Minimum Acceleration with Constraints of Center of Mass: A Unified Model for Arm Movements and Object Manipulation

Abbreviated title: Minimum Acceleration with Constraints of Center of Mass

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Abstract

Daily interaction with the environment consists of moving with or without objects. Increasing interest in both types of movements drove the creation of computational models to describe reaching movements and later to describe a simplified version of object manipulation. The previous suggested models for object manipulation rely on the same optimization criteria as models for reaching movements, yet there is no single model accounting for both tasks which don’t require re-minimization of the criterion for each environment.

We suggest a unified model for both cases: A minimum acceleration with constraints for the center of mass (MACM). For point-to-point reaching movement, the model predicts the typical rectilinear path and bell shaped speed profile as previous criteria. We derive the predicted trajectories for the case of manipulating a mass-on-spring, and show that the predicted trajectories match the observations of a few independent previous experimental studies of human arm movement during a mass-on-spring manipulation. Moreover, the previously reported “unusual” trajectories are also well accounted for by the proposed MACM.

We have tested the predictions of the MACM model in three experiments with 12 subjects where we demonstrated that the MACM model is equal or better (Wilcoxon sign-rank test P<0.001) in accounting for the data than three other previously proposed models in the conditions tested. Altogether, the MACM is currently the only model accounting for reaching movements with or without external degrees of freedom. Moreover, it provides predictions about the intermittent nature of the neural control of movements and about the dominant control variable.

Keywords: reaching movement, object manipulation, minimum acceleration, center of mass.
Introduction

Consider a waiter serving a full cup of coffee, and his colleague reaching to grasp and lift an empty cup. Clearly the tasks are different as the first requires more dexterity than the second, however, the brain may employ similar principles in the motor planning of both tasks. In the motor control research a simplified version of the second task is the well studied unconstrained reaching movement (Abend et al. 1982; Flash and Hogan 1985; Morasso 1981), whereas a simplified version of the first task is the manipulation of mass-on-spring (Figure 1) during reaching movement (Dingwell et al. 2002).

It was hypothesized that smoothness is a primary goal of the motor system. According to this hypothesis, optimization criteria were suggested for reaching, e.g. minimum hand jerk (Flash and Hogan 1985), and later for object manipulation, e.g., Minimum Object Crackle (MOC) (Dingwell et al. 2004), Minimum Hand Jerk (MHJ) and Minimum Hand driving Force Change (MHFC) (Svinin et al. 2005).

Although providing logical solutions for object manipulation, as detailed in the methods section, the minimum object crackle model (Dingwell et al. 2004; Huegel et al. 2009; Svinin et al. 2006) or the hand jerk model (Svinin et al. 2005) don’t fit well to experimental data for some mass and spring values. Moreover, the minimum object crackle model is specific to the mass-on-spring task and cannot be extended to multiple masses (Svinin et al. 2006), while the solution of MHFC for reaching movements may be unstable as it is a numeric solution calculated using iterations scheme.

The common feature for all criteria is the need to re-minimize the criteria under the dynamic constraints imposed by the environment e.g. for reaching, simple object manipulation, or complex object manipulation, i.e. multiple masses. Therefore we developed a single model that covers all these cases without re-minimization of the criterion based on the environment.

Recently, a model based on minimizing hand acceleration was purposed to account for reaching movements. This model suggested minimizing hand acceleration while constraining the maximum value of hand jerk (Ben-Itzhak and Karniel 2008). The minimization results in a three part constant intermittent control signal, which depends on the jerk constraint. The ability to account for experimental results (Berret et al. 2011), the prediction of intermittence control and the biological reasoning of sensing and computing acceleration, make this model a serious candidate to explain the nature of reaching movements.

Most studies of reaching movements concentrated on the hand trajectory. Suzuki et al. (1997) questioned the role of the endpoint during reaching planning; while considering the complex configuration of the hand, comparing reaching movements between different points revealed that while the endpoint trajectory may be altered, the hand center of mass (CoM) trajectory remains invariant emphasizing the important role of the CoM has in planning reaching movements.

In this study we propose a single smoothness minimization criterion to account for reaching movements with or without external degrees of freedom, a minimum acceleration of the center of mass (MACM). We derive the prediction of the MACM for the CoM trajectory of a mass-on-spring. We demonstrate that our model accounts for previously reported trajectories including trajectories unaccountable by other criteria. Moreover, we report new experimental results demonstrating the superiority of our model.

Insert Figure 1 here
Methods

Object Manipulation Task

We used the same task as originally suggested in (Dingwell et al. 2002). The task consists of transporting a mass-on-spring attached to the hand from initial point to a target point in the horizontal plane (Figure 1). This environment sets the following dynamics derived by Newton Second Law:

object motion equation:
\[ k(x_o - x_h) = m_o \ddot{x}_o \]

hand motion equation:
\[ F + k(x_o - x_h) = m_h \ddot{x}_h \]

where \( F \) indicates the forces the hand is generating during movement.

