

# A Model of Venture Capital Screening

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## **A Model of Venture Capital Screening**

### **Abstract**

We consider a model of entrepreneurs compete for venture capital (VC) funding. With asymmetric information, the VC can only judge an entrepreneur by the stage of development which in a separating equilibrium also reveals the quality of the new technology. With limited capital the VC just finances the best project. Thus, having too many entrepreneurs can cause underinvestment by entrepreneurs since effort by losers is wasted. We then give characterization of when more entrepreneurs are better and show how it depends on the shape of the distribution of types. The model also demonstrates that VCs could possibly increase their payoff if they avoid focus on a small number of industries.

**Keywords:** Screening, Contests, Venture Capital, Entrepreneur.

## **A Model of Venture Capital Screening**

### **1. Introduction**

Little-known NEA has an exceptional record in picking young companies. With literally thousands of proposals crossing their desks, how do NEA's partners pick potential winners? "We sit on paradigm beach and look for the next big waves to ride," (Janet Novack, *Forbes*, Nov 4, 1996)

Venture capitalists (VCs) thrive by successfully gambling on companies to invest in. They review a large number of business plans of startups who need financing. This study focuses on whether increasing traffic in the VC firm always has a positive effect. Our model involves entrepreneurs who compete on VC funding providing for an auction-like setting where the VC acts as the auctioneer, in essence, selling a unit of financing to  $n$  entrepreneurs who bid for financing. The surprising answer that we get is that having too many entrepreneurs can cause underinvestment by entrepreneurs since effort by losers is wasted. Moreover, this phenomenon is expected when the industry is very attractive and populated with many high quality entrepreneurs. The reason for this result is since when the number of competitors is high and there are many bidders that likely to have high quality technology, the probability of getting a support from the VC is decreasing as the competition become more aggressive. Since an entrepreneur without financing losing his investment in the development of the technology, he is better off by reducing his investment.

Venture capital financing for early stage companies has dramatically increased in importance in the last two decades and, consequently, so has the academic research on the topic. The majority of the VC literature entails descriptive field and empirical studies

(see, for example, Sahlman (1990), Lerner (1994), Gompers (1995), Gompers and Lerner (1999), Hellmann and Puri (2000), Kaplan and Stromberg (2002)). Some of the theoretical research on the topic has focused on the mechanism of staged investments (see, for example, Neher (1999), and Wang and Zhou (2004)). Others have investigated whether financing should it be provided in the form of debt, equity, or a hybrid instrument (examples of such studies include Bergemann and Hege (1998), Trester (1998), Schmidt (2003), and Elitzur and Gavious (2003)). Several theoretical studies (see for example, Amit et. al (1998) and Ueda (2004)) focus on the *raison d'être* of VCs and argue that VCs exist because of their ability to reduce informational asymmetries. Specifically, banks and other institutional lenders, in contrast with VCs, are less able to distinguish between high and low quality entrepreneurs. As such, VCs are in essence financial intermediaries who thrive because of their superior ability to screen and monitor entrepreneurs. Despite the fact that several studies argue (see, for example, Zacharakis and Meyer (2000)) that screening prospective investments by VCs is crucial for the VC's success, or that the VC's superior ability to do so is the very reason for their existence (Amit et. al (1998) and Ueda (2004), for example). We are not aware of any theoretical study on VC financing that has examined the screening process itself. Another interesting result that we obtain in this study is that VCs could possibly increase their payoff if they avoid spreading into many industries and focus instead on a small number of industries. The study also provides some insights on the effects of multiple investments by VCs and the effects of competition among VCs on the same investments.

Our model is related to the economic literature on private-value contests with incomplete information. The literature in this field (which includes, for example, Weber (1985),

Hillman and Riley (1989), and Krishna and Morgan (1997)) deals with linear cost function and an auctioneer who benefits from the bids (or efforts) made by the players. In this sense, our model is related to Moldovanu and Sela (2001) and Gaviols, Moldovanu and Sela (2002) where a non-linear cost function is assumed. However, in contrast with the traditional literature in this field, our model assumes, in order to fit the venture capital reality, that the auctioneer (the venture capitalist in our model) benefits, in addition to the bid, also from the private value of the winner, which represents firm's quality in our model.

A recent line of literature that is related to our paper in the contest area is Taylor (1995), Fullerton and McAfee (1999) and Moldovanu and Sela (2005). However, the significant difference here is that the VC benefits from the winning bid and the highest technology (i.e.,  $\max(b_i + v_i)$ ) as opposed to the contest literature where the auctioneer receives also a payoff from the losing bids (i.e.,  $\sum_i b_i$ ).

The paper is organized as follows. Section 2 presents the model and the setting that the study uses. Section 3 provides the analysis of the equilibrium bids. In section 4 we make the contracting endogenous and examine the optimal contracting between the parties. Section 5 examines what would happen if there is competition among VCs who make multiple investments. Section 6 concludes.

## 2. The Model

Suppose there are  $n$  entrepreneurs competing over VC financing. We assume that the VC will finance only one entrepreneur where,<sup>1</sup> later on, we deal with more than one. Each entrepreneur  $i$ ,  $i=1, \dots, n$  knows the value of his technology  $v_i$  where  $v_i \in [0,1]$  is private information of entrepreneur  $i$ . The value of each entrepreneur's technology,  $v_i$ , is drawn independently from a twice continuous distribution  $F(v)$  defined over  $[0,1]$ . It is assumed that  $F$  has a strictly positive density  $f(v)$ , with bounded second derivative  $f'$ .

Each entrepreneur is privately informed about the quality of his technology,  $v_i$ . In addition, the entrepreneur reaches a certain stage of development,  $e_i$ ,  $i=1, \dots, n$ , at a cost of  $0.5e_i^2$ ,  $i=1, \dots, n$ , before approaching the VC. Development activity is endogenously determined in our model and is costly to the entrepreneur because it requires investment of his resources (both monetary and non-monetary). Development progress achieved by the entrepreneur,  $e_i$ , is observed by the VC.<sup>2</sup>

Let  $P$  and  $d > 0$  be the VC's investment made by the VC and the VC's discounting parameter respectively. The firm's ex-post value is given by  $(v + e)rP$  where  $r > 0$ . This formulation thus suggests that the value of the firm is positive if either  $v$  or  $e$  is positive, even if the other parameter is zero. The rationale behind having a value to the firm despite having a zero  $v$  is that acquiring knowledge, creating a team, and having a

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<sup>1</sup> In section 4 below we relax this assumption and assume an investment of  $K \geq 1$  units invested in several firms.

<sup>2</sup> Note that the cost function is the same across all entrepreneurs but they are differentiated in their technologies.

research organization is valuable in itself even if the initial research turns out to be worthless.

To simplify notations we assume that  $P=1$  namely, the investment made by the VC is a single monetary unit.<sup>3</sup>

The VC observes development progress,  $e$ , and co-operates with the winner who is the entrepreneur with the highest development progress. If more than one entrepreneur invests the same development level at this high level the VC then chooses randomly among these entrepreneurs. The VC however has the option to reject all proposals if none of them would generate a profit for her.

