Modeling the bistability of barchan and parabolic dunes

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\textbf{A R T I C L E I N F O}

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\item Dunes
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\end{itemize}

\textbf{A B S T R A C T}

Extensive areas in arid and semi-arid regions of the world are covered by sand dunes. Sand dunes can be active, semi-active, or stable, depending on, among other factors, their vegetation and biogenic crust covers. Vegetation growth on barchan dunes can induce a transformation to parabolic dunes with long trailing arms pointed upwind. Here we propose a vegetation dynamical model, coupled to a previously suggested 3D continuous dune morphology model (Durán and Herrmann, 2006). The modeled vegetation reduces sand flux, but it is also suppressed by erosion and by sand deposition. The coupling between the model for bare dunes and the model for vegetation growth is performed by reducing the shear stress due to the presence of vegetation. We use the suggested model to study the effect of vegetation on dune formation, concentrating on the previously simulated barchan-parabolic dune transition. Under the action of sufficiently strong unidirectional winds and a low vegetation growth rate, the model results in a barchan dune. When the vegetation growth rate is increased, the barchan dune turns into a stable parabolic dune. When, however, the growth rate is again decreased, the dune becomes an active barchan dune, only with a much lower growth rate, indicating the existence of bistability and hysteresis behavior of barchan and parabolic dunes under similar climatic (vegetation growth rate or precipitation) conditions.

\section{1. Introduction}

Sand dunes form an important, unique, and complex ecological, geomorphological, and physical system (e.g., Bagnold, 1941; Pye and Tsoar, 1990; Lancaster, 1995; Danin, 1996; Durán and Herrmann, 2006b; Nield and Baas, 2008; Parteli et al., 2009; Reffet et al., 2010; Lorenz and Zimbelman, 2014; Goudie and Viles, 2015). Approximately 20\% of desert areas are dunes (Pye and Tsoar, 1990; Thomas and Wiggs, 2008), and they are considered a menace to human habitations, agricultural fields, roads, etc. (e.g., Khalaf and Al-Ajmi, 1993; Dong et al., 2005; Knight et al., 2004). Yet, fixed dunes have economic value. For example, the inactive Sahel and southern African dunes are used for pastoral and agricultural purposes.

Sand dunes may be either active (such as barchan, transverse, and self dunes) or fixed (mainly parabolic and vegetated linear dunes), depending on, among other factors, vegetation and biogenic crust cover, wind power, precipitation and human activities such as grazing and clear-cutting (e.g., Bagnold, 1941; Lancaster, 1995). Although wind is the dominant dune mobility factor, the amount of precipitation, effects vegetation and crust cover, also plays an important role (e.g., Yizhaq et al., 2009).

Sand dunes exhibit a rich and often complex morphology. Their interesting patterns are rooted in their ability to be formed by the wind as it transports sand grains. Indeed, the wind is the prime force shaping dunes and, to a lesser degree, the vegetation and biogenic crust cover; sand availability and other factors also affect dune shapes. Winds above the threshold speed for sand mobility (around 6 m/s at a height of 10 m) uplift sand particles that then fall again to the ground, triggering the movement of many more particles through saltation and reptation processes. On small temporal and lateral scales, this process leads to the creation of sand ripples (e.g., Bagnold, 1941; Yizhaq et al., 2004) and megaripples in which coarse grains are abundant (e.g., Yizhaq et al., 2012). Sand transport on much larger temporal and lateral scales leads to the formation of dunes. Without the presence of vegetation and biogenic crust and under the action of unidirectional wind, barchan (crescent-shaped) and transverse dunes are formed, depending on sand availability. Barchan dunes gain their shape as a result of the slower migration of the dune’s higher sections, resulting in horns in the downwind direction. When the wind is unidirectional, and in the presence of vegetation, the dunes are parabolic; in this case, the lower parts of the dunes are the first to be stabilized by vegetation, while the higher parts move in the downwind direction, resulting in a dune shape that is basically the mirror image of a barchan dune with respect to the wind (Pye and Tsoar, 1990). Typically, they are some hundreds of...
meters wide, and some have long trailing arms. Under conditions of drought, vegetation removal, and groundwater depletion, they may become activated and transform into transverse and barchan dunes (Goudie, 2011). Parabolic apices have been shown to move with a speed of 0.05–30 m per year (Warren, 2013), depending on the degree of vegetation cover. Some parabolic dunes show a more complex nested pattern with a range of sizes (e.g., in White Sands, New Mexico, USA). Note that the crests of barchan, transverse and parabolic dunes are perpendicular to the wind direction. Under the action of wind with two dominant directions (e.g., when the wind direction changes seasonally), dunes may become linear (longitudinal), in which the crest of the dune is more or less parallel to the mean wind direction within 15° according to (Hunter et al., 1983). Star dunes may form when the wind has more than two dominant directions; in this case, the center of the dune accumulates sand and can grow to heights of several hundred meters (e.g., Pye and Tsoar, 1990; Lancaster, 1995).