The primary objective of transferring both hand and object between points sets twelve boundary conditions which depends on the movement duration, \( T \), and movement length, \( L \):

For the hand:
\[ x_h(t = 0) = 0, \quad \dot{x}_h(t = 0) = \ddot{x}_h(t = 0) = 0 \]
\[ x_h(t = T) = L, \quad \dot{x}_h(t = T) = \ddot{x}_h(t = T) = 0 \]

For the object:
\[ x_o(t = 0) = 0, \quad \dot{x}_o(t = 0) = \ddot{x}_o(t = 0) = 0 \]
\[ x_o(t = T) = L, \quad \dot{x}_o(t = T) = \ddot{x}_o(t = T) = 0 \]

To capture the smoothness character for movement with the object, the problem was defined as an optimization problem in which a cost function of state variables is minimized (Dingwell et al. 2004; Flash and Hogan 1985; Hogan 1984). The minimization is achieved using Euler-Poisson equation result in an expression for object position. To get the hand position, the different expressions for the object position is substitute into the object motion equation (1). Minimum Object Crackle Model (MOC). Dingwell et al. suggested minimizing the fifth derivative of object’s position- the object Crackle (Dingwell et al. 2004):

\[ \int_0^T \left[ \frac{d^5}{dt^5} (x_o) \right]^2 dt \]

The resulting object position model depends on the movement duration and movement length. After substituting the expression for object position into equation (1) we get the resulted hand position model which depends on the ratio between object mass and spring constant as well as on the movement length and duration. The solution for minimizing this criterion indeed satisfies the boundary conditions (3) and can be considered to be a reasonable descriptive model for the object manipulation task.

As discussed in (Dingwell et al. 2004) if the ratio between object mass and spring remain constant, the model predicts identical hand kinematics. As described below we have designed three experimental conditions to test this prediction.

To describe reaching movements we can change the criteria, for example, minimization of hand crackle. However, for more complex environments, like a chain of masses-on-springs, the minimum object crackle doesn’t satisfy the boundary conditions requiring higher order derivatives for the optimization criterion (Svinin et al. 2006). Altogether, the minimum object crackle model seems to be specific to the mass-on-spring task as it needs to be altered according
to the task and environment resulting in different criteria defined in different sets of coordinates (hand or object) while using different derivatives order (e.g. Crackle, Snap or higher derivatives).

**Minimum Hand Jerk Model (MHJ).** Another criterion, suggested by Svinin et al., is the minimum hand jerk model, minimizing the hand position third derivative (Svinin et al. 2006; Svinin et al. 2005):

\[
\int_0^T \left[ \frac{d^3(x_h)}{dt^3} \right]^2 dt
\]

This criterion was used to describe unconstrained reaching movements (Flash and Hogan 1985), however, for object manipulation task, the object motion equation (1) must be used, transforming criterion (5) to:

\[
\int_0^T \left[ \frac{m_o}{k} \frac{d^3(x_o)}{dt^3} + \frac{d^3(x_o)}{dt^3} \right]^2 dt
\]

The resulted hand position model depends on the ratio between object mass and spring constant as well as on the movement length and duration. Similar to the Minimum Object Crackle model, the hand kinematics predicted by this model won’t change as long as the spring-object mass ratio kept constant.

Although the optimization criterion doesn’t change between the two tasks of simple reaching and object manipulation, there is a need for re-minimization of the criterion for each of these tasks and again for more complex objects. Moreover, this criterion couldn’t account for some of the previous experimental results, in the sense of accurate prediction of the number of phases exhibited during movement, as demonstrated in the work of (Svinin et al. 2005).

**Minimum Hand Driving Force Change Model (MHFC).** A third model, suggested by (Svinin et al. 2005), considered the force the hand is producing during movement with the object, \( F \). This variable is an equivalent representation of the joints torque as suggested by (Uno et al. 1989). According to this model the optimal trajectory is found by minimizing the square changes in force:

\[
\int_0^T \left[ \dot{F}^2 \right] dt
\]

The suggested cost function is then transformed using the hand motion equation (2) and object equation (1):

\[
\int_0^T \left[ \frac{m_h m_o}{k} \frac{d^5(x_o)}{dt^5} + (m_h + m_o) \frac{d^3(x_o)}{dt^3} \right]^2 dt
\]

This model also depends on the hand mass value, unlike models resulted from criterion (4) and (5), in addition to the dependency on the object mass, spring constant and movement length and duration. Unlike the previous models, keeping the object mass-spring ratio constant while changing each parameter value will create different predictions of hand kinematic by this model. Criterion (7) can also be used to describe unconstraint reaching movements. Unlike the process of transferring criterion (7) to criterion (8) for object manipulation, for reaching movement there is no need of such transformation. Without an external object the optimization problem need to be resolved according to criterion (7) and not criterion (8) i.e. there is a need to find the optimal trajectory by re-minimization of the criterion. As shown in (Svinin et al. 2005) the solution of minimizing this criterion for unconstraint reaching movement is a numerical one similar to the
iterative scheme solution of the minimum joints torque change model (Uno et al. 1989). As suggested in the same study the solution isn’t guaranteed as it may not converge. Expending the criterion to more complex environments is done in a similar way to the Minimum Hand Jerk model suggesting that there is a need to re-minimize the criterion for each environment and task. The Minimum Acceleration with constraints applied to the Center of Mass (MACM). We propose to examine a state variable that combines both hand and object namely the system center of mass (CoM). As in previous studies, where the hand complex configuration and dynamics is ignored and the hand is considered as a point mass, the system center of mass is defined as:

\[
x_{cm} = \frac{m_h x_h + m_o x_o}{m_h + m_o}
\]

