Dynamic sliding of tetrahedral wedge: The role of interface friction

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SUMMARY

The role of interface friction is studied by slow direct shear tests and rapid shaking table experiments in the context of dynamic slope stability analysis in three dimensions. We propose an analytical solution for dynamic, single and double face sliding and use it to validate 3D-DDA. Single face results are compared with Newmark’s solution and double face results are compared with shaking table experiments performed on a concrete tetrahedral wedge model, the interface friction of which is determined by constant velocity and velocity stepping, direct shear tests.

A very good agreement between Newmark’s method on one hand and our 3D analytical solution and 3D-DDA on the other is observed for single plane sliding with 3D-DDA exhibiting high sensitivity to the choice of numerical penalty value.

The results of constant and variable velocity direct shear tests reveal that the tested concrete interface exhibits velocity weakening. This is confirmed by shaking table experiments where friction degradation upon multiple cycles of shaking culminated in wedge run out. The measured shaking table results are fitted with our 3D analytical solution to obtain a remarkable linear logarithmic relationship between friction coefficient and sliding velocity that remains valid for five orders of magnitude of sliding velocity. We conclude that the velocity-dependent friction across rock discontinuities should be integrated into dynamic rock slope analysis to obtain realistic results when strong ground motions are considered. Copyright © 2011 John Wiley & Sons, Ltd.

KEY WORDS: rate and state friction; tetrahedral wedge; DDA; shaking table; Newmark’s analysis; rock slope stability

1. INTRODUCTION

As rock masses are inherently three-dimensional, stability analysis in rock slope calls for a truly three-dimensional (3D) approach. 3D limit equilibrium analysis for rock slopes has been formulated using stereographical projection coupled with vector algebra [1–4]. In their book on Block Theory, Goodman and Shi [3] address mathematically the removability of a block bounded by an arbitrary number of surfaces and show how to determine the applicable failure mode. Once the failure mode is established, whether single or double face sliding, static equilibrium is formulated to evaluate the factor of safety against sliding.

Considering seismically induced dynamic deformation in rock slopes, a solution for rigid block sliding on a single plane has been suggested both by Newmark [5] and Goodman and Seed [6]. This procedure, largely known as the Newmark’s type analysis, assumes that permanent deformation initiates when earthquake-induced inertial forces acting on a potential sliding block exceed the...
yield resistance of a slip surface [7]. The cumulative displacement of the block is calculated by integrating the acceleration time history twice, while the yield acceleration is used as a reference datum. The original Newmark’s analysis is based on the assumptions that the block is subjected to one component of horizontal input motion (neglecting the other two components of shaking) and that the shear resistance of the interface does not change with ongoing cycles of motion. It should be pointed out, however, that already in the pioneering paper by Goodman and Seed [6] a procedure to account for friction degradation as function of the number of cycles was proposed and demonstrated. Although Yan [8] modified the original Newmark’s procedure to account for vertical accelerations, an analytical approach that takes into full account all the three-dimensional components of vibrations simultaneously has not been proposed to date with respect to the original Newmark’s method. Furthermore, considering double face sliding, or specifically the case of the tetrahedral wedge, a dynamic, analytical, three-dimensional solution has not been proposed to date. In this study, a new analytical solution for dynamic block sliding in 3D is developed based on the original static limit equilibrium formulation presented by Goodman and Shi [3] for both single and double face sliding (Figure 1).

The time dependency of frictional resistance is also very important as rock avalanches formed by large-scale failures of bedrock may be triggered when frictional resistance is diminished with cycles of motion and sliding velocity. Leaving out changes in pore pressure due to climate effects [9] or thermo-poro-mechanical effects [10–12], friction angle degradation during slip may be explained by rock fragmentation [13, 14], subtle anisotropy in grain arrangements on the interface [15], or rate and state effects [16, 17]. Friction degradation during slip requires a modification of Newmark’s analysis [18], by incorporating a shear strength degradation criterion as a function of either displacement [6] or velocity [19, 20]. For rate dependency, Dieterich [16] and Ruina [17] proposed the ‘rate and state’ friction laws where the friction coefficient is a function of both slip rate and a state variable

$$\mu = \mu_0 + A \ln \left( \frac{V}{V_0} \right) + B \ln \left( \frac{\theta V_0}{D_c} \right)$$

where $\mu_0$ is a reference friction coefficient under a constant reference slip rate $V_0$, $V$ is the sliding velocity, $A$ and $B$ are dimensionless empirical fitting parameters which, respectively, characterize the sliding and time dependence of friction, $D_c$ is a characteristic slip distance essential to reach steady-state sliding, and $\theta$ is a state variable.