We assume that the sharing rule between the VC and the entrepreneur stipulates that the entrepreneur receives the percentage,  $\alpha$  where  $0 < \alpha < 1$ , of the firm value while the VC gets  $(1 - \alpha)$  of it. In the first part of this paper we assume that  $\alpha$  (and, thus,  $(1 - \alpha)$ ) is based on what is customary in the market and thus, is an exogenous and known number. Later on, we relax this assumption and determine endogenously the value of  $\alpha$ . The VC announces  $\alpha$  before the contest and commits to this sharing rule.

The VC invests  $P=1$  dollars in the firm ( $P$  is common knowledge). We assume, consistent with the literature (see, for example, Mason and Harrison (2002), and Manigart et al., (2002)), that the VC has a required rate of return,  $d$ , where  $d > 0$ . We assume that  $(1 - \alpha)r \geq 1 + d$ . This assumption ensures that the VC will be involved only in fields where her expected return is strictly positive and that her share in the firm  $1 - \alpha$  is strictly positive.

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<sup>3</sup> We found that assuming an investment of  $P \neq 1$  instead of a single monetary unit does not add much to our analysis.

The utility of entrepreneur  $i$  is given by

$$u_i = \begin{cases} -\frac{1}{2}e^2; & \text{lose,} \\ \alpha r(v+e) - \frac{1}{2}e^2; & \text{win.} \end{cases} \quad (1)$$

Consequently, entrepreneur's  $i$  expected utility is

$$U = \alpha r \text{Prob}(i \text{ wins} \mid \text{development progress } e)(v+e) - \frac{1}{2}e^2. \quad (2)$$

### 3. Analysis of Equilibrium Progress

The VC's utility if she selects a winner is given by

$$V = (1-\alpha)r(v+e) - (1+d) \quad (3)$$

It is clear that if the winning bid results in a loss ( $V < 0$ ) to the VC then, she would reject the winner and thus, the VC hold a constrain on the winner type

$$v+e \geq \frac{1+d}{(1-\alpha)r}. \quad (4)$$

Thus, the VC should have a minimum acceptable level of technology and development level, one that does not entail a loss. In equilibrium the entrepreneur reveals his technology through the development progress  $e(v)$  and thus, VC can set a threshold level of technology  $\underline{v}$  such that  $\underline{v} + e(\underline{v}) = (1+d)/[(1-\alpha)r]$  where  $e(\underline{v})$  is the minimal equilibrium progress made by entrepreneur if he participate in the contest. The VC dictates the minimum level of progress,  $e(\underline{v})$ .

Assuming a monotonic equilibrium function  $e(v)$  for the entrepreneurs, the VC maximizes in equilibrium her profit and, accordingly, her ex-ante expected payoff can be represented by

$$W = \left\{ (1 - \alpha)rE\left(\max_{i=1,\dots,n}(e(v_i) + v_i) \mid \max_{i=1,\dots,n} v_i \geq \underline{v}\right) - (1 + d) \right\} \Pr(\max_{i=1,\dots,n} v_i \geq \underline{v}). \quad (5)$$

The equilibrium progress  $e(v)$  functions and the level of minimum acceptable technology to the VC  $\underline{v}$  in our model entail subgame perfect Nash Equilibrium. Namely, the VC cannot set a minimum level of technology  $v^* > \underline{v}$  which is too high because the bidders know that if the highest level of technology (the winner) is below  $v^*$  but still above  $\underline{v}$ , the VC will accept it nevertheless because she would still end up with a positive expected payoff. Consequently, as we will discuss later on, any demand from the VC for a threshold  $v^*$  that is too high will not be credible. We start with the symmetric equilibrium progress function.

**Proposition 1:** *The symmetric monotonic increasing bid is given by*

$$e(v) = \alpha r F^{n-1}(v) + \sqrt{\alpha^2 r^2 F^{2(n-1)}(v) + 2\alpha r \left( v F^{n-1}(v) - \int_{\underline{v}}^v F^{n-1}(s) ds \right)}, \quad (6)$$

where  $\underline{v}$  is the solution for

$$\underline{v} + e(\underline{v}) = \frac{1 + d}{(1 - \alpha)r}.$$

All proofs are relegated to Appendix 1.

It is easy to verify that (6) is increasing. Denote the VC's minimum acceptable technology, which is a function of the number of entrepreneurs, as  $\underline{v}(n)$ .

**Proposition 2:**  *$\underline{v}(n)$  is monotonically increasing in  $n$ .*

Note that although  $\underline{v}(n)$  is monotonically increasing with  $n$  it is still bounded below 1 by the assumptions that  $(1 - \alpha)r > 1 + d$ . The intuition behind Proposition 2 is that with limited capital the VC just finances the best project and, thus, having too many

entrepreneurs causes underinvestment by low types' entrepreneurs since effort by losers is wasted.<sup>4</sup> Thus, the VC increases the minimum required technology level,  $\underline{v}(n)$ . The VC can observe  $e$  but not  $v$ , and, thus, she evaluates the value of  $v$  from  $e$ .<sup>5</sup> Moreover, as the following result demonstrates,  $\underline{v}(n)$  is bounded by the ratio of the VC's future value coefficient,  $1+d$ , to the share of the VC in the total return on all investments (including development) in the firm,  $(1-\alpha)r$ . Let  $\underline{v}^\infty = \lim_{n \rightarrow \infty} \underline{v}(n)$  then;

**Corollary 1:**

$$\underline{v}^\infty = \frac{1+d}{(1-\alpha)r}. \quad (8)$$

Observe that the assumption  $(1-\alpha)r > 1+d$  guarantees that (8) is bounded below 1.

From equations (A.6) in the Appendix we can write the equation for  $\underline{v}(n)$  as

$$\underline{v} = \frac{1+d}{(1-\alpha)r} - \alpha r F^{n-1}(\underline{v}) - \sqrt{\alpha^2 r^2 F^{2(n-1)}(\underline{v}) + 2\alpha r \underline{v} F^{n-1}(\underline{v})}. \text{ Note that because } \underline{v}(n) < \underline{v}^\infty \text{ is}$$

bounded away from 1,  $F^{n-1}(\underline{v}(n))$  rapidly converges to zero (the rate is exponential).

