

Bosonic Amplification of Noise-Induced Suppression of Phase Diffusion

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We study the effect of noise-induced dephasing on collisional phase diffusion in the two-site Bose-Hubbard model. Dephasing of the quasimomentum modes may slow down phase diffusion in the quantum Zeno limit. Remarkably, the degree of suppression is enhanced by a bosonic factor of order $N/\log N$ as the particle number N increases.

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The interplay between unitary evolution and decoherence has been a central issue of quantum mechanics. A universal formula has been put forward for the dynamical control of quantum systems weakly coupled to a bath [1]. According to this formula, relaxation or decoherence can either be suppressed or enhanced by interventions whose rate is much higher than, or comparable to (respectively) the inverse non-Markovian memory time of the bath response. If these interventions are either projective measurements or, equivalently, stochastic dephasing of the system's evolution [1], the resulting slow-down or speed-up of the relaxation or decoherence coincide with the quantum Zeno effect (QZE) [1,2] or the anti-Zeno effect (AZE) [1,3], respectively.

This universal formula provides simple recipes for the dynamical projection of both single- and multipartite quantum states, provided the bath spectral response is known. Yet an open question remains: can we similarly control or protect quantum states of large multipartite systems from the buildup of *many-body correlations* among its N interacting particles? The analysis of this scenario is tantalizing and nearly impossible for a multimode system with large N . However, useful insights can be gained by exploring few-mode models, for which full numerical solutions may be used to support analytic early-time approximations. At such times, we may describe the slow-down or speed-up of the system's many-body evolution by noisy perturbations, as QZE or AZE, respectively. Since phase diffusion between atomic Bose-Einstein condensates (BEC) [4–7] has a non-Markovian correlation time of ms, it is particularly amenable to the observation of the QZE and AZE [8]. Our main result here, is that for N -boson condensates, the QZE is *Bose amplified*.

Specifically, we consider the noise-induced suppression of phase diffusion in the two-site Bose-Hubbard model, recently used to describe experiments of quantum interference [6,7,9] and tunneling in an array of double wells [10]. This model, under the tight-binding condition for 3D wells, $l \gg N|a|$, where a is the scattering length and $l = \sqrt{\hbar/(m\omega_0)}$ is the characteristic size of a trap with frequency ω_0 , is accurately described by the quantized two-mode Josephson Hamiltonian [11], here rewritten as

$$\hat{H} = -J\hat{L}_x + U\hat{L}_z^2, \quad (1)$$

where $\hat{L}_x = (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)/2$, $\hat{L}_y = (\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1)/(2i)$, and $\hat{L}_z = (\hat{n}_1 - \hat{n}_2)/2$ generate the $SU(2)$ Lie algebra. The mean-field values $\langle \hat{L}_i \rangle$ determine the reduced single-particle density matrix $\rho^{(1)} = \langle \hat{a}_i^\dagger \hat{a}_j \rangle / N$, with mode indices $i, j = 1, 2$. The operators \hat{a}_i and \hat{a}_i^\dagger are bosonic annihilation and creation operators, respectively, for particles in mode i with corresponding particle number operators $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$. The bias potential is here set to zero, J is the intermode coupling, and U is the collisional interaction frequency. We have eliminated c -number terms, proportional to the conserved total particle number $N = \hat{n}_1 + \hat{n}_2$.

The collisional \hat{L}_z^2 term in the Hamiltonian (1) leads to “phase diffusion” [4–7] which degrades the reduced single-particle coherence. Its eigenstates,

$$|l, m\rangle = \frac{1}{\sqrt{(l+m)!(l-m)!}} (\hat{a}_1^\dagger)^{l+m} (\hat{a}_2^\dagger)^{l-m} |0\rangle, \quad (2)$$