According to Lancaster (1995), “the conditions that lead to dune initiation and the processes involved are poorly understood and little studied, but they clearly are of major importance to understanding how dunes and dune patterns develop.” More than 20 years have been passed since this statement was written, and many modeling and underwater experiments (e.g., Hersen et al., 2002; Endo et al., 2004; Hersen and Douday, 2005; Reffet et al., 2010; Worman et al., 2013) have been performed to uncover the underlying processes of sand dune formation and dynamics. Numerical models may be divided into cellular-automaton models (e.g., Werner, 1995; Momjii et al., 2000; Nishimori and Tanaka, 2001; Baas, 2002; Bishop et al., 2002; Nield and Baas, 2008; Pelletier, 2008; Narteau et al., 2009; Pelletier, 2009; Zhang et al., 2010; Malamud and Baas, 2012; Zhang et al., 2012) and continuum models (e.g., Sauermann et al., 2001; Andreotti et al., 2002a,b; Kroy et al., 2002a,b; Lima et al., 2002; Hersen et al., 2004; Elbelhiti et al., 2005; Hersen and Douday, 2005; Katsuki et al., 2005; Zhang et al., 2005; Herrmann, 2006; Parteli et al., 2009; Reffet et al., 2010; Diniegas et al., 2010; Durán et al., 2010; Kok et al., 2012; Durán and Moore, 2013; Parteli et al., 2014a,b).

Field observations (e.g., Tsoar et al., Oct. 2002; Reitz et al., 2010; Barchyn and Hugenholtz, 2012; Barchyn and Hugenholtz, 2015; Zhiwei et al., 2015) and numerical models (Durán and Herrmann, 2006b) have demonstrated the transformation from barchan dunes to parabolic dunes as the result of vegetation fixation under the action of unidirectional wind. There are two main modeling frameworks that have been used to study the barchan-parabolic transition (Barchyn and Hugenholtz, 2015). According to one framework (e.g., Durán and Herrmann, 2006b; Reitz et al., 2010; Barchyn and Hugenholtz, 2012; Barchyn and Hugenholtz, 2015), the transition begins when the sand deposition rate drops below a certain vegetation growth rate after which a positive feedback acts to increase vegetation, eventually stabilizing the dune. A decrease in the slip face deposition rate (or erosion rate) can occur due to climatic changes (an increase in precipitation or a decrease in wind power) or through the prevention of anthropogenic activities such as grazing (Tsoar and Møller, 1986). In the above framework, the dune morphology is important and modeled by a partial differential equation (PDE). A mathematical expression is sought for the local mass transport of sediment over the sand bed. Then, the rate of change of the local bed elevation can be obtained from the divergence of the mass flux of sediments using the Exner equation (also known as the erosion equation).

In the other modeling framework (described by an ordinary differential equation (ODE)), the vegetation, biogenic crust, and sand cover determine the dune’s stability, while the dune’s morphology is ignored (e.g., Hugenholtz and Wolfe, 2005; Yizhaq et al., 2007; Yizhaq et al., 2009; Bel and Ashkenazy, 2014); such models predict the bistability of active (e.g., barchans) and fixed (e.g., parabolic dunes) dunes under the same climatic changes, as well as the hysteresis behavior of dune mobility with respect to wind power, precipitation and human activities.

Previous modeling studies mainly concentrated on the genetics and dynamics of barchan dunes, most likely due to their relatively simple forcing of a unidirectional wind, their absence of vegetation, and their unique crescentic shape. These studies reproduced quite well barchan dune morphology, as well as other observed features. The models were successfully compared with underwater experiments (e.g., Reffet et al., 2010). It is accurate to state that continuum dune models were confirmed to successfully simulate barchan dune dynamics properties, such as the barchan 3D shape and collisions between barchans of different sizes, lending much credibility to these models and to the processes they are based on (Herrmann, 2006; Durán et al., 2010; Kok et al., 2012; Parteli et al., 2014b). In addition, they were used to investigate the dynamics of vegetated parabolic dunes and the transition from a barchan dune to a parabolic dune under the establishment of vegetation (Durán and Herrmann, 2006b).

To our knowledge, there are no dune morphology models (first modeling framework mentioned above) that investigated the bistability of active and stable dunes, although there is much evidence for the coexistence of active barchan dunes and stable parabolic dunes (Jerolmack et al., 2012; Zhiwei et al., 2015). The current study aims to fill this gap. We generalized our previous model for vegetation cover on sand dunes (Yizhaq et al., 2009; Kina et al., 2013; Yizhaq et al., 2013) to account for dune morphology, and we coupled it to the sand dune continuum model of Durán et al. (2010). Note that our model for vegetation growth is substantially different than the one suggested by Durán and Herrmann (2006b) in one main aspect. Our model includes spatial terms in the vegetation cover. These terms describe processes such as seed dispersal, mortality at the edge of the vegetated region (due to burial of vegetation by sand and exposure of roots), and vegetation diffusion, and the individual terms are quite different in the two models (direct and indirect wind effect, human effects, etc.). We show that this coupled dune morphology model exhibits the bistability of active barchan dunes and stable parabolic dunes under the same choice of model parameters representing climate conditions. The effect of biogenic crust on sand dune dynamics plays an important role in the formation of vegetated linear dunes (VLD) (Tsoar, 2013), but the effect is minor in barchan and parabolic dunes which are the main focus of this study. Thus, in this work we concentrated on the effect of vegetation on dune dynamics and leave the effect of biogenic crust for future work.

