

**The Use of a Nonlinear Muscle Model in Explaining the
Relationship Between Duration, Amplitude and Peak Velocity
of Human Rapid Movements**

(Revised Version, 1999)

To appear in the Journal of Motor Behavior Vol.31 No.3 pp. 203-206 September 1999

A. Karniel, G.F. Inbar

Department of electrical engineering, Technion - Israel Institute of
Technology, Haifa 32000, Israel.

Correspondence: karniel@tx.technion.ac.il, Fax - 972 4 8323041

Running Head: nonlinear mechanical model and rapid movements

Keywords: nonlinear muscle model; nonlinear viscosity; reaching movements;
saccadic eye movements; ballistic movements.

Abstract

Rapid human movements exhibit a quasilinear relationship between their amplitude and maximum velocity, and a log-like relationship between their amplitude and duration. We demonstrate that these well-observed relations could be obtained with a simple nonlinear muscle model and a pulse-step control scheme. This result encourages the use of nonlinear musculoskeletal models with simple control schemes for modeling human ballistic movements.

1. Introduction

Rapid human movements have been studied extensively in many different paradigms (For saccadic eye movements, see Robinson 1964 and Bahill et al. 1975; for hand reaching movements, see Karniel and Inbar 1997 and references therein; and also Plamondon 1995 and references therein). These movements have many stereotype features such as a roughly straight-line path and a bell shaped speed profile. In this paper we examine two well-observed features of rapid movements: the quasilinear relationship between amplitude and maximum velocity, and the log-like relationship between amplitude and duration. These two features were found also in fragments of tracking movements (Hanneton et al. 1997, see Fig 1) which suggests that the rapid movements might also be the basis of other slower and more complex movements.

Various reasons, purposes and optimization schemes might account for the stereotypical features of the rapid movements. For example, the bell-shaped speed profile was shown to be consistent with the minimum jerk hypothesis (see Flash and Hogan 1985). We assert that the origin of these features can be the electromechanical properties of the musculoskeletal system rather than a computation process in the nervous system. The role of the muscle properties in creating a smooth bell-shaped speed profile was demonstrated in Karniel and Inbar 1997, where a nonlinear Hill type muscle model was used. In this paper we show that a simple nonlinear musculoskeletal model with a simple pulse-step control scheme can reproduce the observed relationship between amplitude, duration and maximum velocity. First the model is described, and then the simulations are shown and discussed.

