The Limits of Shape Constancy: Point-to-Point Mapping of Perspective Projections of Flat Figures

Simone Moran and David Leiser

Ben-Gurion University of the Negev, Beer-Sheva, Israel.

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* Please address correspondence to Simone Moran, School of Management, Ben Gurion University of the Negev, P. O. Box 653, Beer Sheva 84105, Israel. email: simone@bgumail.bgu.ac.il
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Abstract

The present experiments investigate point-to-point mapping of perspective transformations of 2-D outline figures under diverse viewing conditions: binocular free viewing, monocular perspective with 2D cues masked by an optic tunnel, and stereoptic viewing through an optic tunnel. The first experiment involved only upright figures, and served to determine baseline accuracy in point-to-point mapping, which was found to be very good. Three shapes were used: square, circle, and irregularly round. The main experiment, with slanted figures, involved only two shapes: square and irregularly shaped, showed at several degrees of slant.

Despite the accumulated evidence for shape constancy when the outline of perspective projections is considered, metric perception of the inner structure of the shapes of such projections was quite limited. Systematic distortions were found, especially with the more extreme slants, and attributed to the joint effect of several factors: anchors, 3-D information, and slant underestimation. Contradictory flatness cues did not detract from performance, but stereoptic information improved it.
Introduction

Shape constancy is an important and well-known aspect of perception. This study deals with a particular case of shape constancy: the case of perspective transformations of slanted outline figures. A common assumption is that the visual system has access to projective congruence (i.e. various retinal images of a single object are nearly congruent in projective geometry) and that the visual system is therefore able to compensate for shape distortions caused by figure orientation (see Niall and Macnamara, 1990 for a review). This ability -- called by Niall and Macnamara (1989) the ‘projective thesis’ -- has nearly always been studied by evaluating observers’ ability to reconstruct or identify the general upright shape corresponding to a slanted image (e.g. Epstein, 1978; Hochberg, 1972). In their studies, Niall and Macnamara (1989, 1990) undermined the projective thesis. In a series of experiments, they demonstrated that observers do not have reliable access to the equivalence of shapes in projective geometry. Pizlo and his colleagues (Pizlo, 1994; Pizlo and Salach-Golyska, 1995) analysed the differences between projective and perspective transformations in experiments relying on judgments about identification and matching of shapes, and showed that perspective (rather than projective) invariants underlie 3-D shape perception.

The present study extends the work on shape constancy. Using a point-to-point mapping task, we investigate the metric perception of the inner structure of the figures, that is, how well observers understand the transformation of the inner space of an outlined shape. Taking for instance Figure 1, are observers able to match corresponding points on the upright and the slanted shape?
The importance of a correct understanding of 3D space, and of explicitly being able to match corresponding points in 3D space and in their 2D projections, is essential in many tasks, such as Air Traffic Control (ATC) and piloting, where displays frequently involve perspective views of planes. Interpreting graphic presentations of data, commonly used as a method for data analysis and for decision-making (Schmid and Schmid, 1979; Tufte, 1990; Meyer, Shinar and Leiser, 1997; Chambers, Cleveland, Kleiner and Tukey, 1983), particularly interpreting 3D graphic presentations, such as 3D MDS (multi-dimensional scaling), would be another such task. The ability to grasp the exact and specific positions of objects in space on the basis of point-to-point mapping, although essential for many such tasks, has not yet received much attention. The present study addresses this issue by examining point-to-point mapping of figures presented in 3D displays.

In recent years, human factors considerations have become central to the design of 3D information displays (Wickens, Todd and Seidler, 1989). The development of graphic display techniques make it feasible to present synthetic information about spatial layouts in many desired ways. Hence, finding effective modes of presentation, is an important challenge, and is also considered in this study, by examining the point-to-point mapping of the perspective transformations under a variety of viewing conditions.