2. The model

In the past ten years or so, some of us have developed a simple model for the dynamics of vegetation and biogenic crust cover on sand dunes (Yizhaq et al., 2007; Yizhaq et al., 2009; Ashkenazy et al., 2012; Yizhaq et al., 2013; Kina et al., 2013; Bel and Ashkenazy, 2014; Yizhaq and Ashkenazy, 2016). This model basically describes the dynamics of vegetation and biogenic crust in a sand dune environment. The state of the dune is determined according to the vegetation cover—a vegetated dune is fixed, while a bare dune is active. Vegetation growth in the model is based, in most cases, on the standard logistic growth term, while vegetation decay is due to either direct or indirect wind action through sand movement that suppresses the growth of regular vegetation through root exposure and/or vegetation burial. One of the model’s basic assumptions is that there is a critical vegetation cover amount above which the vegetation blocks the wind, leading to a very sharp decrease in sand transport. The assumption of critical vegetation cover is based on the fact that vegetation cover modifies the air flow pattern above the sand. Three types of air flow were previously identified according to the degree of vegetation cover: isolated-roughness flow, wake-interference flow, and skimming flow (Wolfe and Nickling, 1993). In the isolated-roughness and wake-interference flows, the wind can act directly on the surface, whereas for skimming flow, while some amount of bare surface may be present, the sand is protected from the direct action of the wind by the surrounding vegetation. It was shown by Lee and Soliman (1977), who used wind tunnel experiments to...
investigate the densities of roughness elements, that a ground cover \( v_c \) of 0.4 provides sufficient blocking for skimming flow. This critical vegetation cover, \( v_c \), further depends (Bullard, 1997) on plant shape, porosity, stem flexibility, vegetation height and wind velocity, as plant canopies also can be penetrated by strong winds. Thus, the interaction between vegetation and wind is quite complicated and nonlinear, making it difficult to estimate the value of \( v_c \). Moreover, \( v_c \) varies for different geographical locations, from a measured \( v_c \approx 0.14–0.16 \) for the Kalahari Desert (Wiggs et al., 1995; Wiggs, 2005) to an observed \( v_c \approx 0.35 \) for the Australian deserts (Ash and Wasson, 1983). Thus, the critical vegetation cover is a simple mean field approach to model the complex interaction between wind and vegetation without going into details of vegetation types and height. These details are embedded in the specific value of the vegetation cover.

The dynamics of this model yields a hysteresis behavior (for example, as a function of precipitation and wind power) and multiple states of sand dune activity under the same climatic conditions; the model’s multiple states comport with observations (Yizhaq et al., 2007; Yizhaq et al., 2009). This model was also used to study transgressive dune dynamics (Yizhaq et al., 2013) and the effect of psammophilus plants on sand dune stability (Bel and Ashkenazy, 2014), and to predict the oscillatory behavior of dune dynamics on time scales of centuries to millennia (Yizhaq and Ashkenazy, 2016). The above model is based on the assumption of a mean field approach in which the morphology of the sand dunes is ignored.

Here we extend the above vegetation model to account for 3D dune dynamics and couple it to the 3D sand dune continuum model of (Kroy et al., 2002a; Kroy et al., 2002b; Durán and Herrmann, 2006a; Parteli et al., 2009). The model we propose is composed of two parts, sand dynamics (un-vegetated dune) and vegetation dynamics. Below we briefly describe each of these model components and then provide results of the model.

2.1. A model for un-vegetated dunes

This part is essentially the model description of (Parteli et al., 2009). The model can be described in four steps.

1. The wind over the dune: Based on detailed studies (Jackson and Hunt, 1975; Weng et al., 1991; Kroy et al., 2002a; Kroy et al., 2002b), it is possible to find the wind field over the dune surface. This is based on the Fourier transform of the dune’s height. The Fourier transform of the perturbation to the wind field due to the presence of the dune is given by

\[
\nabla^2 \tilde{v}_l = \frac{2}{k^2} \left[ -1 + \frac{1}{k_0^2} \right] K_0(2\sigma k) K_0(2\sigma k),
\]

where \( k \) and \( \sigma \) are the directions parallel and perpendicular to the wind direction, respectively, \( \sigma = \sqrt{\ln(k_0/k_0)} \), \( k_0 \), \( K_0 \) are the modified Bessel functions, \( k_0, k \) are components of the Fourier space vector, \( \tilde{v}_l \) is the Fourier transform of the dune height, \( U \) is a non-dimensionalized vertical velocity profile, \( l \) is the inner layer depth, \( L \) is the length scale of the dune, and \( k_0 \) is the surface roughness (typically 1 mm). \( \sim \) represents the Fourier transform. The shear stress at each location \( x, y \) is

\[
\tau_0 = \tau_0(\widetilde{\sigma} / \sqrt{\tau_0^2 + \widetilde{\sigma}^2}),
\]

where \( \tau_0 \) is the unperturbed shear stress (i.e., the shear stress over a flat surface).

