Sand dune dynamics and climate change: A modeling approach

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Abstract. We provide several examples for the coexistence of active and fixed sand dunes under similar climatic conditions, namely, with respect to wind power and precipitation rate. A model is developed for dune vegetation cover that includes wind power, precipitation rate and anthropogenic effects, such as grazing and wood gathering. The model reproduces the observed dune’s bi-stability and shows that under intense human pressure and prolonged droughts, the fixed dunes may turn active. Moreover, the model shows that the dune reactivation process is almost irreversible, as a fixed dune will become active only under the action of very strong winds and can then return to the fixed state only when wind power decreases far below the levels under which the initial dune maintained its stability. Similar hysteretic behavior of dune mobility is predicted by the model with respect to changing precipitation and human pressure parameters.

1. Introduction

Among the geomorphological landscapes, sand dunes exhibit interesting forms and dynamics. They cover vast areas of the world’s deserts (~20%) [Pye and Tsoar, 1990] (see Fig. 1) and usually are associated with unique biological and ecological activity [Darin, 1996]. Dune shapes and dynamics have attracted the attention of scientists interested in grain dynamics, dune formation and dune movement [Bagnold, 1941; Wiggs, 2001; Andreotti et al., 2002; Herrmann, 2006]. Aside from theoretical considerations, dune activity has a more prosaic interest, as in many places around the world (e.g. in northern China, Morocco, and Mauritania), sand dunes are an imminent threat to human property [Dong et al., 2005]. Sand dunes may be fixed, active or partially active, mainly as a function of wind power and vegetation cover. Thus, ongoing climatic changes may turn some currently fixed dunes into active ones and vice versa [Muhs and Maat, 1993; Thomas et al., 2005; Alexander, 2007].

Geomorphologists usually quantify dune mobility in terms of mobility indexes [Lancaster, 1997]. These are usually based on parameters such as wind power, precipitation rate and potential evapotranspiration. According to conventional beliefs, dunes in the same geographical area can be active, fixed or partially active dependent on changing climatological conditions that determine mobility indexes [Knight et al., 2004]. However, it was recently shown that active and fixed dunes may coexist under similar climatic conditions [Muhs and Maat, 1993; Tsoar, 2005; Yizhaq et al., 2007; Arens et al., 2007]. This fact raises questions about the validity of some of the mobility indexes in these bistable regions (see Figs. 1, 2) and even seems to contradict the concept of a climatically determined index of dune mobility. This problem has been raised by Hugenholtz and Wolfe [2005], who gave as an example the juxtaposition of fully active dunes and relict stable dunes in the Kelso dune-field, California. Lancaster [1994] suggested velocity amplification as a possible answer, but the phenomenon can also be explained by the fact that active and fixed dunes coexist under the same climatic conditions, as found by others [Muhs and Maat, 1993; Tsoar, 1995; Yizhaq et al., 2007; Arens et al., 2007]. Another problem with mobility indexes is that they take into account only current climatic parameters (e.g. precipitation, evapotranspiration and wind power). But dune response is known to lag behind climatic change [Hugenholtz and Wolfe, 2005]. For this reason, using changes in dune activity as an indicator of climate change is not decisive. The concept of dune mobility becomes even more complex when we have to consider vegetation cover as a stabilizing element. Because the rate of change of vegetation cover with climatic parameters (e.g., precipitation) is inherently nonlinear, dune dynamics is hysteretic in nature as shown in Fig. 3.

We recently suggested [Yizhaq et al., 2007] a simple model for dune vegetation cover that exhibits dune bi-stability over a wide range of wind-power regimes. Here, we generalize this dune dynamics model to include the effects of human activity (such as overgrazing, wood gathering and clear cutting) and precipitation. There is evidence for dune reactivation due to prolonged drought, for example, in the northern Sinai [Tsoar, 2005] and southern Californian deserts [Lancaster, 1997]. Based on our model results, we suggest that, in some cases, once a sand dune becomes active (either due to increasing wind power, prolonged drought, or intense human activity), it will stay active even under the conditions under which it was formerly fixed: in this situation, dune reactivation is an almost irreversible process.

Baas [2007b, a] have lately proposed a discrete (cellular automata) model of vegetated dune landscape, in which the space is represented by a uniform grid of cells. The state of a given cell at a given (discrete) time is determined by a set of rules applied to the state of this cell and its neighbors at the previous time step. The advantages of such discrete models include their straightforward implementation, their low computational cost, and their ability to easily incorporate ecological and geomorphological aspects [Fonstad, 2006]. Alternatively, it is possible to model vegetation growth on sand dunes using continuum models [Durán...
that describe the mean dynamics over scales that are larger than a single shrub but small enough relative to the system size to allow convenient calculations. Continuous models are, in most cases, more amenable to mathematical analysis than discrete models and are more favorable for dynamical analysis [Guckenheimer and Holmes, 1983]. The model presented here involves an implicit-space equation expressed as an ordinary differential equation, a formulation that allows simple mathematical treatment.

