Spatiotemporal model for the progression of transgressive dunes

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Abstract

Transgressive dunes, which are active sand areas surrounded by vegetation, exist on many coasts. In some regions like in Fraser Island in Australia, small dunes shrink while large ones grow, although both experience the same climatic conditions. We propose a general mathematical model for the spatiotemporal dynamics of vegetation cover on sand dunes and focus on the dynamics of transgressive dunes. Among other possibilities, the model predicts growth parallel to the wind with shrinkage perpendicular to the wind, where, depending on geometry and size, a transgressive dune can initially grow although eventually shrink. The larger is the initial area the slower its stabilization process. The model’s predictions are supported by field observations from Fraser Island in Australia.

Keywords: Transgressive dunes, Fraser Island, mathematical modeling, climate change

1. Introduction

Sand dunes cover a vast area of Earth land surface (\(\sim 10\%\)) and are also important in many coastal regions [1]. While several mechanisms/models for the development of dunes have been proposed [2, 3, 4, 5, 6, 7], the dynamics

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of transgressive dunes have received less attention, in spite of their ecological, climatological, and anthropogenic importance [8].

Transgressive dunes are relatively large sand bodies, which can migrate perpendicular to the shore, obliquely landwards or alongshore [9](see Fig. 1). These dunefields have different dune types including blowouts (trough-shaped depressions formed by wind erosion of a sandy substrate) and parabolic dunes (U-shaped vegetated dunes), barchans (active crescentic dunes), barchanoid chains (undulated transverse dunes), transverse dunes (active dunes formed perpendicular to prevailing winds), and depositional lobes (where sand from the blowout walls is being deposited). The dunefield margins are often formed by ridges of sand “precipitation” into vegetation [10]. Basically, sand “rains” down onto the adjacent vegetation by grain-fall, avalanche, and saltation, forming a steep slope, often lying at the angle of repose (∼33°).
Transgressive dunes can be formed after plant destruction by for example fire [11], tsunami, intense tropical storms [8] or human activity [12]; as a result, bare sand is exposed to wind erosion or very intensive storms which overwash the foredune [8]. Transgressive dunes can also develop when the climate gets drier, hotter or windier. Since many climate change models predict that the climate will become more drier and windier in some countries (such as those in Western Europe [13]), this study can have important implications for how the landscape may change. The typical transgressive dune area is $100-10^5 \, \text{m}^2$. They usually propagate downwind and the vegetation on the downwind side is inundated with sand and often completely smothered and killed. Field observations indicate [14] that, despite the same climatic conditions, large transgressive dunes grow while small ones shrink and some show sign of narrowing after an initial phase of widening.

Below we present a spatiotemporal model for sand dune dynamics, where the dependant variable is the vegetation cover. This variable is associated with the stability of dune—active/fixed dunes have small/large vegetation cover. We use this model to study the dynamics transgressive dunes. However, the proposed model can be used to study other sand-vegetation temporal dynamics problems such as blowouts and vegetated dunes.

One of the complex behaviors associated with dune dynamics is dune bistability; i.e., under similar climatic conditions it is possible to find both active and fixed dunes [15, 16]. The propagation of the border between active and fixed dune areas (which we denote as a front) depends mainly on rainfall, wind power, and wind direction. Transgressive dune dynamics may be determined by the dynamics of the fronts separating it from the vegetated background (see, e.g., Fig. 2d,e). These fronts are the transition zones that spatially separate alternative stable uniform states. The dynamics of the whole pattern mainly depend on the dynamics of individual fronts and on the interactions between adjacent fronts. Along the wind direction two fronts may be identified: (i) the fixed-active (FA) front which connects the upwind fixed (vegetated) area with the downwind active (bare) area and the (ii) active-fixed (AF) front which connects the active upwind area with the downwind fixed area.

The crosswind fronts separate the active dunes from the vegetated dunes perpendicular to the wind direction. In the following discussion we refer to transgressive dunes that are not connected to the beach, which can only grow downwind. There are four possibilities of transgressive dunes dynamics, depending on the fronts propagation velocities: (i) an initial transgressive
Figure 2: Location map of Fraser Island (a) within Australia and (b) with respect to the other sand masses in south-eastern Queensland (dunefields are marked in red; urban areas are shown in grey scale based on their density); (c) SPOT satellite image showing all the dunefields (in white) on Fraser Island, mostly located along the eastern coast of the island. The yellow rectangle marks the location of the dunefields shown in subpanels (d) and (e); (d) zoom-in on sand blows in the north of Fraser Island. The lower-right and upper-left sand blows are the ones seen in sub-figure e; (e) an aerial photo taken on August 17th, 2011, looking downwind from the southeast to the northwest. The different fronts (FA front, AF front and CW fronts, see text for more details) which separate the transgressive dune from the vegetation background are indicated in the picture.
Figure 3: Time evolution of transgressive dunes on Fraser Island, Australia, between the years 1958-2005. Fraser Island is the world’s largest sand island. The different colors are explained in the legend.
dune grows both along and perpendicular to the wind direction, (ii) an initial transgressive dune grows along the wind but shrinks perpendicular to the wind, (iii) an initial transgressive dune grows perpendicular to the wind but shrinks along the wind, and (iv) an initial transgressive dune shrinks in both directions. The third possibility does not seem to exist in nature. We show below what are the conditions that determined each of the above cases.