The most commonly used state evolution law is known as ‘Dieterich law’:

$$\frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c}$$

Figure 2 shows schematically the observed frictional response to a suddenly imposed change in sliding velocity. The $A$ parameter, known as the direct velocity effect, is related to the change in
rate, and the $B$ parameter is related to the change in state. As illustrated in Figure 2, the friction coefficient at steady state is:

$$\mu = \mu_0 + (A - B) \ln \left( \frac{V}{V_0} \right)$$  \hspace{1cm} (3)$$

Thus, steady-state friction exhibits velocity weakening if $B$ is greater than $A$, and velocity strengthening otherwise. The rate and state friction laws have been used to address various geophysical problems, for a comprehensive review see Scholz [21].

The classic Coulomb–Mohr friction criterion has been modified to incorporate rate and state effects using a double-direct-shear apparatus [22] or more recently a conventional (single) direct shear system for rock interfaces [23]. In our study, the single direct shear apparatus is used to determine the dynamic friction law for the studied interfaces.

Application of any analytical solution, sophisticated or accurate as it may be, is only valid for a single, rigid block for which the failure mode must be assumed in advance. In order to study the dynamic behaviour of a rock slope consisting of multiple and interacting rigid blocks, however, a discrete numerical approach such as the distinct element method (DEM) [24] or the numerical discontinuous deformation analysis (DDA) method [25, 26] is required. Accurate performance of the numerical method does require rigorous validation studies using comparisons to analytical solutions and/or physical models of simplified problem geometries. In this study, 3D-DDA is used for numerical analysis.

Only a limited number of attempts to check the validity and accuracy of 3D-DDA have recently been published [27–33] perhaps due to the difficulty in developing a complete contact theory that governs the interaction of many 3D blocks [34]. Considering 3D validations, Shi [25] reports very high accuracy for two examples of block sliding modelled with 3D-DDA, subjected to gravitational load only. Moosavi et al. [31] compare 3D-DDA results for dynamic block displacement with an analytical solution originally proposed by Kamai and Hatzor [35] of dynamic sliding in 2D. Yeung et al. [33] studied the tetrahedral wedge problem using physical models and field case histories and reported good agreement with 3D-DDA with respect to the obtained failure mode and the block displacement history, although no quantitative comparison between 3D-DDA and lab test results was reported.

In this paper, we propose an original analytical solution for the dynamic sliding of a tetrahedral wedge and use it to validate 3D-DDA. We begin with dynamic single plane sliding problems where the block is free to slide in any direction along the sliding plane, and proceed to the dynamic sliding of a tetrahedral wedge where the sliding direction is controlled by the orientation of the two boundary planes. In the case of single plane sliding our results are compared with the classical Newmark’s solution. In the case of a tetrahedral wedge both the analytical and numerical results are compared with dynamic shaking table experiments performed on a physical model of a tetrahedral wedge.
In the current formulation of both 2D as well as 3D-DDA codes a constant friction angle value is assumed for the sliding interface regardless of the intensity or duration of shaking. In the course of our shaking table experiments, we clearly observed friction degradation of the sliding interface during dynamic shaking, leading to block ‘run out’ after a certain number of cycles of motion. To obtain a quantitative measure of the amount of dynamic frictional degradation we used the shaking table results in conjunction with our analytical solution so that the velocity dependency of the sliding interface could be determined.

2. ANALYTICAL SOLUTION FOR DYNAMIC SLIDING OF WEDGE

2.1. Limit equilibrium equations

The static limit equilibrium equations formulated for each time step are discussed in this section for both single and double face sliding. Naturally, the expected failure mode must be known in advance to formulate the equilibrium equations. We note furthermore that in the cases discussed here the resultant forces are applied to the centroid of the sliding block, slightly in contrast to the physical reality where the input motion is applied to the foundation upon which the block rests.

2.1.1. Single face sliding. A typical three-dimensional model of a block on an incline is illustrated in Figure 1(a). The dip and dip direction angles are $\alpha = 20^\circ$ and $\beta = 90^\circ$, respectively. Although it is a simple 2D problem, the model is plotted as if it were 3D to demonstrate the advantages of the new solution. For this purpose, a Cartesian coordinate system $(x, y, z)$ is defined, where $X$ is horizontal and points to east, $Y$ is horizontal and points to north, and $Z$ is vertical and points upward. The normal vector of the inclined plane is $\hat{n} = [n_x, n_y, n_z]$, where:

\[
\begin{align*}
    n_x &= \sin(\alpha) \sin(\beta) \\
    n_y &= \sin(\alpha) \cos(\beta) \\
    n_z &= \cos(\alpha)
\end{align*}
\]  