Specifically, we study the effect of several experimental manipulations on the perceived structure: depth cues (e.g. monocular viewing, stereopsis), the effect of different outline shapes (squares, circles, irregular), and the degree of slant of the figures. These manipulations will now be explained and described in more detail.
Degree of slant. Previous studies have already shown that when observers are presented with a perspective display of an object, systematic localization distortions are found (McGreevy and Ellis, 1986; Farber and Rosinsky, 1978; Grunwald and Ellis, 1988, Niall and Macnamara, 1989; Niall and Macnamara, 1990). Ellis and his collaborators (McGreevy and Ellis, 1986; Grunwald and Ellis, 1988) offer a geometrical explanation for these spatial distortions by referring to two geometrical factors, one of which is relevant to the present study – ‘The 2-D effect’. This effect is the geometrical difference between the original 3-D stimuli and its 2-D projection. The present study examines the 2-D effect by manipulating the slant size. In particular, in line with the 2-D effect, we expect point-to-point mapping to become less accurate as the slant increases.

Depth cues. The type of depth cues available in a perspective display is also known to influence spatial interpretation (Coren, Ward and Enns, 1994; Hochberg, 1972; Goldstein, 1984). The present study examines the effect of adding contradicting cues, and of adding appropriate stereoptic cues to the perspective display by comparing three different viewing conditions: (a) a perspective binocular display, viewed freely and therefore including contradicting flatness cues (such as screen frame, binocular disparity and binocular parallax). Braunstein (1976) showed how these cues might adversely affect depth perception. Further, Van der Meer (1979) showed that in case of conflicting perspective and stereoptic (binocular disparity) information, the latter tends to dominate judgments; (b) a perspective display viewed monocularly, in darkness, through an optic tunnel, that greatly reduces the contradicting cues; and (c) a stereoscopic display in which appropriate stereoptic cues were added to the perspective information. In many studies, additional appropriate stereoptic cues improved depth judgments, although the degree of improvement was
dependent on the effectiveness of other monocular cues available (Yeh and Silverstein, 1992; Kim, Ellis, Tyler, Hannaford, and Stark, 1987). However, these studies were concerned with relative depth evaluations only. Results concerning the effect of additional stereoptic cues on metric tasks are contradictory. Todd and Bressan (1990) and Lappin and Love (1992) found stereopsis to be an ineffective source of metric information, whereas Leiser and Todd (1992) showed stereopsis to be effective for metrical evaluations. In all these studies, observers viewed the display freely. Additional flatness cues were therefore present and may have influenced results.

To summarize, while the specific findings on metric information are not entirely clear, the literature suggests that adding appropriate depth cues (particularly stereoscopic ones) improves spatial interpretation, whereas the presence of contradicting cues may be harmful. The present study compares a perspective monocular display binocular display viewed in darkness, through an optic tunnel, that is, without contradicting flatness cues with two other displays: a stereoscopic display, viewed under the same conditions, without contradicting flatness cues, and a binocular perspective display, viewed freely (and therefore including contradicting flatness cues). We will compare the precision of point-to-point mapping under these conditions. Specifically, we expect mapping precision to be highest in the stereoscopic display in which appropriate stereoptic cues are added to the perspective information, and lowest in the perspective binocular display in which contradicting flatness cues are present.

Outline shape. According to the Gibsonian theory (Gibson, 1950), perceived shape is based on the projective invariants of the shape, i.e. the geometrical properties that are preserved in the projective image of the shape. As mentioned above, Pizlo and his
colleagues (Pizlo, 1994; Pizlo and Salach-Golyska, 1995) more recently showed that perspective (rather than projective) invariants underlie 3-D shape perception. When a shape has more identifiable points, it should be easier to compute an invariant relationship involving the target point, and therefore accuracy of point-to-point mapping should improve.

Point-to-point mapping tasks, investigating the understanding of the inner structure of outline figures, have been used in the context of memory. These studies indicate that memory for spatial information of this kind is distorted and that landmarks have an influence (Huttenlocher, Hedges, and Ducan, 1991; Attneave, 1955; Stevens and Coupe, 1978; Nelson and Chaiklin, 1980). Attneave (1955), for example, who investigated localization of points within a circle, concluded that objective and subjective landmarks influence the magnitude and the direction of distortions. Several later studies (Stevens and Coupe, 1978; Nelson and Chaiklin, 1980) found that the accuracy of location memory increased as the distance from the border decreased.

Our study contrasts point-to-point mapping of shapes with very few identifiable points (e.g., a square) and with many identifiable points (the irregular rounded shape illustrated in Figure 1). We hypothesize point-to-point mapping to be more accurate for the irregular shape with many identifiable points.