Such dynamics of transgressive dunefields has been observed on Fraser Island [8] (where they were termed as sand blowouts). Fraser Island is the northernmost and largest sand island in southeast Queensland (Australia), located about 250 km north of Brisbane (see Figs. 2, 3). While most of Fraser Island’s dunes are fixed and covered by forests (mostly vegetated by eucalyptus forests), there are dozens of active transgressive dunes [8]. Using aerial photos and satellite images, changes (since 1940) in the area of 70 active dunes were measured by [8]. The overall trend (Fig. 4) is of dune stabilization, i.e., reduction in transgressive dune area, which is more pronounced in the smaller dunes and in transgressive dunes which were detached from the beach and had no fresh supplies of sand. Although the overall sand drift potential (DP, defined below) by the wind in this area is high (DP~490 for 1997-2006, based on: the NCDC hourly database (www.ncdc.noaa.gov) and the NCEP/NCAR [17] and ECMWF [18] reanalysis), the dunes are stabilizing. This stabilization was attributed to a reduction in wind power and tropical cyclone activity [8].

The goal of this paper is to explain the dynamics associated with transgressive dunes, including the observation of faster stabilization of smaller transgressive dunes as shown in Fig. 4 and the influence of climatic changes on their evolution. More specially our work is concentrated on the study of the dependence of the front velocities on the climatic conditions (rainfall and wind power), and how these conditions affect transgressive dune spatial dynamics. This method is new in the context of vegetated dune modeling studies [4, 7, 19] which do not give a complete analysis of the dynamics of active sand patches. To this end we develop a physically-motivated spatiotemporal model for transgressive dunes dynamics which is based on our previous works [15, 16]. The proposed model predicts parameter values that allow both growth and shrinkage of transgressive dunes, depending on their initial area and geometry [20] and allows a conceptual study of a long-term dynamics. The model can be used to study the various feedbacks between vegetation, climate and transgressive dunes dynamics which may help geomorphologists to better understand these complex phenomena.
Figure 4: The difference in transgressive dunes area in Fraser Island between 1982 and 2005; y-axis shows a normalized change in the area during this period. Negative (positive) values denote transgressive dunes whose areas increased (decreased) in time. Note that $a = 1$ means complete stabilization whereas $a = 0$ means no change in the transgressive dune area. The overall trend revealed is of dune stabilization, most probably due to a significant decrease in wind power during the last three decades. Note the overall negative slope of the graph, indicating that small transgressive dunes shrink faster than larger ones.
2. The Model

Empirical studies have shown that dune activity can be modeled successfully using wind power and vegetation cover [21] as wind power may be associated with the potential sand transport (known as erosivity [1]), while vegetation cover may be linked to the amount of the sand available for transport (known as erodibility [1]). Biogenic crust is not taken into account in the current version of the model as it is less common in the temperate climate usually associated with coastal dunes [22]. Recently, we developed a simple model for the development of vegetation cover on dunes as a function of wind power and rainfall [15, 16]. That model has only temporal dynamics and exhibits bistability and hysteresis mainly with respect to wind power and rainfall. The model proposed here has both spatial and temporal dynamics, including wind directionality. The wind magnitude and direction play an important role in the development of the model’s variable, as is the case for dune formation [23].

Below, based on our previous studies [15, 16], we propose a model with a single variable, the vegetation dune cover density $v$:

$$\frac{\partial v}{\partial t} = \alpha(p)(v + \eta) \left(1 - \frac{v}{v_{\text{max}}} \right) - \varepsilon D P g(v_c, v)v - \gamma D P^{2/3} v - \mu v + \beta D P \frac{\partial g(v_c, v)}{\partial v} \left| \nabla v \cdot \vec{k} \right| v - \kappa D P^{2/3} \left( \nabla v \cdot \vec{k} \right) v + \delta \nabla^2 v.$$  (1)

The vegetation cover $v(x, y, t)$ depends on time $t$, and space, $x$ and $y$, and is bounded between 0 (bare dune) and $v_{\text{max}} \leq 1$. The first term on the RHS (Right Hand Side) represents vegetation logistic growth. This term includes a spontaneous growth rate $\eta$ that represents spontaneous growth even for bare dunes due to soil seed banks, underground roots, seeds carried by wind, animals, etc. The growth, $\alpha$, positively depends on rainfall

$$\alpha(p) = \begin{cases} 
\alpha_{\text{max}}(1 - \exp(-(p - p_{\text{min}})/c)) & p \geq p_{\text{min}} \\
0 & p < p_{\text{min}}
\end{cases},$$  (2)

where $p_{\text{min}}$ is $\sim$50 mm/yr [24]. For high rainfall the growth rate converges to a maximal value $\alpha_{\text{max}}$ in accordance with observations [25]. As in other logistic equations, growth rate decreases as vegetation cover approaches its
maximal value; this is modelled by including in the first term, \((1 - v/v_{max})\), which is the exposed area.

The next term in Eq. (1), \(-\varepsilon DP g(v_c, v)v\), represents the destructive effect of sand transport on vegetation, mainly due to root exposure, plant burial and erosion of plants in the seedling stage [26]. DP is the potential drift potential and \(g(v_c, v)\) is a step-like function represents the masking effect of the vegetation on sand transport are described in details below. Because saltating particles carry most of the mass and momentum, they can have considerable physical effects on existing vegetation, such as the exposure of below-ground plant tissue (pedastaling), abrasion of plant tissue, and leaf stripping [22]. The high mobility of sand may lead to the burial of the entire plant in sites of sand accumulation or to the exposure of much of the root system in sites of deflation [22]. This effect has been shown to indirectly lead to reduced plant growth and mortality [27]. Note that this mortality term is the same for sand deposition and for sand erosion as it is proportional to sand transport in general. In the current model, sand transport has only destructive effect on vegetation growth. This assumption is not accurate for special plants that have developed unique adaptations to sand erosion or deposition (such as adventitious roots) and have the ability to develop new stems when covered by sand [28, 22, 29, 30, 31] 1.

