Shot noise in superconducting junctions with weak link formed by Anderson impurity

Y. Avishai[1], A. Golub[1] and A. D. Zaikin[2,3]

[1] *Department of Physics, Ben-Gurion University of the Negev - Beer-Sheva, Israel*
[2] *Forschungszentrum Karlsruhe, Institut für Nanotechnologie*
76021 Karlsruhe, Germany
[3] *I. E. Tamm Department of Theoretical Physics, P. N. Lebedev Physics Institute*
117924 Moscow, Russia

Abstract. – A theory is developed to study shot noise in superconducting (SAS) and hybrid (SAN) junctions with singly occupied Anderson impurity (A) as a weak link. The zero-frequency dc component of the shot noise spectral density is calculated at zero temperature as a function of the bias at different Coulomb repulsion strengths (U), and shows a remarkable structure resulting from combination of electron-electron interaction and Andreev reflections.

Motivation and main results. – Following the first theoretical work on noise in superconducting point junctions [1] the underlying physics attracted a considerable theoretical [2–7] and experimental [8–12] attention (see review article [13] for a more complete list of references). In most of these works, the quantum point contacts have been considered in both the linear and nonlinear regimes. Tunneling junctions in which the electrodes are separated by a resonant double barrier were studied in refs. [3,14]. Novel experimental techniques now enable the study of transport and shot noise in quantum dots located between either normal (N) or superconducting (S) electrodes. So far, however, the physics of shot noise in superconducting junctions with strong electron correlations has not been exposed. Investigation of this fundamental aspect is carried out in this work. The shot noise spectral density in SAS and SAN junctions is calculated below for the case where the Anderson type impurity level (A) is singly occupied. For an SAN junction at small bias voltages V, doubling of the normal Poisson noise to current ratio (Fano factor) is preserved, although its dependence on the electron-electron interaction is quite essential. In an SAS junction, the main process contributing to the current and shot noise power is multiple Andreev reflections (MAR). However, in spite of the fact that for low V the number $n \approx 2\Delta/eV$ of MAR is large (Δ is the superconductor gap), the current and shot noise density are rather weak. This is due to the low effective transparency Γ of the

junction as a consequence of interaction (Coulomb blockade). Large-n processes are therefore damped as Γ^n.

Model and effective action. – The junction is represented by two half-planar electrodes on the left (L) and the right (R) separated by a quantum dot. The dot (located at the origin) is modeled as an Anderson impurity A with level position $\epsilon_0 < 0$ and Hubbard repulsion U under the condition of single occupancy $U > -\epsilon_0 > 0$. This can be justified by noticing that in recent experiments [15, 16] on semiconductor quantum dots it was shown that tunneling takes place through a separate state with the characteristics of a Kondo behavior (the tunable Kondo effect).

The starting point is the tunnel Hamiltonian $H = H_L + H_R + H_d + H_t$ in which H_j ($j = $ L, R) are lead Hamiltonians (usually of BCS form) defined in terms of electron field operators $\psi_{j\sigma}(x, y, t)$, $H_d = \epsilon_0 \sum_\sigma c_\sigma^\dagger c_\sigma + U n_\uparrow n_\downarrow$ and $H_t = \mathcal{T} \sum_{j\sigma} c_\sigma^\dagger \psi_{j\sigma}(\mathbf{0}, t) + \text{hc}$. Formally, the whole physics is contained in the partition function $Z \equiv \int d[F] \exp[iS]$, where the path integral Grassman integration is carried out over all fermion fields $[F]$ and the action S is obtained by integrating the Lagrangian pertaining to the Hamiltonian H along a Keldysh contour. In a recent work [17] we studied I-V characteristics in SAS and SAN junctions, and developed a formalism to carry out integration over the fields $\psi_{j\sigma}(x, y, t)$, combined with the dynamical mean-field approximation to account for the quartic term $U n_\uparrow n_\downarrow$. At the end of this procedure one arrives at an effective action S_{eff} which depends on the parameter γ_- whose physical meaning is the difference between spin-up and spin-down energies of the dot (to be determined self-consistently in terms of the effective action). The latter is defined in terms of lead Green functions, $g_j^{R/A/K}$ where $j = $ L, R and R/A/K denotes advanced, retarded and Keldysh, respectively. Explicitly, dropping the lead index,

$$\hat{g} = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix} \tag{1}$$

is a matrix in Keldysh space with

$$\hat{g}^{R/A}(\epsilon) = \frac{(\epsilon \pm i0)\tau_z + i|\Delta|\tau_y}{\sqrt{(\epsilon \pm i0)^2 - |\Delta|^2}}, \tag{2}$$

$$\hat{g}^K(\epsilon) = (\hat{g}^R(\epsilon) - \hat{g}^A(\epsilon)) \tanh(\epsilon/2T). \tag{3}$$