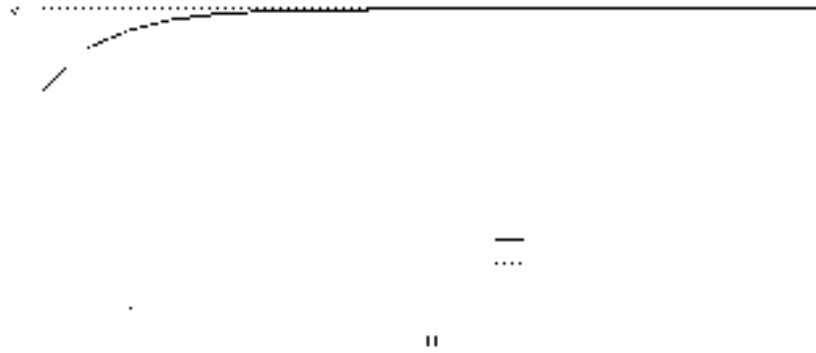
Thus, if the industry is such that the distribution over  $v$  is skewed toward high value technology then the minimum required technology level,  $\underline{v}^\infty$ , gets close to the limit with only a few entrepreneurs. Figure (1) depicts the value of  $\underline{v}(n)$  as a function of  $n$  when the distribution is  $F(v) = v^4$ ,  $r=2$ ,  $d=0$  and  $\alpha = 0.25$ .

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<sup>4</sup> When  $n$  increases development progress decreases for low levels of technology and increases for high level of technology.

<sup>5</sup> The VC knows how to calculate the equilibrium  $e(v)$  and, hence, she can extract  $v$  from  $e$  as an inverse function.

**Figure 1 - The Value of  $\underline{v}(n)$  as a Function Of  $n$**



As we can see from the figure,  $\underline{v}^\infty$  is a good approximation for the minimum technology level required by the VC with as few entrepreneurs as five or six. Moreover, the limit value  $\underline{v}^\infty$  is independent of the shape of the distribution (although the convergence is faster for positively skewed distributions). From (5), the VC expected payoff is given by<sup>6</sup>

$$W = (1 - \alpha)rn \int_{\underline{v}}^1 [e(v) + v]F^{n-1}(v)f(v)dv - (1 + d)(1 - F^n(\underline{v})). \quad (9)$$

Let us find the optimal minimum technology level that maximizes the VC's expected payoff. Observe that this minimum level, although desirable by the VC, is not supported by the sub-game perfect Nash equilibrium. Denote by  $v^*$  the optimal minimum technology level that the VC would like to dictate.

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<sup>6</sup> Observe that in equilibrium if  $e(v)$  is increasing then,  $\max_i (e(v_i) + v_i) = \max_i e(v_i) + \max_i v_i$ .

**Proposition 3:**  $v^* > \underline{v}$ .

Thus, the VC will ideally increase the minimum required level of technology to eliminate weak entrepreneurs above the minimum level that guarantee non-negative payoffs. At the same time, the VC would take in this case the risk that she could end up with nothing if the best entrepreneur is between  $v^*$  and  $\underline{v}$ , the interval where it is still profitable to support the firm. This choice of  $v^* > \underline{v}$  by the VC is not credible (and, thus, essentially is ‘cheap talk’) because nothing prevents her from changing her mind ex-post because she would prefer to invest in a firm with technology level  $v$  such that  $v^* > v \geq \underline{v}$  if this happens to be the maximum she gets from the  $n$  entrepreneur. Thus, if the VC has no way to guarantee that she will not accept technology below  $v^*$ , an entrepreneur with technology  $v^* > v \geq \underline{v}$  may still participate in the contest despite the requirement by the VC, hoping that he will be the one with the highest  $v$  and the VC will still invest in his idea because it is above her breakeven threshold level,  $\underline{v}$ .

Next, we give some characterization of when more entrepreneurs are better (i.e. when the optimal number of entrepreneurs is finite) and show how it depends on the shape of the distribution of types. Denote reverse hazard rate<sup>7</sup> as  $Rhr(v) = \frac{f(v)}{F(v)}$ .  $Rhr(v)$  would be

non-increasing at the maximum technology level if  $Rhr'(1) = \left( \frac{f(v)}{F(v)} \right)'_{v=1} \leq 0$ , which is

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<sup>7</sup> The reverse hazard rate is used in statistics to denote the ratio between the density function to the distribution function and is commonly denoted as  $\sigma_F(v)$ . The ratio is also known as inverse Mills’ ratio.

equivalent to  $\frac{f'(1)}{f^2(1)} \leq 1$ . In the following Proposition we investigate the optimal number

of participating entrepreneurs in the contest for VC funding.

**Proposition 4:** *If the density of types at the maximum technology is large and in addition*

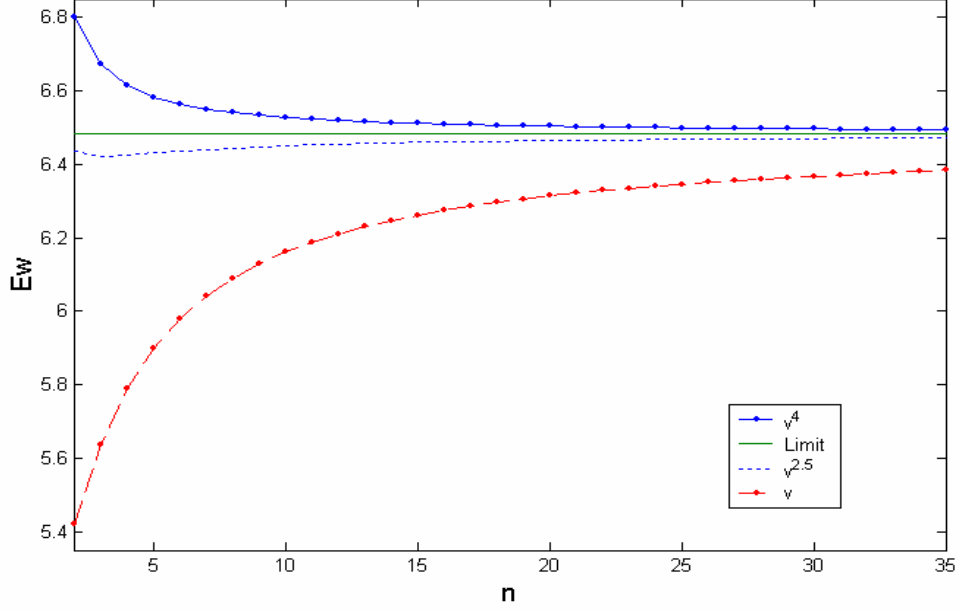
*$Rhr'(1) \leq 0$  then, the optimal number of entrepreneurs in the auction will*

*be finite.*

The above proposition in effect shows that if the density of high level of technology  $f(1)$ , is likely to be high then the optimal number of entrepreneurs is finite (for instance,  $n$  could be 2). Observe that since the distribution is continuous, a large  $f(1)$  implies that the distribution over technology carries great weight near  $v=1$ . A distribution of the form  $F(v) = v^\beta$ ,  $\beta > 1$  has this feature. Sometimes in auctions and contests the revenue for the seller does not monotonically increase with the number of bidders (see for example Moldovanu and Sela (2001)). This, however, is not straightforward in the current model. The firms value in equilibrium depends on the sum of  $e(v)+v$  where the VC takes the maximum over all  $n$  bidders. When  $n$  is increasing, the equilibrium progress function,  $e(v)$ , is decreasing for low  $v$  and increasing for large  $v$ .