constitute a preferred basis set resilient to this process [12]. On the other hand, the most sensitive states to phase diffusion are the spin coherent states $|\theta, \phi\rangle = e^{-i\phi\hat{L}_z} e^{-i\theta\hat{L}_y} |l, -l\rangle$, with $\theta = \pi/2$, corresponding to equal populations of the two sites, and a well-defined relative phase ϕ . For fully separated modes ($J = 0$), the single particle coherence of these states is lost as $\exp[-(t/t_d)^2]$ with a characteristic decay time $t_d = (U\sqrt{l})^{-1}$ and revives after $t_r = \pi/U$ [4,6]. For finite J and $U > 0$ (repulsive interaction), the fastest phase diffusion occurs for the anti-symmetric coherent state

$$|\pi/2, \pi\rangle = \frac{1}{2^l} \sum_{m=-l}^l (-1)^{l+m} \binom{2l}{l+m}^{1/2} |l, m\rangle, \quad (3)$$

i.e., the state with all particles populating the excited, odd superposition of the modes. This experimentally realizable state [7] will be used as the initial condition throughout this Letter. Its evolution with $U > 0$ is identical to the evolution of the state $|\pi/2, 0\rangle$ with $U < 0$, which for $|UN| > J$ drives the system towards a macroscopic cat state [13]. Since $[\hat{L}^2, \hat{H}] = 0$, we fix $l = N/2$.

Control of the phase diffusion of the state (3) may be attained by noise-induced dephasing of the odd- and even

two-mode superpositions, as illustrated in Fig. 1. This control can be introduced by any noise source that does not distinguish between the sites, e.g., an off-resonant incoherent light source focused at the barrier between them [Fig. 1(a)]. Such \hat{L}_x noise, implemented perpendicular to the depletion axis \hat{L}_z [Fig. 1(b)], affects the stochastic (noisy) modulation of the splitting J , modifying the system Hamiltonian as

$$\hat{H}_S(t) = [J + \hbar\delta_S(t)]\hat{L}_x + U\hat{L}_z^2. \quad (4)$$

The depletion-dephasing interplay is described by the general non-Markov (Zwanzig-Nakajima) second-order master equation (ME) for the reduced density matrix of the system. The resulting noise-controlled, normalized decoherence rate R conforms to the universal formula [1],

$$R(t) = 2\pi \int_{-\infty}^{\infty} F_t(\omega) G_{\text{depl}}(\omega + J/\hbar) d\omega, \quad (5)$$

$$F_t(\omega) = (2/N)^2 \langle |\epsilon_t(\omega)|^2 \hat{L}_x^2 \rangle,$$

$$G_{\text{depl}}(\omega + J/\hbar) = \frac{UN}{4\pi} \int_{-\infty}^{\infty} \Phi_{\text{depl}}(t) e^{i(\omega + J/\hbar)t} dt,$$

expressing $R(t)$ as the convolution of the finite time spectral intensity of the noise-control (where $\epsilon_t(\omega) = \int_0^t dt' e^{i\omega t'} + \int_0^{t'} \delta_s(t'', \dots) dt''$) is the Fourier transform of the stochastic phase factor), and the Fourier transform of the depletion correlation function $\Phi_{\text{depl}} = (2/N)^2 \times \langle e^{iJ\hat{L}_x(t-t')/\hbar} \hat{L}_z(t) e^{-iJ\hat{L}_x(t-t')/\hbar} \hat{L}_z(t') \rangle$. Equation (5) provides a general recipe for controlling quantum depletion by