This assumption overlooking the complex dynamics of the hand and treating it as a point mass is not new (Svinin et al. 2005) and serves as a first approximation of the hand. Moreover, since the feedback given to the subject in this and in previous studies is a visual point at the location of the hand, the subject is not required to control its hand configuration. To achieve smooth trajectory of the hand and object we minimize the CoM acceleration from an initial to a final position in a given time \(T\):

\[
\int_0^T [\dddot{x}_{cm}]^2 dt
\]

To solve the optimization problem we suggest using the solution of the minimum acceleration with constraints criterion (MACC) for reaching movements (Ben-Itzhak and Karniel 2008). This will provide us with a description of the CoM trajectory thus solving the optimization problem for the mass-on-spring system. By using this solution, derived for unconstraint reaching movements, we don’t need to resolve the optimization problem given the new environment. The position of the CoM is given by:
This analytic solution provides a descriptive model of the CoM trajectory. This description is divided into three time intervals. The position of the CoM in each time interval is given by a polynomial and its related coefficients $c_j$, where $i$ is time interval index and $j$ is the coefficient number index. As discussed in details in the original paper, this model suggested minimizing the acceleration while constraining the maximum value of the jerk, $u$. The model is derived using Pontryagin’s minimum principle resulted in a three part constant intermittent control signal, which depends on the jerk constraint, $u$, providing a prediction about the intermittence nature of the control signal.

Extracting the object position from equation (9) and substituting in equation (1) we obtain the following ordinary differential equation linking the hand position and its derivatives to the CoM position and its derivatives.

\[
\dot{x}_h + \beta \ddot{x}_h = \alpha \dddot{x}_c + \beta \dddot{x}_c
\]

\[
\beta = k \left( \frac{m_o + m_h}{m_h m_o} \right) = \omega^2
\]

\[
\alpha = \left( 1 + \frac{m_o}{m_h} \right)
\]
To get the hand position we simply need to solve the above ordinary differential equation. Since
the CoM position is divided into three time intervals (11), the solution of the hand position will
also be divided into three time intervals:

\[
\begin{align*}
\varphi_1(t) &= A_1 \cos(\omega t) + A_2 \sin(\omega t) + \sum \left( \frac{1}{6} c_0 t^3 \right) \frac{c_1 t^2}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} , \quad 0 \leq t \leq t_1 \\
\varphi_2(t) &= A_1^2 \cos(\omega t) + A_2^2 \sin(\omega t) + \sum \left( \frac{1}{6} c_0 t^3 \right) \frac{c_1 t^2}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} , \quad t_1 \leq t \leq t_2 \\
\varphi_3(t) &= A_1^3 \cos(\omega t) + A_2^3 \sin(\omega t) + \sum \left( \frac{1}{6} c_0 t^3 \right) \frac{c_1 t^2}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} + \sum \left( \frac{1}{2} c_1 t^2 \right) \frac{c_3 t}{\beta} , \quad t_2 \leq t \leq T
\end{align*}
\]

where:

\[
\begin{align*}
A_1^1 &= 0 \\
A_2^1 &= -\frac{1}{c_0} \left( \frac{\alpha - 1}{\beta \omega} \right) \\
A_1^2 &= \left( a - 1 \right) \left( 3 c_0 \sin(\omega T - \omega t_2) \sin(\omega t_1) + 1 c_0 \sin(\omega t_2) \sin(\omega t_1) \right) \\
A_2^2 &= \left( a - 1 \right) \left( 3 c_0 \sin(\omega t_2) \sin(\omega t_1) \sin(\omega t_2) \sin(\omega t_1) \right) \left( a - 1 \right) \\
A_3^3 &= \left( a - 1 \right) \left( 3 c_0 \sin(\omega T) \sin(\omega t_2) \sin(\omega t_1) \right) \left( a - 1 \right) \\
A_4^4 &= \left( a - 1 \right) \left( 3 c_0 \sin(\omega T) \sin(\omega t_2) \sin(\omega t_1) \right) \left( a - 1 \right)
\end{align*}
\]

The model depends on the object mass, spring constant and movement length and duration as
well on the hand mass value.

We have derived this solution by standard methods of finding the homogenous solution by
solving the characteristic equation and then for each time interval, using the method of
undetermined coefficient to find particular solution (explanation of the method see e.g. (Boyce
and DiPrima 1970)). It is interesting to note that during this procedure we had initially 6
boundary conditions and 6 continuity conditions, however 5 were redundant, namely identical to
one of the other 7 conditions and therefore we are left with 7 equations and 7 parameters to
obtain the single solution in (13). Nevertheless, since ordinary differential equation has unique
solution, one can simply substitute our solution into the equation (12) to prove it. These 7
parameters are the six undetermined coefficients \{A_1^1, A_2^1, A_2^2, A_3^3, A_4^4\} and the Jerk constraint,
\(u\). The expression for the last parameter is not given explicitly since it is a complex expression
which contains many terms; however one can simply solve the high order polynomial equation
by factoring the polynomial or using numeric methods such as Newton method.
A basic requirement of any model is to provide logical description for one degree of freedom object manipulation while using extreme values of the mass and spring. This can be achieved while changing the spring and mass value to create different manipulation scenarios. For example, when the movement is performed without the additional mass, i.e. \( m_o = 0, k = 0 \), the model describing the hand trajectory converge to the MACC as expected in order to describe this simple reaching movement. Another case is when \( k \to \infty \) which can be considered as moving while holding a rigid object, the model again converges to the MACC model as expected since the object position coincides with the hand position.