(4)

The force equations presented below refer to a block with a unit mass. Hence, these equations can be discussed in terms of accelerations. The resultant force vector that acts on the system at each time step is $\bar{r} = [r_x, r_y, r_z]$. The driving force vector that acts on the block ($\bar{m}$), namely, the projection of the resultant force vector on the sliding plane, at each time step is:

\[
\bar{m} = (\hat{n} \times \bar{r}) \times \hat{n}
\]  

(5)

The normal force vector that acts on the block at each time step is:

\[
\bar{p} = (\hat{n} \cdot \bar{r}) \hat{n}
\]  

(6)

At the beginning of a time step, if the velocity of the block is zero then the resisting force vector due to the interface friction angle $\phi$ is

\[
\bar{f} = \begin{cases} 
    -\tan(\phi) |\bar{p}| \hat{m}, & \text{if } |\bar{p}| < |\bar{m}| \\
    -\bar{m}, & \text{else}
\end{cases}
\]  

(7)

where $\hat{m}$ is a unit vector in the direction $\bar{m}$.

If at the beginning of a time step the velocity of the block is not zero, then

\[
\bar{f} = -\tan(\phi) |\bar{p}| \hat{v}
\]  

(8)

where $\hat{v}$ is the direction of the velocity vector.

In an unpublished report [36], Shi refers only to the case of a block subjected to gravitational load, where the block velocity and the driving force have always the same sign. The same is true for the original equations published in the block theory text by Goodman and Shi [3]. However, in the case of dynamic loading the driving force can momentarily be opposite to the block velocity.
2.1.2. Double face sliding. Double face sliding, or as often referred to as the wedge analysis, is a classic problem in rock mechanics that has been studied by many authors [1, 2, 4, 37]. A typical model of a three-dimensional wedge is shown in Figure 1(b). The normal-to-plane 1 is \( \hat{n}_1 = [n_{x1}, n_{y1}, n_{z1}] \) and the normal-to-plane 2 is \( \hat{n}_2 = [n_{x2}, n_{y2}, n_{z2}] \). Consider a block sliding simultaneously on two boundary planes along their line of intersection \( \hat{I}_{12} \), where:

\[
\hat{I}_{12} = \hat{n}_1 \times \hat{n}_2
\]

The resultant force in each time step is as before \( \vec{r} = [r_x, r_y, r_z] \), and the driving force in each time step is:

\[
\vec{m} = (\vec{r} \cdot \hat{I}_{12}) \hat{I}_{12}
\]

The normal force acting on plane 1 in each time step is \( \vec{\rho} = [p_x, p_y, p_z] \), and the normal force acting on plane 2 in each time step is \( \vec{\varrho} = [q_x, q_y, q_z] \), where:

\[
\vec{\rho} = ((\vec{r} \times \hat{n}_2) \cdot \hat{I}_{12}) \hat{n}_1
\]

\[
\vec{\varrho} = ((\vec{r} \times \hat{n}_1) \cdot \hat{I}_{12}) \hat{n}_2
\]

As in the case of single face sliding, the direction of the resisting force (\( \vec{f} \)) depends upon the direction of the velocity of the block. Therefore, as before, in each time step:

\[
\vec{f} = \begin{cases} 
-(\tan(\phi_1) |\vec{\rho}| + \tan(\phi_2) |\vec{\varrho}|) \vec{\dot{m}}, & \vec{V} \neq 0 \text{ and } (\tan(\phi_1) |\vec{\rho}| + \tan(\phi_2) |\vec{\varrho}|) < |\vec{m}| \\
-\vec{\dot{m}}, & \vec{V} \neq 0 \text{ and } (\tan(\phi_1) |\vec{\rho}| + \tan(\phi_2) |\vec{\varrho}|) \geq |\vec{m}| \\
-(\tan(\phi_1) |\vec{\rho}| + \tan(\phi_2) |\vec{\varrho}|) \vec{\dot{v}}, & \vec{V} \neq 0 
\end{cases}
\]

2.2. Dynamic equations of motion

The sliding force, namely, the block acceleration during each time step, is \( \vec{s} = [s_x, s_y, s_z] \) and is calculated as the force balance between the driving and the frictional resisting forces:

\[
\vec{s} = \vec{m} + \vec{f}
\]

The block velocity and displacement vectors are \( \vec{V} = [V_x, V_y, V_z] \) and \( \vec{D} = [D_x, D_y, D_z] \), respectively. At \( t = 0 \), the velocity and displacement are zero. The average acceleration for time step \( i \) is:

\[
\ddot{S}_i = \frac{1}{2} (\vec{S}_{i-1} + \vec{S}_i)
\]