2. Model formulation

Empirical studies have shown that dune activity can be modeled successfully using wind power and vegetation cover [Hugenholzt and Wolfe, 2005] as wind power determines sand transport capacity, while vegetation cover determines the amount of the sand available for transportation. This study seeks to model vegetation cover under the effect of wind, precipitation and human activity, and to relate dune mobility to different vegetation cover values. Recently, we developed a simple model that describes the growth of vegetation cover on sand dunes as a function of wind power [Yizhaq et al., 2007]. The model showed bistability and hysteresis with respect to wind power. Building on this early approach, the improved model we now propose is given as:

\[
\frac{dv}{dt} = \alpha(p)(v + \eta) \left(1 - \frac{v}{v_{\text{max}}}\right) - e DP (v/c) v - \gamma DP^{2/3} v - \mu v. \tag{1}
\]

Here, \(v\) is the dynamical variable representing areal vegetation cover density, which takes values between 0 (bare dune) and \(v_{\text{max}} \leq 1\). Vegetation dynamics is modeled using an implicit-space logistic equation [Baudena et al., 2007] which assumes the area may be subdivided into neighboring sites that are vegetated or empty. The vegetation colonization rate, \(dv/dt\), is represented by \(\alpha(p)(v + \eta) \left(1 - \frac{v}{v_{\text{max}}}\right)\), a logistic growth function (growth \(\alpha(p)(v + \eta) \times \) times the competition term \((1 - v/v_{\text{max}}))\). \(\eta\) is a spontaneous growth cover parameter that describes an average cover due to spontaneous growth for even bare dunes due to soil seed banks, underground roots, seeds carried by wind, animals, etc. \(v_{\text{max}}\) denotes the maximum vegetation cover that the area can support, which depends on the type of vegetation and dune type. \(v_{\text{max}}\) may also be associated with the vegetation carrying capacity of the system.

In the model adopted here, we assume that the parameter \(\alpha\), which denotes vegetative net growth, is a function of the annual precipitation (\(p\)). Field studies [Bullard, 1997] have shown positive relationships between precipitation and above-ground primary production, although the exact dependence is still unknown, especially for sand [Noy-Meir, 1973; Sala et al., 1988; Tsoar, 1996]. For larger amounts of rainfall the value of \(\alpha(p)\) becomes saturated, as the amount of water that can be taken up by vegetation is limited. Moreover, since in our model \(\alpha(p)\) stands for an areal growth rate and not for biomass production, one expects maximal areal vegetation above a certain level of precipitation. Following observations, we assume that \(\alpha(p)\) converges to a maximum value \(\alpha_{\text{max}}\) for sufficiently high precipitation rates [Tsoar, 2005]. On the other hand, we assume that vegetation cannot sustain itself below a minimal precipitation rate \(p_{\text{min}}\), as observed in extremely arid areas [Danin, 1996]. \(p_{\text{min}}\) may depend also on the potential evapotranspiration, but for simplicity we assume a constant value here. Without loss of generality, we can assume an exponential transition between these two limits. Thus, \(\alpha(p)\) may be expressed as:

\[
\alpha(p) = \begin{cases} 
\alpha_{\text{max}} & p \geq p_{\text{min}} \\
0 & p < p_{\text{min}} 
\end{cases} \tag{2}
\]

The estimated value of \(p_{\text{min}}\) is between 50 and 80 mm/yr in the Nizzana (in the western Negev of Israel) area [Tsoar, 2005; Danin [1996] similarly defined 50 mm/yr as the threshold precipitation needed for perennial growth. We will therefore use \(p_{\text{min}} = 50 \text{ mm/yr}\) in the rest of this work. \(c\) quantifies the precipitation dependent transition range between \(\alpha = 0\) and \(\alpha_{\text{max}}\)—the smaller the value of \(c\), the steeper the curve and the shorter the transition range is. \(c\) is related to the vegetative response to precipitation: larger values of \(c\) characterize shrubs, whereas smaller values are typical of herbaceous vegetation.

The second term in Eq. (1), \(-e DP (v/c) v\), represents the effect of sand transport on vegetative growth, mainly due to root exposure and plant burial. We assume here that there is enough sand in the system which is available for wind transport. The effect of limited sand supply due to vegetation cover can be modelled by multiplying this term by \((1 - v)\), which is the exposed area. This correction has only a minor effect on the results presented in the current work. The vegetation cover also modifies the air flow pattern above the sand. Three types of air flow were previously identified according to the degree of vegetation cover, isolated-roughness flow, wake-interference flow, and sweeping flow [Wolfe and Nickling, 1993]. In the isolated-roughness and wake-interference flows, the wind can act directly on the surface, whereas for sweeping flow, while some amount of bare surface may be present, the sand is protected from the direct action of the wind by the surrounding vegetation. It was shown by Lee and Soliman [1977], who used wind tunnel experiments to investigate the densities of roughness elements that a ground cover \(v_c\) of 0.4 provides sufficient blocking for sweeping flow. This critical vegetation cover, \(v_c\), further depends [Bullard, 1997] on plant shape, porosity, stem flexibility, and wind velocity, as plant canopies can also be penetrated by strong winds. Thus, the interaction between vegetation and wind is quite complicated and nonlinear, making it difficult to estimate the value of \(v_c\). Moreover, \(v_c\) varies for different geographical locations, from measured \(v_c \approx 0.14 - 0.16\) for the Kalahari desert [Wiggs et al., 1995; Wiggs, 2005] to observed \(v_c \approx 0.35\) for the Australian deserts [Ash and Wasson, 1983].