**Phase analysis of the MACM velocity profiles.** In order to design the experiment and test the predictions of the various models we have analyzed the expected velocity profiles and in particular the local minima and maxima values. Changes between local maximum and minimum speed values or vice versa are referred as phase transitions (Svinin et al. 2006). This analysis is performed on simulation results of the model predictions provided by equation (13). Since the MACM model predictions depends on the system parameters: spring constant, external mass, hand mass, movement length and duration we couldn’t create a single illustration capturing the effect each parameter have on the velocity profile. To find how changes in a particular constant value affect the number of phases, one can construct a two variables function of the hand velocity which depends on time and the constant in question. For example, we illustrated the velocity profiles as a function of the hand mass while keeping a fixed value of the other constants (Figure 2).

Finding the saddle points in such illustrations will provide the critical value in which phase transition occurs. These saddle points satisfy the condition where both the acceleration and jerk become zero (Dingwell et al. 2004; Svinin et al. 2005)

\[
\begin{align*}
\ddot{x}_h(t) &= 0 \\
\dddot{x}_h(t) &= 0
\end{align*}
\]

The velocity extremum is found by setting its derivative to zero, i.e. finding in which time points the acceleration is equal to zero. After finding these critical points we want to decide whether there is a local maximum, minimum or saddle points. This is done by calculating the second derivative of velocity at these points. If the jerk is zero this is a saddle point otherwise it is a local minimum or maximum (Apostol 1967).

This phase analysis can be performed using the second and third derivative of the simulated hand position provided in expression (13).

Insert Figure 2 here
Extension of the MACM model to more complex environments.

To further examine the abilities of the MACM model and its advantage over previous models, we consider two parallel mass-on-spring objects attached to the hand (Figure 3) and demonstrate that the minimum hand jerk and minimum object crackle cannot be extended to account for this case. This environment sets the following dynamics:

The motion equation of the first mass, $m_1$:

$$m_1\ddot{x}_1 = k_1(x_h - x_1)$$

The motion equation of the second mass, $m_2$:

$$m_2\ddot{x}_2 = k_2(x_h - x_2)$$

The motion equation of the hand, $m_h$:

$$m_h\ddot{x}_h = F + k_1(x_1 - x_h) + k_2(x_2 - x_h)$$

where boundary conditions for the task are as follows:

for the hand

$$x_h(t = 0) = 0, \dot{x}_h(t = 0) = 0, \ddot{x}_h(t = 0) = 0$$

$$x_h(t = T) = L, \dot{x}_h(t = T) = 0, \ddot{x}_h(t = T) = 0$$

for the first object

$$x_1(t = 0) = 0, \dot{x}_1(t = 0) = 0, \ddot{x}_1(t = 0) = 0$$

$$x_1(t = T) = L, \dot{x}_1(t = T) = 0, \ddot{x}_1(t = T) = 0$$

for the second object

$$x_2(t = 0) = 0, \dot{x}_2(t = 0) = 0, \ddot{x}_2(t = 0) = 0$$

$$x_2(t = T) = L, \dot{x}_2(t = T) = 0, \ddot{x}_2(t = T) = 0$$

Minimum Hand Jerk Model. As suggested by (Svinin et al. 2006) in order to create a descriptive model using the Minimum Hand Jerk criterion (5) the two objects problem should be separated to two independent one degrees of freedom problems. However, solving the problem for one object determines the movement of the other objects which does not necessarily satisfies the boundary conditions. Therefore the Minimum Hand Jerk model cannot be extended to account for this task.

Minimum Object Crackle. If we use the model to describe the first mass, the model will satisfy the boundary conditions for the first object and for the hand leaving yet again the problem of the unsatisfied boundary conditions for the second object. Applying the criterion on both masses can create conflict as to the hand trajectory in case the ratios between each external mass and its spring isn’t equal, i.e. $m_1/k_1 \neq m_2/k_2$.

Therefore the Minimum Object Crackle model cannot be extended to account for this task.

Minimum Center of Mass Acceleration. As done for simple object manipulation, the MACM model is based on describing the trajectory of the system’s CoM using the MACC model, i.e. the CoM trajectory can be described as straight line with bell shape velocity profile. We can calculate the link between the CoM and the first mass position:
\[ x_i^{(4)} + \left[ \frac{m_h + m_1}{m_h m_1} + \frac{k_2}{m_h} + \frac{k_2}{m_2} \right] \ddot{x}_i + \dot{k}_1 \left( \frac{m_1 + m_2 + m_h}{m_2 m_h m_1} \right) x_i = \]

\[ \frac{k_1}{m_h m_1} (m_1 + m_2 + m_h) \dddot{x}_{cm} + k_2 \dot{k}_1 \left( \frac{m_1 + m_2 + m_h}{m_2 m_h m_1} \right) x_{cm} \]

Since the position, velocity and acceleration of the first object and hand are continuous and differentiable, the third and forth derivatives of first object position exists, i.e. \( \dddot{x}_i \) and \( x_i^{(4)} \). This can be proven by calculating the limit of the difference quotient while using equation (15) (Apostol 1967).