The velocity for time step \( i \) is therefore:

\[
\vec{V}_i = \vec{V}_{i-1} + \dot{\vec{S}}_i \Delta t
\]

It follows that the displacement for time step \( i \) is:

\[
\vec{D}_i = \vec{D}_{i-1} + \vec{V}_{i-1} \Delta t + \frac{1}{2} \dot{\vec{S}}_i \Delta t^2
\]

Owing to the discrete nature of the suggested algorithm, sensitivity analyses were performed to discover the maximum value of the time increment for the trapezoidal integration method without compromising accuracy. The results are found to be sensitive to the time interval size as long as the friction angle is greater than the slope inclination. We find that the time increment cannot be larger than 0.001 s to obtain accurate results.

2.3. Comparison to classical Newmark’s approach

The validity of the newly developed 3D analytical formulation presented above is tested using the classical 2D Newmark’s solution for the dynamics of a block on an inclined plane. The typical Newmark’s solution requires condition statements and is solved using a numerical time step algorithm as discussed for example, by Kamai and Hatzor [35]. We relate here to Newmark’s procedure
Figure 3. Dynamic block displacement for single face sliding: (a) comparison between 2D Newmark solution, our 3D analytical solution, and 3D-DDA for horizontal input acceleration parallel to the X axis. Relative error with respect to Newmark solution is plotted in the lower panel and (b) comparison between our 3D analytical solution and 3D-DDA for 2D horizontal input acceleration parallel to X and Y axes simultaneously. The relative error is calculated with respect to our 3D analytical solution.

as the ‘exact solution’, to distinguish between the existing approach and the analytical solution proposed here. Figure 3(a) shows a comparison between Newmark, analytical, and 3D-DDA solutions for a plane with dip and dip direction of $\alpha = 20^\circ$ and $\beta = 90^\circ$, respectively, and friction angle of $\phi = 30^\circ$. For dynamic loading we use a sinusoidal input motion in the horizontal X axis, hence the resultant input acceleration vector is $\vec{r} = [r_x, r_y, r_z] = [0.5 \sin(10t) 0 0]g$. The accumulated displacements are calculated up to 10 cycles ($t_f = 2\pi$). The input horizontal acceleration is plotted as a shaded line and the acceleration values are shown on the right-hand side axis. The theoretical mechanical properties as well as the numerical parameters for the 3D-DDA simulations are listed in Table I. For both the Newmark’s and analytical solutions the numerical integration is calculated using a time increment of $\Delta t = 0.001\text{s}$. For the 3D approaches (our analytical solution and 3D-DDA), the calculated displacement vector is normalized to one dimension along the sliding direction.

An excellent agreement is obtained between our analytical and Newmark’s solutions throughout the first two cycles of motion. There is a small discrepancy at the end of the second cycle which depends on the numerical procedures and will decrease whenever the time increment decreases.
Table I. Numerical parameters for 3D-DDA forward modelling simulation.

<table>
<thead>
<tr>
<th></th>
<th>Block on an incline model</th>
<th>Tetrahedral wedge</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mechanical properties</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic modulus (MPa)</td>
<td>20</td>
<td>200000</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>1000</td>
<td>1700</td>
</tr>
<tr>
<td>Friction angle (°)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td><strong>Numerical parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic control parameter</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>628</td>
<td>8000</td>
</tr>
<tr>
<td>Time interval (s)</td>
<td>0.01</td>
<td>0.005</td>
</tr>
<tr>
<td>Assumed max. disp. ratio (m)</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Penalty stiffness (MN/m)</td>
<td>10</td>
<td>20000</td>
</tr>
</tbody>
</table>

The relative error of the new analytical solution and 3D-DDA method with respect to the existing Newmark’s solution is shown in the lower panel of Figure 3(a), where the relative error is defined as:

\[ E_{\text{rel}} = \frac{|D_{\text{Newmark}} - D_{\text{compared solution}}|}{|D_{\text{Newmark}}|} \times 100\% \] (18)

The relative errors for both our analytical solution and 3D-DDA are found to be less than 3% in the final position.