In Figure 2 we show that for  $\alpha = 0.2, d = 0, r = 4, F(v) = v^\beta$ , the expected revenue for the VC as a function of  $n$  for  $\beta = 1$  is increasing with the number of entrepreneurs and strictly decreasing with  $n$  if  $\beta = 4$ . Moreover, in the last case the optimal number of entrepreneurs is two. When  $\beta = 2.5$  the expected revenue is not sensitive to the number of entrepreneurs although it starts by decreasing and then increasing with  $n$ .

**Figure 2 – The Expected Payoff of VC as a Function of  $n$**



#### 4. Optimal Contracting

In this section we relax our previous assumption that the sharing rule between the VC and the winning entrepreneur's  $\alpha$ , is exogenously given by the market and let the VC dictate  $\alpha$  before the contest. We also assume that the VC guarantees this level of  $\alpha$  and cannot change her mind later on. We look for sub-game perfect Nash equilibrium assuming that in the forthcoming stage the entrepreneur will play according their equilibrium strategies given the sharing rule  $\alpha$ . In Corollary 2 below we characterize the optimal  $\alpha$ .

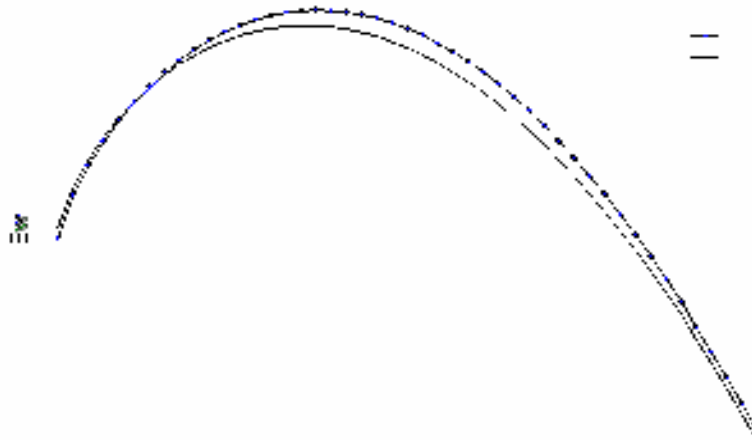
**Corollary 2:** the optimal sharing rule  $\alpha$  satisfy the equation

$$\int_y^1 \left[ (1 - \alpha) \frac{de(v)}{d\alpha} - (e(v) + v) \right] F^{n-1}(v) f(v) dv = 0.$$

Finding a closed-form solution for  $\alpha$  is too complex and thus instead we use the VC's payoff,  $W$ , from (9) to solve numerically for the optimal  $\alpha$ . Obviously, the solution

depends on the distribution  $F(v)$ . However, since for large  $n$  the expected profit for the VC is close to the limit value we can use the limit and obtain an approximate solution, which is independent of the distribution and is still close to the optimal value.<sup>8</sup> Figure 3 depicts the VC's expected profits as a function of the entrepreneur share,  $\alpha$  (dotted line), for 5 entrepreneurs,  $r = 4, d = 0, F(v) = v^4$  where the solid line represents the expected profit of the VC at the limit when the number of entrepreneurs approaches infinity. Figure 3 demonstrates that there is a maximum  $\alpha$  above which there will be diminishing incremental returns for the VC and that the optimal  $\alpha$  is close to the optimal one if we use the limit instead

**Figure 3 – The Expected Payoff of the VC, EW, as a Function of  $\alpha$**



<sup>8</sup> It seems from the proof of Proposition 4 that when the industry is rich with entrepreneurs holding high quality technology (i.e., high density near  $v=1$ ), the convergence is even faster and thus, the approximation is good even for a relatively small number of entrepreneurs.

## 5. Multiple Investments and Competition among VCs

In this section we relax our previous assumption that there is one VC who makes only one investment in a firm. Instead we now assume that the VC has the resources to invest in more than one firm and the amount is identical for all firms. Namely, the VC has  $K$  identical units of resources  $P=1$  and she invest in the  $K$  entrepreneurs with the highest level of progress. A winning entrepreneur obtains, as before,  $\alpha$  of the firm value where  $\alpha$  is pre-announced and identical for all winners. In this case we have a multi-unit auction model but since the demand for each entrepreneur is only for a single unit of investment, the model is similar to the one with a single investment and the equilibrium is given by the following proposition. We assume as before that the VC will not invest if she loses.

**Proposition 5:** *In case of  $K$  identical investment the equilibrium bid function is given by*

$$e(v) = \alpha r G(v) + \sqrt{\alpha^2 r^2 G^2(v) + 2\alpha r \left( vG(v) - \int_{\underline{v}}^v G(s) ds \right)}$$

where  $G(v) = \sum_{j=1}^K \binom{n-1}{j-1} F^{n-j}(v)(1-F(v))^{j-1}$ , is the probability that an

entrepreneur will receive VC funding and  $\underline{v}$  is given in Proposition 1.

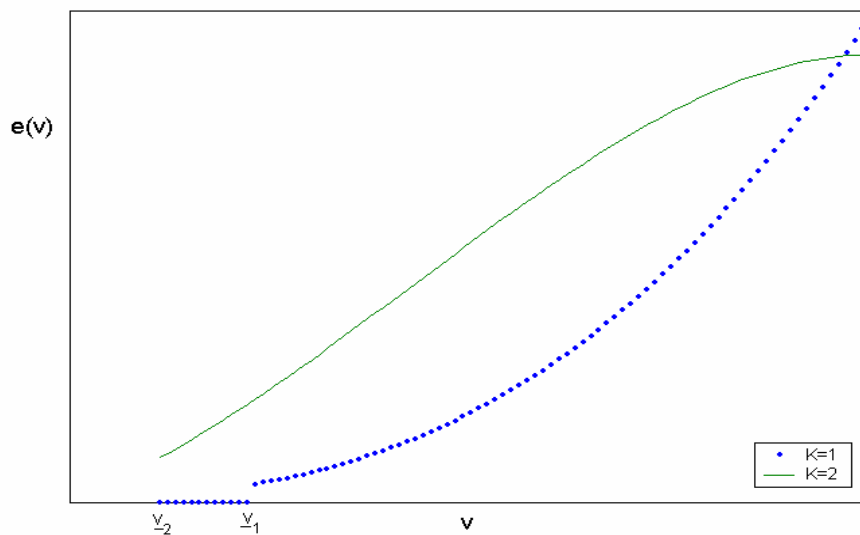
Because the probability of winning for every given technology level  $v$  is increasing with the number of investments,  $K$ , one could expect the level of progress made by an entrepreneur to decrease since the competition on VC funding is less fierce. The answer, however, is not that straightforward because while the entrepreneur with high level of technology (i.e.,  $v$  close to 1) reach a lower development stage when the number of investments  $K$  increases by 1, an entrepreneur with low technology (i.e.,  $v$  close to  $\underline{v}$ )

will make greater progress. Moreover, the minimum technology level required by the VC,  $\underline{v}$ , will be lower in this case.