Solving the ordinary differential equation of (21) will generate a solution which satisfies the boundary conditions of (19). Using this solution we can calculate the trajectory of the hand while satisfying the boundary conditions at the beginning and end of the movement. Once the hand, first mass and CoM trajectory are set, there is no need for special calculation of the second mass trajectory as it automatically given from the CoM definition:

\[ x_{cm} = \frac{m_h x_h + m_1 x_1 + m_2 x_2}{m_h + m_1 + m_2} \]

Similarly the MACM can be further extended to multiple objects and various external degrees of freedom.

Insert Figure 3 here
Subjects, Apparatus and Protocol

Twelve subjects (7 males, 5 females aged between 23-28) participated in three experiments (A, B and C) after signing the informed consent form as stipulated by the Institutional Helsinki Committee, Beer-Sheva, Israel. Subjects were seated and held with their dominant hand the handle of a PHANTOM® 1.5™ haptic device (SensAble Technologies, Inc.) which was used to generate real-time forces. The subject looked at a projection screen displaying the virtual environment from a projector placed horizontally above it (Figure 1A). Movements were performed in 1-D by setting limitations on the subject hand using forces in the direction orthogonal to the movement line. The orthogonal forces were generated according to:

\[ F_\parallel (t) = -200 \cdot z_h (t) \, [N]; \]  

where \( z_h \) represent the hand position in the lateral direction (Figure 1B). These forces created an infinitesimal channel for subjects to move ensuring a straight line movement. After a few trials all subjects generated straight line movements so the forces generated orthogonal to moving direction were unnoticeable. The experiment started with 50 reaching movements between the initial position and the target- the “reaching stage”- to familiarize the subject with the system, followed by 900 movements performed while attached to the mass-on-spring - “object manipulation stage”. The mass and spring constants changed during the experiment every 300 trials: the subject interacted with 2kg object and spring with 60 N/m constant (Exp A) which switched to 3 kg object and spring with 90 N/m constant (Exp B) which switched to 4 kg object and spring with 120 N/m (Exp C). The order of experiments appearance was randomized between subjects (Table 1). In all three experiments the desired movement duration (\( T \)) was 1.5 seconds, while the desired movement length (\( L \)) was set to 15 cm, i.e. the initial position and target position were 15 cm apart. Subjects could not see their hand but their hand location was represented by a grey square. To start the movement subjects needed to set their hand at the initial point by moving the grey square to a blue disk representing the initial position. Once at rest at the initial point a go signal was given and the subject moved his/her hand to the target, represented by a green disk. During the “reaching stage” the visual display of the experiment consisted of these three elements: the hand, the initial point and the target point. During the object manipulation stage subject received additional visual feedback of the mass location using additional red square setting the total number of elements to four: the hand (grey square), the object (red square), the initial point (blue disk) and target point (green disk), as depicted in Figure 1A. The position of the external mass was calculated online by solving the object motion equation (1). The solution depends on the subject hand position and was calculated using 4th order Runge-Kutta method. Hand position was sampled at 1 kHz therefore the object position was calculated and forces were rendered at the same rate.

During the “object manipulation stage” a trial was considered to be successful once: (a) the subject hand and mass reached the target (\( L \pm 0.006 \) m); (b) at the target point the velocity of the hand and the mass were very close to complete stop (speed<0.006 m/s); (c) the movement was performed within a time limit of 1.3-1.7 seconds. Subjects were given visual feedback as to whether their movements were appropriate, too slow if they didn’t reach the objective on the desired time, or too fast if they did reach the objective but again not on desired time. Once the subject was given visual feedback about their movement speed, both haptic feedback and visual feedback of the virtual object position were turned off until a new trial was presented.

This procedure was reproduced on two consecutive days setting the total number of movement each subject perform to 1900 trials- 100 reaching and 1800 object manipulation- 600 trials in each experiment. Subjects were not aware of the changes in spring and mass values and none of them reported a feeling of such changes after the experiment ended. Note that although the mass...
and spring changed between experiments the ratio between them, i.e. \( m_o/k \), didn’t change and was always equal to 1/30.

Insert Table 1 here

Data analysis

Model comparison with the current experiments
To compare the suggested models to experimental data we took the last 10 successful hand velocity profiles executed by each subject in each experiment, i.e. A, B and C during the second day, and calculated the average velocity profile. To generate models prediction we used all the parameters set in each of the experiments i.e. object mass, spring constant and movement length, identical to the process done in (Dingwell et al. 2004). Two of the suggested models, the minimum hand force change model and the CoM minimum acceleration model, depend on the mass of the hand. We used predictive regression equations based on anthropometric measures meaning segment lengths, circumferences, breadths and skin folds (Arthurs and Andrews 2009) in order to estimate the mass of the hand for each subject. These regression equations we used were tested in (Arthurs and Andrews 2009) against actual mass values obtained using Dual-energy X-ray Absorptiometry scans and showed high success in predicting the hand mass.