In our previous study [Yizhaq et al., 2007], we chose a step function for \(g(v_c, v)\). However, it is more appropriate to choose a smoother transition function as it was noted recently for the Simpson desert in Australia [Hesse and Simpson, 2006] that there is no simple vegetation cover threshold below which sand transport begins. Here we use a hyperbolic function for \(g(v_c, v)\)

\[
g(v_c, v) = 0.5(\tanh(b(v_c - v)) + 1), \tag{3}
\]

where parameter \(b\) quantifies the steepness of the curve, the larger the value of \(b\), the steeper is \(g(v_c, v)\). \(b\) can be estimated from field and wind tunnel experiments. Note below that we also use a more simplified linear form for \(g(v_c, v)\) to enable analytical treatment.

The DP term is a measure of wind power, usually expressed by aeolian geomorphologists as a drift potential (DP) [Fryberger, 1979; Tsoar, 2005], which is proportional to the potential sand volume that can be transported by the wind through a 1 m wide cross section per unit time (usually, it is given per year). DP is defined as

\[
DP = (U^2(U - U_t)). \tag{4}
\]

where \(U\) is the wind speed (in knots: 1 knot = 0.514 m/s) measured at a height of 10 m and \(U_t\) is a minimal threshold velocity (= 12 knots) necessary for sand transport [Fryberger, 1979]. There are both theoretical and empirical linear relations between the drift potential and the rate of sand...
transport [Bullard, 1997b]. Still, the DP is a potential sand drift, as the actual drift further depends on the mean grain diameter, the degree of surface roughness, the amount and type of vegetation or crust cover, the amount of moisture in the sand, and the uniformity of wind direction. According to Fryberger [1979] wind energy can be classified into three classes: DP < 200, low energy; 200 ≤ DP < 400, intermediate energy; and DP ≥ 400, high energy. DP values in arid regions can vary from year to year (as does precipitation), as determined by the seasonal activity pattern of storms. To avoid misconceptions, it should be understood that the DP in our model represents an average of several years and ignores the temporal variability.

The parameter ε in Eq. (1) stands for the vegetation tolerance to sand erosion and deposition. As ε decreases, tolerance increases. In the model we do not distinguish between sand erosion and deposition, and the vegetation establishment is considered to depend on sand transport alone which is proportional to DP. It is also possible to include wind directionality in the model by replacing DP here with the resultant drift potential RDP, which is the resultant vector of DP values in each major wind direction (RDP ≤ DP); for simplicity, however, we prefer to keep the present form of the model.

The third term in Eq. (1), γDP²/v, stands for a reduction in vegetation cover due to direct wind action, which increases evaportranspiration, as well as uproots, erodes, and suppresses vegetative growth [Hesp, 2002]. Generally, wind drag is proportional to the square of the wind speed [Bagnold, 1941], while DP is proportional to its cube. Thus DP²/v is used to represent wind drag on vegetation and thereby vegetative growth suppression due to direct wind action. This term is proportional to v, as it is basically a mortality term. The parameter γ is a proportional constant that depends on vegetation types. It is important to note that, unlike the previous terms, this term acts even on maximally vegetated dunes.

The last term in Eq. (1), −µv, stands for an extinction rate due to continuous human activity, such as grazing, clear-cutting or burning. The intensity of this mortality term is governed by the value of the parameter µ (negative values of µ represent stabilization activities).

It is possible to rewrite Eq. (1) in a simpler form with four parameters in the following form:

\[
\frac{dv}{dt} = \alpha_1 \bar{v} - \alpha_2 \bar{v}^2 - \alpha_3 g(\bar{v}, \bar{v})\bar{v} + 1,\]

where the new parameters are defined as follows: \(\alpha_1 = (1 - \mu/v_{\text{max}} - \gamma DP^{2/3} - \mu)/(\eta b)\), \(\alpha_2 = (\eta v_{\text{max}} b^2)^{-1}\), \(\alpha_3 = -\varepsilon DP/(\eta b)\), \(\alpha_4 = v_{\text{max}}\), and \(\bar{v} = v - b\). \(\bar{v}\) is a new net vegetation growth rate, \(\alpha_2\) is a mortality term due limited resources and \(\alpha_3\) is a mortality term due sand transport. We prefer to keep the current form of Eq. (1) since it allows easier interpretation of the results.

2.1. Approximate model

To obtain analytic solutions to the model in Eq. (1), it is useful to approximate \(g(v, v)\) [Eq. (3)] by the linear function \(f(v, v)\) as follows,

\[
f(v, v) = \begin{cases} 
1 & v < v_1 \\
1 + b(v - v_1)/2 & v_1 \leq v \leq v_2 \\
0 & v > v_2 
\end{cases}
\]

where we choose \(b = b/2.35\) with the 2.35 factor being an empirical factor chosen to fit the hyperbolic tangent function. Since for this value the linear approximation crosses the 99% (or 1%) value of the original function \(g(v, v)\). \(v_{1,2}\)

are given by:

\[
v_1 = -1/\bar{b} + v_c, \quad v_2 = 1/\bar{b} + v_c. \tag{7}
\]