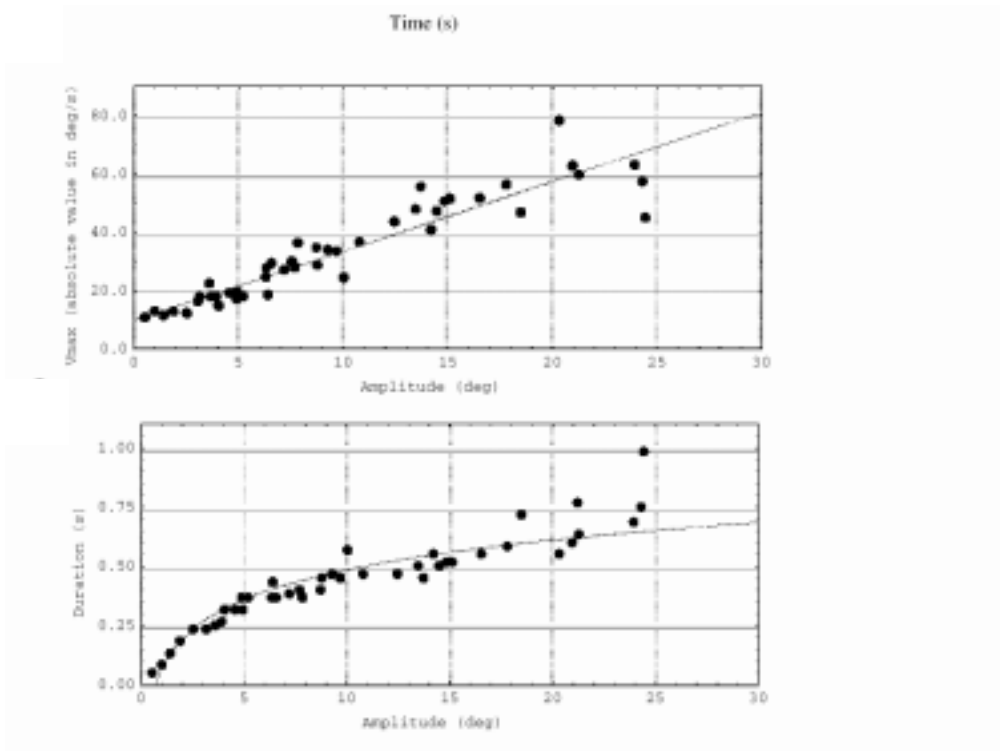


Fig 1. An example of the relationships between the maximum velocity and the movement amplitude (upper graph), and between the movement duration and the movement amplitude (lower graph). Each dot is one short-stereotyped periods of arm movement. The lines are linear and logarithmic fit to the data. For further details on this specific experiment see Hanneton et al 1997. For other examples of these quasilinear and log-like relationship see for example Bahill et al 1975 and Robinson 1964. This figure is adopted with permission from Figure 3 of Hanneton S, Berthoz A, Droulez J, and Slotine JJE. Does the brain use sliding variables for the control of movements? *Biol. Cybern.* 77, 381-393. © Springer-Verlag 1997

2. The model

There are many muscle models of various types and complexities (see Hill 1938, Winters and Stark 1987, Gielen and Houk 1987). In this work we choose the model of Wu et al. (1990) which is a simplified abstract model that incorporates nonlinear properties and is able to produce a fast and damped movement with a simple rectangular neural excitations.

Wu et al. (1990) found that human wrist movement could be qualitatively modeled by the following differential equation that is referred to as the one-fifth power law

$$m \cdot \ddot{x} + b \cdot \dot{x}^{\frac{1}{5}} + k \cdot (x - x_{eq}) = 0. \quad (1)$$

In this equation, x is the position (in meters) of the controlled object and x_{eq} is the resting, or equilibrium position. The mass of the object is m (kg), the stiffness coefficient is k (N/m) and the damping coefficient is b (N/m*(s/m)^{1/5}). The values of the parameters for the simulations were chosen to be $m=1$, $b=4$, $k=60$, which were found to produce qualitatively similar trajectories to those observed in human wrist movements (Wu et al 1990).

The control signal x_{eq} , which represents the neural command, was chosen to be a pulse-step control signal as modeled by Barto et al. 1995 (see Fig 2, and a full paper of Barto et al. 1999).

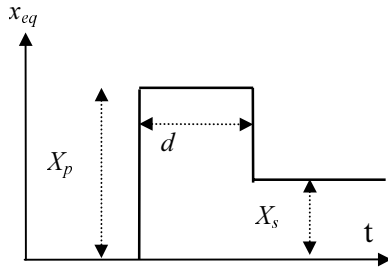


Fig 2. The pulse-step control signal and the parameters that define it.

This command produces a fast movement, which is sometimes called step-and-hold movement. There are three parameters that define the control command, the pulse amplitude X_p , the pulse duration d , and the step amplitude X_s . At the end of the rapid movement, the position of the object is not necessarily X_s , because the system tends to "stick" at non-equilibrium position and then to drift very slowly to the equilibrium

position. Following Barto et al 1995, we define the termination of the movement, the position where the object sticks, as the position at which its absolute velocity falls and remains below 0.005 [m/s]. We denote by X_t , the value of x at the termination of the movement, and by T_t , the termination time of the movement. We also observed the maximum velocity during the movement and denote it by V_m .

The simple model and the simple control strategy described above are used in the next section in order to simulate rapid human movements and examine the relationship between their parameters.

3. Simulations

We begin this section with a simulation of a typical rapid movement (see Fig 3) and discussion of its properties. We then chart the relationships between the parameters of such movements.