To fit the models to the averaged trajectory we had to obtain the movement duration and movement onset time. As mentioned in (Dingwell et al. 2004) subjects may attempt to move with an internal “desired” \( T \) that may be shorter than the required time and therefore obtain more time to dampen extraneous oscillations at the end of the movement, and thus still complete the task “successfully”. Therefore, it was necessary to vary movement time \( (T) \) in fitting the models to the data.

There are many methods for movement onset detection (Botzer and Karniel 2009; Georgopoulos et al. 1982). Here, instead of detecting the onset of movement, for each model, the best fit for each averaged trajectory, in the sense of highest explained variance \( (R^2) \), was obtained by selecting the best temporal translation and best movement duration. Therefore all the predicted trajectories from each model compared in this study are the result of fitting these two parameters. After obtaining the best fitted trajectory the variance accounted for (VAF) were calculated across subjects and experiments and compared.

The values of VAF are bounded in \([0,100]\), regardless to the specific experiment. Therefore, we used the nonparametric Friedman’s test (Friedman 1937) in order to determine whether the difference between the VAF values of the models is statistically significant. We used the Wilcoxon sign-rank test for multiple comparisons in order to perform the comparisons between the individual models.

Model comparison with previous results
In comparing the MACM model predictions to previous results (Fig. 2 of Dingwell et al. (2004)) we used the same parameters fitted originally for all the models (movement onset and movement duration) and graphically placed these predictions on top of the original velocity profile figure reprinted here. Since there was no documentation regarding the hand mass in the original paper we choose the hand mass based on the mean value reported in (Arthurs and Andrews 2009).
Results

A new model for trajectory formation: the Minimum Acceleration with constraints applied to the Center of Mass (MACM)

We have derived a new model for trajectory formation (13) analytically proved (see Methods) accounting for reaching as well as object manipulation. This model describes the optimal trajectory of the hand and is derived from the MACC model for reaching movements without the need to minimize additional criteria considering the new system which consist of the hand and mass-on-spring.

The model well accounts for previous experimental results

Comparing to previous reported experimental results, the MACM can account for reaching movements without an external object. The MACM model converges to the MACC model when setting the object mass and spring constant to zero and the ability of the MACC model to accurately predict the characters of simple reaching movements was demonstrated in (Ben-Itzhak and Karniel 2008; Berret et al. 2011; Yazdani et al. 2012). Therefore, the prediction by the MACM is also consistent with the experimental data as reported in these studies.

In addition to reaching movements the MACM model can account for previous simplified object manipulation task. We examined the results of the experiment B in (Dingwell et al. 2004). This movement was performed while transferring a 3kg object with 120N/m spring between points located 12.5cm apart. As seen in Figure 4A, the MACM model predicts the typical subject velocity profile for the 1.7 seconds duration as reported in (Dingwell et al. 2004). Additionally, the model can account for the one subject who wasn’t “typical” and showed irregular velocity profile as shown in figure 4B.

The model well accounts for new experimental results

All three experiments tested the MACM model predictions of the object manipulation task compared to previous suggested models while keeping the ratio between the external mass and spring constant. In experiment A all 12 subjects exhibited bi-phasic hand velocity profile. The predictions of the MACM model were not significantly different than the MOC or MHFC models (Wilcoxon sign-rank test P>0.1) and significantly better than the MHJ model (Wilcoxon sign-rank test P<0.001). The VAF values for the fitting for each model are presented in Figure 5B and an example of a velocity profile and fitting is presented in Figure 5A. In experiment B and C again all 12 subjects exhibited bi-phasic hand velocity profile. The predictions of the MACM model were significantly better than the MOC, MHJ and MHFC models (Wilcoxon sign-rank test P<0.001). The VAF values for the fitting for each model are presented in Figure 5D,F and an example of a velocity profile and fitting is presented in Figure 5C,E. The success rates of each subject were also similar across experiments ranging between 34% and 72%. Fitted movement times (T) were similar across experiments for each model (in seconds): MHJ- mean 1.424 (SD 0.184), MOC- mean 1.336 (SD 0.160), MHFC- mean 1.417 (SD 0.182), MACM-mean 1.531 (SD 0.197). The differences between actual movement duration performed by subjects and fitted movement duration for each model (in seconds): MHJ- mean 0.122 (SD 0.015), MOC- mean 0.132 (SD 0.023), MHFC- mean 0.118 (SD 0.01), MACM- mean 0.089 (SD 0.026).

Next we tested whether the hand mass is related to kinematic features exhibited by the subjects.

We tested the correlation between the hand mass and local minimum and maximum values of the velocity profile. We found negative correlation between hand mass and mean local maximum value of the velocity profile exhibited in the 3 experiments (Pearson r=-0.743 P value=0.021)
and positive correlation between hand mass and mean local minimum value of the velocity profile exhibited in the 3 experiments (Pearson r=0.717 P value=0.045). These results show that kinematic features of the movement changes with hand mass as predicted by the MACM and MHFC models.