The proposed model, i.e., Eq. (1), has two stable stationary states, A and F, representing active and fixed dunes, respectively. These solutions are:

\[
v_{A,F} = -\Gamma_{A,F} + \sqrt{\Gamma_{A,F}^2 + 4\eta v_{\text{max}}}, \tag{8}
\]

where

\[
\begin{align*}
\Gamma_A &= {\frac {v_{\text{max}}}{2}} (1 + \eta/v_{\text{max}} + \gamma DP^{2/3} + \mu/\alpha) \\
\Gamma_F &= {\frac {v_{\text{max}}}{2}} (1 + \eta/v_{\text{max}} + \gamma DP^{2/3} + \mu/\alpha + \varepsilon DP/\alpha). \tag{9}
\end{align*}
\]

The active dune state \(v_A\) is valid for \(v < v_1\), while the fixed dune state \(v_F\) is valid for \(v > v_2\). For \(v_1 \leq v \leq v_2\) we used the linear approximation Eq. (6) to get the following expression for \(v_1\) which is the unstable solution:

\[
v_1 = -g_1 - \sqrt{g_1^2 + g_2}, \tag{10}
\]

where

\[
\begin{align*}
g_1 &= \frac {v_{\text{max}}}{2} (1 + \eta/v_{\text{max}} + \gamma DP^{2/3} + \mu/\alpha - \varepsilon DP/2\alpha) \\
g_2 &= \frac {v_{\text{max}}}{\alpha} [\alpha \eta - \varepsilon DP(1 + \eta)/2]. \tag{11}
\end{align*}
\]

Fig. 3 shows the stable stationary states as a function of DP for the exact and the approximate models; it can be seen that, indeed, the linear approximation for \(g(v, v)\) is quite good for the choice of parameters given in the figure caption.

It is also possible to find the transition points from the active dune state to the fixed state (denoted by subscript 1 below) and from the fixed state to the active state (denoted by subscript 2 below) as follows. The fixed-active state transition occurs when \(v = v_2\) for a steady state condition \(dv/dt = 0\). Using these two restrictions in Eq. (1) we obtain an equation for the fixed-active transition point:

\[
(\gamma DP^{2/3} + \mu)v_2 = \alpha(p)(v_2 + \eta)(1 - v_2/v_{\text{max}}) \tag{12}
\]

where the transition can be found for any of the model’s parameters. For example, when the control parameter is DP, the fixed-active dune transition is given by:

\[
DP^{2/3} = \alpha(v_2 + \eta)(1 - v_2/v_{\text{max}})/(\gamma v_2 - \mu/\gamma). \tag{13}
\]

Similarly, it is possible to find the fixed-active dune state transition points \(p_2\) and \(p_2\).

To estimate the transition from the active dune state to the fixed one we assume that the transition occurs at \(v = v_c\). Similar to Eq. (12) we now obtain for the active-fixed transition

\[
(\gamma DP^{2/3} + \varepsilon DP/2 + \mu)v_c = \alpha(p)(v_c + \eta)(1 - v_c/v_{\text{max}}) \tag{14}
\]

where the vegetation suppression due to sand transport is now also taken into account (via the \(\varepsilon\) term). Using this equation one may easily find the parameter values \(p_1\) and \(p_2\) at which the active-fixed transitions occur. The DP transition point is more difficult to evaluate, but it may be further approximated by

\[
DP^{1/3} \approx (2\varepsilon\gamma/3)^{1/3} - 2\gamma/(3\varepsilon) \tag{15}
\]
where \( \psi \) is given by
\[
\psi = \alpha (v_c + \eta) (1 - \frac{v_c}{v_{\text{max}}}) / (\gamma v_c) - \mu / \gamma .
\]

3. Results

To further understand the model’s solutions, we choose to use as control parameters, the wind drift potential \( DP \), the annual precipitation \( p \), and the human impact parameter \( \mu \). The first two parameters may represent scenarios of climatic processes (i.e. windiness and rainfall) and the last describes anthropogenic activities, such as grazing and clear-cutting. Although the model has 9 parameters, it is possible to show that there are only 3 independent parameters; we prefer to apply the model using 9 parameters to allow easier interpretation of the model’s results. Moreover, most of the parameters of the model can be estimated from previous works. For example concerning the spontaneous growth rate \( \eta \), in a dune field in Nitzanim in Israel (located along the Mediterranean Sea), a vegetated dune was totally removed (including roots) and started to recover after a few years. The growth rate for bare ground and when no significant winds are present \( (DP = 0) = \alpha \gamma \) and thus \( \eta \) can be estimated as \( \Delta \psi / \Delta \alpha \). Thus, if for example, the vegetation cover after one year is 1.5%, \( \eta = 0.015/1/0.1 = 0.15 \). This estimation is obviously rough and precise field experiments should be carried out in order to approximate this parameter for different plants and different dune types around the world.

Similarly, a range of values can be estimated for the other parameters. Table 1 includes a range of values for each of the model’s parameters, based on the literature published on the subject. Except for the phenomenological parameters \( \varepsilon \) and \( \gamma \), all the other parameters can be estimated. The range within which these two parameters fall can be indirectly found from the ranges of values of \( DP_1 \) and \( DP_2 \), the transition points from the fixed to the active dune state and vice versa. The upper limit for \( DP_2 \) can be estimated from the place with the highest DP value where the dunes are still fixed. Our estimation for this value is \( DP \sim 2000 \), but we overestimate it as 3000. An upper limit for \( DP_1 \) is related to the location where only the fixed state exists but with lowest amount of rainfall. A reasonable approximation is \( DP_1 \approx 200 \). These arguments stand behind the range of values for \( \varepsilon \) and \( \gamma \) shown in Table 1. Direct estimations of these two parameters will require more careful and prolonged field or wind tunnel experiments.