2.4. Comparison between 3D analytical and numerical approaches

The agreement between the 3D analytical and 2D Newmark’s solutions has been established in the previous section; therefore, the analytical solution will be used next as a reference for numerical dynamic simulations with 3D-DDA. Figure 3(b) shows a comparison between the analytical and 3D-DDA solutions for the case of a block on an inclined plane (see Figure 1(a)) subjected to two components of a dynamic, horizontal, input loading function. The resultant input acceleration vector is \( \vec{r} = [r_x, r_y, r_z] = [0.5 \sin(10t), 0.5 \sin(5t) - 1]g \), and the friction angle is again \( \phi = 30° \). The two components of the input horizontal acceleration are plotted as shaded lines and the acceleration values are shown on the right-hand side axis. Note that the relative error presented in the lower panel is now with respect to the 3D analytical solution, defined as:

\[ E_{\text{rel}} = \frac{|D_{\text{analytical}} - D_{\text{3D-DDA}}|}{|D_{\text{analytical}}|} \times 100\% \] (19)

The relative error in the final position in this simulation is approximately 8%.

The comparisons thus far were between analytical and numerical solutions, both of which incorporate significant assumptions regarding material behaviour and boundary conditions. In the following section, physical model test results will be used to study the applicability of both the numerical and analytical approaches employed above.

3. EXPERIMENTAL DETERMINATION OF INTERFACE FRICTION

Both numerical and analytical solutions require a definition of the peak and residual friction angles of the interface along which the dynamic sliding takes place. Peak and residual friction angles for the tested interface were determined experimentally using direct shear tests performed at Ben-Gurion University rock mechanics laboratory. Special cubic concrete samples were cast specifically for that purpose using B30 Portland cement with water/cement ratio of 0.3 in PVC moulds. First, the peak friction angle for very low normal stress was determined using simple tilt tests by slowly increasing the inclination of the base block and measuring the inclination when sliding initiated. Next, direct shear tests were performed on the same cubic concrete blocks...
utilizing a hydraulic, closed-loop, servo-controlled system (Figure 4) where both axial and shear pistons could be operated using either load or displacement outputs as the control variable. Two types of direct shear tests were performed: (1) Velocity stepping under a constant normal stress of 5 MPa and variable slip rates of 10, 0.5, 1, and 3 μm/s, to explore velocity dependency of the concrete interface and (2) Five-segment direct shear tests under constant normal stress values of 1, 2, 3, 4, and 5 MPa and constant displacement rates of 0.002, 0.020, and 0.100 mm/s, to determine steady-state friction coefficient as a function of sliding velocity.

4. DYNAMIC SHAKING TABLE EXPERIMENTS

A 13 cm width by 20 cm long concrete block was attached to the hydraulic, single-degree-of-freedom, horizontally driven, shaking table of the Earthquake Simulation Laboratory at UC Berkeley. The base block was set at an inclination of 28° below horizontal to the North such that the inclinations of the two boundary planes were 51.4/065.8 and 51.4/295.2, arranged symmetrically about the shaking axis. The concrete blocks for the dynamic shaking table experiments were made using the same preparation procedures described above with a static interface friction angle of 36°.

A well-fitted tetrahedral concrete block was placed on the wedge-shaped slab such that under gravitational pull only the block remained stationary. The table was driven by an MTS closed-loop servo-controlled hydraulic actuator, where a Hewlett Packard 33120A arbitrary function generator produced the table command signal. Two 1D linear accelerometers were fixed to the table and to the fixed block. Two displacement transducers measured the relative displacement of the sliding block and shaking table, see Figure 5.

A third linear accelerometer was attached to the upper sliding block in order to measure the natural frequency of the experimental construction. A fast Fourier transform (FFT) performed on the free vibration signal due to a hammer blow in parallel to the shaking table axis yielded a natural frequency for the experimental set up in the range of 70–90 Hz.

Sinusoidal input motions with frequencies ranging between 2 and 5 Hz and amplitudes of 0.20–0.28 g were induced by the function generator. The sinusoidal input motions were ramped up
linearly for 5 s, followed by full amplitude for several tens of seconds, to the time that the block completed 10 cm of sliding, the largest travelling distance allowed by the system configuration.

5. RESULTS

5.1. Determination of friction angle from tilt and direct shear experiments

The average friction angle obtained from 20 tilt tests for the concrete interface was 36.2° with standard deviation of 3.4°. Gentle polishing yielded average friction angle of 31.6° with standard deviation of 2.1°.

A representative result of a direct shear velocity stepping test is presented in Figure 6 where the response of the interface to changes in the imposed sliding velocity is shown. Induced velocity decreases from 10 to 0.5 μm/s results in immediate reduction in friction coefficient followed by steady-state sliding at a higher friction coefficient, whereas induced velocity increases from 0.5 to 1 μm/s and from 1 to 3 μm/s both result in immediate increase in the friction coefficient followed by steady-state sliding at a lower friction coefficient suggesting that the tested interface exhibits a ‘velocity weakening’ behaviour. The immediate response to the change in the induced sliding velocity before steady-state sliding is attained (open circles in Figure 6) is known as the direct velocity effect and is proportional to the A parameter of the rate and state friction law as explained in the introduction (see also [21]).

