_{fr} is a friction coefficient. Then Equation (A4) can be rewritten as follows:

$$D(h,v) = \frac{1}{2}\Omega_n(1,v)\int_0^{2\pi}\Phi^2(0,\phi)d\phi + \int_0^{\Theta(h)} dx \int_0^{2\pi} \Omega_n\big(u(x,\phi),v\big)U(x,\phi)d\phi, \tag{A8}$$

where

$$U(x,\phi) = [u + \mu_{fr}\sqrt{1-u^2}]u_0 = \Phi\Phi_x + \mu_{fr}\sqrt{\Phi^2+\Phi_\phi^2}. \tag{A9}$$

Equation of motion of the impactor having mass m reads:

$$mv\frac{dv}{dh} + D(h,v) = 0. \tag{A10}$$

Let $v = V(h; v_{imp})$ be the solution of Equation (A10) with the initial condition, $v(0) = v_{imp}$, where v_{imp} is the impact velocity. The depth of penetration (DOP) for a given impact velocity, H, that is defined in penetrator shape optimization problems as the depth at which the impactor is brought to rest, can be found from the equation $0 = V(H; v_{imp})$.

The resistance force in Equationa (A4) and (A8) depends upon the instantaneous penetration depth because the projectile-shield contact area varies at the initial stage of the incomplete immersion of the projectile. This dependence renders the model quite involved. The latter shortcoming is usually eliminated by replacing the upper integration limits by constant integration limits in the expression for the drag force, $\Theta(h) = L$. The latter assumption is valid when the DOP is much larger than the length of the nose of a projectile. For such "averaged model", $D = D(v)$, and the DOP is given by the following formula:

$$H = m\int_0^{v_{imp}} \frac{v\,dv}{D(v)} \tag{A11}$$

The most commonly used models in penetration mechanics, and, consequently, widely applied in formulating shape optimization problems, are two-term models:

$$\Omega_n = a_2 v_n^2 + a_0, \quad v_n = uv, \tag{A12}$$

where coefficients a_i depend of mechanical properties of the material of the shield, v_n is normal component of the local velocity of penetrator. Three-term models with $\Omega_n = a_2 v_n^2 + a_2 v_n + a_0$ are used to a lesser extent.

In the case of the "averaged" two-term model, Equation (A8) yields:

$$D(v) = A_2 v^2 + A_0, \tag{A13}$$

where

$$\frac{A_i}{a_i} = \frac{1}{2}\int_0^{2\pi}\Phi^2(0,\phi)d\phi + \int_0^L dx \int_0^{2\pi} \frac{(\Phi\Phi_x)^i(\Phi\Phi_x + \mu_{fr}\sqrt{\Phi^2+\Phi_x^2})}{[\Phi^2(\Phi_x^2+1)+\Phi_\phi]^{i/2}} d\phi, \tag{A14}$$

$i = 0, 2$, and integral in Equation (A11) can be easily calculated:

$$H = \frac{m}{2A_2}\ln\left(1 + \frac{A_2}{A_0}v_{imp}^2\right) \tag{A15}$$

In the case of a body of revolution, $\Phi = \Phi(x)$ and Equation (A15) reads:

$$\frac{A_i}{\pi a_i} = \Phi^2(0) + 2\int_0^L \frac{\Phi\Phi_x^i(\Phi_x+\mu_{fr})}{(\Phi_x^2+1)^{i/2}}dx, \quad i = 0, 2 \tag{A16}$$

Along with LIMs in the canonical form, LIMs are often used as a sub-models, *e.g.* for penetration into concrete shields. As sub-models, LIMs are mainly used for shape optimization of penetrators against finite-thickness shields. Comprehensive description of the models used for shape optimization of high-speed penetrators can be found in [1].

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