Fig. 4 depicts the stationary solutions as a function of \( DP \) (panel a), \( p \) (panel b) and \( \mu \) (panel c). The figure shows hysteresis behavior, which means that the system responds differently to increasing or decreasing the control parameter from its extreme values, respectively. Each of these hysteresis diagrams can be interpreted as a desertification process, which is almost irreversible. For example, panel a of Fig. 4 shows the model’s response to changes in wind drift potential \( DP \). For a wide range of \( DP \) values, both the fixed and active dune states coexist, indicating the hysteresis and bistability of the model. This dune behavior may be described as follows: starting from a very low \( DP \), only the fixed-dune solution \( v_f \) exists; when the \( DP \) slowly increases, this solution persists until the very high \( DP \) of \( DP_2 \) is reached. Beyond this point, the system shifts into the active dune state, \( A \). When \( DP \) is then slowly decreased, the solution continues to show active dune state until \( DP_1 \) is reached, beyond which the solution reverts back to the fixed dune state. Note that, although the system has two stable solutions for the same parameters, a unique set of initial conditions determines only one possible final state. Only initial conditions that are scattered around the unstable branch can evolve to both final states. A large enough perturbation can switch the system from one stable state, across the unstable branch, to the other stable state. Several scenarios can lead to bistability of active and fixed dunes under the same climatic conditions. Among them are climate change and human impact. An example of such a transition is the clear-cutting of vegetation on dunes in North Holland that took place in December 1998 \( (DP = 1706) \) [Arens et al., 2007], which led to the dunes remaining bare and active until today.

The behavior for \( p \) is opposite to that of \( DP \), as increasing the value of \( p \) shifts the system into the fixed state, whereas decreasing \( p \) beyond a threshold value \( p_l \) shifts the system into the active-state solution. The system’s response to changes in the value of \( \mu \) is similar to that of \( DP \), as increasing the value of \( \mu \) beyond \( \mu_2 \) shifts the system from the fixed state to the active one.

More important, even without climatic changes, overgrazing can shift the system from the fixed state to the active one. The opposite scenario is also possible, i.e., stopping grazing can stabilize the dunes. This processes was observed at the coastal dunes of Ashdod-Nizzanaim in Israel [Levin and Ben-Dor, 2004], where grazing and cutting practices of Bedouins between the late 1960s and the late 1970s led to dune activation and after these practices were prohibited, the vegetation recovered and stabilized the dunes. Similar re-stabilization occurred in southern Israel, after the establishment of the Israel-Egypt border in 1982 [Karnieli and Tsakos, 1995].

The typical time scales for convergence to the steady states can be found from the analytic solution of Eq. (1) for \( g(v(v), v) = 0 \) and \( g(v(v), v) = 1 \), which can be written as
\[
v(t) = (1/\beta) (a - \sqrt{A} \tanh B),
\]
where \( \beta = \alpha(p)/v_{\text{max}} \).
\[
a = \begin{cases} 
\alpha(p) - \alpha(p)/v_{\text{max}} - \varepsilon DP - \gamma DP^{2/3} / A - \mu & v < v_c \\
\alpha(p) - \alpha(p)/v_{\text{max}} - \gamma DP^{2/3} / A - \mu & v \geq v_c 
\end{cases}
\]
and \( A = a^2 + 4bc \), and \( c_1 = \alpha(p) \eta; B = (\sqrt{A}/2)(C - t) \), with \( C \) is an integration constant that depends on the initial condition. The characteristic convergence time scales as \( 2/\sqrt{A} \). Fig. 5 shows the typical time \( \tau \) as a function of the control parameters \( DP, p \) and \( \mu \) for a domain where the solution (Eq. 17) is valid. This characteristic time was termed by Gaylord and Stettler [1994] the aeolian response time, which depends on vegetation type. Fig. 5 shows that the stabilization process is much slower than dune activation, especially for high values of \( DP \). This asymmetry in the lengths of the activation time compared to the stabilization time is in accordance with field observations [Hugenholtz and Wolfe, 2005]. It is important to note that the stabilization process can be very fast in cases where there is a biogenic crust as in the Nizzana dunes in the western Negev of Israel. In the current work we do not take into account the biogenic crust.

The values of \( DP_1 \) and \( DP_2 \), as a function of precipitation rate, \( p \), were computed numerically for different values of \( v_f \), and are shown in Fig. 6. As \( v_f \) increases, the fixed-active and active-fixed dune transitions occur for weaker winds (lower \( DP \) values). Above a certain value of precipitation (which depends on choice of parameters), the amount of rainfall is no longer the limiting factor [Tsakos, 2005] and wind power takes over. Below this value (~800 mm/yr for the present choice of parameters), the amount of rainfall is important for understanding dune dynamics as was observed in many studies that relate dune activity to dry conditions.
and ignore the wind effects (see e.g., [Hugenholtz and Wolfe, 2005; Arens et al., 2004]). Fig. 6 shows that there is a good agreement between observations and model predictions using values within the limits given in Table 1. Thus, field observations can help to tune the parameters and partially validate the model’s predictions.