2. Calculation of local sand flux: Given the shear stress, it is possible to calculate the shear velocity \( u_s = \sqrt{\frac{\tau_0}{\rho_{s\text{fluid}}}} \) where, in the case of dunes, \( \rho_{s\text{fluid}} \) is the air density. Then, it is possible to calculate the sand flux \( q(x, y) \) as follows:

\[
\nabla \cdot \vec{q} = (1 - \frac{\tau_0}{|q|}) \vec{q} / \epsilon,
\]

where \( \epsilon \) is the saturated sand flux given by

\[
\epsilon = \left( \frac{2\sigma}{\sqrt{\ln(k_0/k_0)}} \right) / ([u_s / \epsilon_s] - 1),
\]

\( \sigma = 0.43 \) and \( \gamma = 0.2 \) are empirically chosen parameters, \( \rho_0 \) is the gravity constant, and \( \epsilon_s \) is the minimal threshold velocity. \( v_0 \) is the mean velocity of sand particles that can be calculated numerically see Zhang et al. (2005) and Durán and Herrmann (2006a).

3. Calculation of local height: The surface height \( h(x, y) \) is then calculated based on the sand flux \( \vec{q} \) and on the mass conservation (Exner equation) as follows:

\[
\frac{\partial h}{\partial t} = -\frac{1}{\rho_{s\text{fluid}}} \nabla \cdot \vec{q},
\]

\( \rho_{s\text{fluid}} = 1650 \text{ kg m}^{-3} \) is the sand bulk density.

4. Angle of repose and the separation bubble: The local slope on the lee side of the dune cannot exceed the angle of repose \( (34^\circ) \), and once this situation occurs, locally separated streamlines are introduced on this side; in practice, the streamlines are third-order polynomials that are fitted to connect the brink with the ground, composing the separation bubble within which there is no sand flux.

The dynamics of the model is achieved by iterating the above four steps. A constant influx of sand and periodic boundary conditions are assumed.

2.2. Modeling of vegetation

Below, we suggest a model for the growth of vegetation on sand dunes, which is coupled to the un-vegetated dune model described in the previous subsection. The model is based on our previous modeling works (Kinast et al., 2013; Yizhaq et al., 2013). Given the dune height \( h(x, y) \) and shear stress \( \tau(x, y) \), which were calculated using the model described in the previous subsection, the development of local vegetation \( v(x, y) \) cover may be formulated as follows:

\[
\begin{align*}
\frac{\partial v}{\partial t} & = \alpha_v(p) \left( v + \eta_\Theta B \right) s - \rho_s \left[ \frac{\alpha_s(p)}{\pi} \left( \frac{1}{\Theta} - v \right) \right] \\
& - \varphi_u \nu^2 \mu - \nu \delta \nu^2
\end{align*}
\]

(6)

where \( \Theta = 1 - \Theta (d_i / \alpha) \) and \( \Theta \) is the Heaviside step function; \( \Theta \) equals 1 when there is local erosion \((d_i / \alpha < 0)\) and is zero when there is local deposition \((d_i / \alpha > 0)\). \( s \) is bare sand cover, i.e., \( s = 1 - v \). \( \alpha_v(p) \) is the growth rate coefficient of vegetation, which depends exponentially on precipitation \((\text{Kinast et al., 2013})\): it is zero when the precipitation rate is below a certain minimum, and for a higher precipitation rate, it is \( \alpha_v, \max (1 - \exp(-[\rho_p - \rho_{\text{min}}] / c_n)) \). The first term on the RHS is a logistic growth term that includes a spontaneous growth parameter \( \rho_s \); it is assumed here that there is no spontaneous growth under sand deposition as the initial growth of vegetation is suppressed since it is buried by the sand. A more accurate process will enable spontaneous growth only when the sand movement is weak (i.e., small \( d_i / \alpha \)). Obviously, this is a simplistic assumption as psammophilous plants actually flourish when the sand is active (Bel and Ashkenazy, 2014). However, we restrict ourselves here to regular non-psammophilous plants that are suppressed in the presence of active sand.