The drift potential, DP, is \(\text{DP} = \langle U^2(U - U_t) \rangle\), where \(U\) is the wind speed (in knots: 1 knot = 0.514 m/s) at 10 m height and \(U_t\) is the minimal threshold velocity (= 12 knots) necessary for sand transport [23]. There are both theoretical and empirical linear relations between the DP and the rate of sand transport [32]. Previous studies have shown that DP values can be linked to the temporal variability in coastal dune activity [33, 34]. In addition, due to its third power on the wind speed, DP is drastically affected by strong winds, which are more relevant for mobilizing sand grains.

\(v_c\) is the critical vegetation cover above which the sand is protected from the wind [35, 36]. \(v_c\) is associated with a skimming flow pattern and it varies, typically from 0.14 to 0.35, for different geographical locations and for different vegetation types since plants are porous objects [37] and each plant has a different porosity to wind flow.

To model the effect of vegetation cover on sand transport [38] we use a

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1Plants which are adapted to desert dunes can be modeled by smaller value of \(\varepsilon\) or even by \(\varepsilon < 0\)
(step-like) hyperbolic function for $g(v_c, v)$ in Eq. (1)

$$g(v_c, v) = 0.5(\tanh(b(v_c - v)) + 1). \quad (3)$$

$g(v_c, v)$ mimics the effect of the critical vegetation cover described above (i.e. $g(v_c, v) = 1$ for $v = 0$ and $g(v_c, v) \rightarrow 0$ for $v > v_c$). It is important to note that using other forms of $g(v_c, v)$ will not change the main qualitative results of the model as long as the above destructive effect decreases with vegetation cover or when the erodible gaps between plants decrease in size [37]. Studies based on wind tunnel experiments with live plants suggested an exponential decrease in total sediments flux with increasing vegetation cover [39]. This exponential decrease is similar to our mathematical representation of $g(v_c, v)$ in Eq. 3.

The third term in Eq. (1), $-\gamma DP^{2/3}v$, stands for vegetation suppression due to direct wind action (not related to sand transport), which increases evapotranspiration and also uproots, erodes, and suppresses vegetation growth [40]. The two-thirds power aims to represent the wind drag on vegetation which is proportional to the square [2] of the wind speed $U$ (while $DP \propto U^3$). This term is proportional to $v$, as it is a mortality term. $\gamma$ is a proportionality constant that may depend on vegetation types.

The fourth term in Eq. (1), $-\mu v$ stands for general mortality term which may simulate effects like extinction rate due to human activity, such as grazing [26], clear-cutting or burning, and off-road vehicles, governed by the value of the parameter $\mu$.

The last three terms in Eq. (1) describe the spatial dynamics of the vegetation cover acting in regions with spatial variability in vegetation cover, like at the fronts separated between active and vegetated domains and also introduce the coupling with the wind direction: (i) a mortality term due to sand erosion or deposition, (ii) a term representing the direct effect of the wind on vegetation, and (iii) a Fickian diffusion term representing spatial spreading of vegetation cover. These three terms which are absent in our previous model [15, 16] allow us to model the spatial dynamics of the transgressive dunes which are localized sand active areas in a background of vegetated dunes. Both models share the same homogenous stationary solutions as we show in the next section.

The fifth term on the RHS of Eq. (1) describes the vegetation response to sand flux in regions where $\nabla v \neq 0$ i.e at the borders between bare and vegetated areas. Sand flux through an area element, $\Delta x \times \Delta y$, is proportional
to $\text{DP}_g(v_c,v)$. We first develop the net sand flux (due to sand erosion or deposition) along the $x$ direction—it is proportional to $\Delta y [\text{DP}_g(v_c,v)]_x + \Delta x - \text{DP}_g(v_c,v)$. Taking the limit $\Delta x \to 0$ and invoking the definition of the partial derivative leads to $\Delta x \Delta y \partial_x [\text{DP}_g(v_c,v)]$, we use the absolute value as this term is always associated with decrease in vegetation. When also including the $y$ direction this term is generalized to $\Delta x \Delta y \text{DP} \left| \frac{\partial g}{\partial v} \right| \nabla v \cdot \vec{k}$, where $\vec{k}$ is a unit vector along the wind direction. Below we assume spatially unidirectional uniform wind (magnitude and direction) and thus DP is not affected by the differentiation operation. Using the chain rule on $g(v_c,v)$, the fact that $g(v_c,v)$ is a monotonic decreasing function of $v$, and applying the erosion/deposition effect only along the wind direction, $\vec{k}$, lead to $\Delta x \Delta y \text{DP} \left( \frac{\partial g}{\partial v} \right) \nabla v \cdot \vec{k}$. Finally, using a proportionality constant, $\beta$, that quantifies the strength of the vegetation response to deposition or erosion of sand at the dune front per unit area $^2$, and assuming that this suppression term is proportional to $v$, we obtain $\beta \text{DP} \frac{\partial g}{\partial v} \left| \nabla v \cdot \vec{k} \right| v$. Note that $\beta$ includes the spatial resolution and thus its value is resolution dependent. Using $g(v_c,v)$ as given in Eq. (3) we get

$$\frac{\partial g(v_c,v)}{\partial v} = -\frac{b}{2} \text{sech}^2 \left( b(v_c - v) \right). \tag{4}$$

This term is (i) zero where $\nabla v = 0$, i.e., for uniform vegetation cover, (ii) negative at the windward side of a transgressive dune where there is sand erosion (iii) negative at the leeward slope where there is sand deposition. Thus, this term mimics transgressive dune expansion along both sides of the prevailing wind direction and is proportional to the wind power. The reason for that is that in our model the vegetation response is negative both to sand erosion and deposition. Thus, we exclude in the current version of the model the effect of pioneer species which need sand deposition for their growth [7].