Almost all the bistable dune areas that we found are along coast (see Figs. 1, 2). On one hand, the coexistence of active and fixed dunes requires relatively high precipitation to support vegetation growth, while on the other hand, coexistence requires high DP values to erode/bury vegetation, which maintains dune activity. Such areas are commonly found along coasts [Hesp, 2004]. In addition to strong wind power and high precipitation, a large sand supply is also necessary [Arens et al., 2007] for dune reactivation. The available sand comes either from the beach or from bare sand sheets and is controlled by vegetation cover [Lancaster, 1999]. In places with limited sand supply the dunes will eventually stabilize.

It is interesting to study the transition state wind power, DP, p, and μ, which may be associated with climate change and human impact, are summarized in Figs. 7, 8, and 9. Hysteretic behavior can be seen in the function of each of these parameters, but each graph depicts a different scenario. Fig. 7 shows the active-to-fixed dune-transition curve DP1 (blue line) and the fixed-to-active dune-transition curve DP2 (red line), which are computed numerically by solving Eq. (1) as a function of precipitation p (Fig. 7a) and μ (Fig. 7b). More rainfall (up to 800 mm/yr) means that more wind power is needed to shift dunes from the fixed to the mobile state, and vice versa. In contrast, increased μ values, i.e., increased human impact, decreases the values of the transition points, especially DP2, indicating that the system becomes more vulnerable for remobilization.

In Fig. 8, we show transition points p1,2 as a function of DP and μ. It is clear from Fig. 8a that the higher the value of DP, the higher the value of precipitation needed to trigger the transition. For example, in locales where wind power is great, the vegetative growth rate has also to be large in order to overcome wind erosion—higher values of p2 will support a larger growth rate. Fig. 8b depicts p2,1 curves as a function of μ. Note that the area above the p1 curve is very small, i.e., for higher values of μ only the active state is stable independent of precipitation rate.

Fig. 9 completes the description and shows the active-to-fixed dune-transition point μ1 (blue line) and fixed-to-active dune-transition point μ2 (red line) as a function of DP and p. μ1 and μ2 decrease monotonically for higher values of DP (Fig. 9a) and behave in the obverse manner for increasing values of p (Fig. 9b). This suggests that increased wind power enhances dune reactivation in the sense that even minor human activity may switch a fixed dune into the active state. The opposite happens for increased precipitation, namely when precipitation increases, more massive human activity is needed for dune remobilization.

4. Dune remobilization under climate change and human activity

The model described above allows us to investigate the response of vegetation to climate change (such as prolonged drought) or human intervention. Fig. 10 shows two drought scenarios that differ in duration. In the first, the vegetation cover recovers and returns to its initial state, whereas in the second the drought is long enough to allow the dunes to remain active despite precipitation returning to its original value. For the prolonged drought case, the vegetation cover is reduced and reaches the locus of attraction of the active-dune state alone, and it therefore converges to this state. This scenario represents desertification, which is defined as an irreversible decrease in biological productivity induced by an environmental change, and is captured in the model, which demonstrates bistability ranges of active and fixed dune states. Transitions of this kind are often referred to in the ecology literature as “catastrophic regime shifts” [Rietkerk et al., 2004; Solé, 2007]. In cases where the fixed state is the only stable state, the decline of vegetation cover will be continuous and will not involve sharp transitions.

An additional scenario that we tested is the response of vegetation to periodic droughts. Fig. 11 shows two such cases, one with a 4 year drought every 20 years and the other with an 8 year drought every 20 years. For the short duration drought, the dune system remains in the fixed state although the vegetation cover fluctuates; in this case the vegetation wasn’t reduced sufficiently to enter the locus of attraction of the active state. For the longer duration droughts, after a transient time, the system shifts to the active state after the vegetation is reduced sufficiently to enter the locus of attraction of the active dune state. During the transition, the vegetation shows growth phases in accordance with the high values of rainfall. Note that only in the third cycle of drought does the system converge to the the active state. The time needed to displace the dune systems to another state by applying perturbation is termed the response time [Hugenholtz and Wolfe, 2005] and depends on values of the parameters.

Another scenario for desertification due to grazing or clear cutting is illustrated in Fig. 12. In the model, vegetation mortality (due to human activity) is represented by μ. Increasing the value of μ from 0.1 to 0.11 during a period of 10 years reactivates the dunes irreversibly. Grazing and wood gathering are major problems in many countries (e.g. Somalia), which can be even greater during droughts. Even small changes in vegetation mortality as a result of anthropogenic factors such as grazing, agricultural burning or off-road traffic can irreversibly shift sand dunes into the active state. An example of such human-induced desertification was reported by Khalaf for the Kuwait Desert [Khalaf and Al-Ajmi, 1993]. By comparing satellite images from 1977 and 1989, it was clear that the southern limit of the mobile sand sheets had moved 35 km in a southeasterly direction. This resulted from deterioration of the vegetated sand sheet due to human activities, which included overgrazing and woody shrub collecting. Our model shows that dunes or sand sheets may remain active even though human perturbation ceases.

5. Discussion

The model we present here extends our previous work [Yizhaq et al., 2007] in several ways: (i) Here we take into account the effect of precipitation on vegetation growth rates; i.e., α is a function of p and is not constant. (ii) We replace the step function with a continuous function for the vegetation response to the sand transport term. (iii) The current model also includes a grazing/clear-cutting term. The above modifications allow us to use the model to explore different activation/stabilization scenarios under climatic changes as well as anthropogenic activities. The model can be used to investigate dune mobility in more informative ways than are available using indices of mobility regularly applied by aeolian geomorphologists.