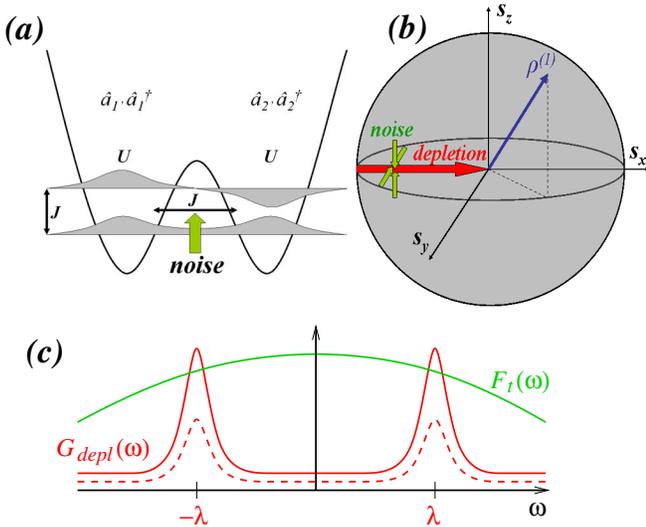


FIG. 1 (color online). Bose-enhanced QZE suppression of two-mode depletion: (a) Schematic diagram of the two-site Bose-Hubbard model. Depletion of the excited quasimomentum state is slowed down by a nonlocal noise source. (b) N -partite Bloch sphere, with \hat{L}_z -dependent depletion and \hat{L}_x -dependent noise. (c) Suppression of the depletion rate R as the overlap of $G_{\text{depl}}(\omega)$ and $F_t(\omega)$ noise decreases with N [double-peaked solid line to dashed line—Eqs. (5) and (10)].

noise. It yields the QZE limit of $R(t)$ suppression when $F_t(\omega)$ is spectrally much broader than G_{depl} [Fig. 1(c)], i.e., when the inverse width of these spectral functions (their memory times) satisfy $(t_c)_{\text{noise}} \ll (t_c)_{\text{depl}}$. Conversely, it yields the AZE limit of $R(t)$ enhancement when the two spectral widths or memory times are comparable, and their spectral centers are mutually shifted.

In what follows, we focus on the QZE limit, neglecting $(t_c)_{\text{noise}}$ altogether, i.e., taking the broadband noise to be Markovian. In atomic BECs, this limit is obtained for $1/(t_c)_{\text{noise}} \gg \Gamma_x \gg 1/(t_c)_{\text{depl}} \gtrsim \text{kHz}$, Γ_x being the rate of the Markovian dephasing. In this limit, we can use the Markovian quantum kinetic ME

$$\dot{\hat{\rho}} = i[\hat{\rho}, \hat{H}] - \Gamma_x [\hat{L}_x, [\hat{L}_x, \hat{\rho}]]. \quad (6)$$

Exact solutions of Eq. (6) can be found by expansion in the $|l, m\rangle$ basis set and numerical integration. In order to analytically approximate the initial phase diffusion, we truncate the hierarchy of dynamical equations for the \hat{L}_i operators at second-order correlations to obtain the Bogoliubov Backreaction (BBR) equations [5,14], for the mean-field single-particle Bloch vector $\mathbf{s} = 2\langle \hat{\mathbf{L}} \rangle / N$ and the correlation functions $\Delta_{ij} = 4(\langle \hat{L}_i \hat{L}_j + \hat{L}_j \hat{L}_i \rangle - 2\langle \hat{L}_i \rangle \langle \hat{L}_j \rangle) / N^2$. Linearizing these equations around the $|\pi/2, \pi\rangle$ state, we obtain the initial dynamics of the normalized