The second and third terms of the RHS of Eq. (6) stand for mortality due to erosion and deposition of sand. The fourth and fifth terms of Eq. (6) are mortality terms that simulate suppression due to direct action of the wind on the vegetation. As in previous studies (Hesp, 2002), vegetation experiences wind stress even when there are no local changes in the vegetation cover (i.e., the term \( \chi_{\nu} (\tau) \)). Vegetation is assumed to
experience either growth or decay when there are local changes in the vegetation cover as wind can suppress/erode vegetation on the upwind and the sandier regions and can enhance growth on the downwind and the sandier regions due to seed transport (the fifth term in Eq. 6). The sixth term of the RHS of Eq. (6) is a general mortality terms that simulate general suppression processes such as grazing, clear-cutting, burning and trampling, which mainly affect crust cover (Danin, 1996). Since the proposed model is formulated based on vegetation cover and not by vegetation height, the grazing acts to decrease vegetation cover when in reality, it can decrease both vegetation cover and vegetation height. Thus, the action of grazing in our formulation is similar to clear-cutting. The effect of trampling can be seen along the Israel-Egyptian border (see Fig. 1b in Yizhaq et al. (2007) where there is a clear visual difference between the active dunes on the Egyptian side and the vegetated and almost fully stabilized dunes on the Israeli side (Meir et al., March 1996). This difference is attributed to wood gathering, over-grazing, and trampling of the sand crust on the Egyptian side, practices that have been prohibited on the Israeli side since the establishment of the border in 1982. Once this anthropogenic pressure ceased on the Israeli side, the dunes converged to their natural state of vegetated fixed dunes. The last term in Eq. (6) is a diffusion term that represents an isotropic seed dispersal.

It is possible to constrain most of the parameters of the above model based on observations (Yizhaq et al., 2009, as in and Yizhaq et al. (2013)), and sensitivity tests have been performed for the unconstrained parameters. The list of the parameters and their value ranges are presented in Table 1. The above model is simplistic and ignores many other vegetation processes. However, we believe that it can serve as an initial model for the growth of vegetation on sand dunes. This vegetation model constitutes the main difference between our model and the previous model of Durán and Herrmann (2006b) who used a different equation that describes the rate of change of the vegetation height on the dune surface. Unlike the model of Durán and Herrmann (2006b), our model includes spatial terms for the development of the vegetation cover. These terms (Eq. (6)) describe processes such as seed dispersal, mortality at the edge of the vegetated region (due to burial of vegetation by sand and exposure of roots), and vegetation diffusion, which represents an isotropic seed dispersal (e.g., by animals and winds). Although the wind can induce long distance seed dispersal, most of the seeds fall at a short distance from the canopy (Nathan et al., 2002), which justifies the use of a simple diffusion term.

We assume that the two main factors that affect vegetation growth on sand dunes are wind and precipitation. For simplicity, other factors, such as light, temperature, moisture and nutrients, are not included in the model, since they are of secondary importance. Due the inverse

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>Annual mean precipitation</td>
<td>mm/yr</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Vegetation cover growth rate</td>
<td>5-35 yr^{-1}</td>
</tr>
<tr>
<td>$P_{min,v}$</td>
<td>Minimal rainfall needed for vegetation growth</td>
<td>50 mm/yr</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Vegetation growth response for $\tau$</td>
<td>50-300 mm/yr</td>
</tr>
<tr>
<td>$\eta_s$</td>
<td>Spontaneous vegetation growth</td>
<td>0-0.2</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Critical vegetation cover</td>
<td>0.1-0.4</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Human impact parameter on vegetation</td>
<td>0-0.1</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Vegetation tolerance to sand erosion</td>
<td>3.10^7 yr^{-1}U^{-1}</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Vegetation tolerance to sand deposition</td>
<td>$\varepsilon_c/100$ yr^{-1}U^{-1}</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Vegetation vulnerability to shear stress</td>
<td>30-100 yr^{-1}U^{-1/2}</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Vegetation vulnerability</td>
<td>Values in the range of $\xi$</td>
</tr>
<tr>
<td>$\delta_r$</td>
<td>Vegetation diffusion coefficient</td>
<td>10-100 m^2/yr</td>
</tr>
</tbody>
</table>

Texture effect (Tsoar, 2013), sand exhibits deep infiltration, and moisture can be protected from evaporation during the long, dry and hot summers through water storage below the upper 30-60 cm. It was shown that when the wind power is very low, desert dune sand is a relatively more favorable ecological habitat for vegetation than non-sandy soils with a better holding capacity, such as loamy soils. The model does not include the effect of annuals and grasses on dune stabilization, which may be important especially in rainy years. In principle, the effect of annuals can be included in the model by adding another equation with a seasonally variable growth rate and a high mortality rate. This model modification would need to include intraannual rainfall (variable growth rate), and not a uniform rainfall such as in the current model, to cope with the faster response of annuals to short rainfall events.

To conclude, the model we propose includes the dynamics of vegetation cover, as well as several important processes that were not part of Durán and Herrmann (2006b) model. Our model allows us to study the effects of different terms (processes such as grazing and the direct effect of the wind on vegetation) in the vegetation equation on dune development.