In sum, the present experiments were designed to examine point-to-point mapping of perspective projections of varied 2-D outline shapes under a variety of viewing conditions.
Experiment 1: Baseline

The first experiment involves only upright figures, and serves to determine baseline accuracy in point-to-point mapping.

Method

Participants

Thirteen undergraduate students from Ben-Gurion University, all had normal or corrected vision.

Apparatus

Stimuli were generated on a 14" VGA (640 x 480 pixel), controlled by a PC. The observers' field of view was constrained by an optical tunnel, 60 cm long. The tunnel also included a diaphragm that masked the screen boundaries, at a distance of 20 cm from the screen. There were two oculars at the observer's end, which could be independently covered. In this experiment, vision was monocular, using only the dominant eye. Each trial began with the presentation of a point inside a model figure, and the observer’s task was to move a cursor to the equivalent location inside a target figure, both figures being simultaneously in view on the screen. The general organization of the display is illustrated in Figure 1, though in Experiment 1, both figures (model and target) were upright, perpendicular to the observer's line of sight, and therefore undistorted. The model point was indicated by a cross within the model figure. The observer controlled a cursor (also cross-shaped) within the target figure by using the arrow keys on the keyboard. Three
figures were used: a square (side=7.6cm), a circle (diameter=7.6cm), and the irregular rounded shape illustrated in Figure 1 (diagonal dimension=7.6cm).

**Design and Procedure**

Stimuli were organized in three blocks, one for each shape. Block order was randomized for each participant. The specific target points were selected as follows. For each figure, a grid was drawn (Cartesian for the square and the irregular figure, polar for the circle), and points selected at the intersections of the grid. We exploited symmetry wherever possible, to limit the number of points. In all, there were 31 different points for the square, 25 for the circle and 25 for the irregular shape. Each point was presented twice, yielding 162 trials per participant. The order of the trials was also randomized, and the experiment was self-paced.

**Results**

For every participant and every point, we averaged the two responses. In preparing the data for graphic display, we dealt with the distorting influence of outliers by computing median X and Y positions across participants, rather than means. Figure 2 illustrates the median performance. All the units are pixels (1mm=3.07 pixel), since this is the resolution afforded by the apparatus. Cursory inspection reveals that precision is high, while the errors do not seem to fall in any systematic pattern. In contrast to the results on mapping from memory (Huttenlocher et al., 1991; Attneave, 1955), participants are quite accurate in copying locations between two upright figures.
We next computed three errors: the absolute error along the X-axis, the absolute error along the Y-axis, and the diagonal error (the root of the sum of square errors along the Y-axis and the X-axis, i.e. a measure unrelated to a specific direction). The absolute median errors along the X-axis are 2.5, 2.0, and 1.0 pixels for the square, circle, and irregular shape respectively. The absolute median errors along the Y-axis are 2 pixels for each of the three figures, while the diagonal errors are: 4.1 pxl for the square, 4.1 pxl for the circle, and 2.2 pxl for the irregular shape. We ran a Median Test of the diagonal error that confirmed that precision was significantly higher for the irregular shape than for the others (Chi-Square = 45.7, p = .000, where the value of the error for the irregular figure was below and that for the other two was above expected).

**Experiment 2**

Following the findings of Experiment 1, we now turn to explore how point-to-point mapping is affected when the model shape is presented at a slant, as well as to examine the effect of the viewing conditions and of the shape of the model on such mapping.

**Method**

**Participants**

Ten undergraduate students from Ben-Gurion University, with normal or corrected vision, participated in the experiment. The participants were screened by a commercial test that consists in identifying raised circles in artificial stereoscopic stimuli viewed through polarizing glasses. Only observants able to identify a target with a disparity of 50" of arc participated in the experiment.
Apparatus

The apparatus was identical to that used in the first experiment, with two exceptions. First, the target was upright (as in Experiment 1), but the model was projected at a slant (see Figure 1). Three slants were used: 40°, 55°, and 70° around the X-axis. The central perspective corresponded to an observer viewing the display from a distance of 60 cm from the screen, at the mid-screen height. The stereoscopic stimuli were presented as red/green anaglyphs, also computed for a distance of 60 cm.