The sixth term on the RHS of Eq. (1), $-\kappa \text{DP}^{2/3} (\nabla v \cdot \vec{k}) v$, represents the spatial direct wind effect on vegetation. When vegetation cover increases along the wind direction (i.e $\nabla v > 0$; at the downwind part of the transgressive dune), vegetation cover is locally suppressed due to the stronger

$^2$Note that generally the suppression effect can be different for erosion and deposition such that two parameters $\beta_1$ and $\beta_2$ will be needed. For the sake of simplicity we choose these effects to be symmetrical.
wind stress effect on the less-protected (vegetated) region and since seeds are carried away by the wind from this region. The opposite (vegetation enhancement) occurs for regions in which the vegetation density decreases along the wind direction ($\nabla v < 0$; at the upwind part of the transgressive dune)—wind stress effect in this region is smaller since vegetation at the edge is protected by the more dense region on the upwind side and since more seeds are transported from the upwind vegetated side. Sand is also protected from wind action in this area due to increased surface roughness from upwind vegetation [41]. The sharper the vegetation gradient the larger this term is [42]. As the effect of this term is negative on the windward side of a transgressive dune and positive on the downside it will enable a net downwind migration of the transgressive dune.

The last term of Eq. (1), $\delta \nabla ^2 v$, represents an isotropic seed dispersal (e.g., by animals and winds) where $\delta$ is a diffusion coefficient. Although the wind can induce long-distance seed dispersal, most of the seeds fall at short distance from the canopy [43], which justifies the use of a simple diffusion term. The diffusion term acts both parallel and perpendicular to the wind direction whereas the former two terms act only along the wind direction. The diffusion term acts to smooth the vegetation cover at the borders between bare and vegetation areas. Thus, in addition to its logical representation of vegetation growth it prevents numerical instabilities due to unbounded vegetation gradients at the transition zones between bare and vegetation areas. As the diffusion term works also perpendicular to the wind direction it allows the elongation of a transgressive dune during its progress due to vegetation colonization at the dunes lateral edges. Thus, all new terms we introduced in the model (compared the nonspatial version [16]) are needed for studying the evolution of transgressive dunes.

Note that the model does not explicitly account for evapotranspiration which is not a decisive factor in dune stabilization and mobilization due to the unique properties of sand. The permeability of dune sand is 2,500 times higher than that of soil composed of smaller particles of silt and clay [24]. Hence, most of the rain in humid areas will infiltrate to the ground water and the available moisture will be at or close to the low field capacity. Perennial vegetation on sand dunes can obtain its necessary moisture from wetting zones at depth of 60 to 180 cm where enough water can be found even in very dry years [26].

The model parameters are summarized in Table 1. It is possible to reduce the number of independent parameters to 8. Still, we prefer to keep
the model in its present form to allow easier interpretation of the results. Most of the model’s parameters have been estimated in previous works (see [16] for more details); Table 1 includes a range of values for each of the model’s parameters, based on the literature published on the subject. For the homogenous part of the model (the first four terms on the RHS of Eq.1), except for the phenomenological parameters $\varepsilon$ and $\gamma$, all the other parameters can be estimated. The values of $\beta$ and $\kappa$ must be close to the values of $\varepsilon$ and $\gamma$ respectively as they have a similar effect on vegetation. The value of the diffusion coefficient $\delta$ which describes the lateral expansion of vegetation due to seed dispersal and vegetative growth, can be estimated from the change in the distribution of vegetation cover and it varies with vegetation types. The values of the last three parameters were chosen to give reasonable migration speed (2-5 m/s) for Fraser island transgressive dunes [8].

Validation of the model’s parameter is a very difficult task and we do not pretend that the model can be easily used to simulate a specific dune environment under specific climatic conditions. Our main goal is to study the main conceptual processes and feedbacks between climate (mainly wind power and rainfall) and transgressive dune dynamics, and our results should be only qualitatively compared with real data.

3. Results

The model described above can be used to study the development of different vegetation patterns such as vegetated spots within background of sand, sand-vegetation fronts, and sand spots within background of vegetation. First, we discuss typical stationary and uniform solutions of the model (i.e $\nabla v = 0$) and then we show the results of transgressive dune dynamics which are the main focus of this work. The stationary and homogenous solutions under fixed and variable climatic conditions aim to help the readers understand the model main results without the spatial terms (which we presented in details in our previous work [16]) and its sensitivity to the climatic variables. Fig. 5 depicts the stationary solutions as a function of DP for different values of $p$.