2.2.1. The coupling between the vegetation model and the 3D dune morphology model

The dune height [Eq. (5)] calculated by the 3D dune morphology model affects the growth of vegetation explicitly in the corresponding equation [Eq. (6)]. Vegetation affects the sand dune model by weakening the shear stress $\tau$ [Eq. (3)]:

$$\tau^* = \tau^*/[1 + (\tau^*/\tau_s-1)/\nu/\nu_s].$$

where $\tau^*$ is the effective shear stress, $\tau^*_0$ is the shear stress over a flat bed, $\tau_s$ is the threshold shear stress necessary for sand transport, $\nu$ is the local vegetation cover, and $\nu_s$ is the critical vegetation cover above which the effect of wind stress on sand transport is drastically reduced see Yizhaq et al. (2009) and therein. Eq. (7) is applicable when $|\tau^*_0| < \tau_s$ and when $|\tau^*_0| > \tau_s$ then $\tau^* = \tau^*_0$. This form of suppression is based on Eq. (1) of Durán and Herrmann (2006b), which yields a small effective shear stress when the vegetation cover exceeds the critical vegetation cover $\nu_s$. The coupling is performed by replacing $\tau^*$ by $\tau^*$ in steps 2-4 of the un-vegetated dune model (Section 2.1). We note that there are other possible forms of the effective shear stress $\tau^*$ including the step-like function form as used in Yizhaq et al. (2009).

3. Results

It is important to note that biogenic crust is crucial for the development of vegetated linear dunes (VLD), but it is usually absent in transverse dunes due to their higher mobility. Thus, in the current work which is devoted to the barchan-parabolic transition, we focus only on the coupling with vegetation and exclude the biocrust dynamics. We first validated our own newly developed (Matlab) code without the coupling with vegetation by simulating a barchan dune and comparing its features and properties to the published knowledge. Fig. 1 depicts the evolution of a barchan dune, starting from a Gaussian heap of sand that then evolved under the action of unidirectional wind. As expected, the heap developed into a barchan dune, resembling this dune type’s actual shape.

To examine the vegetation component of the model, we repeated the numerical experiment of Durán and Herrmann (2006b) in which they demonstrated the transition from a barchan dune to a parabolic dune, taking vegetation dynamics into account. In Fig. 2, we demonstrate such a transition when starting from a barchan dune under a precipitation rate and shear stress $\tau^*_0$ that allow vegetation growth; the wind is unidirectional. After several hundred days of simulation, the barchan dune indeed converted into a parabolic dune. The basic mechanism (Zhiwei et al., 2015, model 2 in) is that the barchan’s horns are anchored by vegetation, while the rest of the barchan continues
migrating downwind, leaving behind the trailing arms. The dune maintains a barchan shape for a while until it becomes smaller and thinner, finally stabilizing in a parabolic shape with long upwind trailing arms.

The rate of stabilization and the morphological details of the transformation sequence depend on the model’s parameters and strongly on \( \alpha_v \) (the vegetation growth rate, which is related precipitation) as shown in Fig. 3. For a small value of \( \alpha_v \), the vegetation is unable to stabilize the dune, and the barchan preserves its typical form and downwind migration. For intermediate values of \( \alpha_v \), the barchan is transformed into a parabolic dune with upwind trailing arms as shown in Fig. 2. Note that for \( \alpha_v = 10, 15 \), the model predicts that the parabolic dune may break into two separate stabilized linear dunes. For larger values of \( \alpha_v \), the stabilization process is rapid, and the original barchan shape is preserved. Such quick stabilization was observed in the Mu Us dune field in north-central China (Zhiwei et al., 2015) where some barchan and barichaoonoid dunes were totally stabilized in less than 10 years.

Another factor that affects the morphodynamics of dunes is the vegetation vulnerability to shear stress dictated by the parameter \( \gamma_v \). For higher values of \( \gamma_v \), the mortality of vegetation due to the direct action of the wind is larger, which means that the vegetation is less adapted to wind stress. Fig. 4 shows the dune dynamics for different values of \( \alpha_v \) (as in Fig. 3) but for a larger value of \( \gamma_v = 100 \), which means a larger wind suppression. Here, a larger growth rate is needed to stabilize the barchan dune.

The effect of \( \gamma_v \) can be clearly shown by fixing the value of \( \alpha_v \) (\( \alpha_v = 20 \)) and studying the dynamics for different values of \( \gamma_v \). Fig. 5 shows the stabilization process of a barchan dune that is partially covered by vegetation on its lee slope for different values of \( \gamma_v \), which mainly depends on the vegetation type. For \( \gamma_v = 0 \), the stabilization process is fast, and the barchan shape is almost unchanged since the dune is covered by vegetation. For intermediate values of \( \gamma_v \), the barchan is transformed into a parabolic dune with long trailing arms. For higher values of \( \gamma_v \), the vegetation cannot become established, resulting in a mobile and active barchan.