The second exception is that in this experiment, the circle was excluded and only two figures were used: the square and the irregular shape. This was done to increase the number of measurements for each condition, without unduly increasing the total number of trials. We excluded the circle since in a pilot study that involved point- to-point mapping of figures presented at a slant of 40°, the diagonal error was not significantly different for circles and squares. The remaining two figures still enable us to examine the difference between a shape with few identifiable points (a square) compared to one with many identifiable points (our irregular shape).

Design and Procedure

As in Experiment 1, each trial began with the presentation of a point inside the model figure, and the observer’s task was to move a cursor to the equivalent location inside the target figure, both figures being simultaneously in view on the screen. Every participant was involved in three experimental sessions, each devoted to one of the display methods described in the introduction: perspective binocular display, perspective monocular display (using the dominant eye) and stereoscopic display. The order of display methods was randomized between participants. Each session was divided into 6 blocks: 2 shapes x 3
slants. Since our main interest is in determining the distortion pattern of mapping from model to target, we used interior points of the figures only, after the pilot experiment showed accuracy to be far better (almost perfect) for points on the outline of the figure than for interior points. In all, there were 17 different points for the square and 13 for the irregular shape. Each point was presented twice. There were therefore 180 trials in each of the three sessions. The order of the trials within the blocks was randomized for each participant, as was the order of the blocks within sessions. The experiment was self-paced.

Results

As in Experiment 1, preparatory to the analysis, we averaged the two responses of each participant for each point and prepared the data for graphic display by computing median X and Y positions across participants. Results are summarized in Figures 3 and 4.

Insert Figures 3 and 4 about here

It is readily seen that when observers map model figures presented at a slant, errors are much larger than when they map model figures that are upright, as in Experiment 1 (compare Figures 3 and 4 to Figure 2). Further, point-to-point mapping appears to be more accurate with the irregular shape than with the square. Finally, the greater the slant of the model, the worse is the observers’ performance. These observations were subjected to an ANOVA, taking again as dependent variable the median diagonal error, and as dependent variables the shape (square vs. irregular), the slant, and the viewing condition. The shape variable produced a significant main effect ($F_{1,9}=18.7$, $p<.002$); the mean error for squares was 13.5 pxl, and that for the irregular shape was 6.5 pxl only. There was also a significant main effect of slant ($F_{2,18}=9.8$; $p<0.002$). Mean errors were 8.6, 9.5 and 11.9 pxl for slants of 40°, 55° and 70° respectively. LSD post hoc comparisons established that only the most
extreme slant (70°) differs significantly from the others: (p<.005 for the difference from the 55°, and p<.0005 for the difference from the 40° slant). Lastly, the interaction between shape and viewing condition was significant too (F=3.8; p<.04): stereoptic conditions enabled more accurate performance with the square, but this was not true for the irregular shape, where precision was good throughout.

The better accuracy of mapping with the irregular shape may be explained by the many identifiable points that characterize the specific shape we used. The identifiable points (the several protrusions on the shape) may serve as anchors, especially after the figure has undergone distortion. Even if the observers do not see the target figure as a perspective projection of the upright shape but merely as a deformed version, these anchors can assist them in their task. As mentioned in the introduction section, the influence of landmarks, i.e. anchors, was found in previous experiments on spatial memory as well (Nelson and Chaiklin, 1980). However, with extreme slants, when distortion is very large, there are errors despite the anchors.

Let us now focus on the positional errors of the observer’s judgments within each shape. Comparison of Figures 3 and 4 reveals a different pattern of errors for the two shapes. While no systematic pattern of errors is apparent when mapping the irregular shape, for the square, one can identify definite error patterns along each axis (an inward error along the horizontal axis and an upward error on the vertical axis). Accordingly, we computed additional error measures: the median inward error and the error along the vertical axis. We will analyse these separately.