The proposed model can suggest possible scenarios for future dune mobility by using climatic data from GCM models. The main factor that affects dune mobility is wind
power, though the amount of precipitation is also important for a certain range of values. First, there is the lower threshold of precipitation (taken as 50 mm/yr in our simulations) which is necessary for vegetation growth. An upper threshold (taken as \(\sim 800 \text{ mm/yr}\)) also exists because above this point rainfall will not enhance vegetative cover, as ground water has already reached its maximal capacity. Between these two thresholds the amount of precipitation can affect dune mobility, and increased aridity can be related to higher dune mobility and vice versa. Prolonged droughts have a greater impact on dune reactivation when precipitation values are close to the lower threshold, i.e., in more arid environments.

Our model also shows that mobile and fixed dunes can co-exist under the same climatic conditions and that the processes of activation and stabilization are not symmetrical; once the stabilized dunes become active in response to climatic change or human intervention, much weaker wind power or much higher precipitation rates are required to re-stabilize the dunes. Therefore, as dune remobilization intensifies, steps such as reduced grazing and enhanced seeding [Wolfe et al., 1997] must be taken to prevent vegetative cover decline from reaching its critical threshold. Once degradation reaches this state, reversal is difficult and costly. According to our model, prolonged droughts and increasing windiness due to climatic change, combined with grazing or clear-cutting, may turn currently stable, vegetated dunes into mobile, bare dunes. From the point of view of dune management, it will be useful to identify the critical vegetation cover below which transition to the fully activated state occurs. Fig. 4 can give some insight into how these critical values depend on the different model parameters.

In the present model, we consider the interaction between vegetation, wind and sand, but in many dune fields the biogenic crust plays a crucial role in dune stabilization. Biogenic crusts are commonly found on sand dunes in the dry

![Figure 1](image1.png)

**Figure 1.** The global distribution of aeolian sand dunes according to mobility. The map follows [Thomas, 1997], modified and supplemented with a few examples of locations where active and fixed dunes coexist. We emphasize that the map provides only a rough description of the location of the main sand seas.

![Figure 2](image2.png)

**Figure 2.** Areas in which active and fixed dunes coexist under similar climatic conditions (Google Earth images). The locations of the sites are indicated by the panel’s labels on the global map in Fig. 1.

![Figure 3](image3.png)

**Figure 3.** Stable states diagram showing the vegetation cover \(v\) vs. DP for \(p = 80 \text{ mm/yr}\). The approximated solution (dashed-dotted) is very close to exact numerical solution (solid). The unstable solution is marked with a dashed line. Parameter values used are: \(v_{\text{max}} = 1, v_c = 0.3, \epsilon = 0.001, \gamma = 0.0008, \eta = 0.2, \mu = 0, \alpha_{\text{max}} = 0.15, p_{\text{min}} = 50 \text{ mm/yr}, c = 100\) and \(b = 15\).

![Figure 4](image4.png)

**Figure 4.** Stable states diagrams showing vegetation cover \(v\) vs. DP, \(p\) and \(\mu\). (a) Vegetation cover vs. DP for \(p = 100 \text{ mm/yr}\) (solid) and \(p = 200 \text{ mm/yr}\) (dashed) and \(\mu = 0\); (b) Vegetation cover vs. precipitation \(p\) for DP = 200 (dashed), DP = 100 (solid) and \(\mu = 0\); (c) Vegetation cover vs. \(\mu\) for DP = 100 and \(p = 150 \text{ mm/yr}\) (solid) and \(p = 200 \text{ mm/yr}\) (dashed). Parameter values used are: \(v_{\text{max}} = 1, v_c = 0.3, \epsilon = 0.001, \gamma = 0.0008, \eta = 0.2, \alpha_{\text{max}} = 0.15, p_{\text{min}} = 50 \text{ mm/yr}, c = 100, b = 15\).
regions of the world (Africa, Australia, and Asia); they can resist long periods of drought and recover biological activity following rainfall [West, 1990; Benlap and Lange, 2001]. A continuous crust cover can withstand even the strongest wind intensities. By adding a second dynamic variable for the biogenic crust to our model, we will be able to reconstruct with greater accuracy the mutual interactions between crust-vegetation-wind and sand.

6. Conclusions

The model presented here for vegetation-cover dynamics of sand dunes takes wind power, precipitation and human pressure as its major variables. The model shows the coexistence of active and fixed dunes over a large range of drift potentials and precipitation values. Areas in which active and fixed dunes coexist are characterized by high values of DP and precipitation, in agreement with the model’s prediction. The model shows that the transition from one model state to another can be irreversible and discontinuous in the bistability regime of active and fixed dunes.