**Proposition 6:** *Increasing the number of investments  $K$  by the VC would increase the development progress made by a low technology entrepreneurs and decrease the development made by high technology entrepreneurs. Moreover,  $\underline{v}$  decreases with  $K$ .*

The value of  $\underline{v}$  decreases with  $K$  since the development  $e(v)$  increases for low technology levels and thus, the VC can reduce the minimum technology required to guarantee non-negative profits. In Figure 4 we can see the equilibrium progress function for 1 and 2 investments for  $r = 4, \alpha = 0.2, n = 4, d = 0$  and uniform distribution. In this example the progress function for the two investments scenario is above the one relating to a single investment except when the technology parameter,  $v$ , is very close to 1.

**Figure 4 – Development progress for  $K=1,2$**



Using the same example, if the VC has two investments she may decide to invest in two different industries. Assuming that the industries are independent with respect to the entrepreneurs behavior and that she may find  $n=4$  entrepreneurs in each market. We would like to compare the VC's expected profits from two investments in different industries might be lower than focusing on a single industry.<sup>9</sup> This phenomenon however is confusing. On one hand, we have two investments in one industry with 4 entrepreneurs, which should boost the entrepreneurs' willingness to develop since there are more investments available to them (see Figure 4). However, splitting into two industries introduces a total of 8 entrepreneurs, consequently, increasing the possibility for high technology. In our example the expected revenue from one investment in one industry with 4 entrepreneurs is 5.786 and thus, the VC total expected revenue from two industries is  $5.786 \cdot 2 = 11.572$ . When the VC invest in one industry his expected revenue from the first winner is 7.189 and from the second one 5.28 what sums to higher revenue. Observe that in the example the  $f(1)=1$  is not high and thus, the phenomenon is not caused by the increases in  $n$  as we have found in Proposition 4. The practical implication of this result is that VCs could possibly increase their payoff, if they avoid spreading into many industries.

Consider competition among  $K$  VCs with a constant exogenous  $\alpha$  in the same industry each with a single unit of investment. Every entrepreneur in this case would

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<sup>9</sup> This setting is different from the common models in contests. Usually, in contests the focus is on dividing the  $n$  competitors into subgroups where the total number is fixed. Here, the alternative is many groups with the same size, which increases the total number of entrepreneurs.

approach all VCs and thus,<sup>10</sup> leading to a situation of a single VC with  $K$  investments (where  $K$  is the total number of investments available by all VCs) and the analysis above still holds. In this setting, the  $K$  entrepreneurs with the highest progress win. This assumption seems to make sense since the submission is the level of progress. The only piece still missing is the matching between winning entrepreneurs and the VCs (i.e., which VC gets the entrepreneur with the highest progress made, which one gets the second highest and so forth). The mechanism of market clearing in this setting, however, is not covered in our analysis. We learn from the previous example that the total expected profits of all VCs might be higher than if each VC become a monopolist in a different industry. Thus, competition might be beneficial for the VCs.

## 6. Conclusions

Venture capitalists' success depends on their deal flow and the quality of the firms that they invest in. An important insight of this study is that having a large number of entrepreneurs compete simultaneously for the funds of the VC could be dysfunctional. The reason for this is that this situation leads to a lower average development investment by entrepreneurs because they perceive their chances of winning the auction to be relatively slim and their investment in development is costly. The last section in this paper demonstrates that increasing the number of investments by VCs would increase (decrease) the development progress made by low (high) technology entrepreneurs. Moreover, the minimum acceptable technology that is required by the VC decreases with the number of investments made by the VC. In the last section of the paper we also show that competition among VCs on entrepreneurial firms does not affect our previous results.

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<sup>10</sup> We assume that the entrepreneurs submit the same proposal to all VCs.

A possible extension to this paper could involve further investigation of VCs investments in different industries and examine what should be the optimal number of industries that VCs would get into and their characteristics.

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## APPENDIX

### PROOF OF PROPOSITION 1

Assuming that there is an symmetric equilibrium development function  $e(v)$  which is monotonic and differentiable than, the sum of  $v+e(v)$  is monotonic in equilibrium and thus, the winner is the one with the highest level of technology. The probability that entrepreneur  $i$  wins in equilibrium is  $F^{n-1}(v)$ . Thus, from (2), an entrepreneur utility function becomes  $U = \alpha r F^{n-1}(v)(v + e(v)) - 0.5e^2(v)$ . If an entrepreneur diverges and gets to the stage of development  $e = e(\hat{v}) \neq e(v)$  then, his utility is  $U(\hat{v}; v) = \alpha r F^{n-1}(\hat{v})(v + e(\hat{v})) - 0.5e^2(\hat{v})$ . Differentiating  $U$  with respect to  $\hat{v}$  and setting it as zero yields

$$\alpha r(n-1)F^{n-2}(\hat{v})f(\hat{v})(v+e(\hat{v})) + \alpha r F^{n-1}(\hat{v})e'(\hat{v}) - e(\hat{v})e'(\hat{v}) = 0 \quad (\text{A.1})$$

where  $e'(\hat{v}) = \frac{d}{d\hat{v}}e(\hat{v})$ . In equilibrium  $\hat{v} = v$  and thus we obtain the following

differential equation

$$\alpha r(n-1)F^{n-2}(v)f(v)(v+e(v)) + \alpha r F^{n-1}(v)e'(v) - e(v)e'(v) = 0. \quad (\text{A.2})$$

Solving this equation with the initial condition  $U(v) = 0$  obtains the proposition.

For consistency sake we note that the equilibrium bids are monotonic with respect to the technology  $v$ . It is simple to calculate the second order condition

$$\left. \frac{\partial^2 U(\hat{v}; v)}{\partial \hat{v}^2} \right|_{\hat{v}=v} = -\alpha r(n-1)F^{n-2}(v)f(v) < 0 \text{ that verifies that we indeed obtain}$$

equilibrium.  $\square$

## PROOF OF PROPOSITION 2

By the definition of  $\underline{v}(n)$  we have  $e(\underline{v}(n)) + \underline{v}(n) = \frac{1+d}{(1-\alpha)r}$  for every  $n$  and thus the

following obtains (we omit the variable  $n$  from  $\underline{v}(n)$ ),

$$\underline{v} + \alpha r F^{n-1}(\underline{v}) + \sqrt{\alpha^2 r^2 F^{2(n-1)}(\underline{v}) + 2\alpha r \underline{v} F^{n-1}(\underline{v})} = \frac{1+d}{(1-\alpha)r}. \quad (\text{A.5})$$

Observe that the left hand side of (A.5) is increasing with  $\underline{v}$  for fixed  $n$  and decreasing with  $n$  for fixed  $\underline{v}$ . Thus, increasing  $n$  and fixing  $\underline{v}$ , decreases the left hand side of (A.5). To preserve the equality in (A.3) we need to increase  $\underline{v}$ .  $\square$