Fig. 5 depicts hysteresis behavior, which means that the system responds differently to increasing or decreasing the control parameter from its extreme

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3 Note that the value of $\beta$ is inversely depended on the squared of the spatial resolution.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Range</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\text{max}}$</td>
<td>Maximum vegetation cover growth rate</td>
<td>0.1-0.2</td>
<td>[21]</td>
</tr>
<tr>
<td>$p_{\text{min}}$</td>
<td>Minimal rainfall needed for vegetation growth</td>
<td>50 mm/yr</td>
<td>[22]</td>
</tr>
<tr>
<td>$c$</td>
<td>Vegetation growth response to rainfall</td>
<td>50-300 mm/yr</td>
<td>[25]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Spontaneous growth cover</td>
<td>0-0.2</td>
<td>[44]</td>
</tr>
<tr>
<td>$v_{\text{max}}$</td>
<td>Maximum vegetation cover</td>
<td>0-1</td>
<td>$v_{\text{max}} = 1$ in [21]</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Critical vegetation cover</td>
<td>0.1-0.4</td>
<td>$v_c = 0.14$ in Kalahari dunes [45], $v_c = 0.35$ in Australian dunes [46]</td>
</tr>
<tr>
<td>$b$</td>
<td>Steepness of transition between interference flow and skimming flow</td>
<td>5-20</td>
<td>Fig. 1 in [46]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Human impact parameter</td>
<td>0-0.1</td>
<td>Can be estimated as 0.0008 for Sinai dunes between 1986-1989 [47]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Vegetation tolerance to sand transport</td>
<td>0.005-0.02</td>
<td>Control the value of $D_1$ (Fig. 5) $D_1$ values are between 30 and 200</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Vegetation vulnerability to wind shear stress</td>
<td>0.005-0.02</td>
<td>Control the value of $D_2$ (Fig. 5) $1500 \leq D_2 \leq 3000$</td>
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<td>$\beta$</td>
<td>Vegetation tolerance to sand transport per unit area</td>
<td>0.0001-0.02 m$^{-2}$</td>
<td>Values in the range of $\varepsilon$</td>
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<tr>
<td>$\kappa$</td>
<td>Vegetation vulnerability to wind shear stress at the fronts</td>
<td>0.005-0.02</td>
<td>Values in the range of $\gamma$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Vegetation diffusion coefficient</td>
<td>0.1-15 m$^2$/yr</td>
<td></td>
</tr>
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</table>

Table 1: Definition and typical values of the main model parameters discussed in the text.
Figure 5: Stable states diagram showing the vegetation cover $v$ vs. DP for $p = 100$, $p = 200$ and $p = 700$ mm/yr (which is used for most of the model simulations). The arrows indicate the hysteresis loop along which the system responds differently to increasing or decreasing the control parameter DP from its extreme values. The transition points between the two solutions defined as DP$_1$ and DP$_2$ are shown for $p = 700$ mm/yr diagram. The parameter values used are: $v_{\text{max}} = 1$, $v_c = 0.3$, $\epsilon = 0.001$, $\gamma = 0.0008$, $\eta = 0.2$, $\mu = 0$, $\alpha_{\text{max}} = 0.15$, $p_{\text{min}} = 50$ mm/yr, $c = 100$ and $b = 15$. 
values. The hysteresis diagrams show the model’s response to changes in wind drift potential DP. For a wide range of DP values, both the fixed (the upper branch) and active (lower branch) dune states coexist, indicating the hysteresis and bistability of the model. This dune behavior may be described as follows: starting from a very low DP, only the fixed-dune solution $v_f$ exists; when the DP slowly increases, this solution persists until the very high DP is reached (defined as $DP_2$). Beyond this point, the system shifts into the active dune state solution $v_a$. When DP is then slowly decreased, the solution continues to show active dune state until a certain value of DP is reached (defined as $DP_1$), beyond which the solution reverts back to the fixed dune state. Note that, although the system has two stable solutions for the same parameters, a unique set of initial conditions determines only one possible final state. Several scenarios can lead to bistability of active and fixed dunes under the same climatic conditions. Among them are climate change and human impact. An example of such a transition is the clear-cutting of vegetation on dunes in North Holland that took place in December 1998 ($DP = 1706$) [44], which led to the dunes remaining bare and active until today. For lower values of rainfall the hysteresis diagram shifts left and the domain of bistability shrinks. Similar hysteresis diagrams can be found for $p$ and $\mu$ as shown in [16].

3.1. Spatial dynamics

Below we use the spatial version of the model to focus on the dynamics of transgressive dunes and link the results to geomorphological observations, most of them from Fraser Island. In order to simulate transgressive dune dynamics we run the model for a range of parameters in the bistable regime (see Fig. 5) since a transgressive dune is a domain characterized by the active state solution which invades the background of the fixed dune state. Thus, our interpretation is that, for modeling a transgressive dune, these two states must coexist. The boundaries of transgressive dunes (fronts) are not stationary and can propagate in time. The spatial dynamics of a transgressive dune is a result of the fronts (which connect the two stable states) different velocities. For lower values of DP where only the stabilized state is stable, the initial transgressive dune will quickly become stable as the vegetation will grow everywhere and the transgressive dune will vanish. At the other extreme, for a higher value of DP where only the active state is stable, the entire area will become active, as the vegetation will die everywhere.
Figure 6: Three hundred years model evolution of a sand spot (yellow square) in the background of vegetation (green). The initial transgressive dune (a) propagates downwind with a constant speed that is much higher than the front speed perpendicular to the wind. Panels (b, c, d, e) depict the transgressive dune evolution at successive 60 year intervals. In Fig. 5 we show in details the spatial dynamics of this transgressive dune. The parameter values used: $\alpha_{\text{max}} = 0.15$, $\beta = 6.64 \cdot 10^{-4}$, $\gamma = 0.0008$, $\delta = 0.2$, $\eta = 0.2$, $\kappa = 0.1$, $\mu = 0$, $b = 15$, $c = 100$, $v_c = 0.3$, $v_{\text{max}} = 1$ where for this simulation, $p = 700$ mm/yr, $\text{DP}=425$, and the initial dimension of the domain is $50 \times 800$ m.