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References


Figure 6. Numerical computation of the active-to-fixed dune-transition point DP1 and the fixed-to-active dune-transition point DP2 for different values of \( \gamma \), as a function of precipitation rate \( p \). In the regime below the DP1 curves, dunes are fixed whereas in the regime above the DP2 curves, the dunes are active. In the intermediate regime, active and fixed dunes can coexist. Here \( \mu = 0 \) and the other parameters are as given in Fig. 4. The labeled points indicate dune fields from different locations, which are marked according to their mobility (see the inset box). The data were collected from the International Station Meteorological Climate Summery (ISMCS), version 3.0. The letters adjacent to the bistable dunes indicate dune locations given in Fig. 1, and the precipitation values for these points are those of NCEP/NCAR reanalysis EAR40 database [Kalnay et al., 1996]. The parameter values are within the ranges given in Table 1 and were further chosen to tune the data: \( \nu_{\text{max}} = 1, \epsilon = 0.001, \gamma = 0.0008, \eta = 0.2, \alpha_{\text{max}} = 0.15, \mu = 0, p_{\text{min}} = 50 \text{ mm/yr}, c = 100, b = 15 \).

Figure 7. Active-to-fixed-dune-transition point DP1 (solid line) and fixed-to-active-dune-transition-point DP2 (dashed-dotted line), computed numerically by solving Eq. (1). (a) DP1 and DP2 as a function of precipitation; (b) DP1 and DP2 as a function of the human-impact parameter \( \mu \).


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Figure 9. Active-to-fixed dune-transition point $\mu_1$ (solid line) and fixed-to-active dune-transition point $\mu_2$ (dashed-dotted line) computed numerically by solving Eq. (1). (a) $\mu_1$ and $\mu_2$ as a function of the drift potential $DP$; (b) $\mu_1$ and $\mu_2$ as a function of precipitation $p$.

Figure 10. Vegetation response to prolonged droughts. (a) Two scenarios of precipitation sequences are mapped, one with a reduction of the rainfall from 200 mm/yr to 100 mm/yr for a period of 10 years (dashed-dotted) and the other for a period of 20 years (solid). (b) For each scenario in (a), the evolution of vegetation cover is presented in (b). In the case of 10 years of drought (black curve), vegetation recovers and the fixed dune state returns. However, for a prolonged 20 year drought (blue curve), the dunes converge to the active state and remain there despite the return of precipitation to its initial value. Parameters: $DP = 200$ and other parameters as in Fig. 4.

Figure 11. Vegetation response to periodic droughts. (a) Two scenarios of periodic precipitation sequences are presented, one with a reduction of the rainfall from 200 mm/yr to 100 mm/yr over a period of 4 years (dashed-dotted) and the other for a period of 8 years (solid) every 20 years. (b) For each scenario in (a), the evolution of vegetation cover with time is presented. For the shorter-duration droughts the vegetation cover remains in the fixed state with small fluctuations, while for the longer-duration droughts the vegetation cover drops to the active dune state and remains there. Parameters: $DP = 200$, with other parameters as given in Fig. 4.
Figure 12. Vegetative response to grazing for the bistable case. (a) Two scenarios of overgrazing are represented with \( \mu \) going from 0.05 to 0.1 (dashed-dotted) or to 0.11 (solid), for 10 years. (b) Vegetation cover time evolution in response to the overgrazing sequences in (a). After 10 years of grazing at \( \mu = 0.1 \), the vegetation recovered and returned to the fixed state, while for higher intensity grazing at \( \mu = 0.11 \) it converges to the active state and remains there even when the grazing returns to its initial value of \( \mu = 0.05 \). Parameters: DP = 200, with other parameters as given in Fig. 4.

Table 1. Definition and typical values of the main parameters discussed in the text.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Range</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{\text{max}} )</td>
<td>maximum vegetation cover growth rate</td>
<td>0.1-0.2</td>
<td>Hugenholtz and Wolfe [2005]</td>
</tr>
<tr>
<td>( p_{\text{min}} )</td>
<td>Minimal precipitation needed for vegetation growth</td>
<td>50 mm/yr</td>
<td>Danin [1996]</td>
</tr>
<tr>
<td>( c )</td>
<td>Vegetation growth response to precipitation</td>
<td>50-300 yr(^{-1} )</td>
<td>Shmida et al. [1986]</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Spontaneous growth cover</td>
<td>0-0.2</td>
<td>Arens et al. [2007]</td>
</tr>
<tr>
<td>( v_{\text{max}} )</td>
<td>Maximum vegetation cover</td>
<td>0-1</td>
<td>( v_{\text{max}} = 1 ) in Hugenholtz and Wolfe [2005]</td>
</tr>
<tr>
<td>( v_{c} )</td>
<td>Critical vegetation cover</td>
<td>0.1-0.4</td>
<td>( v_{c} = 0.14 ) in Kalahari dunes Wiggs [2005], ( v_{c} = 0.35 ) in Australian dunes Ash and Wasson [1983], ( v_{c} = 0.17 ) in Nizzana dunes Allgaier [2008].</td>
</tr>
<tr>
<td>( b )</td>
<td>Steepness of transition between interference flow and skimming flow</td>
<td>5-20</td>
<td>Fig. 1 in Ash and Wasson [1983]</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Human impact parameter</td>
<td>0-0.1</td>
<td>Can be estimated as 0.0008 for Sinai dunes between 1988-1989, Meir and Tsoar [1996]</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Vegetation tolerance to sand transport</td>
<td>0.005-0.02</td>
<td>Control the value of DP(_1), DP(_1) values are between 30 and 200</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Vegetation vulnerability to wind shear stress</td>
<td>0.005-0.02</td>
<td>Control the value of DP(_2), 1500 ( \leq ) DP(_2) ( \leq ) 3000</td>
</tr>
</tbody>
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