## PROOF OF COROLLARY 1

Because  $\underline{v}(n) + e(\underline{v}(n)) = \frac{1+d}{(1-\alpha)r}$  and  $(1-\alpha)r > 1+d$  it is easy to verify that  $\underline{v}(n)$  cannot

be equal to 1 and it is strictly bounded below 1. Consequently,  $e(\underline{v}(n))$  is approaching zero when  $n$  increasing, thus, obtaining the corollary in the limit.  $\square$

## PROOF OF PROPOSITION 3

Substituting  $v^*$  instead of  $\underline{v}$  in (9) and differentiate with respect to  $v^*$  provides

$$\begin{aligned} \frac{\partial W}{\partial v^*} = n F^{n-1}(v^*) & \left\{ (1-\alpha)\alpha r^2 \int_{v^*}^1 \frac{\partial e(v)}{\partial v^*} F^{n-1}(v) f(v) dv \right. \\ & \left. - f(v^*) [(1-\alpha)r(e(v^*) + v^*) - (1+d)] \right\}. \end{aligned} \quad (\text{A.6})$$

Observe that at  $v^* = \underline{v}$  the second component is equal to zero and  $\frac{\partial e(v)}{\partial v^*} > 0$ , and, hence,

$$\left. \frac{\partial W}{\partial v^*} \right|_{v^* = \underline{v}} = n F^{n-1}(\underline{v})(1-\alpha)\alpha r^2 \int_{\underline{v}}^1 \frac{\partial e(v)}{\partial v^*} F^{n-1}(v) f(v) dv > 0. \quad (\text{A.7})$$

$\square$

#### PROOF OF PROPOSITION 4

From (9), let us write the VC's expected payoff when she has  $n+1$  bidders;

$$\begin{aligned}
 W &= (1-\alpha)r(n+1) \int_{\underline{v}}^1 [e(v)+v]F^n(v)f(v)dv - (1+d)(1-F^{n+1}(\underline{v})) = \\
 &= (1-\alpha)r(n+1) \int_{\underline{v}}^1 e(v)F^n(v)f(v)dv + (1-\alpha)r(n+1) \int_{\underline{v}}^1 vF^n(v)f(v)dv - (1+d)(1-F^{n+1}(\underline{v})).
 \end{aligned} \tag{A.8}$$

We look for series expansion in  $1/n$ . Thus we will have the following relation

$$W = W_0 + \frac{1}{n}W_1 + O\left(\frac{1}{n^2}\right),$$

where  $W_0 = \lim_{n \rightarrow \infty} W$ . We will show that for  $f(1)$  sufficiently large,  $W_1 > 0$ , which, in turn,

proves that for a large enough  $n$ ,  $W$  is decreasing with  $n$ . We start with the second and third components of (A.8). We integrate the second components by part and use the following lemma.<sup>11</sup>

**Lemma 1** [Fibich et. al. 2004]:  $\int_{\underline{v}}^1 F^{n+1}(y)dy = \frac{1}{n} \frac{1}{f(1)} + O\left(\frac{1}{n^2}\right)$ .

After integrating by parts the second component of (A.8) using Lemma 1 and summing with the third component of (A.8) we have,

$$\begin{aligned}
 &(1-\alpha)r(n+1) \int_{\underline{v}}^1 vF^n(v)f(v)dv - (1+d)(1-F^{n+1}(\underline{v})) = \\
 &= [(1-\alpha)r - (1+d)] - \frac{1}{n+2} \frac{(1-\alpha)r}{f(1)} + O\left(\frac{1}{n^2}\right) \\
 &= [(1-\alpha)r - (1+d)] - \frac{1}{n} \frac{(1-\alpha)r}{f(1)} + O\left(\frac{1}{n^2}\right).
 \end{aligned} \tag{A.9}$$

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<sup>11</sup> For more details on the method see De Bruijn (1981) and Fibich et. al. (2004)).

Observe that  $O(1/n^2)$  contains elements such as  $F^{n+1}(v)$  which, relative to  $1/n^2$ , are exponentially small.<sup>12</sup> After substituting the first component in (A.8) we have

$$\begin{aligned} & \frac{1}{(1-\alpha)r(n+1)} \int_{\underline{v}} e(v) F^n(v) f(v) dv = \\ & = (1-\alpha)r(n+1) \int_{\underline{v}} \left( \alpha F^n(v) + \sqrt{\alpha^2 r^2 F^{2n}(v) + 2\alpha r \left( v F^n(v) - \int_{\underline{v}}^v F^n(y) dy \right)} \right) F^n(v) f(v) dv. \end{aligned}$$

The first component in the integral gives (using again the same approach as in Lemma 1):

$$\begin{aligned} & (1-\alpha)\alpha r^2 (n+1) \int_{\underline{v}} F^{2n}(v) f(v) dv = (1-\alpha)\alpha r^2 \frac{(n+1)}{2n+1} + O\left(\frac{1}{n^2}\right) = \\ & = \frac{1}{2}(1-\alpha)\alpha r^2 + (1-\alpha)\alpha r^2 \frac{1}{4n} + O\left(\frac{1}{n^2}\right). \end{aligned} \tag{A.10}$$

For the other part we are going to apply the Laplace method (see De Bruijn (1981)).

$$\begin{aligned} A &= (1-\alpha)r(n+1) \int_{\underline{v}} \left( \sqrt{\alpha^2 r^2 F^{2n}(v) + 2\alpha r \left( v F^n(v) - \int_{\underline{v}}^v F^n(y) dy \right)} \right) F^n(v) f(v) dv = \\ &= (1-\alpha)r(n+1) \int_0^{1-\underline{v}} \sqrt{\alpha^2 r^2 F^n(1-s) + 2\alpha r \left( (1-s) - \frac{\int_{\underline{v}}^{1-s} F^n(y) dy}{F^n(1-s)} \right)} F^{1.5n}(1-s) f(1-s) ds = \\ &= (1-\alpha)r(n+1) \int_0^{1-\underline{v}} \sqrt{\alpha^2 r^2 F^n(1-s) + 2\alpha r \left( (1-s) - \frac{1}{n} \frac{F(1-s)}{f(1-s)} \right)} + O\left(\frac{1}{n^2}\right) \times \\ & \quad \times F^{1.5n}(1-s) f(1-s) ds = \tag{The} \\ &= (1-\alpha)r(n+1) \int_0^{1-\underline{v}} \sqrt{\alpha^2 r^2 e^{n \ln F(1-s)} + 2\alpha r - 2\alpha r \left( s + \frac{1}{n} \frac{F(1-s)}{f(1-s)} \right)} + O\left(\frac{1}{n^2}\right) \times \\ & \quad \times e^{1.5n \ln F(1-s)} f(1-s) ds. \end{aligned}$$

first equality follows from taking out  $F^n(v)$  from the square root and substitution of

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<sup>12</sup> By the assumption,  $\underline{v}$  is bounded below 1. Else, the VCs' profits are identically zero and thus, all the analysis is meaningless.