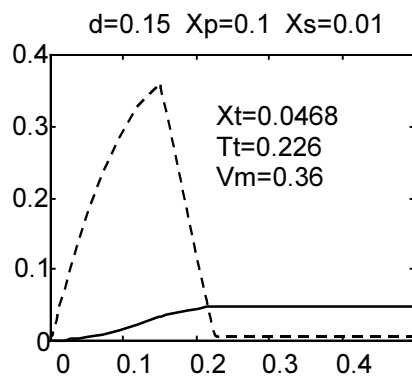


Fig 3. A typical simulated rapid movement. Position (solid line), speed (dashed line).

The nonlinear viscosity in the model facilitates the execution of rapid movement that stops without oscillations. The same phenomenon appears in other nonlinear muscle models, such as the Hill model where the damping coefficient becomes larger at small velocities, and also in models with nonlinear stiffness. A Hill-type model is used in

Karniel and Inbar (1997) where the role of the nonlinear properties of the muscles is demonstrated, see also Karniel and Inbar (1999). A nonlinear muscle model is essential to produce rapid movement with a smooth stop in response to simple pulses or in response to a pulse-step excitation regime. The model used in Karniel and Inbar (1997) contains more elements and therefore produces smoother movements. The one-fifth power law was chosen for this work due to its simplicity.

The one-fifth power law (1) was formulated for positive velocities. However, at the end of the movement small negative velocities might occur. Therefore a reflection about the origin is assumed. That is, the following differential equation is solved:

$$m \cdot \ddot{x} + b \cdot \text{sign}(\dot{x}) \cdot |\dot{x}|^{\frac{1}{5}} + k \cdot (x - x_{eq}) = 0 \quad (2)$$

The simulation was repeated for various values of the command parameters X_s, d, X_p , and the values of the movements parameters V_m, X_b, T_t , were examined. A salient tendency towards the stereotype relationships was observed over a large part of the command parameter space. An example is illustrated below for a various pulse amplitude values, see Fig 4 and Fig 5.

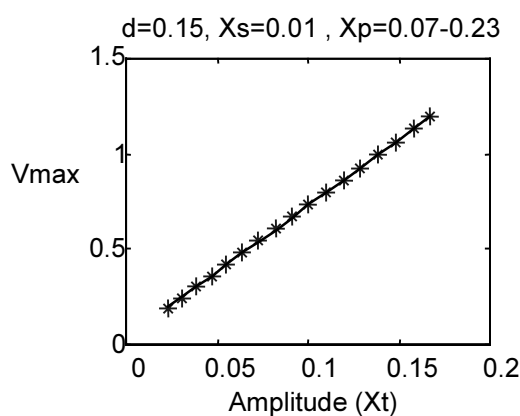


Fig 4. Maximum velocity against movement amplitude of 17 simulated movements with different pulse amplitudes. The solid line is a linear fit to the data.

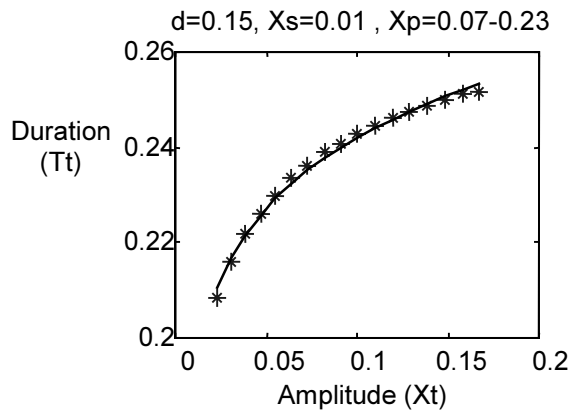


Fig 5. Movement duration against movement amplitude of 17 simulated movements with different pulse amplitudes. The solid line is a logarithmic fit to the data.

This simple control scheme of changing the pulse amplitude was described and studied extensively by Gottlieb et al. 1989, who called this mode of control ‘speed-sensitive’. We conclude that with a physiologically plausible control strategy and nonlinear muscle model one can reproduce the well-observed relationships between amplitude duration and maximum velocity.