The set of Pauli matrices $\tau_{x,y,z}$ acts in spin (Nambu) space, and the lead dependence enters through the corresponding superconducting gaps Δ. It is now possible to define the kernel

$$\hat{G}^{-1}(\epsilon, \epsilon') = \delta(\epsilon - \epsilon')(\epsilon + \gamma_- - \tau_z\tilde{\epsilon}) + i\Gamma\hat{g}_+(\epsilon, \epsilon')\tau_z \tag{4}$$

(where $\hat{g}_+ \equiv (\hat{g}_L + \hat{g}_R)/2$) as an operator in Nambu $\otimes$ Keldysh product spaces (here $\tilde{\epsilon} \equiv \epsilon_0 + U/2$ and $\Gamma \propto \mathcal{T}^2$ is the usual transparency parameter). The effective action is a functional of the Grassman fields $c, \bar{c}$ which are fourtuples in this space,

$$S_{\text{eff}} = \int \frac{d\epsilon}{2\pi} \int d\epsilon' \bar{c}\hat{G}^{-1}(\epsilon, \epsilon')c. \tag{5}$$

The self-consistency equation for γ_- is,

$$\gamma_- = -i\frac{U}{2} \int dt \, \text{Tr}\,(G\sigma_x), \tag{6}$$

where the set of Pauli matrices $\sigma_{x,y,z}$ acts in Keldysh space.

The physical justification for using the mean-field approximation is related to the fact that the Kondo resonance is suppressed by the superconducting order parameter. Such suppression occurs due to strong attenuation of the density of states in an energy region of order Δ near the Fermi energy. Thus, the number of low-energy electrons which are able to screen the local impurity spin is small [18, 19]. Therefore, we can expect that ineffective screening results in a larger domain in parameter space (ϵ_0, U) for which the single occupancy (doublet state) becomes the ground state of the whole system. For our choice of parameters, the Kondo temperature $T_K \approx \sqrt{2\Gamma U/\pi} \exp[-\pi\tilde{E}/(2\Gamma)]$ (where $\tilde{E} = (U - |\epsilon_0|)|\epsilon_0|/U$) is $\leq 0.2\Delta$. In this case the ground state of the system is a doublet, and the Kondo singlet is suppressed. Therefore, we can look for a doublet solution which can be obtained from the mean-field equation (6).

Explicit expression for the shot noise. – The noise spectrum measures the current fluctuations in the junction. It is defined by the symmetrized current-current correlation function which, in terms of current operators in Nambu $\otimes$ Keldysh space reads

$$K(t_1, t_2) = \hbar[\langle \hat{T} I^{(1)}(t_1) I^{(2)}(t_2) \rangle + \langle \hat{T} I^{(1)}(t_2) I^{(2)}(t_1) \rangle - 2\langle I \rangle^2], \qquad (7)$$

where $\hat{T}$ is the time ordering operator and $\langle ... \rangle$ denotes quantum-mechanical thermodynamic averaging with respect to the total Hamiltonian H. Starting from the general definition of the current operator for tunneling through a quantum dot [20], in the present case it reads

$$I^{(1,2)} = \pm \frac{ie}{\hbar} \sum_k \mathcal{T} \left[\bar{c} \frac{1 \pm \sigma_x}{2} \psi_{Rk}(0) - \text{h.c.} \right], \qquad (8)$$

where $\psi_{Rk}(0)$ is the Fourier transform of $\psi_R(0, y, t)$ (the so-called surface field operator) with respect to y. Substituting the explicit form for the current operators (8) into eq. (7), we then obtain an expression for $K(t_1, t_2)$ which involves Grassman quadrature over surface fields and dot electron operators. The first integration (involving the Green function matrix $\hat{g}$) is Gaussian and can be done exactly. Integration over the dot fermion fields is achieved within the dynamic mean-field approximation. It is then possible to express the power spectrum $K(\omega)$ and γ_- in terms of the Green functions of the entire system $\hat{G}^{R,A}$ (inverse of the kernel defined in eq. (4)). In the present work, attention is focused on the zero-frequency dc component of the shot noise $K = K(0)$ at zero temperature. The first novel result of our study then consists of a workable expression for the noise power density function $K = K_1 + K_2$ supported by a self-consistent equation for the energy occupancy parameter γ_-. For a symmetric dot ($\Gamma_L = \Gamma_R$) it reads