$v = 1 - s$  (observe that  $dv = -ds$  what inverse the integral boundaries). The second equality

follows from the relation  $\int_{\underline{v}}^{1-s} F^n(y) dy = \frac{1}{n} \frac{F^{n+1}(1-s)}{f(1-s)} + O\left(\frac{1}{n^2}\right)$  that is obtained similarly

to the one in Lemma 1 (see Fibich et. al. (2004)). Observe that  $F^{1.5n}(1-s)$  rapidly decreases for positive  $s$ . Thus, most of the mass of the integral obtains near  $s=0$  where the exponent obtain its maximum. We use the following expansions near  $s=0$ ;

$$\begin{aligned} \ln F(1-s) &= -sf(1) + O(s^2), \\ \frac{F(1-s)}{f(1-s)} &= \frac{1 - sf(1) + O(s^2)}{f(1) - sf'(1) + O(s^2)} = \frac{1 - sf(1) + O(s^2)}{f(1)} \frac{1}{1 - \frac{sf'(1)}{f(1)} + O(s^2)} = \\ &= \frac{1 - sf(1) + O(s^2)}{f(1)} \left( 1 + \frac{sf'(1)}{f(1)} + O(s^2) \right) = \frac{1}{f(1)} - s + s \frac{f'(1)}{f^2(1)} + O(s^2). \end{aligned}$$

Expanding the limit from  $1-\underline{v}$  to infinity makes only a very small difference since all the mass is near zero thus, we can shift the difference to the  $O(1/n^2)$ . Since the mass is near zero, we can include the  $O(s^2)$  terms in the exponent in the  $O(s^2)$  and write

$e^{n \ln F(1-s)} = e^{-snf(1) + O(s^2)}$ . Thus, using  $(n+1) \int_0^\infty e^{-1.5snf(1)} O(s^2) ds = O(1/n^2)$  we

have,

$$\begin{aligned} A &= (1-\alpha)r(n+1) \times \int_0^\infty \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r - 2\alpha r \left( \frac{1}{nf(1)} + s \left( 1 - \frac{1}{n} + \frac{1}{n} \frac{f'(1)}{f^2(1)} \right) \right)} + O(s^2) + O\left(\frac{1}{n^2}\right) \times \\ &\times e^{-1.5snf(1)} (f(1) - sf'(1) + O(s^2)) ds = \\ &= (1-\alpha)r(n+1) \times \int_0^\infty \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r - 2\alpha r \left( \frac{1}{nf(1)} + s \right)} + O(s^2) + O(s/n) + O\left(\frac{1}{n^2}\right) \times \\ &\times e^{-1.5snf(1)} (f(1) - sf'(1)) ds + O\left(\frac{1}{n^2}\right). \end{aligned}$$

For large  $n$  and small  $s$ , the term  $2\alpha r \left( \frac{1}{nf(1)} + s \right)$  is arbitrarily small and we can use the

expansion  $\sqrt{a-x} = \sqrt{a} - \frac{x}{2\sqrt{a}} + O(x^2)$  for small  $x$  namely,

$$\begin{aligned} & \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r - 2\alpha r \left( \frac{1}{nf(1)} + s \right) + O(s^2) + O(s/n) + O\left(\frac{1}{n^2}\right)} = \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r} - \\ & \frac{\alpha r \left( \frac{1}{nf(1)} + s \right) + O(s^2) + O(s/n) + O\left(\frac{1}{n^2}\right)}{\sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r}} + O(s^2) + O\left(\frac{1}{n^2}\right) + O\left(\frac{s}{n}\right) = \\ & = \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r} - \frac{\alpha r \left( \frac{1}{nf(1)} + s \right)}{\sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r}} + O(s^2) + O\left(\frac{1}{n^2}\right) + O\left(\frac{s}{n}\right). \end{aligned}$$

It is easy to verify that all the terms of order  $s^2, s/n, 1/n^2$  yield  $O\left(\frac{1}{n^2}\right)$  after integration

and thus, we have

$$\begin{aligned} A &= (1-\alpha)r(n+1) \times \int_0^\infty \left( \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r} - \frac{\alpha r \left( \frac{1}{nf(1)} + s \right)}{\sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r}} + O(s^2) + O\left(\frac{s}{n}\right) + O\left(\frac{1}{n^2}\right) \right) \times \\ & \times e^{-1.5snf(1)} (f(1) - sf'(1)) ds + O\left(\frac{1}{n^2}\right) = (1-\alpha)r(n+1) \times \int_0^\infty \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r} e^{-1.5snf(1)} (f(1) - sf'(1)) ds - \\ & (1-\alpha)r(n+1) \times \int_0^\infty \frac{\alpha r \left( \frac{1}{nf(1)} + s \right)}{\sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r}} e^{-1.5snf(1)} f(1) ds + O\left(\frac{1}{n^2}\right). \\ & (1-\alpha)r(n+1) \times \int_0^\infty \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r} e^{-1.5snf(1)} f(1) ds = \\ & = (1-\alpha)r \frac{\left( (\alpha\alpha+1)\sqrt{\alpha^2 r^2 + 2\alpha r} - \ln\left(1 + \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r}\right) \right)}{2\alpha r} + \\ & + (1-\alpha)r \frac{1}{n} \frac{\left( (\alpha\alpha+1)\sqrt{\alpha^2 r^2 + 2\alpha r} - \ln\left(1 + \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r}\right) \right)}{2\alpha r} + O\left(\frac{1}{n^2}\right). \end{aligned}$$

Since  $e^{-snf(1)} < 1$  we can bound

$$\begin{aligned} (1-\alpha)r(n+1) \int_0^\infty \sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r} e^{-1.5snf(1)} sf'(1) ds &< (1-\alpha)r(n+1) \sqrt{\alpha^2 r^2 + 2\alpha r} \int_0^\infty e^{-1.5snf(1)} sf'(1) ds = \\ &= \frac{(1-\alpha)r \sqrt{\alpha^2 r^2 + 2\alpha r}}{1.5^2 n} \frac{f'(1)}{f^2(1)} + O\left(\frac{1}{n^2}\right). \end{aligned} \quad (\text{A.12})$$

Since  $e^{-snf(1)} > 0$  we can bound  $\frac{\alpha r \left( \frac{1}{nf(1)} + s \right)}{\sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r}} < \frac{\alpha r \left( \frac{1}{nf(1)} + s \right)}{\sqrt{2\alpha r}}$  and thus,

$$\begin{aligned} (1-\alpha)r(n+1) \int_0^\infty \frac{\alpha r \left( \frac{1}{nf(1)} + s \right)}{\sqrt{\alpha^2 r^2 e^{-snf(1)} + 2\alpha r}} e^{-1.5snf(1)} f(1) ds &< \frac{(1-\alpha)r(n+1) \sqrt{\alpha r} f(1)}{\sqrt{2}} \int_0^\infty \left( \frac{1}{nf(1)} + s \right) e^{-1.5snf(1)} ds = \\ &= \frac{(1-\alpha)r \sqrt{\alpha r}}{\sqrt{2}} \int_0^\infty e^{-1.5snf(1)} ds + \frac{(1-\alpha)rn f(1) \sqrt{\alpha r}}{\sqrt{2}} \int_0^\infty s e^{-1.5snf(1)} ds + O(1/n^2) = \frac{(1-\alpha)r \sqrt{\alpha r}}{\sqrt{2}} \frac{1}{nf(1)} \frac{10}{9} + O(1/n^2). \end{aligned} \quad (\text{A.13})$$