Towards the end of each constant velocity sliding segment frictional resistance seems to apparently increase (see Figure 6). We have no observations of the actual interface condition during each sliding segment, but visual inspections of the tested interface at the end of each complete test reveal that the interface was damaged during the entire testing. We therefore attribute the slight increase in shear strength detected at the end of each steady-state sliding segment to the apparent increase in the contact area due to interface fragmentation.

The induced velocity ratio between the two segments \( V_2 / V_1 \) allows us to evaluate the rate and state A and B coefficients [16, 17]. Assuming steady-state sliding is reached in each segment, we obtain \( A \approx 0.027 \) and \( B \approx 0.035 \) for the tested concrete interface.

To determine the classical Coulomb–Mohr failure envelopes for different sliding rates we changed the normal stress while steady-state sliding was maintained, as shown for example in Figure 7(a) for sliding velocity of 0.020 mm/s. The resulting Coulomb–Mohr failure envelopes for three different values of sliding velocity are plotted in Figure 7(b), where highly linear trends are indicated. We find that the Coulomb–Mohr friction coefficient clearly exhibits velocity dependence, decreasing the tested interface from \( \mu = 0.6547 \) (corresponding to \( \phi = 33.2° \)) to \( \mu = 0.6220 \) (corresponding to \( \phi = 31.9° \)) as sliding velocity increases from 0.002 to 0.100 mm/s, suggesting again a ‘velocity weakening’ interface as inferred from the series of velocity stepping tests discussed above.
Figure 6. Representative result of a velocity stepping test performed with the direct shear system shown in Figure 4.

Figure 7. Direct shear test results for determination of the effect of imposed sliding velocity on Coulomb–Mohr friction: (a) representative example of a complete stress–displacement history for a typical five-segment test \( (v=0.020 \text{ mm/s}) \) and (b) Coulomb–Mohr envelopes for the tested concrete interface obtained with three values of shear rate.

5.2. Comparison between dynamic shaking table experiments and 3D-DDA results

Comparison between 3D-DDA and shaking table experiments for the dynamic sliding of a tetrahedral wedge is shown in Figure 8. The input motion is sinusoidal at a frequency of 2 Hz and amplitude of 0.21 g (Figure 8(a)). Two different input motion modes are modelled: (1) ‘loading mode’—application of the dynamic force at the centre of mass of the (upper) sliding block, (2) ‘displacement mode’—application of the dynamic displacement into the (lower) foundation block. To allow meaningful analysis in ‘displacement mode’ we filter out high-input frequencies using a low-pass Butterworth filter of 2.5 Hz. To preserve similarity between the original loading functions used in DDA simulations we perform the same filtering procedure also for analysis in ‘loading mode’.

Both DDA simulations are carried out using a constant value of friction angle on both planes, namely \( \phi_1 = \phi_2 = \phi \). To determine the appropriate input friction angle for 3D-DDA simulations, we use the displacement data for the wedge in the following manner. Consider Figure 8(b) where actual block displacement initiates at a time \( t_i \) after the beginning of the experiment. The corresponding level of input acceleration at \( t_i \) is used to recover the limiting value of friction angle for the two boundary planes of the wedge by inversion, using a pseudo-static limit equilibrium analysis for a tetrahedral wedge. The limiting friction angle value thus obtained is \( \phi = 30^\circ \) and this is the value used here in 3D-DDA simulations. This limiting value of the friction angle is confirmed by our 3D analytical solution discussed above for the given wedge geometry and level of induced shaking.
We find that in both loading modes the computed displacement results are highly sensitive to the choice of the numerical penalty stiffness parameter. In ‘loading mode’ accurate results are obtained when the numerical penalty stiffness value is increased up a maximum value of 40,000 MN/m, beyond which the numerical solution will not converge. The upper limit penalty value for the ‘displacement mode’ is found to be 20,000 MN/m, beyond which the numerical solution again does not converge. Otherwise, no significant differences are found between the two different modes of input motion. A small discrepancy is detected between the measured and computed block response at the beginning of the simulation in both loading modes (Figure 8(b)). We attribute this behaviour to the high sensitivity of the numerical code to the choice of the numerical penalty stiffness value.

The numerical control parameters for 3D-DDA simulations are listed in Table 1. Note that in contrast to a previous study [38] no kinetic damping is applied here.