$$K_1 = \frac{ie^2\Gamma}{2\hbar} \int \frac{d\epsilon}{2\pi} \text{Tr} \left\{ \tau_z[(\hat{g}_R^R - \hat{g}_R^A)(\hat{G}^R - \hat{G}^A) - \hat{g}_R^K F] \right\}, \qquad (9)$$

$$K_2 = \frac{e^2\Gamma^2}{8\hbar} \int \frac{d\epsilon}{2\pi} \text{Tr} \left\{ \frac{1}{2}(F\tilde{g})^2 - \tau_z \tilde{g} \tau_z \hat{G}^A \tilde{g} \hat{G}^R - \right.$$
$$\left. -2\tilde{g}\hat{G}^R \hat{g}_R^K F + (\hat{G}^R \hat{g}_R^K)^2 - \tilde{g}^R \hat{G}^R \tilde{g}^R \hat{G}^R + \text{h.c.} \right\}, \qquad (10)$$

$$\gamma_- = -i\frac{U}{2} \int \frac{d\epsilon}{2\pi} \text{Tr} \, F, \qquad (11)$$

where

$$F = \frac{-i\Gamma}{2} \hat{G}^R (\hat{g}_R^K + \hat{g}_L^K) \tau_z \hat{G}^A, \quad \tilde{g} = \tau_z(\hat{g}_R^R + (\hat{g}_R^R)^+)\tau_z, \quad \tilde{g}^{R,A} = \tau_z \hat{g}_R^{R,A} \tau_z - \hat{g}_R^{R,A}. \qquad (12)$$

We arrive now at our main goal, namely, using expressions (9)-(12), to analyze the noise in SAN and SAS junctions. Note that eqs. (9), (10), (11) can be easily generalized to the asymmetric case $\Gamma_{\mathrm{R}} \neq \Gamma_{\mathrm{L}}$.

Shot noise in SAN junctions. – In this case only one Andreev reflection takes place and K_2 can be written explicitly as an integral over the entire energy domain (including the sub-gap region), while for K_1 integration is effected only above Δ. As a by-product we get, from that part of $K_1 + K_2$ related to integration with $|\epsilon| > \Delta$, the shot noise for NAN junction (tunneling between normal metal electrodes through an Anderson-impurity center). It assumes a standard form [1], although it depends on the interaction through the parameter γ_-, that is

$$K = \frac{e^3|V|}{\pi\hbar}[T_-(1 - T_-) + T_+(1 - T_+)], \qquad (13)$$

where $T_\pm = \Gamma^2/((\tilde{\epsilon} \pm \gamma_-)^2 + \Gamma^2)$. In a perfect resonance condition ($\tilde{\epsilon} = 0$ and $U = 0$) we get $K = 0$ as expected for a pure point junction [1,13]. When $U \neq 0$ we have to solve the self-consistent equation (11) to find γ_- and substitute the solution for a given voltage into eqs. (9), (10).

Another limiting situation for which a simple analytical expression exists is that of small bias $eV < \Delta$. For completely transparent point junctions the current-noise spectral density vanishes [1]. For nonresonant transport there is a finite shot noise in the sub-gap region which, in the limit of low effective transparency, yields the Fano factor to be equal to 2. The principal contribution to the noise density in the regime $eV \ll \Delta$ comes from K_2. Therefore, integrating over the sub-gap energies in eqs. (9), (10), (11) results in

$$K = \frac{2e^3|V|}{\pi\hbar} \frac{\Gamma^4[\Gamma^2\tilde{\epsilon}^2 + (\gamma_-^2 - \tilde{\epsilon}^2)^2]}{[\Gamma^2\gamma_-^2 + (\tilde{\epsilon}^2 + 0.5\Gamma^2 - \gamma_-^2)^2]^2}. \qquad (14)$$

This formula, together with the self-consistency equation (11), yields the shot noise density for $T = 0$ and $eV \ll \Delta$. For $\Gamma \ll \tilde{\epsilon}, \gamma_-$ it leads to twice the normal Poisson noise-current ratio. Numerical calculations using the exact expressions (9), (10) for specific values of U indicate that the deviation from the approximate relation (14) starts already at $eV \sim \Delta/2$. Therefore, even in the sub-gap regime, the nonlinear V-dependence of the shot noise is important and exact expressions for the spectral density should be employed. The shot noise of an SAN junction is displayed in fig. 1 (hereafter, all the energies are expressed in units of Δ). For the parameters employed in the figure, the noise in the sub-gap regime is rather small, starting to grow near the superconducting gap. It is strongly dependent on the Coulomb interaction which, as a general rule, suppresses it in comparison with its value appropriate for simple tunneling through a noninteracting impurity. It is interesting to note that the role of U in an SAN junction (in the regime of single occupancy) is similar to that of an exchange term in a contact between a superconductor and a ferromagnetic metal. Therefore, our results might be relevant for this junction as well. Note also that for clean point junctions there is a saturation value for the shot noise spectrum density at large voltages [1]. This value is readily reproduced from our formulas (9), (10) if we consider the limit $\Gamma \gg \Delta$ at resonance ($\tilde{\epsilon} = 0$) for bias voltage $eV > \Delta$. The only contribution to the correlation function comes from energy integration above the gap in eqs. (9), (10), yielding the saturated value