From (A.9)-(A.13) we find that

$$\begin{aligned} W_1 &> -\frac{(1-\alpha)r}{f(1)} + (1-\alpha)\alpha r^2 \frac{1}{4} + (1-\alpha)r \frac{\left( (\alpha r + 1) \sqrt{\alpha^2 r^2 + 2\alpha r} - \ln \left( 1 + \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r} \right) \right)}{2\alpha r} \\ &\quad - \frac{(1-\alpha)r \sqrt{\alpha^2 r^2 + 2\alpha r}}{1.5^2} \frac{f'(1)}{f^2(1)} - \frac{(1-\alpha)r \sqrt{\alpha r}}{\sqrt{2}} \frac{1}{f(1)} \frac{10}{9} = -(1-\alpha)r \left( 1 + \frac{\sqrt{\alpha r} 10}{\sqrt{2} 9} \right) \frac{1}{f(1)} + \\ &\quad + (1-\alpha)r \left( \frac{\alpha r}{4} + \frac{\left( (\alpha r + 1) \sqrt{\alpha^2 r^2 + 2\alpha r} - \ln \left( 1 + \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r} \right) \right)}{2\alpha r} - \frac{\sqrt{\alpha^2 r^2 + 2\alpha r}}{1.5^2} \frac{f'(1)}{f^2(1)} \right) \end{aligned}$$

For sufficiently large  $f(1)$  the first term is arbitrarily small. It left to show that the second

term is positive. Observe that even for large  $f(1)$  the relation  $\frac{f'(1)}{f^2(1)}$  still might be

significant. Thus we need the assumption  $\frac{f'(1)}{f^2(1)} \leq 1$  or  $Rhr'(v)|_{v=1} \leq 0$ .<sup>13</sup> The second

component gives

$$\begin{aligned} & \frac{\alpha r}{4} + \frac{\left( (\alpha r + 1)\sqrt{\alpha^2 r^2 + 2\alpha r} - \ln\left(1 + \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r}\right) \right)}{2\alpha r} - \frac{\sqrt{\alpha^2 r^2 + 2\alpha r}}{1.5^2} \frac{f'(1)}{f^2(1)} > \\ & > \frac{\sqrt{\alpha^2 r^2 + 2\alpha r}}{2} - \frac{\sqrt{\alpha^2 r^2 + 2\alpha r}}{2.25} \frac{f'(1)}{f^2(1)} + \frac{\left( \sqrt{\alpha^2 r^2 + 2\alpha r} - \ln\left(1 + \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r}\right) \right)}{2\alpha r} > \\ & > \sqrt{\alpha^2 r^2 + 2\alpha r} \left( \frac{1}{2} - \frac{1}{2.25} \right) + \frac{\left( \sqrt{\alpha^2 r^2 + 2\alpha r} - \ln\left(1 + \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r}\right) \right)}{2\alpha r}. \end{aligned}$$

Observe that the first component is positive and thus, it left to show that the second is

also positive. Define  $y(x) = \sqrt{x^2 + 2x} - \ln(1 + x + \sqrt{x^2 + 2x})$ , it is simple to verify that

$$\lim_{x \rightarrow 0} y(x) = 0 \text{ and } y'(x) > 0 \text{ for } x > 0.$$

We have found that  $W_1 > 0$  and thus,  $W$  is decreasing with  $n$  for large  $n$ .  $\square$

## PROOF OF COROLLARY 2

Note that  $\underline{v}$  is a function of  $\alpha$ . By differentiation of  $W$  (see (9)) with respect to  $\alpha$  and

using the minimum technology level rule  $e(\underline{v}) + \underline{v}(n) = \frac{1+d}{(1-\alpha)r}$  we obtain the result.  $\square$

## PROOF OF PROPOSITION 5

The proof is similar to Proposition 1 in the appendix where we replace the probability of

wining  $F^{n-1}(v)$  by the probability of winning with  $K$  investments made by the VC,  $G(v)$ .

Observe that the value of the minimum technology level  $\underline{v}$  is dictated by the same

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<sup>13</sup> We can give a little bit weaker assumption but it will not make any significant difference.

equation as before,  $\underline{v} + e(\underline{v}) = (1 + d)/[(1 - \alpha)r]$  since the VC can and will avoid any investment that will cause losses.  $\square$

### PROOF OF PROPOSITION 6

Define  $\underline{v}_K$  as the minimum technology when the number of investments made by the VC

is  $K$ . From Proposition 5 define  $g(x; K) = e(\underline{v})|_{\underline{v}=x} = \alpha r G(x) + \sqrt{\alpha^2 r^2 G^2(x) + 2\alpha r x G(x)}$

and since  $G$  increasing with  $K$  we find that  $g(x; K) < g(x; K + 1)$  in addition,  $g(x; K)$

increasing with  $x$ . Since  $\underline{v}_K + e(\underline{v}_K) = \underline{v}_{K+1} + e(\underline{v}_{K+1}) = (1 + d)/(1 - \alpha)r$  it follows that

$\underline{v}_K + g(\underline{v}_K; K) = \underline{v}_{K+1} + g(\underline{v}_{K+1}; K + 1)$ . Thus  $\underline{v}_K + g(\underline{v}_K; K) < \underline{v}_K + g(\underline{v}_K; K + 1)$  and by

the monotonicity of  $x + g(x; K)$  with  $x$  it follows that  $\underline{v}_K > \underline{v}_{K+1}$ . Observe that it also

follows that  $g(\underline{v}_K; K) < g(\underline{v}_{K+1}; K + 1)$ . Thus,  $e(v)$  is higher for  $K + 1$  investments for all  $v$

in  $[\underline{v}_{K+1}, \underline{v}_K]$  ( $e$  is zero in this range for  $K$  investments) and by continuity, from

$g(\underline{v}_K; K) < g(\underline{v}_{K+1}; K + 1)$  the result is followed. For  $v$  slightly above  $\underline{v}_K$  For  $v=1$ ,

$e(1) = \alpha r + \sqrt{\alpha^2 r^2 + 2\alpha r \left(1 - \int_{\underline{v}}^1 G(s) ds\right)}$  and thus, since  $G$  increases with  $K$ ,  $e(1)$

decreases. Again, by continuity  $e(v)$  is lower for values that close to 1.  $\square$