4. Discussion

Many works describe the features and invariance of rapid movements (see for example Gottlieb et al. 1989). A recent work by Plamondon (1995) suggests a kinematic theory of rapid human movement, which asserts a linear system and describes the movements via seven parameters of their ΔA law. We suggest that a physiologically plausible nonlinear model can produce similar results with the same order of the number of parameters and with the great advantage of being closely related to physiological and neurological measurements. The parameters of the ΔA law have no physiological meaning while the parameters of the pulses in a proper muscle model can be related to the integrated muscle electrical activity.

Latash and Zatsiorsky (1993) wrote about the extensive abuse of the term stiffness and suggested new terms such as apparent stiffness and quasi stiffness. Zatsiorsky (1997) wrote a similar paper on viscosity. We agree with their observation. However, instead of introducing new terms we suggest using the notion of impedance and physiological nonlinear models in order to describe the system behavior.

Goodman and Gottlieb (1995) wrote that the fast reaching movement plays a role similar to the impulse response in linear systems studies. We agree with the importance of the rapid human movement as a building block of complex movements. However, we wish to stress that the musculoskeletal system is clearly not a time-invariant linear system. Therefore the superposition property does not hold and the nonlinear properties of the muscles must be considered. In a previous article (Karniel and Inbar 1997) a six-muscle Hill-type model was used to demonstrate the role of nonlinear muscle properties in producing a bell-shaped speed profile with simple control signals. In this paper we used a simple nonlinear muscle model to demonstrate two well-known relationships between properties of rapid movements, again with simple control signals.

References

Bahill AT, Clark MR, and Stark L (1975) The main Sequence, A tool for Studying Human Eye Movements. *Mathematical Biosciences* 24:191-204.

Barto AG, Buckingham JT and Houk JC. (1995) A predictive Switching Model of Cerebellar Movement Control. *NIPS 8*, MIT press, pp. 138-144.

Barto AG, Fagg AH, Sitkoff N, and Houk JC (1999) A cerebellar model of timing and prediction in the control of reaching. *Neural Computation* 11:565-594.

Flash T, Hogan N (1985) The coordination of arm movements: an experimentally confirmed mathematical model. *J. Neurosci* Vol 5 No. 7: pp.1688-1703.

Gielen CCAM, Houk JC (1987) A model of the motor servo: Incorporation nonlinear spindle receptor and muscle mechanical properties. *Biol. Cybern.* 57:217-231.

Goodman SR, Gottlieb G (1995) Analysis of kinematic invariances of multijoint reaching movement. *Biol. Cybern.* 73:311-322

Gottlieb GL, Corcos DM, Agarwal GC (1989) Strategies for the control of voluntary movements with one mechanical degree of freedom. *Behavioral and brain sciences*12:189-250

Hanneton S., Berthoz A, Droulez J, Slotine JJE (1997) Does the brain use sliding variables for the control of movements? *Biol. Cybern.* 77,381-393

Hill A. V. (1938) The heat of shortening and dynamic constants of muscle. *Proc. Royal Society of London Vol. B*126:136-195

Karniel A, Inbar GF (1997) A Model for Learning Human Reaching-Movements, *Biological Cybernetics* 77:173-183

Karniel A, Inbar GF (1999) Human motor control: learning to control a time-varying non-linear many-to-one system. *IEEE Transactions on SMC-C*, to appear in 1999.

Latash ML and Zatsiorsky VM (1993) Joint Stiffness: Myth or reality? *Human Movement Science* 12:653-692

Plamondon R. (1995) A Kinematic theory of rapid human movements Part I. Movement representation and generation. *Biol. Cybern.* 72:295-307.

Robinson DA (1964) The Mechanics of Human Saccadic Eye Movement. *J. Physiol* 174:245-264.

Winters JM, Stark L (1987) Muscle Models: What is Gained and What is Lost by Varying Model Complexity. *Byol. Cybern.* 55:403-420

Wu C, Houk JC, Young KY and Miller LE (1990) Nonlinear Damping of Limb Motion. In Winters JM and Woo SL-Y (Eds) *Multiple Muscle Systems*. Springer-Verlag: 214-235

Zatsiorsky VM (1997) On muscle and joint viscosity. *Motor Control* 1:299-309.