3D-DDA in its current stage of development assumes a constant friction angle value for the sliding interfaces. Our shaking table test results, however, suggest that the dynamic displacement of the physical block departs from classical Newark’s type displacement and actually exhibits ‘run out’ behaviour (see Figure 9) that can only be explained by means of frictional degradation as a function of the number of cycles of motion. In the following section, we employ our analytical solution to determine quantitatively, by back analysis, the rate and amount of friction degradation during dynamic slip.

5.3. Dynamic friction degradation

Two representative tests exhibiting ‘run out’ behaviour are shown in Figure 9. The obtained time-dependent sliding function can be approximated by three linear segments. To reproduce the physical test results analytically we therefore introduce three different values of friction angle into
Figure 9. Best fit between analytical solution and shaking table results allowing for friction degradation due to dynamic slip. Input frequency 2 Hz, input acceleration amplitude: (a) 0.21 g and (b) 0.22 g.

Table II. List of back calculated tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>$\phi_1$ (°)</th>
<th>$\phi_2$ (°)</th>
<th>$\phi_3$ (°)</th>
<th>Max. shaking table acceleration (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CU</td>
<td>29.5</td>
<td>29</td>
<td>27</td>
<td>0.206</td>
</tr>
<tr>
<td>CV</td>
<td>29.5</td>
<td>29</td>
<td>27</td>
<td>0.206</td>
</tr>
<tr>
<td>CW</td>
<td>29.4</td>
<td>28.7</td>
<td>26</td>
<td>0.208</td>
</tr>
<tr>
<td>CY</td>
<td>29.3</td>
<td>28.8</td>
<td>27</td>
<td>0.205</td>
</tr>
<tr>
<td>CZ1</td>
<td>29.9</td>
<td>29.4</td>
<td>27</td>
<td>0.219</td>
</tr>
<tr>
<td>CZ2</td>
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</table>

our analytical solution to best fit the experimental data, recalling that the friction angles for the two boundary planes of the wedge are assumed equal. The ‘best fit’ friction angles thus obtained for each sliding segment along with maximum shaking table acceleration are listed in Table II for eight different experiments. Inspection of the results shown in Table II reveals the striking similarity between back calculated friction angle values obtained in all the tests. The consistency in the back analyzed friction angle values is demonstrated in Figure 10 where results from the two representative sets of experiments are plotted.
Figure 10. Friction angle (solid lines) and average sliding velocity (dashed lines) as a function of shaking cycles with sinusoidal input frequency of 2 Hz and induced amplitude of 0.21 g (a) and 0.22 g (b), obtained from back analysis of eight shaking table experiments.

Figure 11. Friction coefficient as a function of sliding velocity: open triangles—data obtained from direct shear tests, open diamonds—back calculated results from shaking table experiments.

The number of loading cycles required for onset of friction degradation is found to be between 30 and 50 in the current experimental set up, with input acceleration amplitudes of 0.21–0.22 g and input frequency of 2 Hz. As would be expected, the number of loading cycles required for onset of friction degradation is reduced with increasing acceleration amplitude.

The average sliding velocity for the best fit friction coefficients obtained for all the tests is plotted in Figure 11. Clearly, the tested interface exhibits velocity weakening behaviour that can explain the observed ‘run out’ of the wedge as shown in Figure 9. Interestingly, the friction coefficients obtained from slow direct shear tests plot on the same linear trend obtained from back analysis of shaking table experiments (Figure 11).
6. DISCUSSION

This paper presents the first attempt known to the authors to perform fully dynamic analysis with 3D-DDA and to check the accuracy of the method using both analytical solutions and physical test results. The numerical results obtained with 3D-DDA are found to be in good agreement with both methods of validation, with the numerical results being slightly on the conservative side when compared with the results of shaking table experiments. We find that the dynamic application of 3D-DDA is highly sensitive to the numerical penalty (contact spring stiffness); this sensitivity may lead to observed sliding initiation in 3D-DDA that may precede the actual arrival time of the theoretical yield acceleration.

Two methods of dynamic input are studied, referred to here as ‘loading’ and ‘displacement’ modes, both of which provide similar results within the accuracy resolution sought in this study. The ‘loading mode’ is found to be less sensitive to the numerical penalty and consequently results obtained with this input method are smoother. As in 2D-DDA, we obtain the most accurate results with 3D-DDA with a highest value of numerical penalty, beyond which the numerical solution does not converge. The optimum value of this numerical control parameter is case specific and depends, to a great extent, on the elastic modulus and mass of the modelled block (see Table I).

In the 3D-DDA codes used in this research a constant value of frictions angle is assumed for a given interface, whereas physical test results suggest that frictional degradation does occur with increasing numbers of shaking cycles culminating, ultimately, in wedge ‘run out’ (see Figure 9). We determine here the actual amount of friction loss by best-fitting the shaking table results with our 3D analytical solution.