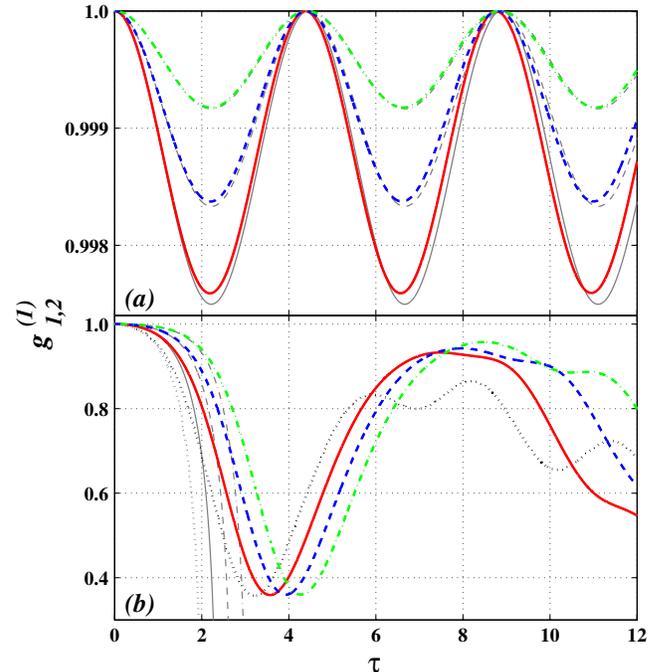


FIG. 2 (color online). Single-particle coherence as a function of rescaled time, starting from the coherent state $|\pi/2, \pi\rangle$ with (a) $\kappa = 0.5$, (b) $\kappa = 2$. Bold lines in (a) correspond to $N = 100$ (solid line), 150 (dashed line), and 300 (dash-dotted line) particles. Bold lines in (b) are $N = 50$ (dotted black line), 100 (solid line), 200 (dashed line), and 400 (dash-dotted line) particles. Gray lines correspond to the analytic forms of Eq. (7) for the same N .

correlation function $g_{1,2}^{(1)} = |\rho_{12}^{(1)}|(\rho_{11}^{(1)}\rho_{22}^{(1)})^{-1/2}$, corresponding to the fringe visibility in interference experiments [6,7]. In the absence of noise ($\Gamma_x = 0$), we find,

$$[g_{1,2}^{(1)}(\tau)]^2 = \begin{cases} 1 - \frac{4\cot^2(2\Theta)}{N} \sin^2(\lambda\tau) & \kappa < 1 \\ 1 - \frac{4\coth^2(2i\Theta)}{N} \sinh^2(\lambda\tau) & \kappa > 1 \end{cases} \quad (7)$$

where $\lambda = |\sqrt{1 - \kappa}|$ and $\tan\Theta = \sqrt{1 - \kappa}$, $\kappa = UN/J$ is the coupling parameter, and $\tau = Jt$ is the rescaled time. Dynamical BEC depletion in the weak-interaction ($\kappa < 1$) regime is thus bound and inversely proportional to the number of particles N . By contrast, for strong interactions ($\kappa > 1$), the phase diffusion rate is independent of the number of particles, but its onset time scales logarithmically with N [5]. This strong interaction instability may account for the rapid heating observed in the merging of two condensates with a π relative-phase, on an atom chip [9].

Numerical results based on the Markovian ME confirm the short-time dynamics of Eq. (7). The weak-coupling behavior [Fig. 2(a)] exhibits the anticipated stable oscillations. The oscillation amplitude decreases with increasing N while keeping κ fixed, so that $\mathcal{O}(1/N)$ initial quantum fluctuations remain small compared to the $\mathcal{O}(1)$ classical mean-field. By contrast, for strong interactions [Fig. 2(b)], quantum fluctuations grow rapidly and single-particle coherence is lost. Equation (7) gives an accurate description of the depletion at short times and a good estimate for the phase diffusion time. At longer times, depletion is significant, and the linearized BBR equations are no longer adequate. The fully nonlinear BBR equations, however, describe the loss of single-particle coherence with good accuracy. Partial and full revivals are observed due to the finite number ($N + 1$) of phase-space dimensions.

We now proceed to explore the effect of noise on phase diffusion. It is evident from Eq. (7) that in the absence of noise, regardless of the interaction strength, $g_{1,2}^{(1)}$ at $\tau < 1/\lambda$ scales *quadratically* rather than linearly in time,

$$g_{1,2}^{(1)}(\tau) = \frac{2\langle\hat{L}_x\rangle}{N} = 1 - (\Omega\tau)^2, \quad (8)$$

where $\Omega = \sqrt{2/N}|\coth(2i\Theta)|\lambda$. When the frequent measurement dephasing condition $\gamma_x \gg \lambda > \Omega$ is satisfied, we can adiabatically eliminate Δ_{yz} in the linearized BBR equations and obtain the QZE behavior,

$$g_{1,2}^{(1)}(\tau) = \exp\left(-\frac{\Omega^2}{2\gamma_x}\tau\right), \quad (9)$$