The previous figures show the transition of a barchan dune into a parabolic dune by vegetation. The opposite process is also possible in which a parabolic dune is transformed into a barchan dune by sharply decreasing the vegetation growth rate, i.e., decreasing the precipitation rate. Such a change may occur due to prolonged droughts (Siegal et al., 2013) or more dramatic climatic changes. Fig. 6 shows the activation process of a parabolic dune caused by decreasing the vegetation growth rate. The initial state is a parabolic dune with a high value of \( \alpha_v \), Decreasing the value of \( \alpha_v \) to zero results in two barchan dunes, whereas, for \( \alpha_v = 15 \), the parabolic dune splits into two vegetated linear dunes. The splitting of a parabolic dune into two linear dune-like arms was observed on the northern Coral Sea coast of Queensland (Warren, 2013).

Active and fixed dunes can coexist under the same climatic conditions depending on their history (Yizhaq et al., 2007; Yizhaq et al., 2009), and this bistability is associated with hysteresis behavior with respect to several parameters that represent, for example, wind power and precipitation. Hysteresis behavior is also observed by the model proposed above and is demonstrated in Fig. 7, where the control parameter is the vegetation growth rate \( \alpha_v \), which is related to precipitation. Panel (a) shows the dune state (expressed by the sand cover) as a function of \( \alpha_v \) starting from a barchan dune that transformed into a parabolic dune. Panel (b) shows the opposite process of the reactivation of a parabolic dune into two small barchan dunes when decreasing the vegetation growth rate. These simulations show that there is a range of \( \alpha_v \) values (precipitation values) for which both barchan and parabolic dunes coexist under the same climatic conditions (mainly wind power and precipitation) and that the final state depends on the history of the system.

4. Discussion

The transformation (stabilization process) of active barchan dunes into parabolic dunes can be triggered by different processes such as a decrease in the wind power, an increase in precipitation and through human intervention (such as the building of barriers that reduce sand flux). The transformation (activation process) of parabolic dunes into barchan dunes is also possible due to the opposite processes, i.e., an increase in wind power, megadroughts and human intervention such as vegetation removal (Arens et al., 2004; Arens et al., 2007) and the use
of fire as in the Brazilian Amazon (Goudie, 2011). It was shown that the rapid stabilization of barchan dunes in the Mu Us dune field in north-central China from the years 2000–2012 was caused by a sharp decrease in wind activity (drift potential); see Fig. 2B of Zhiwei et al. (2015). The decline in the wind drift potential decreased the sand flux, which led to an increase in the vegetation cover over the dunes, eventually leading to dune stabilization. The stabilization of barchan and transverse dunes to parabolic dunes due to anthropogenic processes has been shown in the Mediterranean coastal dunes of Israel (Tsoar et al., Oct. 2002). A decrease in human activity (grazing and wood gathering) allowed the establishment of vegetation on the dune crests, triggering the transformation from a barchan to a parabolic dune. In the White Sands dune fields (New Mexico, USA), the transition from barchan dunes to parabolic dunes was also caused by a decrease in the wind power, but the mechanism was different (Jerolmack et al., 2012) and related to the growth of the internal boundary layer (IBL) thickness due to the dune roughness. This increase in the IBL thickness reduces the shear stress exerted by the wind.

The model and results presented here predict that for a range of vegetation growth rates (or precipitation rates), both barchan and parabolic dunes can coexist. This prediction seems to be indicated by field observations (Tsoar et al., 2009; Zhiwei et al., 2015), but this remains to be verified. The coexistence of both dune types implies hysteresis behavior, which is common in nonlinear systems. Increasing the vegetation growth rate beyond a certain value ($\approx \alpha^{20}$, corresponds to 160 mm/yr) resulted in the stabilization of the initial barchan dune;

Fig. 2. Development of a parabolic dune from a barchan dune when vegetation dynamics is taken into account ($\alpha = 10$). Left column depicts the dune height in meters while the right column depicts the relative vegetation cover (i.e., 0 value indicates no vegetation while 1 indicates full vegetation cover). The horizontal and vertical axes indicate the spatial extent in meters. $u_0 = 0.4$ m/s, and the wind is unidirectional (from left to right). (a) $t = 20$ days; (b) $t = 60$ days; (c) $t = 100$ days; (d) $t = 140$ days; (e) $t = 200$ days; (f) $t = 300$ days.
however, in the opposite direction (i.e., decreasing the growth rate), the dune reactivation from a parabolic to a barchan dune occurred at a lower value of the vegetation growth rate ($\alpha_1$ corresponds to $100 \text{ mm/yr}$). In other words, a much lower precipitation rate is needed to reactivate the parabolic dunes.