Insert Figure 5 here

Discussion
In this study we derived a computational model for object manipulation based on the minimum acceleration with constraints criterion applied to the center of mass (MACM). We used a known paradigm to simulate a mass-on-spring system and showed that minimizing the hand-mass system CoM acceleration can account for our observed experimental data. We considered three additional models and found that the MACM can explain the results better than these previous models. Moreover, we showed that unaccountable previous results, regarded as not typical, are well accounted for by our model.

The criterion of minimum acceleration was found successful for reaching movement and here we show a generalization of this criterion for mass-on-spring manipulation. During our experiments we changed the values of the external mass and spring constant but kept the ratio between these two elements constant. This was performed to check whether the hand kinematic changes, as we did see, but limited our possibilities to learn about the ability to generalize to different mass-on-spring objects. The constant ratio sets the same dynamic equation for the external object making it possible for subject to generalize between the objects we presented but keeping the question of generalizing to different object dynamics still open (Dingwell et al. 2002). To further test our model we can examine the generalization of the present solution to different mass-on-spring dynamics and different tasks (e.g., reaching with a glass full of water, flexible or articulated rods, etc).

As suggested by (Svinin et al. 2006) the ability of a model to be expended to more complex environments can show advantages of one criterion over the other. We showed that the solution for simple point-to-point reaching movement can be used to generate the needed movement for transporting mass-on-spring object without the need for resolving the optimization problem. Using this procedure the model can be extended even further to more complex environments with more degrees of freedom such as chain of masses connected by springs (Svinin et al. 2006) or multiple mass-on-spring connected in a parallel way. The extension of other models to the later mentioned environment is not as trivial, if even possible, as we showed in the Methods section.

Our model is based on three underlying principles: (i) minimum acceleration as the optimization criterion (ii) the center-of-mass as the relevant variable (iii) maximum CoM jerk as a constraint. Since the MACM model, as well as the other models, describes the end point trajectory, it is independent of manipulation goal and other task parameters such as the movement direction. The CoM jerk constraint, $u$, found during the model derivation process in equation (13) is used for satisfying a continuity condition eliminating a possibility of discontinues hand velocity profile. Setting the CoM jerk to a different value will generate discontinues point in the hand velocity profile which implies unbounded, non-smooth and non-physiological hand acceleration profile. It is important to note that the question of the underlying principle on which movements are planned is still open. The previous criteria, such as the MHJ and MHFC, needs to be converted
from hand coordinate to object coordinate in order to be calculated. The equivalency between
these criteria (equation (5) and (6) and equation (7) and (8)) make it harder to determine which
formulation the central nervous system is using. In our system, minimizing the acceleration of
the center of mass is equivalent to minimizing the hand driving force. Similarly, the minimum
jerk of the center of mass is equivalent to the minimum hand driven force change as mentioned
in eq. (7). This is easy to prove examining the definitions of the MACM and minimum hand

driving force cost functions, $\int_{0}^{T} [F]^2 dt$, while keeping in mind the definition for the CoM and the
dynamic equations of the system (see eq. (1), (2) and (9)). The equality between the MACM and
minimum hand driving force makes it difficult to determine whether the hand trajectory is the
outcome of minimizing the CoM acceleration or the CoM trajectory is the outcome of
minimizing the driving forces of the hand during movement. As further discussed below,
planning and executing movements in the task space of CoM coordinates may reflect a general
approach with metabolic benefits which can be extended and tested in more sophisticated object
manipulation tasks.

We propose a model for the arm trajectory after extended training. One should note that in the
practice session subjects have to learn the task using dynamic and kinematic information
(Krakauer et al. 1999). Visual and proprioceptive information about the hand and object states as
well as the forces acting during movement are required in order to form an internal
representation of the external system (Izawa et al. 2008). Since the dynamics of the hand is
changed with the additional external mass and the kinematic planning is changed by the multiple
objectives, subjects need to learn the new environment. The learning process is evident from the
increased success rate of keeping with the demands of the mass-on-spring transporting task
(Svinin et al. 2006) or by the shortening of the movement duration (Dingwell et al. 2002). In this
sense our model suggest that subjects need to both estimate the dynamics changes, i.e. the altered
CoM, and use different kinematic planning i.e. implicitly compute the required hand movement
that will create CoM trajectory with similar characters of the unconstrained reaching movement.

Previous studies supported the ability to estimate the CoM using vision (Hirsch and Mjolsness
1992; Salimi et al. 2003) and using the size of objects in such estimation (Friedenberg and Liby
2002). Proprioceptive information may provide a sense of forces acting on the hand during
movement; however, since trajectory planning is predictive, the nervous system can use sensory
information from the previous trial to predict other variable such as the CoM. Since both hand
and mass were presented as squares with the same size, it is not inevitable that there were
integration of proprioceptive and visual feedback to estimate the CoM.