where $\gamma_x = \Gamma_x/J$. The modified diffusion time in the presence of noise, $\tilde{\tau}_d = 2\gamma_x/\Omega^2 = N\gamma_x \tanh^2(2i\Theta)/\lambda^2 \gg 1/\Omega$, should be compared, in the strong-interaction regime, with the noise-free, finite- J diffusion time, $\tau_d \propto \log[N \tanh^2(2i\Theta)]/\lambda$. Hence, the transition from the hyperbolic growth (7) which only depends on N through its onset time, to the QZE-suppressed depletion (9) at a rate linear in N , introduces a bosonic factor of order $N/\log N \gg 1$ in the diffusion time ratio $\tilde{\tau}_d/\tau_d$. The QZE is thus strongly amplified with increasing particle number. This constitutes the main result of this work.

Similar scaling is obtained from the universal Eq. (5). In the QZE limit of non-Markovian depletion in the presence of Markovian noise, $g_{1,2}^{(1)}(t) \simeq 1 - \int_0^t R(t')dt'$ may be inferred from it by substitution of the stochastic broadband $F_t(\omega) \simeq (2/N)^2(t/2\pi)\text{sinc}^2(\omega t/2)\langle\hat{L}_x^2\rangle$, so that

$$R(t) \simeq \frac{2t}{\pi N^2} \int_{-\infty}^{\infty} G_{\text{depl}}(\omega + J/\hbar)\text{sinc}^2\left(\frac{\omega t}{2}\right)\langle\hat{L}_x^2\rangle d\omega. \quad (10)$$

This form characterizes frequent projective measurements

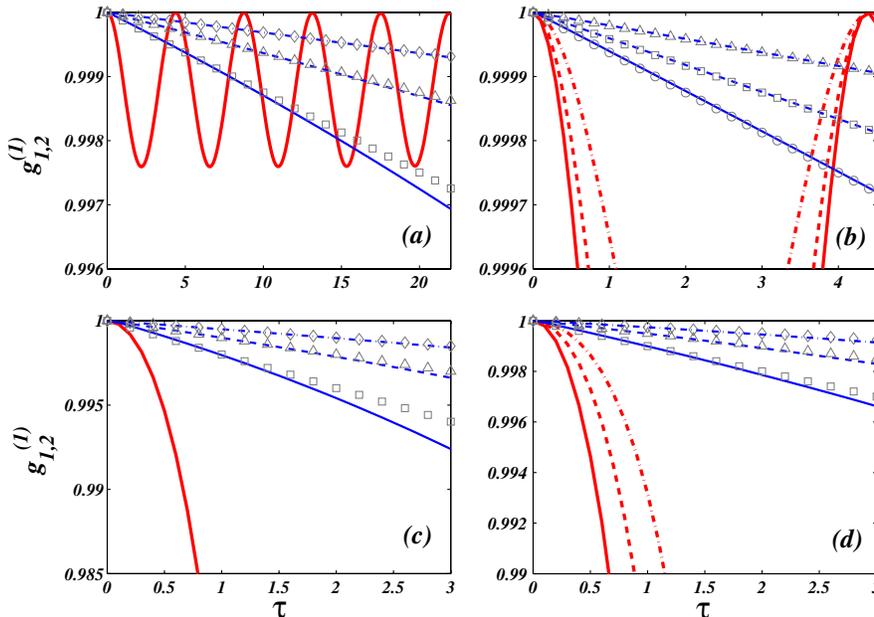


FIG. 3 (color online). Single-particle coherence as a function of rescaled time, with $\kappa = 0.5$ in (a), (b) and $\kappa = 2$ in (c), (d). The Hamiltonian dynamics with $\gamma_x = 0$ (bold lines) is compared to the evolution with nonvanishing γ_x (normal lines). Solid, dashed, and dash-dotted lines in (a) and (c) correspond, respectively, to $\gamma_x = 5, 10, 20$, and $N = 100$. Normal lines in (b) and (d) correspond to $\gamma_x = 10$. Solid, dashed, and dash-dotted lines in (b) correspond to $N = 100, 150, 300$ particles, respectively, whereas solid, dashed, and dash-dotted lines in (d) are $N = 100, 200, 400$ particles, respectively. Gray symbols portray the QZE behavior of Eq. (9).