Fig. 6 depicts the model’s 300 year evolution of a transgressive dune propagating downwind. We choose$^4$ $p = 700$ mm/yr and $\text{DP}=425$ since these values yield bistability of both active and fixed dune states in the absence of spatial dynamics, and hence front dynamics is expected to occur when spatial dynamics is included. The initial squared transgressive dune becomes more elongated in the wind direction. It can be seen that some of the transgressive dunes in Fraser Island (see Fig. 3) have this shape.

The initial transgressive dune area can increase or decrease with time, $^4$The results will be almost the same for $p > 700$ mm/yr since for large rainfall the growth rate converges to a maximal value.
depending, for example, on DP or \( p \). The dynamics in the alongwind direction is different from those of the crosswind direction. For example for DP=425 the alongwind cross section increases with time whereas the crosswind cross section decreases but more slowly, resulting in a total area increase. Still, it is clear that after a sufficiently long time the transgressive dune will vanish as growth in both directions is necessary for transgressive dune expansion. Such elongated form of transgressive dunes along the wind direction is supported by field observations from transgressive dune dynamics study at Lake Huron in Canada [20] and from Fraser Island (see Figs. 3, 4).

The dynamics of transgressive dunefields and transgressive dunes can be explained by the alongwind FA (which connects the fixed and active state) and AF (which connects the active and the fixed states) front dynamics. The FA and AF front velocities, \( v_{FA} \) and \( v_{AF} \), are mainly dictated by the gradient terms in Eq. (1). Fig. 6 depicts the velocities of these fronts as a function of DP for \( p = 700 \) mm/y. Note that \( v_{FA} \) is almost constant for large values of DP whereas \( v_{AF} \) increases almost linearly with DP. This behavior is expected as the front \( v_{AF} \) advances as the sand transported by the wind covers the vegetation. Since the amount of sand is proportional to the DP, \( v_{AF} \) also increases. In contrast, \( v_{FA} \) advances mainly by the spread of vegetation thus, it is mainly dictated by the value of the diffusion coefficient \( \delta \) and also depend on \( DP^{2/3} \) through the sixth term in the model (Eq. 1).

The initial transgressive dune is growing in the alongwind (\( x \)) direction when \( v_{AF} > v_{FA} \) while in the crosswind (\( y \)) direction the condition for growth is \( v_{CW} < 0 \) (\( CW \) is the abbreviation for crosswind). We denote the DP for which \( v_{AF} = v_{FA} \) as \( DP_{W} \) and the DP for which \( v_{CW} = 0 \) as \( DP_{CW} \). The transgressive dune advance rate (\( v_{AF} \)) is \( 5 \) m/s for \( DP \sim 1000 \) which is in rough agreement with data from Fraser Island [8] \(^5\). Note that \( v_{CW} \) is much smaller than \( v_{AF} \) and \( v_{FA} \) thus, the dynamics in the alongwind direction is faster than that of the crosswind direction (assuming unimodal direction wind).

According to Fig. 7a it is possible to classify the transgressive dune dynamics into three categories: (i) the transgressive dune is shrinking in both directions, \( DP < DP_{W} \); (ii) the transgressive dune is expanding in both directions, \( DP > DP_{CW} \), and (iii) the transgressive dune is shrinking in the

\(^5\)This advance rate is inversely dependent on the distance from the beach, probably due to decrease in DP and a limited sand supply [8].
Figure 7: (a) Front velocities $v_{AF}$ (Active-Fixed front velocity), $v_{FA}$ (Fixed-Active front velocity), $v_{CW}$ (Cross Wind front velocity) in meter per year, as a function of DP for $p = 700$. The velocities were computed from a numerical solutions of Eq.1) with an initially square-shaped sand area. The velocities were computes by taking time derivative of the front displacement which causes small scattering of the data. (b) $v_{AF} - v_{FA}$ and $v_{CW}$ as a function of DP. The DP for which $v_{AF} = v_{FA}$ is indicated by DP$_W$ and the DP for which $v_{CW} = 0$ by DP$_{CW}$. Note that direction the condition for growth in the lateral direction is $v_{CW} < 0$. For DP$_W < DP < DP_{CW}$ a transgressive dune may initially grow (depending on its starting area and geometry), although finally it will diminish. The red arrow indicates DP=425 the value used for the simulation of Fig. 6. The parameter values are as in Fig. 6.
crosswind direction by growing in the alongwind direction, \( \text{DP}_w < \text{DP} < \text{DP}_{CW} \). It is clear that after a sufficiently long time category (iii) will also result in a diminished transgressive dune. Yet, within region (iii) when \( v_{AF} - v_{FA} > v_{CW} \) the area of the initial transgressive dune may grow despite the fact that after some time it will shrink. A simple analysis for a rectangular transgressive dune with dimensions \( L_x \) and \( L_y \) yields that \( dL_y/dt = -2v_{CW} \) and \( dL_x/dt = v_{AF} - v_{FA} \) where we assume that the transgressive dune remains rectangular during its evolution. The transgressive dune area \( S = L_xL_y \) can be written as:

\[
S(t) = -2v_{CW}ut^2 + (L_y0u - 2v_{CW}L_x0)t + S_0, \tag{5}
\]

where \( u = v_{AF} - v_{FA} \) and \( L_x0, L_y0 \) and \( S_0 \) are the initial dimensions and area of the transgressive dune; see Fig. 7. According to Eq. 5, the transgressive dune area evolution follows a quadratic growth (see Fig. 8b). The simple analysis of Eq. 5 capture the basic quadratic behavior but differs in the details as it assumes that the transgressive dune remains rectangular where this is not the case. Eq. 5 ignores curvature effects of the fronts which not remain straight lines. The maximum area will occur at \( t_{max} = (L_y0u - 2v_{CW}L_x0)/4v_{CW}u \). The area of the large transgressive dunes increases initially although finally they are predicted to vanish. In addition, small or elongated (alongwind) transgressive dunes shrink faster (compare to squared ones of the same area) [8], as seems to be the case for the transgressive dunes on Fraser Island shown in Fig. 4, and numerically in Fig. 9. Fig. 9 shows the dependence of the stabilization process on transgressive dune aspect ratio \( \Lambda = L_x/L_y \) for different times. It shows that for larger \( \Lambda \) the transgressive dune area shrinks more significantly.

By using a time series of climate data (annual averages) for Fraser Island, the model can show transgressive dune evolution with a variable climate. Our simulations provide a possible qualitative scenario for the evolution of transgressive dunes under real climatic conditions (wind power and precipitation). Fig. 10 shows the annual DP and rainfall measured at Sandy Cape Lighthouse meteorological station (24.73°S, 153.21°E) located at Fraser Island, for the years 1957-2006 [8]. Since the proposed model was written for uniform wind direction, it is better to use RDP (Resultant Drift Potential, [23]) instead of DP for non-unimodal winds which is generically true. The direction of RDP is referred to as the resultant drift direction (RDD), which expresses the net trend of sand drift, namely, the direction in which sand
Figure 8: (a) $x$ (circles) and $y$ (squares) transgressive dune dimensions vs. time for the simulation shown in Fig. 6. The solid lines are the linear regressions, indicating growth along the wind direction and shrinkage in the crosswind direction. (b) Transgressive dune area vs. time showing initial (transient) growth followed by a transgressive dune shrinkage; the solid line is quadratic fit follows the prediction of Eq. 5. The parameter values are as in Fig. 6.
Figure 9: The ratio between the transgressive dune area after 15, 30, 45 years \((S_{15}, S_{30}, S_{45})\) and the initial transgressive dune area \(S_0\) as the function of the transgressive dune aspect ratio \(\Lambda = L_x/L_y\) when the initial transgressive dune area is the same in all simulations (it is three times smaller than the one in Fig. 8, thus it shrinks faster.) For larger \(\Lambda\) the transgressive dune area shrinks more significantly. Note that for the presented short times and for small \(\Lambda\) the initial area increases. DP=425 and other parameter values are as in Fig. 6.
would drift under the influence of winds blowing from various directions. It is calculated by vector summing the DP from the different wind directions. The ratio of RDP to DP (RDP/DP) is an index of the directional variability of the wind (RDP/DP = 1 stands for unidirectional wind whereas RDP/DP = 0 characterizes multidirectional winds that vectorally cancel each other out). There has been a sharp reduction in DP since 1982, probably due to a reduction in tropical cyclone intensity (average DP for 1957-1981 was 1671 compared to 823 for 1982-2008) [8]. Fig. 10c shows RDD values for Fraser Island which are mainly related to south easterly winds. In the following simulations for Fraser Island we use RDP values which better describe the net effect of winds on transgressive dunes and assumed a constant wind direction for all years (see further discussion in the next section).

Fig. 11 shows the reduction of the transgressive dune area between 1982 and 2005 for different transgressive dune area in a log-log plot. According to the model simulations the stabilization process follows a power law $a \propto S^{-\sigma}$.
with $\sigma = 0.441$. We can use the simple analysis of Eq. 5 to find the exponent $\sigma$ for squared transgressive dune by expressing $a(t) = (S_0 - S)/S_0$. Using $L_{x0} = L_{y0} = \sqrt{S_0}$ gives
\[
a(t) = \frac{2v_{cw}ut^2}{S_0} + (2v_{cw} - u)tS_0^{-1/2}. \tag{6}
\]

Since $v_{cw}u \approx 0.001 \text{ m/y}^2$ and $S_0 > 1000 \text{ /rmm}^2$ first term in Eq. 6 is very small and can be neglected, thus for a specific value of $t$ the scaling of $a$ is $a \propto S_0^{-0.5}$. This theoretical value of $\sigma = 0.5$ is a geometric consequence of the transgressive dune shape as the action of the erosive forces which acting on the transgressive dune fronts are proportional to the transgressive dune perimeter which scales with the squared root of the area. According to this scaling the smaller the initial transgressive dune the faster its stabilization and vice versa as we mentioned before. This power law behavior is less pronounced for the data from Fraser Island which shows a large scattering. This scattering can be attributed to inhomogeneity in the climatic conditions among the transgressive dunes and that their shape does not remained rectangular. Another reason for the scattering is the various length to width ratio ($\Lambda$) of transgressive dunes in Fraser Island which seems to be independent on the dune size (see inset in Fig. 11). However, the overall trend is in agreement with the data from Fraser Island (Fig. 4), but the area reduction is smaller. This difference may be mainly attributed to the simplicity of our model and also to our embedded assumption that there is enough sand in the system to be transported by wind and that it is limited only by vegetation cover (via the function $g(v_c, v)$). In addition the spatial variability of the wind field was ignored in our simulations.