With a low amplitude sinusoidal input we find that several tens of seconds of cyclic motion are required to induce run out. The number of cycles required for block run out is found to be inversely proportional to the amplitude of shaking. Few tens of cycles of motion at a frequency range of 2–5 Hz may represent a relatively large earthquake in moment magnitude (Mw) range of 7–8. Kanamori and Brodsky [39] show that the source duration of such an earthquake may reach up to 100 s.

Velocity stepping and classic direct shear tests indicate that the tested concrete interface exhibits velocity weakening. Velocity weakening is also inferred from the results of the dynamic shaking table experiments where friction degradation culminated in block run out. Interestingly, friction degradation leading to block run out is obtained here under dry conditions, in contrast to recent studies on the run out of large landslides [10–12]. Note that there are two fundamental differences between the direct shear and shaking table tests: (1) in direct shear tests the velocity of the sliding block is induced by the hydraulic servo-control command whereas in shaking table tests the velocity of the block is in response to the induced displacement of the underlying table, (2) in direct shear tests friction values are obtained from ‘steady state’ sliding, whereas steady-state conditions are never obtained in the shaking table experiments and friction values are inferred by inversion. While we cannot determine for the case of shaking table tests whether friction degradation is a result or a cause of the sliding velocity of the tested wedge, it is clear that interface friction and sliding velocity are interrelated.

The friction coefficients obtained from slow direct shear tests and rapid shaking table experiments are plotted in Figure 11 on a semi-logarithmic scale, with velocity spanning five orders of magnitude. The results are quite striking. First, the slow rate direct shear tests as well as the fast rate shaking table experiments each plot on a linear trend, and second, both sets of tests can be fitted on the same linear trend with a very good linear regression coefficient of \( R^2 = 0.948 \), confirming in essence the rate and state law of seismology [16, 17] but with different testing methodologies (the rate and state law was originally formulated from analysis of velocity stepping tests). The \((A-B)\) term of the Dieterich–Ruina ‘rate and state’ variable friction law can be recovered from the results of the direct shear tests where steady-state sliding was clearly reached (see Figure 11). The obtained \(A-B\) value \((-0.0079)\) is one order of magnitude lower than a single value obtained from the entire suite of test data \((-0.0174)\). Therefore, extrapolation from slow rate direct shear test data to fast rates proves in accurate, and from engineering stands point—un-conservative.
The classical static or pseudo-static analyses, as well as Newmark’s solution, do not take into account friction degradation along rock discontinuities, and therefore cannot predict run-out phenomena in rock slopes in response to ground vibrations emanating from strong earthquakes of long duration. We conclude that velocity-dependent friction across rock discontinuities should be integrated into dynamic rock slope analysis, either analytical or numerical, to obtain more realistic results when strong ground motions of long duration are considered.

7. SUMMARY AND CONCLUSIONS

A new algorithm for dynamic sliding in three dimensions is presented based on the static limit equilibrium equations suggested by Goodman and Shi [3]. The newly developed algorithm is found to be suitable for analyzing the dynamic response of single block for both single and double plane sliding. Effective application of this algorithm however requires that the failure mode is assumed in advance.

A very good agreement between Newmark’s method on one hand and the new 3D algorithm and 3D-DDA on the other is observed using theoretical dynamic problems with high sensitivity to the choice of numerical penalty indicated in 3D-DDA. Dynamic 3D-DDA is applied in two loading modes (‘load’ and ‘displacement’) both of which provide similar results with the ‘displacement’ mode found to be more sensitive to the choice of numerical penalty.

The friction coefficient of the tested concrete interface exhibits velocity weakening behaviour. Several tens of loading cycles are required for onset of friction degradation when, as in our case, the input amplitude is slightly higher than the static yield acceleration. The number of loading cycles required for onset of friction degradation is found to be inversely proportional to the shaking amplitude.

A logarithmic relationship between friction coefficient and sliding velocity is observed with the velocity spanning five orders of magnitude, the linear trend of which confirms the Dieterich–Ruina ‘rate and state’ variable friction law. The obtained A–B value obtained from slow direct shear tests is one order of magnitude lower than a single value obtained from the entire suite of test data including rapid shaking table experiments. Extrapolation from slow rate direct shear test data to fast sliding rates therefore proves in accurate, and from engineering stands point—un-conservative.

Finally, we conclude that the velocity-dependent friction degradation, as determinate in lab experiments, must be integrated into dynamic rock slope analyses in order to receive more realistic results.

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REFERENCES