The bistability of barchan and parabolic dunes can be represented by other model parameters such as $\mu$, that controls the intensity of grazing and clear-cutting as was shown in Yizhaq et al. (2009). This bistability of barchan and parabolic dunes was not studied in the cellular automata model of Nield and Baas (2008) where only the development of parabolic dunes from blowouts was considered, nor in the crestline-based approach of Barchyn and Hugenholtz (2015) where only...
Fig. 5. Numerical simulation of the model showing the stabilization process for different values of $\gamma_v$ (which represents the vegetation vulnerability to wind shear stress) and for different times (indicated in days on the left). The colors indicate the relative vegetation cover between 0 (yellow) and 1 (green). For low values of $\gamma_v$, the vegetation can survive, and the stabilization process is quite fast. For $\gamma_v = 200$, the vegetation is suppressed, resulting in an active barchan dune. Parameters: $\alpha_v = 20$, $\delta = 100$, $\epsilon = 3 \times 10^{-5}$, $\sigma_v = \epsilon/100$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 6. Numerical simulation of the model showing the activation process of a parabolic dune by decreasing the vegetation growth rate ($\alpha_v$) and for different times (indicated in days in the left). The colors indicate the relative vegetation cover between 0 (yellow) and 1 (green). The initial condition is a parabolic dune with a high value of $\alpha_v$. Decreasing the value of $\alpha_v$ to zero results in two barchan dunes, whereas for $\alpha_v = 15$, the parabolic dune splits into two vegetated linear dunes. Other parameter values are as in Fig. 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
the stabilization of barchan dunes to parabolic dunes was studied. Another difference between our model and the model of Nield and Baas (2008) is that in our model, we take into consideration only one type of vegetation (shrubs), whereas their model for parabolic dunes has two vegetation types: grass and shrubs with different tolerance levels for sand transport. In their simulations, the grass colonizes the top and the lee side of a parabolic dune, and the shrubs colonize the trailing arms. They argued that both vegetation types were needed to reproduce the trailing arms. According to the model presented here, the trailing arms can be formed with only a single vegetation type. However, a two-species vegetation model (e.g., Bel and Ashkenazy, 2014) can be used to consider the different effects of pioneer species in the stabilization process. As we previously showed (Yizhaq et al., 2009), almost all the bistable dune areas on Earth are along coastal regions. On the one hand, the coexistence of active and fixed dunes requires relatively high precipitation to support vegetative growth, but on the other hand, coexistence also requires high wind power to erode vegetation, which maintains dune activity. In addition, a high rate of sand supply is also necessary for dune mobility (Arens et al., 2004), and high sand flux is commonly found in coastal regions. The coexistence of active and fixed dunes under similar climatic conditions is important from a practical ecological management point of view. Biodiversity is richer when an area contains both dune types since different ecosystems are associated with each type. Dune stabilization affects the faunal and floral communities inhabiting the dunes, particularly the psammophilous species, which depend exclusively on a shifting-sand habitat for reproduction and survival. In order to restore the structure and composition of the dune communities, including the rare and endemic psammophilous species, the dunes must be reactivated by complete vegetation removal or through a grazing intervention (Bird et al., 2017).

Such a reactivation has been implemented in the coastal dunes in the Netherlands (Arens et al., 2004; Arens et al., 2007). If the system is bistable, then this reactivation can cause the dunes to become active, but if the system is not bistable, then the system will return after some time to the stabilized dune state. In this case, simple vegetation removal is not sufficient to restore dune mobility, and a more sophisticated management strategy is needed to keep the dunes active (see Fig. 2 in Yizhaq et al. (2007)). The model we propose can also be used to study the effect of other disturbances, such as the effect of prolonged droughts on dune stability (Yizhaq et al., 2009) for example, to investigate how long and severe a drought must be in order to shift the system from parabolic dunes to barchan dunes for a specific initial climatic condition. Such a drastic decrease in precipitation is different from the gradual change that is depicted in the hysteresis behavior in Fig. 7.

5. Conclusions

We present a 3D continuum dynamical model for the morphological development of sand dunes that is coupled to vegetation dynamics. The coupling between the model for bare dunes and the model for vegetation growth is performed by weakening the shear stress due to the presence of vegetation, similar to the form introduced by Durán and Herrmann (2006b); the rate of change of dune height affects the growth of vegetation by increased erosion. The model is used to study the transition from barchan to parabolic dunes (and vice versa) under different vegetation growth rates and different wind stress intensities. For low vegetation growth rates, the barchan dune remains active and preserves its typical crescent form, while for higher growth rate values, the barchan transforms into a parabolic dune with trailing arms. For even higher vegetation growth rates, the vegetation covers the entire barchan, preserving its original form almost unchanged, in agreement with field observations from the Mu Us dune field in China. The model shows, for the first time, that for a range of vegetation growth rates (precipitation), there is a bistability of barchan and parabolic dunes. This prediction should be verified against future field studies.

The model we propose here may be extended to include the dynamics of biogenic crust, to study the genesis and dynamics of other dune types for which biogenic crust plays an important role, such as vegetated linear dunes, which are abundant in the Kalahari and Australian Deserts (Tsoar, 2013; Hesse et al., 2017) and whose formation is still under debate (Tsoar, 2013).

Authors statement

Gershon Hanoeh – Numerical simulations and developing the codes. Hezi Yizhaq – Methodology and writing original draft and revised draft. Yossi Ashkenazy – Investigation; Methodology and supervision.

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Appendix A. Supplementary data

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References