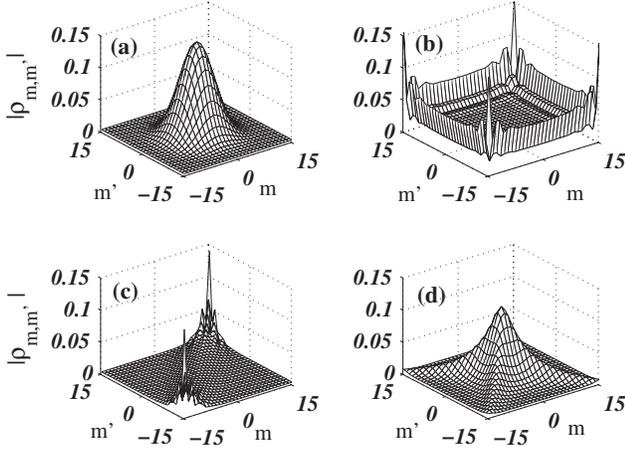


FIG. 4. Absolute values of the elements of ρ in relative Fock basis $|l, m\rangle$, for $N = 30$ and $\kappa = 2$: (a) Initial coherent state, (b) cat state after noise-free evolution at $\tau = 2.4$, (c) incoherent mixture after evolution with local site noise with $\Gamma_z = 0.05J$, at $\tau = 2.3$, (d) protected single-particle coherence after evolution with $\Gamma_x = J$.

[3]. The QZE is obtained when $\text{sinc}^2(\omega t/2)\langle\hat{L}_x^2\rangle$ is much broader than G_{depl} . As N is increased while keeping κ fixed, we have $4\langle\hat{L}_x^2\rangle/N^2 \approx s_x^2 \sim 1$ and $G_{\text{depl}}(\omega + J/\hbar) \propto 1/N$ [Fig. 1(c)] so that $R(t) \propto 1/N$.

The QZE-suppressed phase diffusion is illustrated in the numerical results of Fig. 3, where we compare the initial evolution of $g_{1,2}^{(1)}$ with and without noise, in the weak- and strong-interaction regimes. The weak-interaction oscillations of Eq. (7) are replaced, as γ_x is increased, by the exponential decay of Eq. (9), at a rate proportional to $1/(N\gamma_x)$ [Figs. 3(a) and 3(b)]. The strong-interaction dependence on the dephasing rate γ_x [Fig. 3(c)] and its Bose-amplified suppression [Fig. 3(d)], show a clear transition from $\log N$ dependent diffusion-times followed by N -independent depletion rate, to $1/N$ dependent depletion rates. These numerical results agree well with the appropriate closed form of Eq. (7) and (9).

It is interesting to contrast this noise-induced QZE behavior to the effects of local-site noise, which may be induced by collisions with thermal particles [15]. In Fig. 4, we plot the density matrix elements for the initial coherent state [Fig. 4(a)] and for the macroscopic cat state generated after noise-free evolution in the strong-interaction regime [Fig. 4(b)]. The dynamics leading to this cat state is extremely sensitive to local site noise [13], which destroys the macroscopic coherence, resulting in a 50-50 statistical mixture [Fig. 4(c)]. Phase diffusion in this case is *enhanced* to the extent that the strong-interaction diffusion time is bound at large N [5]. By contrast, the site-indiscriminate (nonlocal) noise considered here protects the single-particle coherence and slows down phase diffusion [Fig. 4(d)]. Whereas any weak local noise will degrade the intricate dynamics leading to a cat state, our nonlocal noise needs to be sufficiently strong to induce the QZE.

To conclude, we have found novel collective features of the QZE, which do not appear in the noise-controlled decay of single particles. The results presented here are generic rather than specific to the two-site Bose-Hubbard model of atomic BECs. There is currently great interest in phase diffusion experiments, enabling the measurement of single-particle coherence via the visibility of interference fringes [6,7,9]. Our predictions may thus be directly verified using current experimental apparatus. Consequently, new avenues may be opened for noise-control of complex multipartite systems.

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