Eq. 6 can be modified to include the dependence on the initial area aspect ratio $\Lambda$. Assume that $L_{x0} = \Lambda L_{y0}$ gives
\[
a(t) = \frac{2v_{cw}ut^2}{S_0} + \frac{L_{y0}(2v_{cw}\Lambda - u)}{S_0}. \tag{7}
\]
The second term can be negative when $\Lambda < u/2v_{cw}$ which means that for smaller enough values of $\Lambda$ and for shorter times (where the first term can be neglected) the initial area can grow as we showed in Fig. 8.
Figure 11: Log of normalized area reduction between 1982 and 2005 as a function of the log of the transgressive dune area using model simulations (with $\Lambda = 4$) forced by the climate data of Fraser Island (note that $a = 1$ stands for complete stabilization whereas $a = 0$ means no change in the transgressive dune area; negative values of $a$ which denote transgressive dune expansion were ignored). Black circles denote data from Fraser Island whereas red squares denote model simulations. Under the conditions described in the text the model exhibit power law behavior—the real data only crudely follow a power-law behavior. The inset shows the ratio of length to width for the Fraser transgressive dunes for 1957 (mean value is 3.41). Note that this ratio is independent on the dune size. The parameter values are as in Fig. 6 with $\delta = 10$ and $\beta = 2.67 \cdot 10^{-4}$. See text for further discussion.
4. Summary and Discussion

We have developed a model for the spatial dynamics of vegetation cover on sand. Based on this model we suggest conditions for the growth and shrinkage of transgressive dunes (depending on wind power and rainfall). We highlight the possibility of transient growth of a transgressive dune that eventually shrink with time—larger transgressive dunes experience a longer transient growth period. In agreement with field observations, the larger the initial transgressive dune is the faster it grows while small and/or elongated dune shrink faster. The transgressive dune propagation is a consequence of the difference in along/crosswind front velocities. Thus, changes in rainfall and wind power can change transgressive dune dynamics. We show that transgressive dune have a long response time and that year to year variability (and obviously inter-annual variability) is filtered out due to this long response time [21]. Prolonged droughts or an increase in DP can lead to the development of transgressive dunes which will increase the overall dune activity [13].

According to the model and in agreement with Levin’s view of the process of transgressive dune development and stabilization (see Fig.17 in [8]), transgressive dune development consists of the following stages. After the transgressive dune initiation, the transgressive dune advances inland and grows in both directions. Then, wind power decreases and the transgressive dune detaches from the beach and its sand supply decreases. The transgressive dune continue to migrate inland however, its movement rate decreases, and its upwind parts are being stabilized ($v_{AF} < v_{FA}$). If the wind power continues to be low, the transgressive dune area will continue to shrink in both directions ($v_{CW} > 0$). Eventually the transgressive dune will be completely stabilized. The stabilization rate depends on the initial size of the perturbation and its initial shape. The larger the transgressive dune area the slower its stabilization rate. Under specific conditions the proposed model predicts that this dependence follows a power law behavior.

The proposed model is a simple representation of transgressive dune dynamics. First, it ignores the transgressive dune topography and assume a 2D geometry. A real transgressive dune or a dune is a 3D structure and its typical shape is irregular as shown in Fig. 2e. Vertical and volumetric change may be of fundamental importance for characterizing sand transport rates and dune morphological response [48]. The model can be extended to include topography using the minimal modeling approach [49] by coupling the
vegetation to the calculated shear stress over the dune topography and by coupling the sand flux to vegetation growth like in [4]. More simplifications are related to the wind representation in the model. In the simulations we used annual DP values for unidirectional winds. It would be more accurate to use monthly RDP values with their directions (RDD) since prevailing wind direction changes throughout the year. Using multi-directional winds will probably result in irregular transgressive dune shapes. We also assume that the DP is spatially uniform (i.e. DP is not a function of space). This assumption can be relaxed by an appropriate derivation of the sixth term in the model equation (Eq. 1). It is known that wind speed (and consequently DP) generally decreases with distance from the coast, due to increased surface roughness inland compared with over-the-sea surface roughness [50]. The surface roughness also increases due to the upwind vegetation [8]. Preliminary simulations with a spatially variable DP show the possibility of the existence of stationary fronts between active and fixed dunes. This is one of several different scenarios that can lead to juxtaposition of active dunes next to stabilized dunes. We leave this subject for a future work.

In the model we also assume that sand transport does not depend on rainfall (the second term in the model does not depend on \( p \)). The rainfall in the proposed model affects only the vegetation growth rate. It is well known that soil moisture can inhibit sand transport by wind since it increases the threshold velocity for sand transport [9]. For strong winds (a high value of DP) sand transport can sustain even if the sand is wet, because the wind dries the upper layer of the dune and the process can continue layer by layer [51]. Thus, in high energy wind environment (DP > 400) like Fraser Island, the assumption that rainfall does not significantly affect sand transport can be justified.

The model we suggest is simple and does not definitively simulate the complicated process of Transgressive dune dynamics and the effects of wind direction variability. Nevertheless, the proposed model may shed light on the dynamical scenarios underlining transgressive dune dynamics and suggests a new way to study their development, their dependence on initial area and aspect ratio and their response to climate change.

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References