$$K_{\mathrm{ex}} = \frac{4e^2}{\pi\hbar} \lim_{eV \to \infty} \left\{ \int_\Delta^{eV} \mathrm{d}\epsilon \frac{N(\epsilon) - 1}{(N(\epsilon) + 1)^2} \right\} = \frac{4e^2\Delta}{15\pi\hbar}, \qquad (15)$$

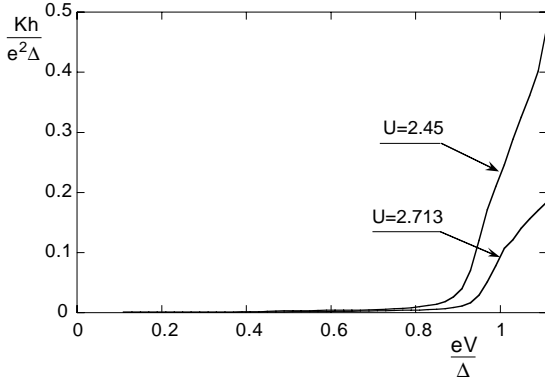where $N(\epsilon) = \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta^2}}$.
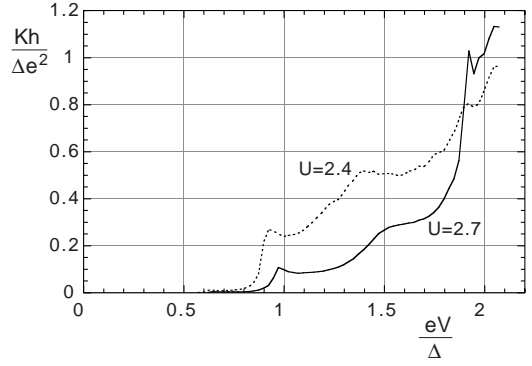
Fig. 1

Fig. 2

Fig. 1 – The shot noise K *vs.* bias V for an SAN junction at sub-gap voltages. The parameters are $\epsilon_0 = -1.5$, $\Gamma = 0.6$, $U = 2.45$ and 2.713.

Fig. 2 – The shot noise K *vs.* bias V for an SAS junction at sub-gap voltages. The parameters are $\epsilon_0 = -1.5$, $\Gamma = 0.6$, $U = 2.4$ and 2.7.

Shot noise in SAS junctions. – The current-noise spectral density of an SAS junction is calculated numerically from eqs. (9), (10). For a constant bias V it is useful to discretize the energy integration [17]. The energy domain in eqs. (9), (10), (11) is divided into slices of width $2eV$ and integration is performed on an interval $[0 < E < 2eV]$. The Green functions become matrices with indices representing the different energy slices. Explicitly

$$(m|\hat{G}^{-1}(\epsilon)|n) = \delta_{m,n}\left[\epsilon_m + \gamma_- - \tau_z\tilde{\epsilon} + \frac{i\Gamma}{2}\hat{g}_{\mathrm{R}}(\epsilon_m)\tau_z\right]\frac{i\Gamma}{2}(m|\hat{g}_{\mathrm{L}}(\epsilon)|n)\tau_z, \qquad (16)$$

$$(m|\hat{g}_{\mathrm{L}}(\epsilon)|n) = (\hat{g}_{\mathrm{L}}^{11}(\epsilon_m^-)P_+ + g_{\mathrm{L}}^{22}(\epsilon_m^+)P_-)\delta_{m,n} + $$
$$+ g_{\mathrm{L}}^{12}(\epsilon_m^-)\tau_+\delta_{n,m-1} + g_{\mathrm{L}}^{21}(\epsilon_m^+)\tau_-\delta_{n,m+1}, \qquad (17)$$

where $\epsilon_m = \epsilon + 2\,\mathrm{meV}$, $\epsilon_m^\pm = \epsilon_m \pm eV$, the superscripts denote the matrix elements in Nambu space and $P_\pm = (1 \pm \tau_z)/2$. We dropped the constant phase difference $\phi_0 = \arg(\Delta_{\mathrm{L}} - \Delta_{\mathrm{R}})$ which does not play a special role here.