**Horizontal axis.** Participants appear to place the points too far inwards. To compare the size of this effect for the various experimental conditions, we computed the median
inward error for each participant and each condition (viewing method, slant, and true horizontal position). These median values were then subjected to a within-subject ANOVA, with viewing method, slant, and $x_{\text{true}}$ (that is the horizontal position of the point on the model figure) as independent variables. This analysis yielded the following results. The main effect of viewing method was significant ($F_{2,18}=161.95; p<0.001$). An LSD post hoc analysis showed that the only significant differences are between stereopsis and ‘free’ binocular ($p<0.0002$) and between stereopsis and monocular viewing ($p<0.001$). The mean values are Stereo: 0.5, ‘free’: 5.5, and monocular: 3.9 pxl. The superiority of stereopsis is evident in Figure 3: the inward tendency all but disappears under stereoscopic viewing conditions. Slant angle also produced a main effect ($F_{2,18}=5.86.4, p<0.02$). The inward tendency is stronger for the 70° slant than for slants of 55° ($p<0.05$) and 40° ($p<0.002$), whereas the difference between the latter two is not significant. The mean error values are 2.1, 3.5, and 4.3 pixels for the 40°, 55°, and 70° slants respectively. The $x_{\text{true}}$ also had a significant effect ($F_{4,36}=33.4; p<0.001$). As can be seen in Figure 5, the inward mapping error increases as the distance from the centre increases. In order to understand this pattern, consider the function that links upright ($true$) and slanted ($persp$) figures as a function of slant (Equation 1):

$$x_{\text{persp}} = \frac{z \cdot x_{\text{true}}}{y_{\text{true}} \cdot \sin(\theta) + z} \tag{1}$$

where: $x_{\text{true}}$ and $y_{\text{true}}$ are the location coordinates in the upright figure; $x_{\text{persp}}$ is the x-coordinate in the perspective figure; $\theta$ is the slant of the perspective figure; $z$ is the viewing distance. Equation 1 implies that if too small a value is used for $\theta$, $x$ will be underestimated.
and that this effect increases with the absolute magnitude of $x_{\text{true}}$, which is the pattern that was found.\textsuperscript{1}

*Insert Figure 5 about here*

**Vertical axis.** The error pattern along the vertical axis is upward (Figure 3) and especially pronounced for the upper part of the figure. To bring this pattern out, we performed a within-subject ANOVA with median $y$-error as the dependent variable, and with three independent variables: viewing method, slant and $y_{\text{true}}$ (that is, the height on the model figure). There was no main effect of the display method or of the slant (Median errors were 4.8, 4.4, and 3.97 pixels for the “free” perspective binocular, the perspective monocular, and the stereoscopic displays respectively, and 3.8, 4, and 5.3 pixels for the 40°, 50°, and 70° slants respectively). However, the $y_{\text{true}}$ had a significant effect ($F_{4,36}=4.10; p<0.007$).\textsuperscript{2}

The function that links the heights in the upright and slanted figure as a function of slant is given by Equation 2:

$$y_{\text{persp}} = \frac{z \cdot y_{\text{true}} \cdot \cos(\theta)}{y_{\text{true}} \cdot \sin(\theta) + z} \tag{2}$$

where: $x_{\text{true}}$ and $y_{\text{true}}$ are the location coordinates in the upright figure; $y_{\text{persp}}$ is the $y$-coordinate in the perspective figure as measured from its lower boundary; $\theta$ is the slant of the perspective figure; $z$ is the viewing distance. Equation 2 implies that if too small a value

\textsuperscript{1} In a pilot experiment not reported here, participants were required to indicate the perceived slant by orienting appropriately a rod, rotating around an axis parallel to the $x$-axis. It was found that, under these conditions, participants indeed underestimate the slant angle.

\textsuperscript{2} The interactions between slant and $y_{\text{true}}$ ($F_{4,72}=17.5; p<.0001$) and between method and $y_{\text{true}}$ ($F_{4,72}=2.55; p=.02$) were statistically significant too, but are of little interest.
is used for $q$, $y$ will be overestimated, and that this effect increases with the magnitude of $y_{true}$, which is the pattern that was found.

**General Discussion**

Is there metric shape constancy? To review the main findings: distortions are found in the point-to-point mapping task. We established that this relatively poor performance is not a mere matter of imprecise pointing, since the baseline data of Experiment 1 are accurate enough, and since the distortions are systematic. Even when the display provided rich stereoscopic information on the 3-D layout, performance, though improved as hypothesized, remained inaccurate, especially along the vertical axis of the square.