The CoM variable appears to be relevant to the motor system for a range of activities. Most
relevant to this work is the possibility of CoM to have fundamental rule in planning reaching
movements (Suzuki et al. 1997). Other examples can be found in studies of limb orientation
perception (van de Langenberg et al. 2007), body orientation perception (Fourre et al. 2009), sit-
stand task (Scholz and Schöner 1999) or object grasping task (Lukos et al. 2007).
Furthermore, controlling the CoM variable may affect metabolic cost, for example during
driving (Gordon et al. 2009). We provide another role for the CoM as the controlled variable
during transporting of flexible objects; our setup focused on 1D movement where there is only
one geometrical solutions corresponding to the CoM position while making the number of
fingers holding the robotic handle or the way it was grasp irrelevant to the task goal. Future
studies could further test our model in 2D and 3D movement and extend it to various hand grip
configurations.
An important prediction of the MACM model is an intermittence control since the solution is divided into three parts. Intermittence is useful for hierarchical systems in particular with delay (Gawthrop et al. 2011). This view of the neural control of movement suggests that the central nervous system sends sparse command to the spinal cord to switch motor command at certain points of time, such as the three points in our solution.

As previously suggested, the internal representation of objects can be related to the cerebellum (Nowak et al. 2007). In the structure of internal models (Kawato 1999), the cerebellum is linked with the role of a forward model (Pasalar et al. 2006) as it is may have the ability to predict movement outcome. In the context of object manipulation the cerebellum may have to acquire the intrinsic dynamics of the object to predict how hand movements and generated forces will affect the object movement, as seen for example during learning to use a new tool (Imamizu et al. 2000). This new model has interesting implications about the possible neural representation of the CoM, together with the intermittence nature of the control and its relation to neural bursts such as the bistability observed in Purkinje cells (Loewenstein et al. 2005).

In this study we derived a model for both reaching and simplified object manipulation task. This new model extends the minimum acceleration with constraints model suggesting that during simple reaching movements and object manipulation, the state variable being controlled is the center of mass. Since the controlled variable and the optimization criteria don’t change between tasks, the suggested solution creates the desirable simple strategy for the brain to implement during daily interaction with the environment.

References


Figure Legends

Figure 1. Experimental setup. A) haptic device setup. The subject looks on a projection of the experiment from a projector located above while holding the handle of robotic haptic device. B) The visual display used in the experiment. Blue disk represent the starting moving point, green disk represent the target point, grey square represent subject hand and red square represent the object. L is the distance between the initial point and the target while the dashed arrows represent the orthogonal forces keeping the movement within a one dimensional channel. C) Diagram of the hand as a point mass, $m_h$, and the simulated mass, $m_o$, attached (virtually) to the hand using a spring with constant $k$. The hand position is denoted by $x_h$ and the object position is denoted by $x_o$. F indicates the forces the hand is generating during movement. The center of mass ($x_{cm}$) is derived as a weighted sum of the hand position and the object position.

Figure 2. Predictions of the minimum acceleration of center-of-mass (MACM) model. A-D are examples for the hand velocity profile predicted by the MACM model for transporting the mass-on-spring where spring stiffness equals to 120 N/m and a 4 kg object while increasing the hand mass from 1kg to 4kg. The movement duration ($T$) increased between the sub-figures: A) $T$ equals to 1.2 seconds B) $T$ equals to 1.3 seconds C) $T$ equals to 1.4 seconds D) $T$ equals to 1.5 seconds. E-F are examples for the hand velocity profiles predicted by the MACM model for transporting the mass-on-spring where the movement duration remain constant while increasing the hand mass from 1kg to 4kg. The spring and mass values are changed between the sub-figures: E) 3kg object and 90 N/m spring F) 2 kg object and 60 N/m spring.

For the selected constants used in this example the phase analysis didn’t show any change in the number of phases which remain two.

Figure 3. Diagram of the hand as a point mass, $m_h$, while attached to two external masses, $m_1, m_2$ using two springs $k_1, k_2$. This setup generate two mass-on-spring connected to the hand in a parallel way. The hand position is denoted by $x_h$, the first object position is denoted by $x_1$ and the second object position is denoted by $x_2$. The center of mass, $x_{cm}$, is derived as a weighted sum of the hand position and the object position. This system demonstrates a situation in which the movement of each object depends on the movement of the hand and there is no direct link between the two objects.

Figure 4. Models fit to previous results. A) Typical velocity profile of subjects performing the object manipulation task while transferring a 3kg object with 120N/m spring between points located 12.5cm apart (Dingwell et al. 2004). Thin light gray lines represent individual trials where thick gray lines represent the average of these trials. Optimal trajectories predicted by the models denoted by black line (minimum object crackle-MOC), pink line (minimum hand jerk-MHJ), green line (minimum hand force change-MHFC) and blue line (minimum acceleration of center of mass-MACM). B) The velocity profile of the exceptional subject reported in (Dingwell et al. 2004). Lines types are the same as in A.

Both A) and B) were modified from (Dingwell et al. 2004)

Figure 5. Models fit to new experimental results. A) Individual Variance Accounted For (VAF) by each model in experiment A. Coding of subject symbols is common to A, C and E panels. *** indicate significant difference P<0.001 (see RESULTS). B) Single subject from experiment A. Thin light gray lines represent individual trials where thick gray lines represent the average of these trials. Optimal trajectories predicted by the models denoted by black line (minimum object crackle-MOC), pink line (minimum hand jerk-MHJ), green line (minimum hand force change-
MHFC) and blue line (minimum acceleration of center of mass-MACM). C) Individual VAF for experiment B. D) Single subject from experiment B. E) Individual VAF for experiment C. F) Single subject from experiment C.

Table 1. Order of presentation of each of the three experiments

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