We are mainly interested in the voltage dependence at the sub-gap region where MAR are important and the way how this noise-voltage characteristic is influenced by Coulomb interaction. The result is displayed in fig. 2. As can be deduced, there is a strong effect of the Coulomb repulsion on the noise spectrum K in the sub-gap region. For voltages not too close to 2Δ the noise is smaller for the higher value of U. This is similar to what happens with the current itself, as can be judged by a glance at the *I-V* curve (fig. 3). And yet, the Fano factor $K/2eI$ is nearly independent of interaction (see fig. 4). This finding stresses the importance of MAR process, analogous to what has been argued in junctions with low transparency [6]. We can suggest a simple explanation for such a behavior of Fano factor with interaction. Indeed, Fano factor shows that the shot noise is larger than the Poisson noise. The ratio of the shot noise to the Poisson noise depends on how many charge quanta are transferred. These additional charge quanta appear in multiple Andreev reflections. The interaction shifts the exact canonical value $eV = 2\Delta/n$ of the voltages when new charge quanta appear, but does not changes the number of these quanta.
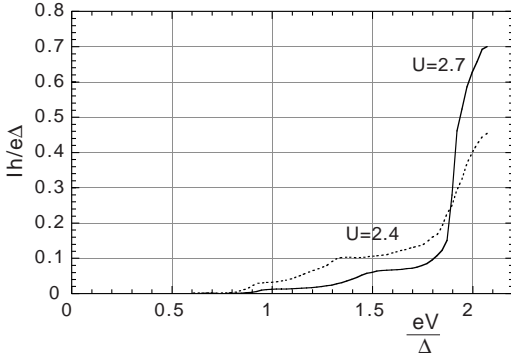
Fig. 3

Fig. 4

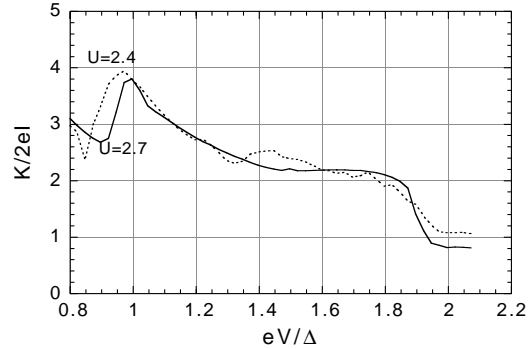Fig. 3 – The *I-V* curves for an SAS junction at sub-gap voltages. Parameters are the same as in fig. 2.

Fig. 4 – The Fano factor $K/2eI$ *vs.* bias V for an SAS junction at sub-gap voltage. The parameters are as in fig. 2.

It is also important to note that in our case we are dealing with the strong interaction that put the energy level deep under the E_F. Therefore we are in the Coulomb blockade regime and the sub-gap current is vanishingly small at $eV < \Delta$ (see figs. 2, 3). It is difficult to follow the MAR processes for this region. However, for larger voltages $eV < 2\Delta$ we clearly see one MAR with a step slightly shifted from 2Δ and two MAR near Δ (see Fano factor, fig. 4).

As in the case of SAN junction, there is a saturated value for the spectral noise density power also for SAS junctions. In the limit $\Gamma \gg \Delta$ this saturated value is twice of that of eq. (15).

In conclusion, based on a theory developed for the study of tunneling in SAS and SAN junctions [17], the shot noise in such systems is calculated. Special attention is devoted to analyzing the implications of the Coulomb repulsion between electrons in the dot on the tunneling process in general and the noise spectrum in particular. The theoretical treatment uses a combination of Keldysh nonequilibrium Green function and path integral formalism, and the interaction is handled within the dynamical mean-field approximation. General expressions for the current noise spectral density correlation function are then derived.

The main results of the present research can be summarized as follows: 1). In SAN and SAS junctions, the Coulomb interaction results in a nonzero value for the occupancy energy (γ_-) and thus always acts as a factor which lowers the transparency. Therefore in SAN junctions the shot noise appears already at $eV < \Delta$ (even if $\tilde{\epsilon} = 0$). The doubled value of the Fano factor emerges in the limit $\Gamma \ll \tilde{\epsilon}, \gamma_-$. 2) In SAS junctions, the shot noise correlation function in the sub-gap regime ($eV < 2\Delta$) is suppressed at higher values of U. The Fano coefficient is nearly interaction independent, and unambiguously manifests the importance of MAR.

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