These findings show that despite all the evidence for shape constancy, when the outline of perspective projections is considered (Hochberg, 1972; Coren et al., 1994; Goldstein, 1984), the metric perception of the **inner structure** of the shapes of such projections is quite limited. Indeed, even with respect to the irregular shape, for which mapping in the baseline condition was especially precise, accuracy was reduced when the 3-D slant was extreme ($70^\circ$). This argument is in line with Cutting’s findings about observer’s acquiescence to shape distortions when seen from the side, as in a cinema when the observer is seated in the front row of a side aisle (Cutting, 1987). The shape appears right, and an illusion of normality is maintained, even when the mapping is seriously distorted.

The distortion patterns we observed are comparable to those found for location memory, with the presence of anchors exerting an attractive influence (Stevens and Coupe, 1978; Nelson and Chaiklin, 1980). The superior performance with the irregular shape may be attributed to the presence of additional identifiable points. This explanation fits our
hypothesis and the invariants theories (Gibson, 1950; Pizlo, 1994; Pizlo and Salach-Golyska, 1995), according to which the perceived shape is based on the geometrical relations that are preserved in the projected image of the shape. Since our irregular shape has more identifiable points, it is easier to compute an invariant relationship involving the target point, yielding a more accurate point-to-point mapping. Another factor affecting the mapping is the quality of 3-D information afforded by the viewing conditions. We found no difference between free binocular viewing, which includes conflicting information, and monocular perspective viewing through an optical tunnel, where conflicting 2-D information was masked. However, where stereoscopic information was available, it improved precision. Lastly, the specific pattern of mapping errors along both axes in the square suggests that underestimation of the slant affects point-to-point mapping as well.

Our results suggest that the perceptual system functions opportunistically, in a manner reminiscent of Wade’s (1982) views on illusion -- performance is not the outcome of a single mechanism at work, but rather the joint effect of several factors, each having a stronger or weaker effect depending on circumstances. The various available information sources (e.g. identifiable points, 3D cues concerning slant) operate jointly, influencing the eventual mapping of each point. While the relative strength of each factor changes according to situational factors (type of display and richness of 3D information, shape structure, specific space in shape and so on), the influence of identifiable points (i.e., anchors) appears to be dominant whenever available.

These findings have important consequences for the use and designs of 3D displays. It cannot be assumed that the inner organization of a perspective display is properly understood, merely from the shape of the outline figure. On the other hand, the multiplication
of anchor points in the display enhances accuracy. Care should therefore be taken to include sufficient such points (e.g. by grid lines and the like) to ensure the required degree of accuracy (Leiser, Bereby, and Melkman, 1995). Adding stereoscopic information also improves accuracy.
References


Figure Captions

**Figure 1.** The point-to-point mapping task: place the cursor in the target figure at the location corresponding to the cross in the model figure.

**Figure 2.** Experiment 1: median responses for the circle, square and irregular shape. Group medians are represented by diamonds. Correct responses are at the nodes of the construction grids for the circle and square, and represented by circles for the irregular shape.

**Figure 3.** Experiment 2: median responses for irregular shape, by slant and viewing method. Group medians are indicated by circles. Correct responses are indicated by dots.

**Figure 4.** Experiment 2: median responses for square, by slant and viewing method. Group medians are represented by crosses. Correct responses are at the nodes of the construction grid.

**Figure 5.** Experiment 2: inward mapping error along the horizontal axis of the square as a function of the actual horizontal position on the model figure ($x_{true}$).
Point-to-point mapping

Figure 2

Circle

Square

Irregular shape
Figure 3

Point-to-point mapping

Slant 70
Slant 55
Slant 40

Free  Perspective  Stereo

Viewing Method
Figure 4

Point-to-point mapping

METHOD

SLANT

70

55

40

Free  Perspective  Stereo

METHOD
Figure 5

Point-to-point mapping
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Simone Moran is a Lecturer at the School of Management, Ben Gurion University of the Negev, P.O.Box 653, Beer Sheva 84105, Israel (simone@bgumail.bgu.ac.il). Her research interests include perception, decision-making, negotiation and organizational behavior.

David Leiser is Associate Professor in Psychology, in the Dept. of Behavioral Sciences, Ben-Gurion University, P.O.Box 653, Beer Sheva 84105, Israel (dleiser@bgumail.bgu.ac.il). His research interests include perception, reasoning and economic psychology.